Computational Mechanics by Isogeometric Analysis
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Exercises April 19, 2016
NURBS–based Isogeometric Analysis: Galerkin method. II

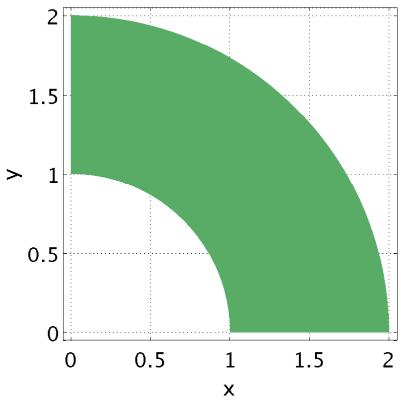
Let us consider the following Poisson problem:

$$\text{find } u \ : \Omega \to \mathbb{R} \quad : \quad \begin{cases} -\nabla \cdot (\mu \nabla u) = f & \text{in } \Omega, \\ u = g & \text{on } \Gamma, \end{cases}$$

being $\Omega \subset \mathbb{R}^d$, for $d = 2, 3$, a bounded open computational domain with regular boundary $\Gamma = \partial\Omega$. The diffusion coefficient $\mu \in L^\infty(\Omega)$ and the source term $f \in L^2(\Omega)$ are assigned functions such that $\mu \geq \mu_0 > 0$ a.e. in Ω; we assume that $g \in H^{1/2}(\Gamma)$. We solve the problem by means of the Galerkin method with NURBS–based Isogeometric Analysis; use the MATLAB functions `test_iga2D.m` and `test_iga3D.m` as templates (the functions are based on the *GeoPDEs* software).

1. Numerically and graphically determine the convergence orders (rates) of the errors in norms L^2 and H^1 under h–refinement for the Poisson problems defined by the following data. Consider different polynomial degrees p and regularities of the basis functions. Also, graphically compare the errors obtained with basis functions of the same degree p but different regularity; plot the errors vs. the characteristic mesh size h and the number of degrees of freedom.

   a) Let us consider the domain Ω reported in the following figure ($\Omega = (1, 2) \times (0, \pi/2)$ in polar coordinates (r, θ)) for which $\mu = 1$, and f and g are such that the exact solution reads $u = \mathrm{x}\,\mathrm{y}\,\sin\left(\frac{\pi}{3}\left(\mathrm{x}^2 + \mathrm{y}^2 - 1\right)\right)$.



   b) Set $\Omega = (0, 1)^3$, $\mu = 1$, $g = 0$, and f such that the exact solution is $u = \sin(\pi \mathrm{x}) \sin(\pi \mathrm{y}) \sin(\pi \mathrm{z})$.

2. Let us consider the Poisson problem with the following data: $\Omega = (0, 1)^2$, $\mu = 1$, and $f = 0$. Solve the problem for different mesh sizes h and polynomial degrees p by approximating the Dirichlet data g with the least squares and the "interpolation at the control points" techniques. Compare the results obtained with the two techniques for the following choices of the data g:

a) $g = \begin{cases} \sin(\pi \mathrm{y}^2) & \text{on } \Gamma_1, \\ 0 & \text{on } \Gamma \backslash \Gamma_1, \end{cases}$

b) $g = \begin{cases} 1 & \text{on } \Gamma_2, \\ 0 & \text{on } \Gamma \backslash \Gamma_2, \end{cases}$

where $\Gamma_1 := \{\mathbf{x} \in \Gamma \ : \ \mathrm{x} = 0\}$ and $\Gamma_2 := \left\{\mathbf{x} \in \Gamma \ : \ \mathrm{x} = 0 \text{ and } \mathrm{y} > \dfrac{2}{3}\right\}$.