APPLIED PHYSICS 155

EXAM 4. Due: 21 May 2018, 10:00AM (via UVLe)

Instructions:

1. Use one ipynb file per problem. Label the files problemX.ipynb, where X is the problem number and submit all five in one compressed file.
2. Work alone - discussing the problem/solution with anyone is to be avoided until after the deadline of submission. Any code you turn in must be your own.
3. Submit your solution via UVLe. Emailed, hardcopy, and other non-UVLe submissions will receive a grade of zero. You may (and are encouraged to) resubmit your solution as often as practicable prior to the deadline. Late/no submissions will receive a grade of zero.

Problem 4.1: Rolling dice

a) Write a program that generates and prints out three random numbers between 1 and 6, to simulate the rolling of two dice.

b) Modify your program to simulate the rolling of three dice a million times and count the number of times you get a triple six. Divide by a million to get the *fraction* of times you get a triple six. You should get something close to, though probably not exactly equal to, $\frac{1}{216}$.

Problem 4.2: Brownian motion

Brownian motion is the motion of a particle, such as a smoke or dust particle, in a gas, as it is buffeted by random collisions with gas molecules. Make a simple computer simulation of such a particle in two dimensions as follows. The particle is confined to a square grid or lattice $L \times L$ squares on a side, so that its position can be represented by two integers $i, j = 0 \ldots L-1$. It starts in the middle of the grid. On each step of the simulation, choose a random direction—up, down, left, or right—and move the particle one step in that direction. This process is called a random walk. The particle is not allowed to move outside the limits of the lattice—if it tries to do so, choose a new random direction to move in.

Write a program to perform a million steps of this process on a lattice with $L = 101$ and make an animation on the screen of the position of the particle. (We choose an odd length for the side of the square so that there is one lattice site exactly in the center.)

Check the example at `https://matplotlib.org/examples/animation/dynamic_image2.html` for matplotlib-based animations.

Problem 4.3: Calculate a value for the integral

$$I = \int_0^1 \frac{x^{-1/2}}{e^x + 1}\,dx,$$

using the importance sampling formula,

$$I \simeq \frac{1}{N}\sum_{i=1}^{N} \frac{f(x_i)}{w(x_i)} \int_a^b w(x)\,dx \tag{1}$$

with $w(x) = x^{-1/2}$, as follows.

a) Show that the probability distribution $p(x)$ from which the sample points should be drawn is given by

$$p(x) = \frac{1}{2\sqrt{x}}$$

and derive a transformation formula for generating random numbers between zero and one from this distribution.

b) Using your formula, sample $N = 1\,000\,000$ random points and hence evaluate the integral. You should get a value around 0.84.