Computational Mechanics by Isogeometric Analysis

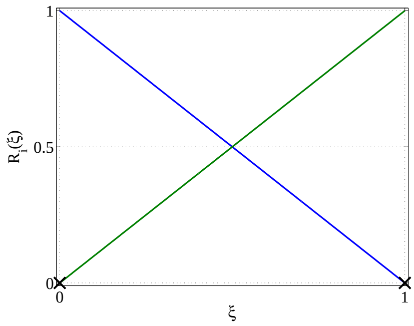
Dr. L. Dedè. A.Y. 2013/14

Exercises March 13, 2014: Solutions

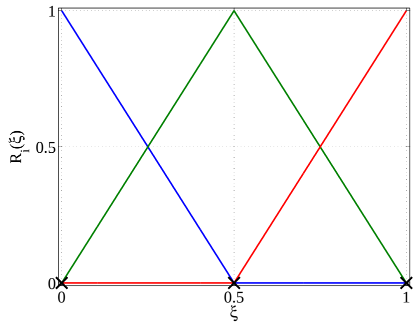
B–splines and NURBS: hpk– refinements

1.

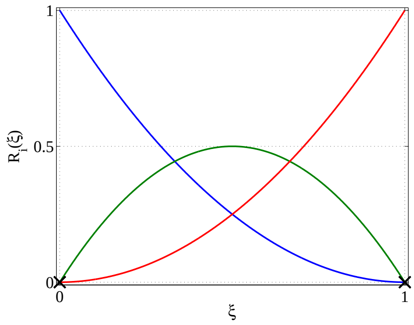
a)



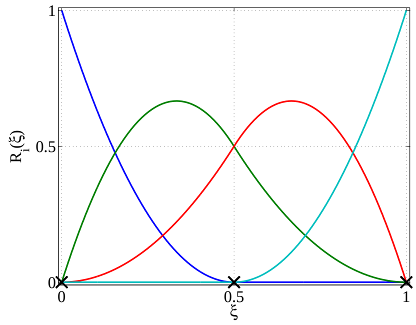
$\Xi = \{\{0\}^2, \{1\}^2\}$



h–ref.: $\Xi = \{\{0\}^2, 1/2, \{1\}^2\}$

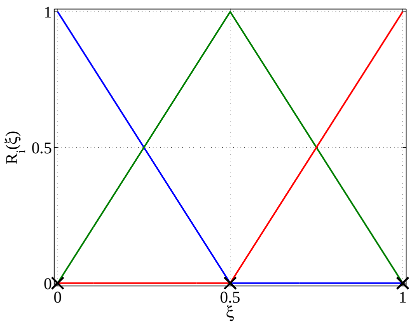


p–ref.: $\Xi = \{\{0\}^3, \{1\}^3\}$

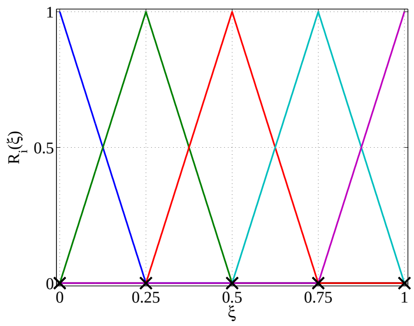


k–ref.: $\Xi = \{\{0\}^3, 1/2, \{1\}^3\}$

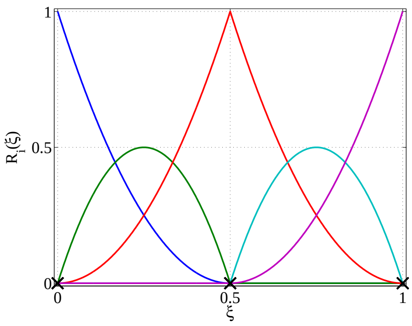
b)



