VaR/ES Calculator System Introduction

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I. Purpose of the System

The system is designed to calculate the 5-day 99% VaR and 97.5% ES of a portfolio consisting of stocks and vanilla options using 3 different method (historical, parametric and Monte Carlo simulation) for the last 10 year trading days. The computed VaR & ES together with VaR backtesting results are saved into 2 csv files. We use 2-year horizon for the 3 methodologies.

II. Scope of the System

i. Assumption

1. The input data used in the system is clean. There are no NAs or outliers in the input data.
2. The net value of input portfolio is always greater than zero.
3. The option always has its underlying stock in the portfolio.
4. For each stock we only have one option in the portfolio.

ii. Application

The application of this system may be offering a guidance for companies and banks to model risk management effectively and automatically. The system is established for market risk capture and control. By calculating the VaR and ES through historical, parametric and Monte Carlo methodologies, we figure out which method works the most accurate to measure the risk. Then by rigorous model validation and diversified model test, we further identify the limitation of the system and make refinement in model development.

iii. Justification of model choice

Early 1990s, due to the improper and sufficient internal controls, many companies failed. Some of the most typical examples are Orange County (1994, losses of US$1.8 billion) and Barings (1995, US$1.3 billion). According to Markowitz's (1952, [1]) modern portfolio theor, the variance of the Profit and Loss (P&L) distribution was one of most popular and dominating risk measures at that time. However, conventional risk measures have two important drawbacks (Susanne Emmer, 2015, [2]). One of the limits is that risks are randomized and have finite variance. The other drawback is that their distributions are almost symmetric around the mean. Therefore corresponding to this issue, new risk measures including Value-at-Risk (VaR) and Expected Shortfall (ES) are proposed and become prevalent.

In 1996, according to a report [3] by the Basel Committee on Banking Supervision (BCBS), the risk capital of a bank is required to cover losses on the bank's portfolio over a 10-day 99% of occasions. 99% VaR thus became the determinant of the market risk capital requirements.

According to the work of Artzner et al. [4], now ES's important advantage – coherence has allowed it to overtake VaR as a better risk measure for institutions and funds allocation Tasche [5]. In the second consultative document [6] issued by Basel Committee on Banking Supervision (2013), the Committee has proposed using stress calibration in checking sufficient regulatory capital during market stress and also reached an agreement to employ 97.5% ES for the internal model-based method instead of VaR. this indicates that the Committee are seeking a transition from general internal models-based approach to a single, stressed metric AKA ES. Concerning the ES's failure in backtesting, a research by Ziegel [7] and Bellini et al. [8] showed that its alternative Expectiles works well in terms of coherence and elicitability.

The three methodologies used to calculate VaR/ES in this project are justified according to recent research. For Gaussian parametric approach, it is used by the researcher when a probability distribution of a risk factor can be identified (Yun Hsing Cheung, [9]). However, in professional, this is vulnerable for 2 reasons: First, it is difficult to supply accurate parameters. Second, VaR is not coherent. Third, the Gaussian assumption does not fit the reality as there might be a fat tail.

For non-parametric approaches (including historical and Monte Carlo methods), data is generally sparse in the tail region and hence proposing a proper model. This is where the non-parametric methods can play a significant role according to Deepak Jadhav (2009, [10]). They are much more favorable than parametric methods in their robustness to the model assumption. Thus bias caused by the use of a misspecified return distribution is avoided.

iv. Limitation

1. No missing value checkups: if some values in the inputs, including stock prices on specific day, miss, the system cannot run smoothly and no mechanism to locate or fix the error

2. One option for each stock: only one option of the stock is employed in the portfolio because of the GBM limitation which allows only constant volatility. The case of volatility smile is way too complicated to capture in this system

3. Only at-the-money option: no in-the-money or out-the-money options are considered in the system

4. Restricted future price: for simplification, we use 1-week-after price as our future price for historical simulation, which is a limited assumption

