The fixed order coefficients

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This note documents the use of the MATHEMATICA program. In this program, we provide the $\Sigma^{(i,j)}_{i\bar{i}\leftarrow ab}$ and the relevant functions.

I. THE HARD AND SOFT FUNCTIONS

The complete analytic results for the hard and soft functions matrices are provided in this programs. The perturbative expansions of the hard and soft functions are in paper. The functions **Hqq(i)** and **Hgg(i)** give the $\mathcal{O}(\alpha_s^i)$ hard function for quark-antiquark annihilation and gluon gluon fusion channels, respectively, where $i = 0, 1$. The functions **softqq(i)** and **softgg(i)** give the $\mathcal{O}(\alpha_s^i)$ soft functions and here $i = 0, 1, 2$, where at NNLO level we only give the scale-dependent soft function. And these fixed order hard and soft functions can be obtained by following the example in **example.nb**. The traces of the product of hard functions and soft functions are written as

$$\mathrm{Tr}\left[H_{i\bar{i}}(M, m_t, \cos\theta, \mu) S_{i\bar{i}}(M, m_t, L_\perp, \cos\theta, \mu)\right] = \frac{8\alpha_s^2}{3d_i} \sum_{i=1} \left(\frac{\alpha_s}{4\pi}\right)^n \mathrm{HS}^{(n)}_{i\bar{i}}(M, m_t, L_\perp, \cos\theta, \mu) \ , \tag{1}$$

where

$$\mathrm{HS}^{(n)}_{i\bar{i}}(M, m_t, L_\perp, \cos\theta, \mu) = \sum_{m=0}^{2n} \mathrm{HS}^{(n,m)}_{i\bar{i}} L_\perp^m \ . \tag{2}$$

In this program, we define

$$\mathrm{HSgg21} = \mathrm{HS}^{(2,1)}_{gg}, \qquad \text{similar for others,} \tag{3}$$

which are used in the $\Sigma_{i\bar{i}\leftarrow ab}$ functions.

II. THE $\Sigma_{i\bar{i}\leftarrow ab}$ COEFFICIENTS

The partonic cross section at NNLO can be written as

$$C_{i\bar{i}\leftarrow ab}(z_1, z_2, q_T, M, \cos\theta, m_t, \mu) = \sum_{n=0} \left(\frac{\alpha_s}{4\pi}\right)^n C^{(n)}_{i\bar{i}\leftarrow ab}(z_1, z_2, q_T, M, \cos\theta, m_t, \mu) \tag{4}$$

The definition of $\Sigma_{i\bar{i}\leftarrow ab}$ have been defined in our paper. The NLO and NNLO results can be expressed as

$$\int \frac{dz_1}{z_1}\frac{dz_2}{z_2}\delta(z - z_1 z_2) C^{(1)}_{i\bar{i}\leftarrow ab}(z_1, z_2, q_T, M, \cos\theta, m_t, \mu) = \Sigma^{(1,0)}_{i\bar{i}\leftarrow ab}\delta(q_T^2) + \Sigma^{(1,1)}_{i\bar{i}\leftarrow ab}\left[\frac{1}{q_T^2}\right]_*^{[\mu^2]} + \Sigma^{(1,2)}_{i\bar{i}\leftarrow ab}\left[\frac{1}{q_T^2}\ln\frac{q_T^2}{\mu^2}\right]_*^{[\mu^2]} \ , \tag{5}$$

$$\begin{aligned}\int \frac{dz_1}{z_1}\frac{dz_2}{z_2}\delta(z - z_1 z_2) C^{(2)}_{i\bar{i}\leftarrow ab}(z_1, z_2, q_T, M, \cos\theta, m_t, \mu) &= (\Sigma^{(2,0)}_{i\bar{i}\leftarrow ab} - 4\zeta_3\Sigma^{(2,3)}_{i\bar{i}\leftarrow ab})\delta(q_T^2) \\ + (\Sigma^{(2,1)}_{i\bar{i}\leftarrow ab} + 16\zeta_3\Sigma^{(2,4)}_{i\bar{i}\leftarrow ab})\left[\frac{1}{q_T^2}\right]_*^{[\mu^2]} + \Sigma^{(2,2)}_{i\bar{i}\leftarrow ab}\left[\frac{1}{q_T^2}\ln\frac{q_T^2}{\mu^2}\right]_*^{[\mu^2]} &+ \Sigma^{(2,3)}_{i\bar{i}\leftarrow ab}\left[\frac{1}{q_T^2}\ln^2\frac{q_T^2}{\mu^2}\right]_*^{[\mu^2]} + \Sigma^{(2,4)}_{i\bar{i}\leftarrow ab}\left[\frac{1}{q_T^2}\ln^3\frac{q_T^2}{\mu^2}\right]_*^{[\mu^2]} \ .\end{aligned} \tag{6}$$

The function **Sigmaqqbar("ab", i, j)** returns the coefficient $\Sigma^{(i,j)}_{q\bar{q}\leftarrow ab}$, where "ab" can be "qqbar", "qg", "gg" and "qqprime". And the function **Sigmagg("ab", i, j)** returns the coefficient $\Sigma^{(i,j)}_{gg\leftarrow ab}$, where "ab" can be "gg", "qg" and "qq". In the Coefficients, the two-loop splitting functions $P^{(2)}_{i\leftarrow j}(z)$ in the $\Sigma^{(i,j)}_{gg\leftarrow ab}$ are labeled as **Pij2(z)**. And the function **pgg2(z), pqg2(z), pqg2(z), pqqV(z), pqqbV(z), pqqS(z)** returns the corresponding expressions of two loop splitting functions.

The kinematic variables used in this program are defined as

$$M^2 = (p_3 + p_4)^2, \qquad t_1 = (p_1 - p_3)^2 - m_t^2, \qquad u_1 = (p_1 - p_4)^2 - m_t^2,$$
$$\beta_t = \sqrt{1 - 4m_t^2/M^2}, \qquad x_s = (1 - \beta_t)/(1 + \beta_t), \qquad \beta_{34} = i\pi + \ln(x_s). \tag{7}$$