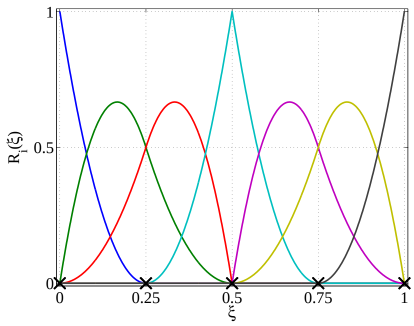
$\Xi = \{\{0\}^2, 1/2, \{1\}^2\}$



h–ref.: $\Xi = \{\{0\}^2, 1/4, 1/2, 3/4, \{1\}^2\}$

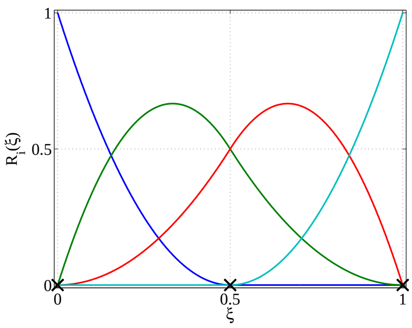


p–ref.: $\Xi = \{\{0\}^3, \{1/2\}^2, \{1\}^3\}$

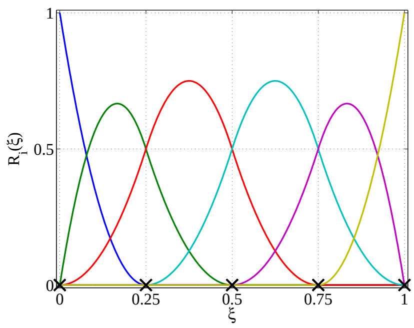


k–ref.: $\Xi = \{\{0\}^3, 1/4, \{1/2\}^2, 3/4, \{1\}^3\}$

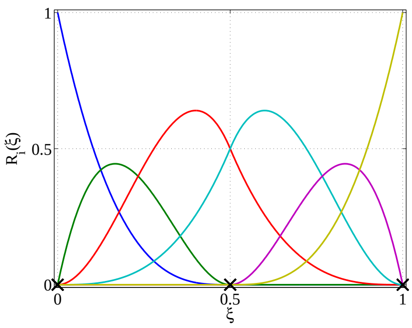
c)



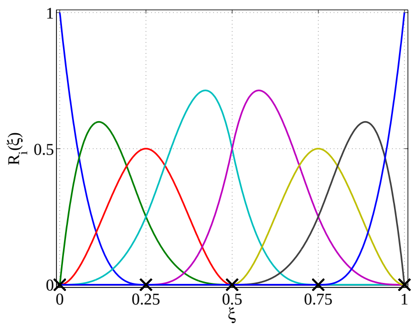
$\Xi = \{\{0\}^3, 1/2, \{1\}^3\}$



h–ref.: $\Xi = \{\{0\}^3, 1/4, 1/2, 3/4, \{1\}^3\}$

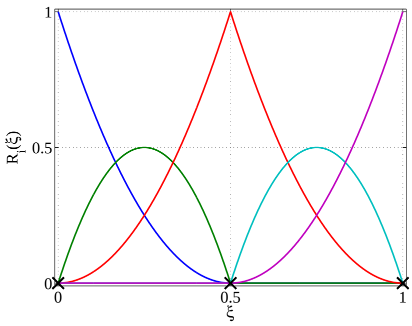


p–ref.: $\Xi = \{\{0\}^4, \{1/2\}^2, \{1\}^4\}$

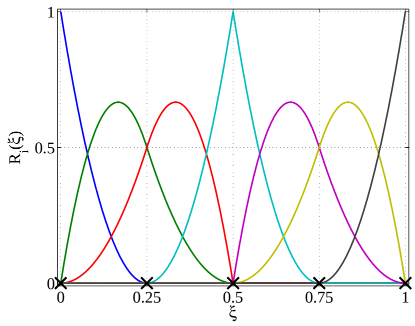


k–ref.: $\Xi = \{\{0\}^4, 1/4, \{1/2\}^2, 3/4, \{1\}^4\}$

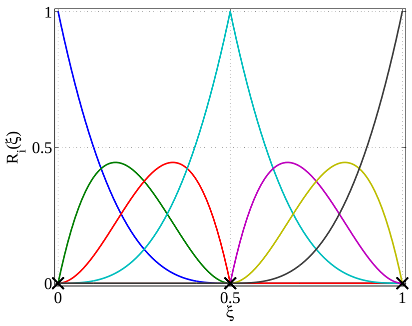
d)



