Computational Mechanics by Isogeometric Analysis
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Exercises April 26, 2016
NURBS–based Isogeometric Analysis: Galerkin method. III

Let us consider the following advection–diffusion–reaction problem:

$$\text{find } u \ : \Omega \to \mathbb{R} \quad : \quad \begin{cases} -\nabla \cdot (\mu \nabla u) + \mathbf{b} \cdot \nabla u + \sigma\, u = f & \text{in } \Omega, \\ u = g & \text{on } \Gamma_D, \\ \mu \nabla u \cdot \widehat{\mathbf{n}} = \phi & \text{on } \Gamma_N, \end{cases}$$

being $\Omega \subset \mathbb{R}^d$, for $d = 2, 3$, a bounded open computational domain with regular boundary $\partial\Omega = \Gamma_D \cup \Gamma_N$ such that $\overset{\circ}{\Gamma}_D \cap \overset{\circ}{\Gamma}_N = \emptyset$ and $\widehat{\mathbf{n}}$ the outward directed unit vector normal to $\partial\Omega$. The diffusion coefficient $\mu \in L^\infty(\Omega)$, the advection term $\mathbf{b} \in [L^\infty(\Omega)]^d$, the reaction term $\sigma \in L^2(\Omega)$, and the source term $f \in L^2(\Omega)$ are assigned functions such that the problem is well posed; we assume that $g \in H^{1/2}(\Gamma_D)$ and $\phi \in L^2(\Gamma_N)$.

Solve the problem by means of the Galerkin method with NURBS–based Isogeometric Analysis for the data reported in the following. Consider different mesh sizes h and polynomial degrees p. Suitably use the MATLAB functions `op_vel_dot_gradu_v_tp.m` and `op_u_v_tp.m` to assemble the advection and reaction terms, respectively.

1. Set $\Omega = (0,1)^2$, $\Gamma_D \equiv \partial\Omega$ ($\Gamma_N = \emptyset$), $\mu = 1$, and $f = 0$. Solve advection–diffusion and diffusion–reaction problems defined in Ω by suitably choosing the advection field $\mathbf{b}$, the reaction term σ, and the Dirichlet data g.

2. Let us consider the domain Ω reported in the following figure ($\Omega = (1,2) \times (0, \pi/2)$ in polar coordinates (r, θ)) for which $\Gamma_N = \{(\mathrm{x}, \mathrm{y}) \in \partial\Omega \ : \ \mathrm{y} = 0\}$, $\Gamma_D = \partial\Omega \backslash \Gamma_N$, $\mu = 1$, $\phi = 0$, $\mathbf{b} = (\sin(\theta), -\cos(\theta))^T$, $\sigma = 0$, and $f = 0$. Solve the advection–diffusion problem by considering different choices of the Dirichlet data g.