Computational Mechanics by Isogeometric Analysis

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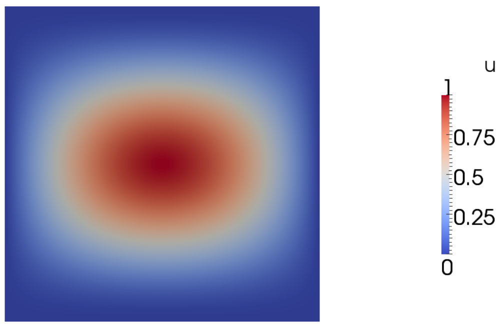
Exercises April 4, 2016: Solutions

NURBS–based Isogeometric Analysis: Galerkin method

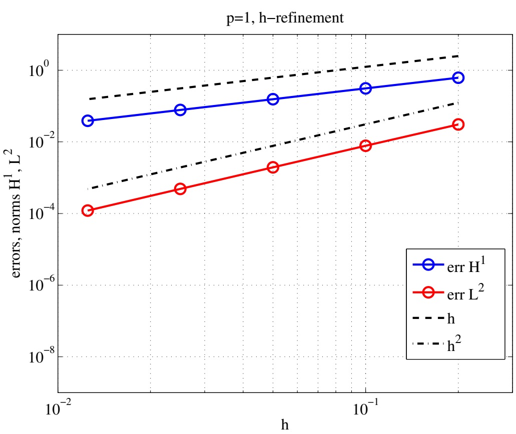
1. Consider the MATLAB file `ex6_1.m`. The exact solution u is reported in the following for which we remark that $u \in C^\infty(\Omega)$.



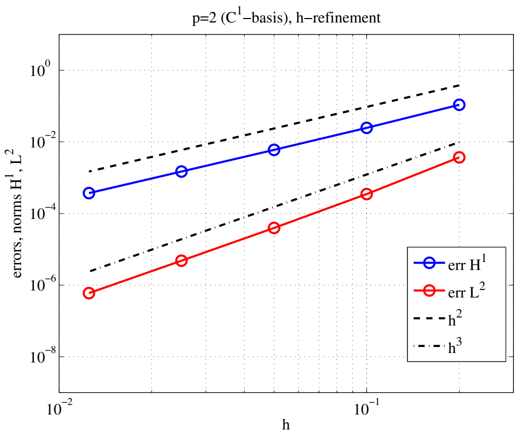
Exact solution, $u = \sin(\pi \mathrm{x})\, \sin(\pi \mathrm{y})^2$

We notice that the computational domain $\Omega = (0,1)^2$ is represented by means of B–splines basis functions of polynomial order $p = 1$ in both the parametric directions with a single mesh element. We consider a h–refinement procedure and compute the errors in norms L^2 and H^1 for different values of the mesh size h; we obtain that the convergence orders in h for the errors in norms L^2 and H^1 are 2 and 1, respectively (see the following figure, left).

As alternative, we perform a one level k–refinement for which we firstly elevate the order of the basis from $p = 1$ to $p = 2$ starting from the geometry with 1 mesh element and then insert the knots without repeating them. Then, we perform consecutive h–refinements while maintaining the basis functions globally C^1–continuous in Ω. We obtain the convergence orders 3 and 2 for the errors in norms L^2 and H^1, respectively (see the following figure, right).

