Computational Mechanics by Isogeometric Analysis
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Exercises March 29, 2016
B–splines and NURBS: function spaces

1. Let us consider the B–splines function space $\mathcal{B}_h := \text{span}\,\{B_i(\xi),\ i = 1,\ldots,n_{\mathcal{B}}\}$, with the B–splines basis $\{B_i(\xi)\}_{i=1}^{n_{\mathcal{B}}}$ obtained from the knot vector $\Xi_{\mathcal{B}} = \{0, 0, 1/2, 1, 1\}$. Determine if the function space $\mathcal{B}_h$ is "nested" into the B–splines function space $\mathcal{N}_h := \text{span}\,\{N_i(\xi),\ i = 1,\ldots,n_{\mathcal{N}}\}$ ($\mathcal{B}_h \subseteq \mathcal{N}_h$) with the basis $\{N_i(\xi)\}_{i=1}^{n_{\mathcal{N}}}$ obtained from the following knot vectors:

   a) $\Xi_{\mathcal{N}} = \{0, 0, 0, 1/2, 1, 1, 1\}$;

   b) $\Xi_{\mathcal{N}} = \{0, 0, 0, 1/2, 1/2, 1, 1, 1\}$.

2. Determine the function $u_h \in \mathcal{B}_h$ which minimizes the errors in the L^2 norm with respect to the function $u(\xi) = \sin(\pi\xi)$. We consider the B–splines function space $\mathcal{B}_h := \text{span}\,\{B_i(\xi),\ i = 1,\ldots,n_{\mathcal{B}}\}$ defined from the B-splines basis $\{B_i(\xi)\}_{i=1}^{n_{\mathcal{B}}}$ obtained from the following knot vectors:

   a) $\Xi_{\mathcal{B}} = \{0, 0, 0, 1, 1, 1\}$;

   b) $\Xi_{\mathcal{B}} = \{0, 0, 0, 1/2, 1, 1, 1\}$;

   c) $\Xi_{\mathcal{B}} = \{0, 0, 0, 1/2, 1/2, 1, 1, 1\}$.

   Hint: determine the control variables $\{d_i\}_{i=1}^{n_{\mathcal{B}}}$ defining the function $u_h(\xi) = \displaystyle\sum_{i=1}^{n_{\mathcal{B}}} B_i(\xi)\, d_i$.

3. Repeat point 2) for $u(\xi) = \begin{cases} 1 & \text{if } \xi \in (0, 1/2), \\ 0 & \text{if } \xi \in [1/2, 1). \end{cases}$