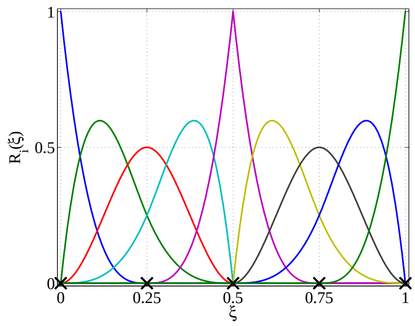
$\Xi = \{\{0\}^3, \{1/2\}^2, \{1\}^3\}$



h–ref.: $\Xi = \{\{0\}^3, 1/4, \{1/2\}^2, 3/4, \{1\}^3\}$

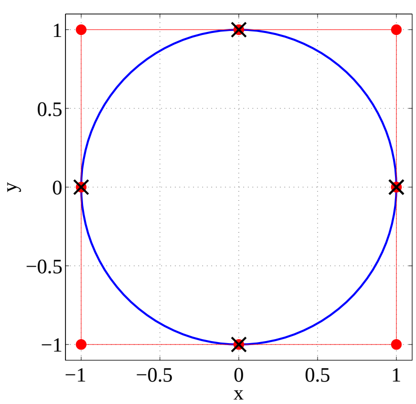


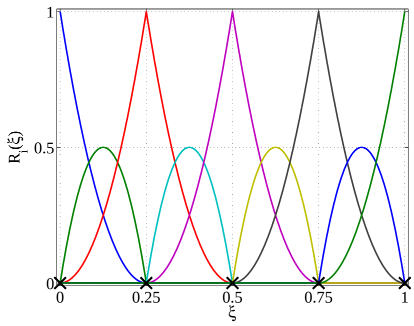
p–ref.: $\Xi = \{\{0\}^4, \{1/2\}^3, \{1\}^4\}$



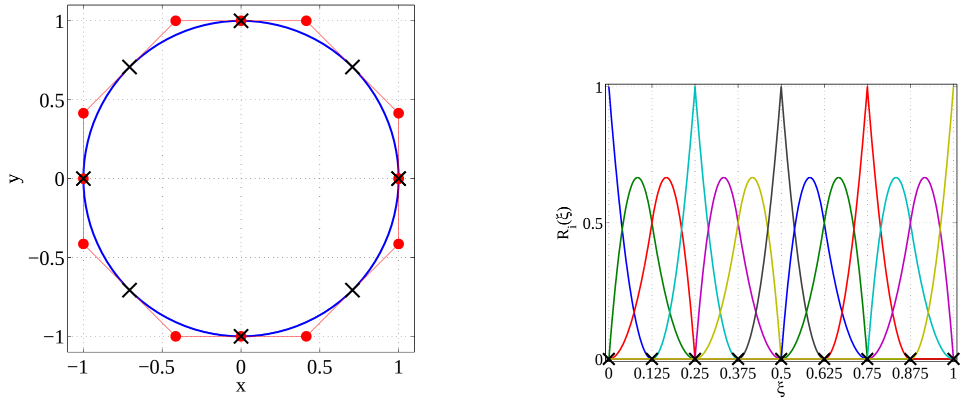
k–ref.: $\Xi = \{\{0\}^4, 1/4, \{1/2\}^3, 3/4, \{1\}^4\}$

2. We consider different levels of hpk–refinements; the control points for the circle are reported in red on the left. The original NURBS basis functions obtained from the knot vector $\Xi = \{\{0\}^3, \{1/4\}^2, \{1/2\}^2, \{3/4\}^2, \{1\}^3\}$ are reported on the right.

