Computational Mechanics by Isogeometric Analysis
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Exercises May 17, 2016
NURBS–based Isogeometric Analysis: Galerkin method. VI
Time dependent problems

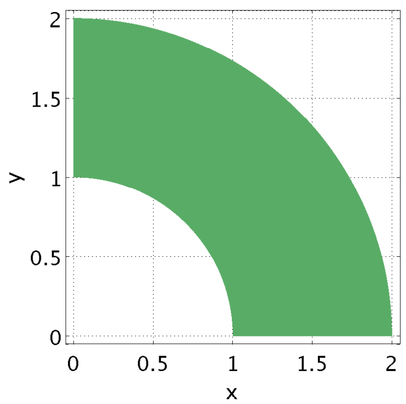
Let us consider the following *time dependent* advection–diffusion problem:

$$\text{find } u\ : \Omega \times (0,T) \to \mathbb{R} \quad : \quad \begin{cases} \rho \dfrac{\partial u}{\partial t} - \nabla \cdot (\mu \nabla u) + \mathbf{b} \cdot \nabla u = f & \text{in } \Omega \times (0,T), \\ u = g & \text{on } \Gamma_D \times (0,T), \\ \mu \nabla u \cdot \widehat{\mathbf{n}} = \phi & \text{on } \Gamma_N \times (0,T), \\ u = u_0 & \text{in } \Omega \times \{t = 0\}, \end{cases}$$

being $\Omega \subset \mathbb{R}^d$, for $d = 2, 3$, a bounded open computational domain with regular boundary $\partial\Omega = \Gamma_D \cup \Gamma_N$ such that $\mathring{\Gamma}_D \cap \mathring{\Gamma}_N = \emptyset$ and $\widehat{\mathbf{n}}$ the outward directed unit vector normal to $\partial\Omega$. The diffusion coefficient $\mu \in L^\infty(\Omega)$, with $\mu > 0$, the advection term $\mathbf{b} \in [L^\infty(\Omega)]^d$, the Dirichlet data $g \in H^{1/2}(\Gamma_D)$, and the Neumann data $\phi \in L^2(\Gamma_N)$ are assigned functions not dependent on time. The source term $f \in L^2(\Omega \times (0,T))$ is time dependent and assumes the following separable form $f = f(t, \mathbf{x}) = f_t(t)\, f_{\mathbf{x}}(\mathbf{x})$.

Specifically, let us consider the domain Ω reported in the following figure ($\Omega = (1,2) \times (0, \pi/2)$ in polar coordinates (r, θ)) for which $\Gamma_D = \{(\mathrm{x}, \mathrm{y}) \in \partial\Omega\ :\ \mathrm{x} = 0\}$, $\Gamma_N = \partial\Omega \backslash \Gamma_N$. We set $T = 4$, $u_0 = 0$, $g = 0$, $\phi = 0$, $\mu = 0.02$, $\mathbf{b} = (\mathrm{y}, -\mathrm{x})^T$, $f_t(t) = \begin{cases} t & \text{if } t < 1, \\ 1 & \text{if } t \geq 1 \text{ and } t < 2, \\ 0 & \text{if } t \geq 2, \end{cases}$

and $f_{\mathbf{x}}(\mathrm{x}, \mathrm{y}) = 100\, e^{-500\left((\mathrm{x}-\mathrm{x}_0)^2 + (\mathrm{y}-\mathrm{y}_0)^2\right)}$, with $\mathrm{x}_0 = 0.5000$ and $\mathrm{y}_0 = 1.4142$.



For the spatial approximation consider NURBS based Isogeometric Analysis within the framework of the Galerkin method. Solve the problem by using the θ–method with the time step $\Delta t = 0.1$ for different values of $\theta \in [0, 1]$. Plot the approximate solution in one point in Ω vs. the time t and compare the results obtained for different values of θ.