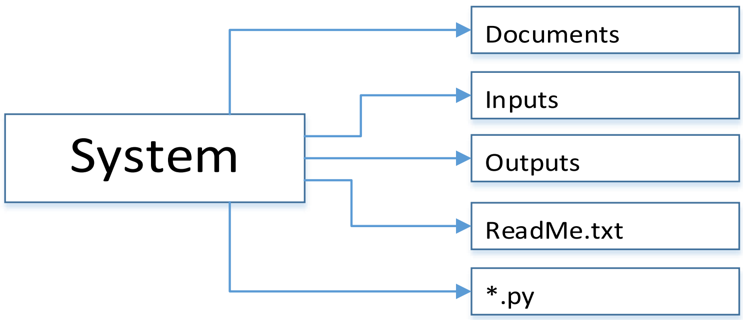
v. Comparison with alternative approach

Pros & Cons	**Value at Risk**	**Expected Shortfall**	**Expentile**
i. Detectable for risk concentration?	No, not subadditive	Yes, subadditive	May not, not comonotonically additive
ii. Sensitive to tail risk?	No, serious problem for diverse risks with different tails	Yes	Yes
iii. Elicitable?	Yes, easier backtesting	No, harder backtesting and more validation data needed	Yes, easier backtesting
iv. Easy to implement?	Yes	Yes	No, less intuitive

III. System Structure

i. Folder System

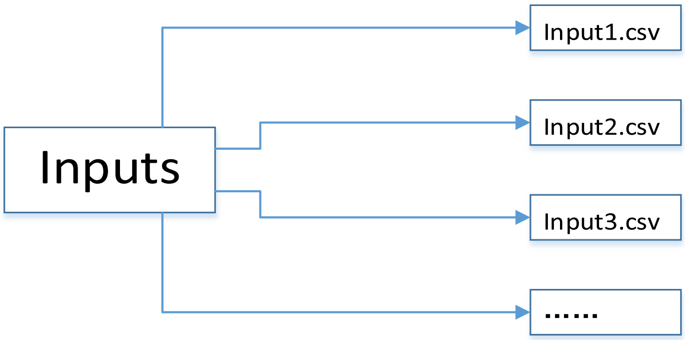
There are 5 types of files in the folder system, i.e. Inputs, Outputs, and Documents. Their structures are displayed below.



1. Inputs

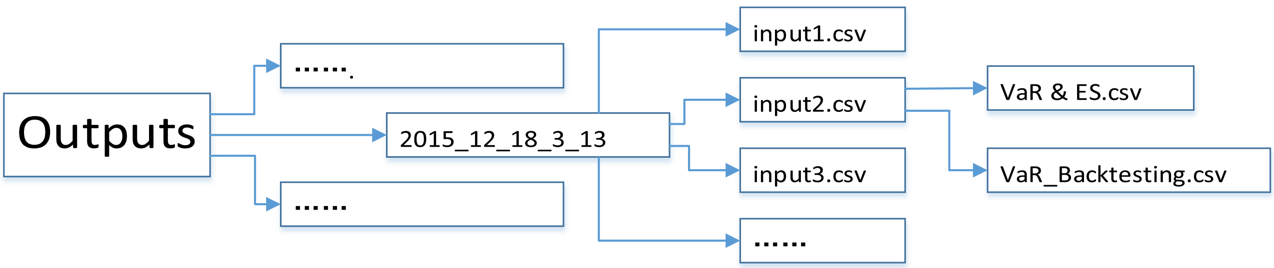
This folder contains many input files which are differentiated from each other. For example, input1.csv can be the portfolio with long positions in only AAPL and IBM (a benchmark same as home assignment);

input2.csv can be the portfolio with long/short positions in only AAPL and IBM; input3.csv can be the portfolio with long/short positions in only stocks; input4.csv can be the portfolio with long/short positions in both stocks and options; and so forth.



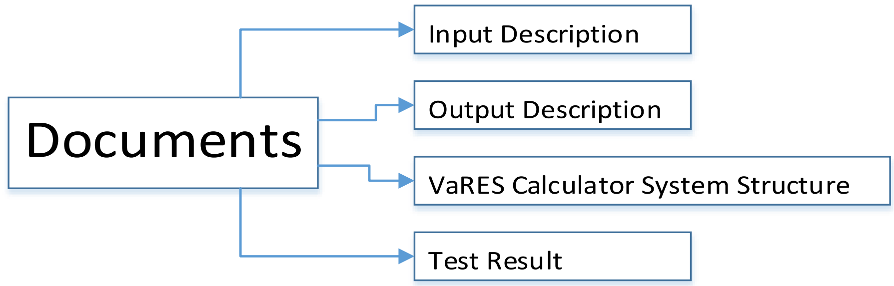
2. Outputs

The final outputs (VaR & ES.csv / VaR_Backtesting.csv) are displayed in the folder corresponding to their inputs and their unique birth time. With successful running, the following results will be generated. There are 3 layers: the first layer showed the birth time of each result package; the second layer consists of folders with exact the same name with the input files in the input folders; the third layer is comprised of two kinds of files – VaR & ES for each input and the corresponding backtesting result.



