

Frenet Frame

Curva Parametrizada:

Position: $x(t): [a, b] \rightarrow \mathbf{R}^3$

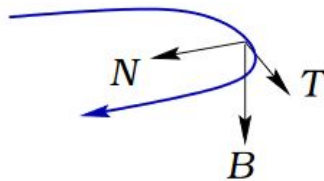
Velocity: $v(t) = x'(t)$

Acceleration: $a(t) = v'(t) = x''(t)$

Longitud de Arco:

$$s'(t) = |x'(t)| = |v(t)| \qquad s(t) = \int_a^t |x'(\tau)| \, d\tau = \int_a^t |v(\tau)| \, d\tau$$

Frenet Frame:



- Tangente:

$$T = \frac{dx}{ds} = \frac{x'}{|x'|} = \frac{v}{|v|}$$

- Principal Normal:

$$N = \frac{dT/ds}{|dT/ds|} = \frac{T'}{|T'|}$$

- Binormal:

$$B = T \times N$$

Curvatura y Torsión:

$$\kappa = \left| \frac{dT}{ds} \right| = \frac{|v \times a|}{|v|^3} \qquad |\tau| = \left| \frac{dB}{ds} \right|$$

Fórmulas Frenet-Serret:

$$\frac{d}{ds} \begin{bmatrix} T \\ N \\ B \end{bmatrix} = \begin{bmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix} \begin{bmatrix} T \\ N \\ B \end{bmatrix}$$

Resolución:

- Función Vectorial:

$$\mathbf{c}(t) = \begin{pmatrix} 2t^3 + t^2 - 3t + 6 \\ t^3 - 2t^2 + 4t \\ -3t^3 + 4t^2 + 2t - 9 \end{pmatrix}$$

- Primera y Segunda Derivada:

$$\mathbf{c}'(t) = \begin{pmatrix} 6t^2 + 2t - 3 \\ 3t^2 - 4t + 4 \\ -9t^2 + 8t + 2 \end{pmatrix}$$
$$\mathbf{c}''(t) = \begin{pmatrix} 12t + 2 \\ 6t - 4 \\ 8 - 18t \end{pmatrix}$$

- Norma de la Primera Derivada:

$$\begin{aligned} \|\mathbf{c}'(t)\| &= \sqrt{(6t^2 + 2t - 3)^2 + (3t^2 - 4t + 4)^2 + (-9t^2 + 8t + 2)^2} \\ &= \sqrt{126t^4 - 144t^3 + 36t^2 - 12t + 29} \end{aligned}$$

- Cálculo de la Tangente:

$$\mathbf{T}(t) = \frac{\mathbf{c}'(t)}{\|\mathbf{c}'(t)\|} = \frac{1}{\sqrt{126t^4 - 144t^3 + 36t^2 - 12t + 29}} \begin{pmatrix} 6t^2 + 2t - 3 \\ 3t^2 - 4t + 4 \\ -9t^2 + 8t + 2 \end{pmatrix}$$

- Se Obtiene el Vector Binormal:

$$\mathbf{c}'(t) \times \mathbf{c}''(t) = \begin{pmatrix} 12t^2 - 84t + 40 \\ 66t^2 - 30t + 28 \\ 30t^2 - 66t + 4 \end{pmatrix}$$

- Se Normaliza el Resultado:

$$\begin{aligned} \|\mathbf{c}'(t) \times \mathbf{c}''(t)\| &= \sqrt{(12t^2 - 84t + 40)^2 + (66t^2 - 30t + 28)^2 + (30t^2 - 66t + 4)^2} \\ &= 24(225t^4 - 414t^3 + 717t^2 - 372t + 100) \end{aligned}$$

- Se Ensambla el Vector Binormal:

$$\mathbf{B}(t) = \frac{\mathbf{c}'(t) \times \mathbf{c}''(t)}{\|\mathbf{c}'(t) \times \mathbf{c}''(t)\|} = \frac{1}{\sqrt{24(225t^4 - 414t^3 + 717t^2 - 372t + 100)}} \begin{pmatrix} 12t^2 - 84t + 40 \\ 66t^2 - 30t + 28 \\ 30t^2 - 66t + 4 \end{pmatrix}$$