Computational Mechanics by Isogeometric Analysis
Dr. L. Dedè. A.Y. 2015/16

Exercises April 4, 2016
NURBS–based Isogeometric Analysis: Galerkin method

Let us consider the following Poisson problem:

$$\text{find } u \ : \Omega \to \mathbb{R} \quad : \quad \begin{cases} -\nabla \cdot (\mu \nabla u) = f & \text{in } \Omega, \\ u = g & \text{on } \Gamma_D, \\ \mu \nabla u \cdot \widehat{\mathbf{n}} = \phi & \text{on } \Gamma_N, \end{cases}$$

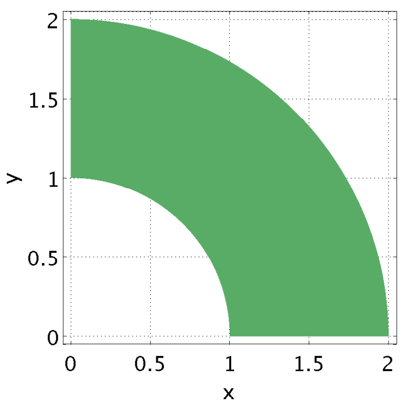
being $\Omega \subset \mathbb{R}^d$, for $d = 2, 3$, a bounded open computational domain with regular boundary $\partial\Omega = \Gamma_D \cup \Gamma_N$ such that $\mathring{\Gamma}_D \cap \mathring{\Gamma}_N = \emptyset$ and $\widehat{\mathbf{n}}$ the outward directed unit vector normal to $\partial\Omega$. The diffusion coefficient $\mu \in L^\infty(\Omega)$ and the source term $f \in L^2(\Omega)$ are assigned functions such that $\mu \geq \mu_0 > 0$ a.e. in Ω; we assume that $g \in H^{1/2}(\Gamma_D)$ and $\phi \in L^2(\Gamma_N)$.

Solve the Poisson problem by means of the Galerkin method with NURBS–based Isogeometric Analysis for the data reported in the following. Consider different h, p, and k–refinement strategies and evaluate the errors in norms L^2 and H^1 when the exact solution is available. Use the MATLAB functions `test_iga2D.m` (see point 1) and `test_iga3D.m` (see point 4) as templates (the functions are based on the *GeoPDEs* software); use *ParaView* to visualize the approximated solution u_h (`http://www.paraview.org/`).

1. Set $\Omega = (0, 1)^2$, $\Gamma_D \equiv \partial\Omega$ ($\Gamma_N = \emptyset$), $\mu = 1$, $g = 0$, and f such that the exact solution is $u = \sin(\pi \mathrm{x}) \, (\sin(\pi \mathrm{y}))^2$.

2. Set $\Omega = (0, 1)^2$, $\Gamma_N = \{(\mathrm{x}, \mathrm{y}) \in \partial\Omega \ : \ \mathrm{x} = 1\}$, $\Gamma_D = \partial\Omega \backslash \Gamma_N$, $\mu = 1$, $g = 0$, and f and ϕ such that the exact solution is $u = \sin\left(\frac{\pi}{6}\mathrm{x}\right) (\sin(\pi \mathrm{y}))^2$.

3. Let us consider the domain Ω reported in the following figure ($\Omega = (1, 2) \times (0, \pi/2)$ in polar coordinates (r, θ)) for which $\Gamma_N = \{(\mathrm{x}, \mathrm{y}) \in \partial\Omega \ : \ \mathrm{x} = 0 \text{ or } \mathrm{y} = 0\}$, $\Gamma_D = \partial\Omega \backslash \Gamma_N$, $\mu = 1$, $\phi = 0$, and f and g are such that the exact solution reads $u = \frac{1}{3}(4 - \mathrm{x}^2 - \mathrm{y}^2) \, e^{\mathrm{x}^2 + \mathrm{y}^2 - 1}$.



4. Set $\Omega = (0,1)^3$, $\Gamma_D \equiv \partial\Omega$ ($\Gamma_N = \emptyset$), $\mu = 1$, $g = 0$, and f such that the exact solution is $u = \sin(\pi \mathrm{x})\,\sin(\pi \mathrm{y})\,\sin(\pi \mathrm{z})$.

5. Let us consider the three dimensional domain Ω reported in the following figure ($\Omega = (1,2) \times (0,\pi/2) \times (0,1)$ in cylindrical coordinates $(\mathrm{r},\theta,\mathrm{z})$) for which $\Gamma_D = \{(\mathrm{x},\mathrm{y},\mathrm{z}) \in \partial\Omega \;:\; \mathrm{x} = 0 \text{ or } \mathrm{y} = 0\}$, $\Gamma_N = \partial\Omega\backslash\Gamma_D$, $\mu = 1$, $f = 0$. Set:

$$g = \begin{cases} 1 & \text{on } \Gamma_{D1}, \\ 0 & \text{on } \Gamma_D\backslash\Gamma_{D1}, \end{cases} \qquad \text{and} \quad \phi = \begin{cases} \sin(\pi \mathrm{z})\, e^{2\mathrm{z}-1} & \text{on } \Gamma_{N1}, \\ 0 & \text{on } \Gamma_N\backslash\Gamma_{N1}, \end{cases}$$

where $\Gamma_{D1} = \{(\mathrm{x},\mathrm{y},\mathrm{z}) \in \Gamma_D \;:\; \mathrm{x} = 0\}$ and $\Gamma_{N1} = \{(\mathrm{x},\mathrm{y},\mathrm{z}) \in \Gamma_N \;:\; \mathrm{x}^2 + \mathrm{y}^2 = 1\}$.