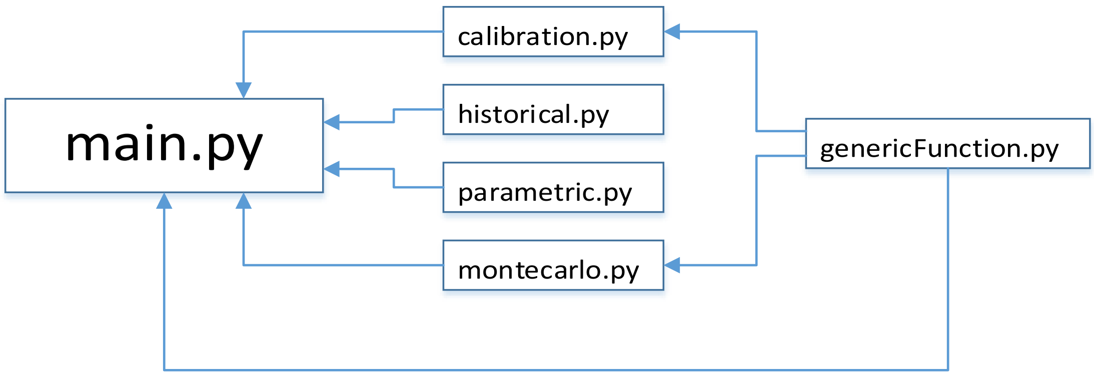
3. Documents

The Documents folder contains 4 kinds of files: Input & Output Description demonstrates clearly the meanings of each item's title in the input and output csv; System Description gives a walk-through of the whole system; User Manual will teach you how to use the risk calculation system; Test Result displays the final result of testing. Note that User Manual and Test Result are also stated in the file Readme.txt.



ii. Relationship of Python Scripts

There are 6 .py files in root folder. Their relationship are given by:



main.py: the main python script that calls all the python scripts

calibration.py: the python script that is used to calibrate the mu and sigma.

historical.py: the python script used to calculate the historical VaR and ES

parametric.py: the python script used to calculate the parametric VaR and ES

montecarlo.py: the python script used to calculate the Monte Carlo VaR and ES

genericFunction.py: the python script used to define the BS model or get the underlying name, which are callable by other codes

IV. Model Methodology

i. Parametric VaR/ES

1. Assumption

Parametric model has two main assumptions. First, the whole portfolio value is following a geometric Brownian motion. Second, the portfolio value is greater than zero.

2. Model Description

By assuming geometric Brownian motion, we could calibrate drift term and volatility term from historical value of portfolio, then plug the parameters into the following formulas:

$$VaR\ (S_0, T, p)\ =\ S_0 - S_0 e^{\sqrt{T}\Phi^{-1}(1-p)+(\mu+\frac{\sigma^2}{2})T}$$

$$ES(S_0, T, p) = S_0 - \frac{S_0 e^{\mu T}\Phi(\Phi^{-1}(1-p) - \sigma\sqrt{T})}{1-p}$$

Where,

$S_0: current\ portfolio\ value$

$\mu: drift\ term\ of\ geometric\ brownian\ motion$

$\sigma: volatility\ term\ of\ geometric\ brownian\ motion$

$T: \frac{n_{day}}{252}$

$p: VaR\ level$

ii. Historical VaR/ES

The Historical VaR/ES model is to simulate assume risk factors by following actual historical distributions.

1. Assumption

Historical model has two main assumptions. First it assumes the asset returns are identically-distributed and independent random variables without a particular distribution and potential volatility clustering. Second it assumes all asset returns are applied with equal weight, which is not the real case since it violates the diminishing predictability of data.