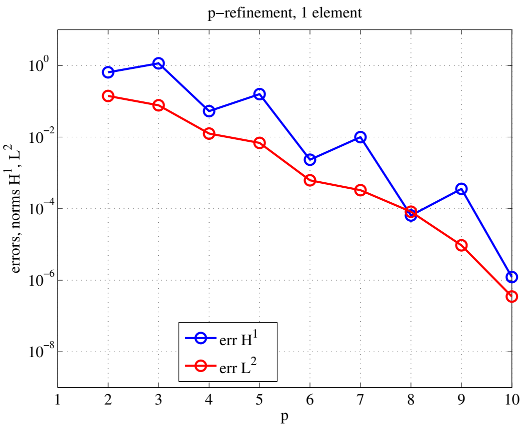


$p = 1$, h–refinement

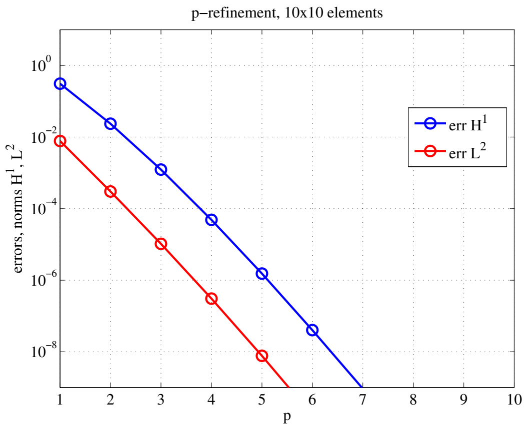


$p = 2$, h–refinement (C^1–cont. fncs.)

We consider now p–refinement procedures. The behavior of the errors in norms L^2 and H^1 vs. p are reported in the following figure starting from 1 mesh element (left) and 10×10 mesh elements (right) for $p = 1$.

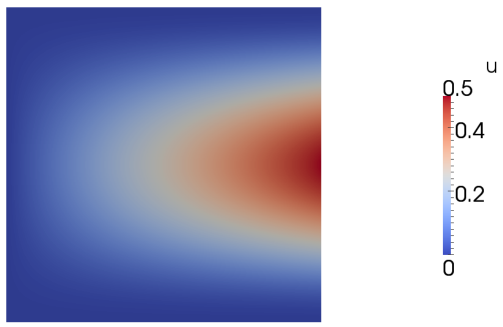


p–refinement, 1 mesh elem.



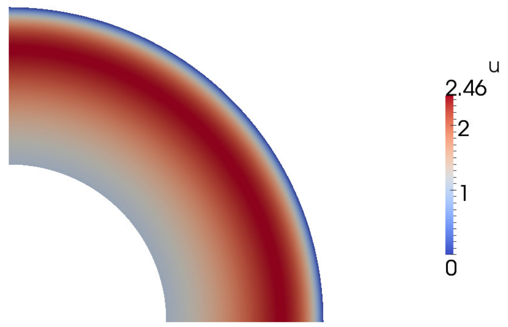
p–refinement, 10×10 mesh elems.

2. Refer to the MATLAB file `ex6_2.m` to obtain the solution of the problem. Similar results of point 1 can be obtained by using suitable refinement procedures.



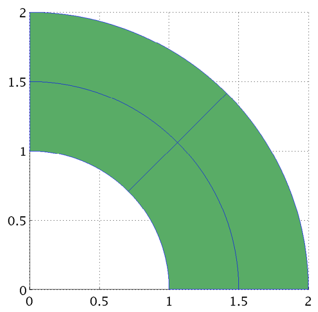
Exact solution, $u = \sin(\pi/6\,\mathrm{x})\,\sin(\pi\mathrm{y})^2$

3. Refer to the MATLAB files `ex6_3.m` and `ex6_display_nurbs_surface_anular.m`.

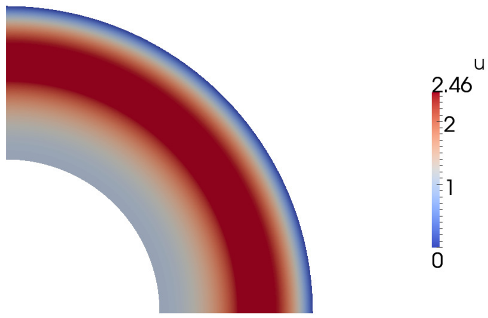


Exact solution, $u = \frac{1}{3}\left(4 - \mathrm{x}^2 - \mathrm{y}^2\right)\, e^{\mathrm{x}^2+\mathrm{y}^2-1}$

We remark that already when considering a coarse mesh, the approximate solution u_h exhibits an asymmetrical behavior similarly to the exact solution u; this follows from the exact representation of the geometry allowed by NURBS at the coarsest level of discretization. See the example below.

