

Simulated data analysis

Example 1. Time series with multiple change points and censored observations (40% of censoring)

In this section, the RJMCMC algorithm is exemplified in a simulated data set. The process considered in this section has 600 observations with change points at $\tau_1 = 200$ and $\tau_2 = 400$ and 40% of censoring. The parameters of the 3 AR segments are $\mu = (12, 12, 12)$, $\sigma^2 = (3, 1.5, 3)$ and $\phi = (0.5, 0.79, 0.5)$. This process can be written as

$$Y_t = \begin{cases} 12 + 0.50(Y_{t-1} - 12) + \varepsilon_t, & 1 \leq t \leq 200, \\ 12 + 0.79(Y_{t-1} - 12) + \varepsilon_t, & 201 \leq t \leq 400, \\ 12 - 0.50(Y_{t-1} - 12) + \varepsilon_t, & 401 \leq t \leq 600. \end{cases} \quad (1)$$

A realization of the model in (1) was draw and censored 40% of the observations. In the Figure 1 a realization of the process in (1) is shown and in the of Figure 2 shows the censored realization.

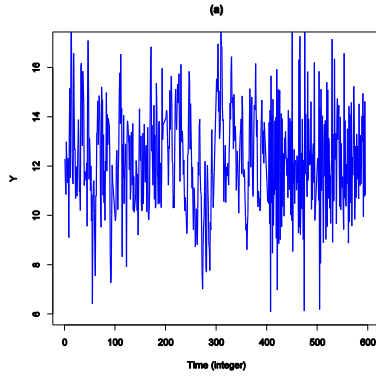


Figure 1.

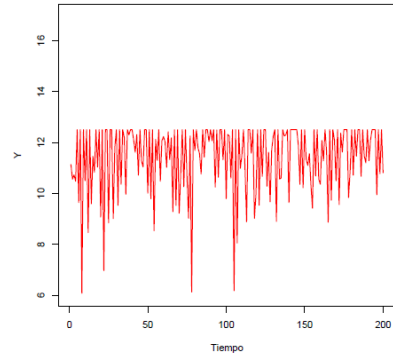


Figure 2.

Through the RJMCMC algorithm, a sample of size 50,000 was simulated, from which the statistics $R_1 = 1.003$ and $R_2 = 1.0018$ to assess the convergence in the chain, as they're higher than 1. According to Castellote (1998), with the 50,000 iterations, the chain has converged to its stationary distribution.

Using the chain, the final distribution of K, the number of change points, is estimated. The estimation is shown in the figure 3. According to this estimation, it is observed that the number of change points can be estimated to 2.

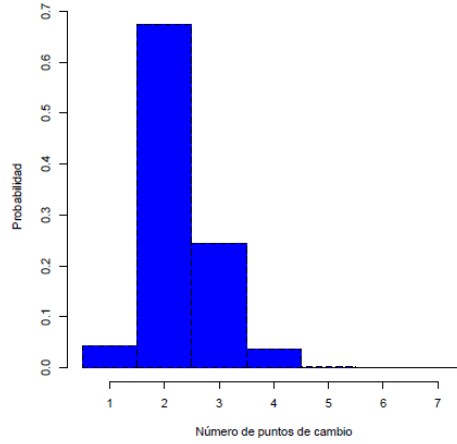


Figure 3. Histogram of the number of change points (40% censoring)

Using the chain, the final distribution of the localizations τ_1 and τ_2 , conditioned to $k = 2$, is estimated. Those estimations are shown in the figure 4.

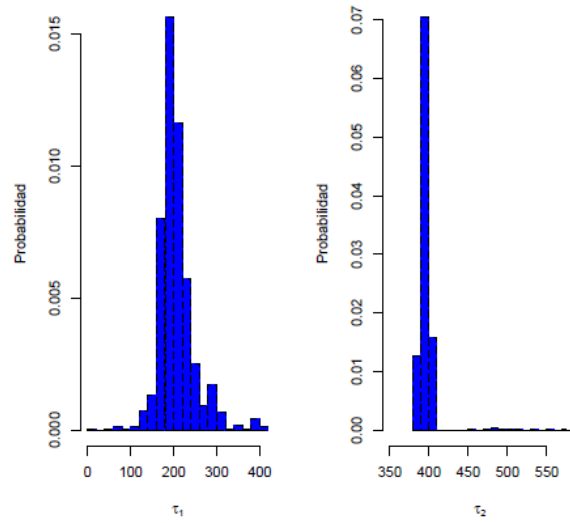


Figure 4. Histogram of the localizations of the change points (40% censoring)

The table 1 shows the real values, estimated values and standard deviation of the change points, the localizations of such points, as well as the means, variances and autoregressive coefficients in each segment that is obtained with the RJMCMC algorithm. Those estimations are conditioned to $k = 2$ (2 change points).