2. Model Description

i. Select relative historical model (reason)

   There's no occasions for absolute crossing zero and also the net portfolio value is far larger than zero

ii. Calculate the relative returns of all stocks / options in the portfolio in each time interval

iii. Calculate VaR/ES

   1. Sort the series of the relative portfolio P&L from the lowest to the highest value
   2. Get the value at the worst 1-p_1 percent position as the p_1 level VaR
   3. Get the average value of the worst 1-p_2 percent tails as the p_2 level ES

iii. Monte Carlo VaR/ES

The Monte Carlo VaR/ES is to simulate the risk factors and use the prices to directly compute the VaR/ES, which is not really a model. Instead, it is more of a numerical method applied to the problem.

i. Assumption

Monte Carlo model has one primary assumption that we assume the individual stock or option price follows Geometric Brownian Motion.

ii. Model Description

i. Simulate adapted Wiener processes for each stock/option

1. Generate n standard Wiener processes (W) with the targeted sample size for every day
2. Calculate the correlation matrix of complements of the portfolio for every day (pho)
3. Use the Cholesky decomposition to decompose the everyday correlation matrix, generating a new component matrix

$$\text{pho} = AA^T$$

Where pho is the everyday correlation matrix which is a Hermitian positive-definite matrix

4. Use the component matrix and the original standard Wiener processes to generate the adjusted Wiener processes ($\widetilde{W}$)

$$\widetilde{W} = AW\sqrt{dt}$$

ii. Calculate portfolio value

1. Simulate the individual stock price by the Geometric Brownian Motion

$$S_t = S_0 e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma \widetilde{W}}$$

2. Calculate the option price O using the Black-Scholes equation
3. Calculate the simulated portfolio value:

$$P_t = \sum_{i=1}^{n} S_{t,i}(\#\ of\ shares) + \sum_{j=1}^{m} O_{t,j}$$

iii. Calculate VaR/ES

1. Sort the simulated values and get the value at the worst 1-p_1 percent position as the p_1 level VaR
2. Sort the simulated values and get the average value of the worst 1-p_2 percent tails as the p_2 level ES

V. Model Comparison

Pros & Cons	Historical Model	Parametric Model	Monte Carlo Model
i. Able to capture the risks of portfolios which include options	Yes, regardless of the options content of the portfolio	No, except when computed using a short holding period for portfolios with limited or moderate oprtions content	Yes, regardless of the options content of the portfolio
ii. Easy to implement?	Yes, for portfolios for which data on the past values of the market factors are available	Yes, for portfolios restricted to instruments and currencies covered by available "off-the-shelf" software. Otherwise, reasonably easy to moderately difficult to implement, depending upon the complexity of the instruments and availability of the data.	Yes, for portfolios restricted to instruments and currencies covered by available "off-the-shelf" software. Otherwise, reasonably easy to moderately difficult to implement.
iii. Computations performed quickly?	Yes	Yes	No, except for relatively small portfolios
iv. Produces misleading VaR estimates when recent past is atypical?	Yes	Yes, except that alternative correlations/standard deviations may be used.	Yes, except that alternative estimates of parameter may be used.
v. Easy to perform "what-if" analyses to examine effect of alternative assumptions?	No	Easily able to examine alternative assumptions about correlations/standard deviations.	Yes

vi. Model nonlinear payoff	Yes	No	Yes

VI. Performance Requirements of the Model

We require the inaccuracy to be less than 2%, meaning that if we set the VaR to be 99% level, then we expect at least 97% of the loss level does not exceed the 99% VaR. Similar arguments should also be applied to ES. Besides, we also expect that historical model should be more precise than the other two models except for portfolio value close to 0 because we developed historical VaR/ES based on relative historical change, so extreme percentage change could happen in such a case. As for parametric and Monte Carlo models, they might underestimate VaR/ES because they both incorporate the assumption of Geometric Brownian Motion distribution of equity prices or portfolio values and such distribution results in thinner tails than the regular distribution of financial return.

VII. System Implementation

i. Software Architecture

Python version: Python 2.7

Package: pandas, numpy, itertools, scipy.stats, heapq, os, time, sys

Operating system: Windows / Linux (currently incompatible with MacOS)

ii. Input Description

See Input Description.pdf in folder "Documents"

iii. Output Description

See Output Description.pdf in folder "Documents"

iv. User Guide

First, put input csv files in the required format explained in Section VII (ii) in Inputs Folder. Second, if using the Windows system, run the program by opening the file "main.py" and click the run button. If using the Linux shell with Python interpreter, use the cd() to enter the VaR/ES Calculation System directory. Then type Python "main.py" in the shell. Third, go to Outputs folder and check latest generated folders, "VaR & ES.csv" and "VaR Backtesting.csv" are final results. The detailed description and all test results are saved in Documents – Test Plan & Result.

VIII. Test Plan & Result

See Test Plan & Result.pdf in folder "Documents"

IX. Reference

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