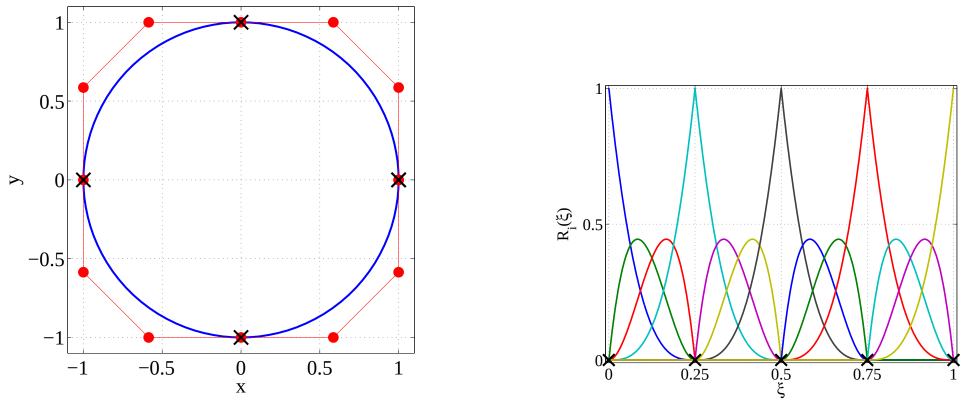




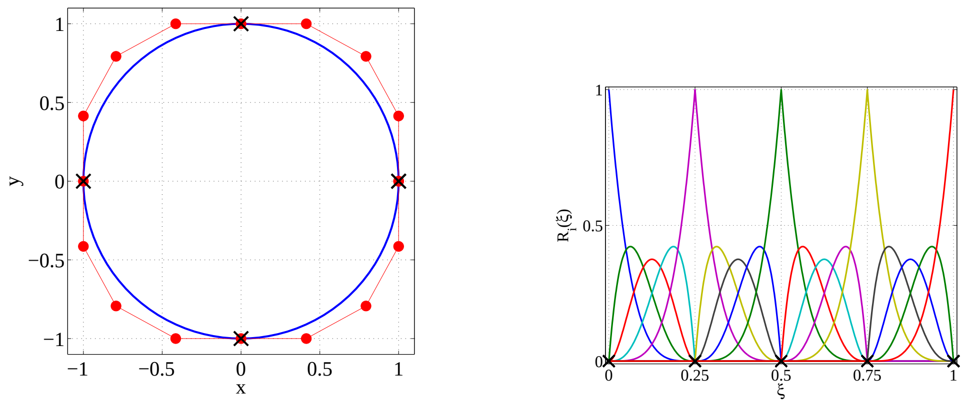
- h–refinement (the knots 1/8, 3/8, 5/8, and 7/8 are inserted), $\Xi = \{\{0\}^3, 1/8, \{1/4\}^2, 3/8, \{1/2\}^2, 5/8, \{3/4\}^2, 7/8, \{1\}^3\}$:



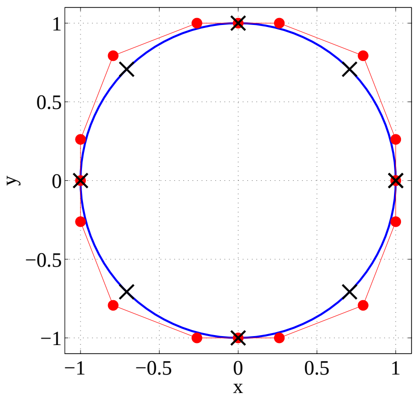
- p–refinement (first level), $\Xi = \{\{0\}^4, \{1/4\}^3, \{1/2\}^3, \{3/4\}^3, \{1\}^4\}$:

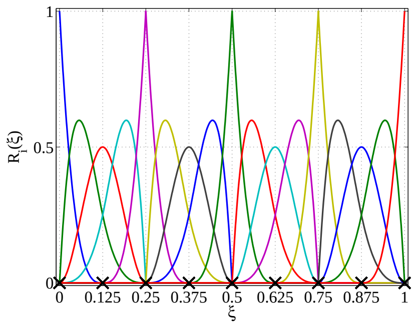


- p–refinement (second level), $\Xi = \{\{0\}^5, \{1/4\}^4, \{1/2\}^4, \{3/4\}^4, \{1\}^5\}$:

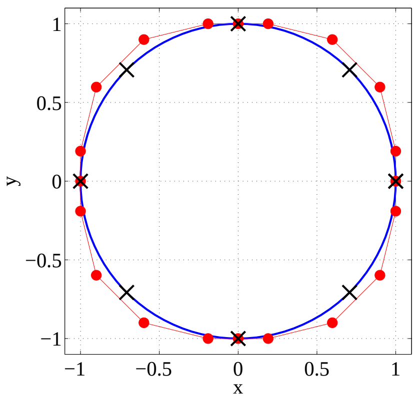


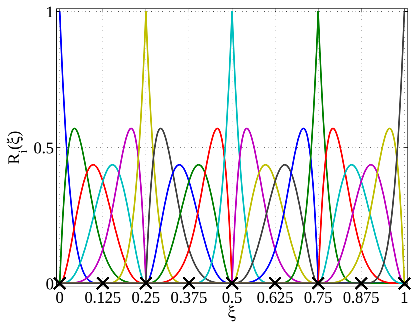
- k–refinement (first level), $\Xi = \{\{0\}^4, 1/8, \{1/4\}^3, 3/8, \{1/2\}^3, 5/8, \{3/4\}^3, 7/8, \{1\}^4\}$:





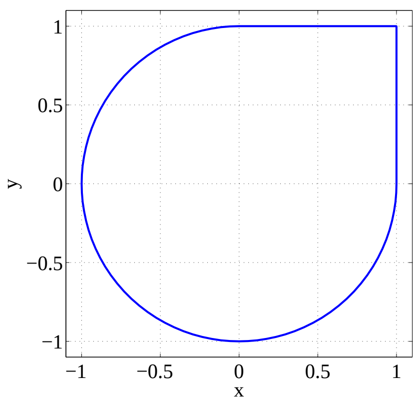
- k–refinement (second level), $\Xi = \{\{0\}^5, 1/8, \{1/4\}^4, 3/8, \{1/2\}^4, 5/8, \{3/4\}^4, 7/8, \{1\}^5\}$:



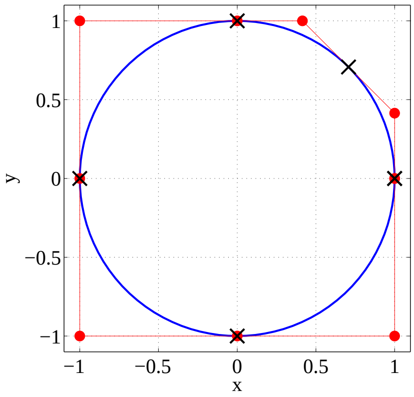


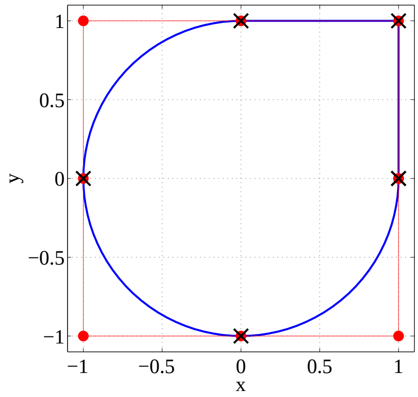
Also, refer to the MATLAB file `ex4_2.m`.

3. We aim at obtaining the following curve by modifying the circle.



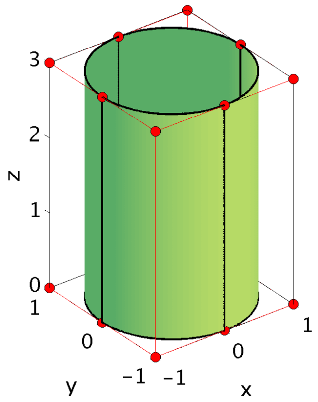
We start by inserting the knot 1/8 in the knot vector which introduces an additional NURBS basis function and a new control point; the new positions of the control points are highlighted on the left. By modifying the positions of the two control points in the upper–right quadrant such that they coincide at the coordinate $(1, 1)$, we obtain the desired curve.

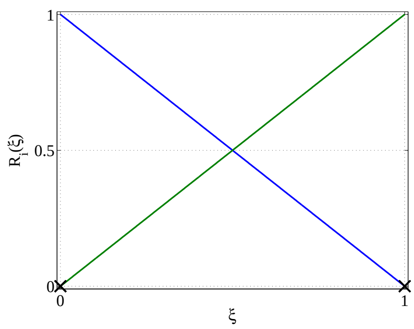




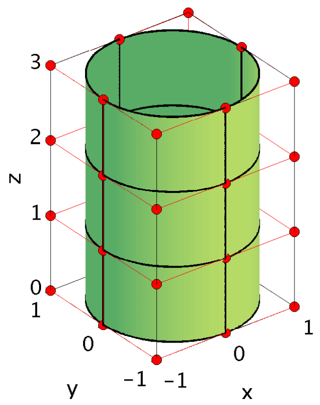
Refer to the MATLAB file `ex4_3.m`.

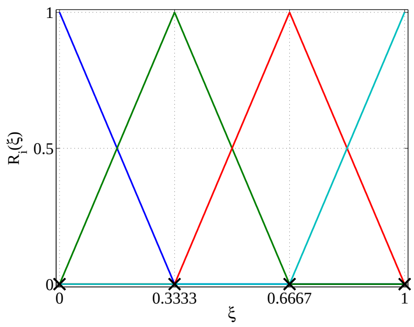
4. We consider different levels of *hpk*–refinements along the parametric direction corresponding to the centerline of the cylindrical shell. The original univariate B–splines basis functions obtained from the knot vector $\Xi = \{0, 0, 1, 1\}$ are reported on the right; the control points are reported in red on the left.