Parameters	True value	Estimated value	Estimated standard deviation
τ_1	200	199	41.38
τ_2	400	398	12.47
μ_1	12	11.69	0.12
μ_2	12	11.76	0.27
μ_3	12	11.47	0.06
σ_1	1.73	1.02	0.09
σ_2	1.22	0.67	0.1
σ_3	1.73	1.20	0.12
ϕ_1	0.5	0.34	0.079
ϕ_2	0.79	0.76	0.137
ϕ_3	-0.5	-0.35	0.059

Table 1. Estimations for the case of two change points (40% censoring)

Example 2. Time series with multiple change points and censored observations (0% of censoring)

In this section, the RJMCMC algorithm is exemplified in a simulated data set. The process considered in this section has 600 observations with change points at $\tau_1 = 200$ and $\tau_2 = 400$ and 0% of censure. The parameters of the 3 AR segments are $\mu = (12, 12, 12)$, $\sigma^2 = (3, 1.5, 3)$ and $\phi = (0.5, 0.79, 0.5)$. This process can be written as

$$Y_t = \begin{cases} 12 + 0.50(Y_{t-1} - 12) + \varepsilon_t, & 1 \leq t \leq 200, \\ 12 + 0.79(Y_{t-1} - 12) + \varepsilon_t, & 201 \leq t \leq 400, \\ 12 - 0.50(Y_{t-1} - 12) + \varepsilon_t, & 401 \leq t \leq 600. \end{cases} \quad (2)$$

A realization of the model in (2) was draw and censored 0% of the observations. In the Figure 5 a realization of the process in (2) is shown without censoring.

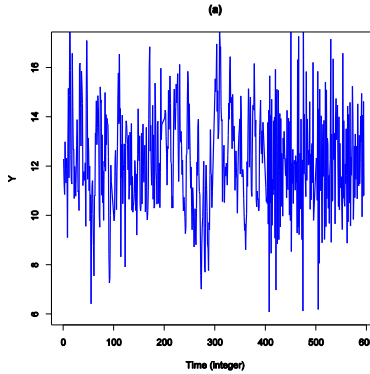


Figure 5

Through the RJMCMC algorithm, a sample of size 50,000 was simulated, from which the statistics $R_1 = 1.20$ and $R_2 = 1.19$ to assess the convergence in the chain, as they're higher than 1. According to Castellote (1998), with the 50,000 iterations, the chain has converged to its stationary distribution.

Using the chain, the final distribution of K was estimated and the number of change points changed, as shown in the figure 6. According with this estimation, it was observed that the number of change points can be estimated to 2.

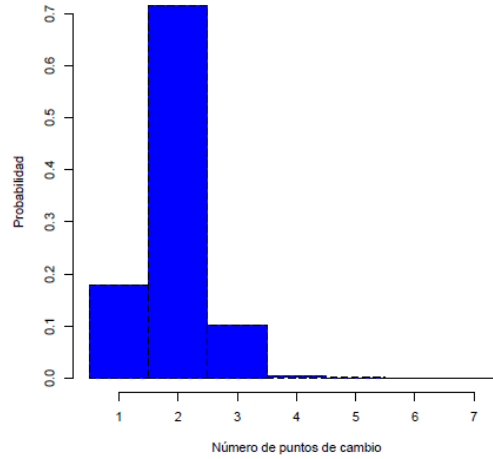


Figure 6. Histogram of the number of change points

Using the chain, the final distribution of the localizations τ_1 and τ_2 , conditioned to $k = 2$, is estimated. Those estimations are shown in the figure 7.

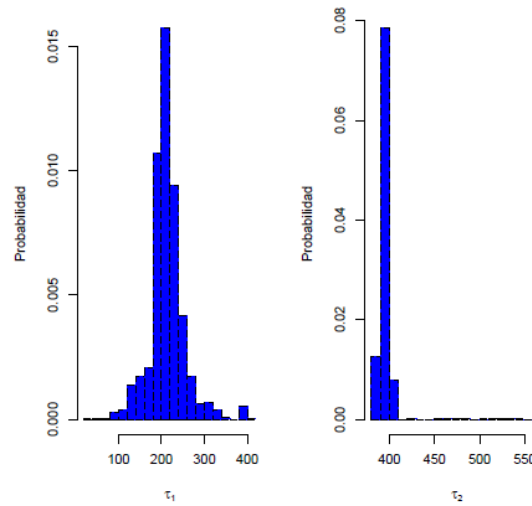


Figure 7. Histogram of the localizations of the change points.

The table 2 shows the real values, estimated values and standard deviation of the change points, the localizations of such points, as well as the means, variances and autoregressive coefficients in each segment that is obtained using the RJMCMC algorithm. Those estimations are conditioned to $k = 2$ (2 change points).

Parameters	True value	Estimated value	Estimated standard deviation
τ_1	200	209	41.24
τ_2	400	391	13.01
μ_1	12	12.28	0.21
μ_2	12	12.22	0.41
μ_3	12	12.10	0.10
σ_1	1.73	1.58	0.18
σ_2	1.22	1.21	0.20
σ_3	1.73	1.60	0.21
ϕ_1	0.5	0.44	0.06
ϕ_2	0.79	0.77	0.15
ϕ_3	-0.5	-0.52	0.05

Table 2. Estimations for the case of two change points (0% censoring)