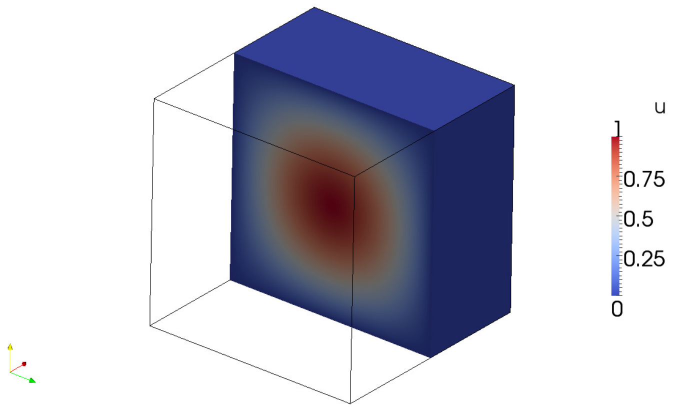


Mesh with 2×2 elems.



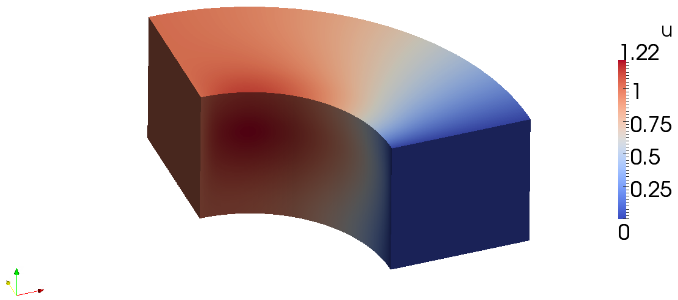
Corresponding approximate solution u_h, $p = 2$

4. Refer to the MATLAB file `ex6_4.m`.

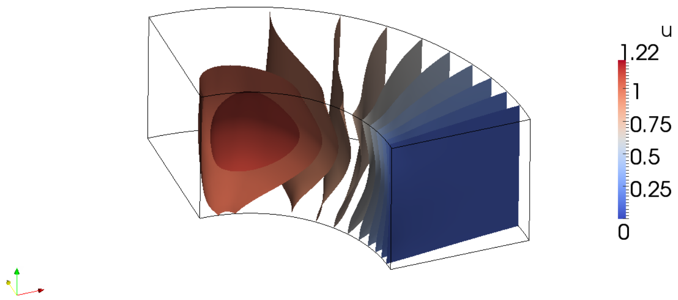


Exact solution, $u = \sin(\pi \mathrm{x})\,\sin(\pi \mathrm{y})\,\sin(\pi \mathrm{z})$

5. Refer to the MATLAB files `ex6_5.m` and `ex6_display_nurbs_solid.m`. The approximate solution u_h obtained with polynomial order $p = 3$ and $5 \times 10 \times 5$ mesh elements(the basis functions are globally C^2–continuous in Ω) is reported in the following figures.

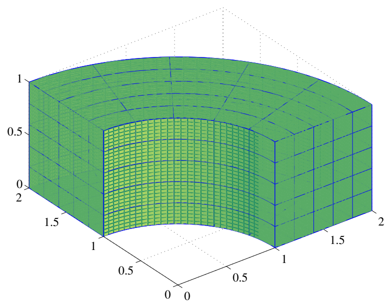


Solution u_h

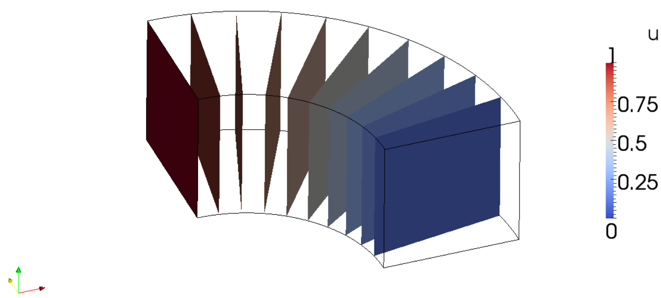


Contour surfaces of the solution u_h

Let us consider the problem with the Neumann data $\phi = 0$, yielding the exact solution $\widetilde{u}$. We observe that already for a coarse mesh and polynomial order $p = 2$ the approximate solution $\widetilde{u}_h$ is able to capture the main features of the exact solution $\widetilde{u}$ by taking advantage of the exact geometric representation. See the example below.



Mesh with $5 \times 5 \times 5$ elems.



Approximate solution $\widetilde{u}_h$ for $\phi = 0$, $p = 2$; contour surfaces