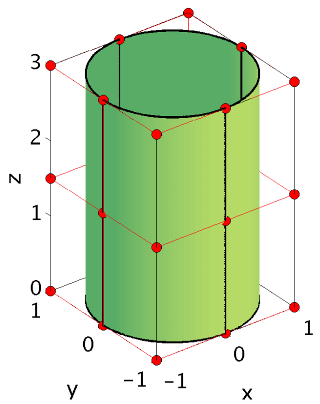


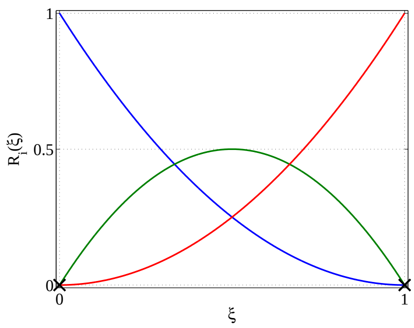
- h–refinement (the knots 1/3 and 2/3 are inserted), $\Xi = \{0, 0, 1/3, 2/3, 1, 1\}$:



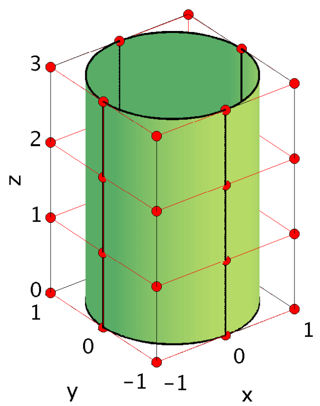


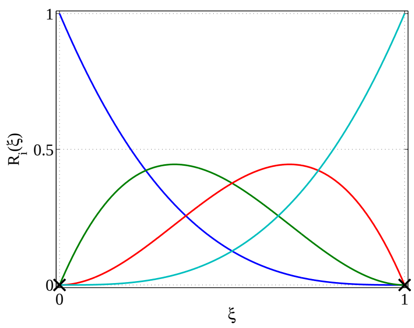
- p–refinement (first level), $\Xi = \{0, 0, 0, 1, 1, 1\}$:



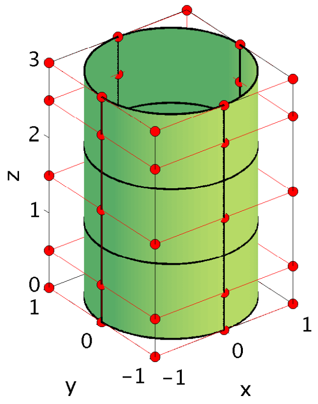


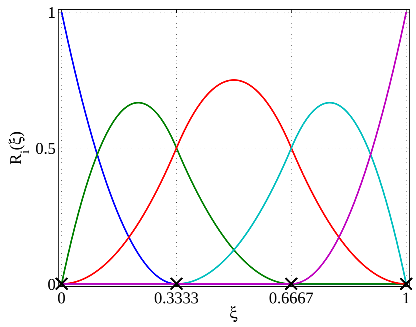
- p–refinement (second level), $\Xi = \{0, 0, 0, 0, 1, 1, 1, 1\}$:



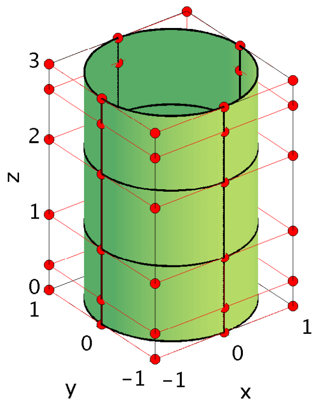


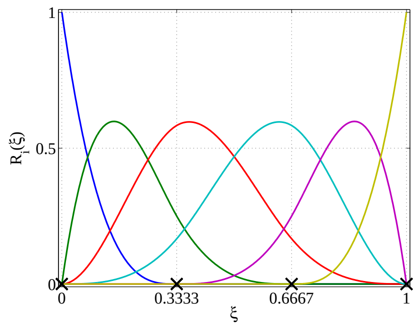
- k–refinement (first level), $\Xi = \{0, 0, 0, 1/3, 2/3, 1, 1, 1\}$:





- k–refinement (second level), $\Xi = \{0, 0, 0, 0, 1/3, 2/3, 1, 1, 1, 1\}$:





Also, refer to the MATLAB file `ex4_4.m`.