Computational Mechanics by Isogeometric Analysis
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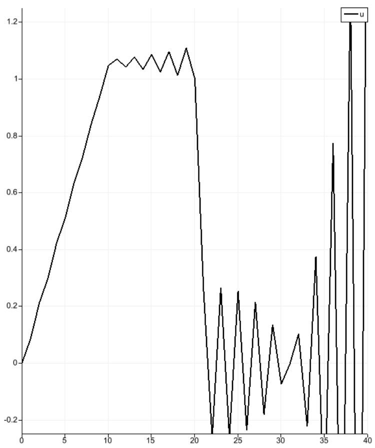
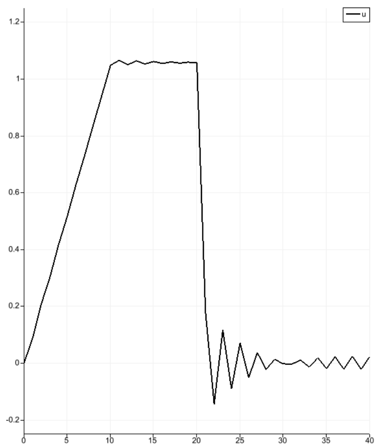
Exercises May 17, 2016: Solutions
NURBS–based Isogeometric Analysis: Galerkin method. VI
Time dependent problems

For the spatial approximation we consider NURBS basis functions of degree $p = 2$ which are globally C^1–continuous in the computational domain Ω; a mesh comprised of 60×30 elements is used.

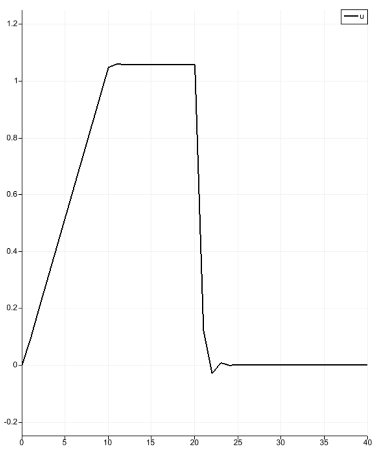
Let us consider the MATLAB file `ex11.m` for which, for the time discretization, the θ method is used; the time step $\Delta t = 0.1$ is considered. We report the results obtained for different values of the parameter $\theta \in [0, 1]$ in the following figure where we highlight the evolution of the numerical solution $u_h(\mathbf{P})$ in a point $\mathbf{P} = (1, 1)^T$ (approximately) vs. the time step index k. We recall that for $\theta = 0$ we obtain the forward Euler method, for $\theta = \frac{1}{2}$ the Crank–Nicholson method, while for $\theta = 1$ the backward Euler method.



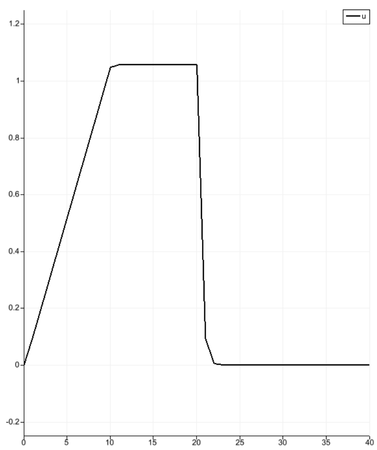
$\theta = 0.45$



$\theta = 0.5$ (Crank–Nicholson)



$\theta = 0.75$



$\theta = 1$ (backward Euler)

We observe that the solution obtained for $\theta = 0.45$ exhibits oscillations increasing in time, yielding an unstable numerical solution for $t \to T$; the result was expected since the θ–method is conditionally stable for $\theta < \frac{1}{2}$ and a smaller value of $\Delta t = 0.1$ should have been chosen to ensure absolute stability. Conversely, all the numerical solutions obtained for $\theta \geq \frac{1}{2}$ are absolutely stable. Diminishing oscillations on the numerical solution $u_h(\mathbf{P})$ are still present for $\theta \geq \frac{1}{2}$, larger for $\theta = \frac{1}{2}$ and absent for $\theta = 1$.