Principe - Physics 265 PS5

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1 Physics 265 Problem Set 5

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[16]:
```
import numpy as np
import matplotlib.pyplot as plt
```

1.1 $\mathbf{n_1(\lambda)}$ of BaSF10

$$n_1^2(\lambda) = 2.6531250 - 8.1388553 \times 10^{-3}\lambda^2 + 2.2995643 \times 10^{-2}\lambda^{-2} + 7.3535957 \times 10^{-4}\lambda^{-4} \tag{1}$$
$$-1.3407390 \times 10^{-5}\lambda^{-6} + 3.6962325 \times 10^{-6}\lambda^{-8} \tag{2}$$

[17]:
```
def BaSF10_n(lambda_):
    term1 = 8.1388553*10**-3 * lambda_**2
    term2 =  2.2995643*10**-2 * lambda_**-2
    term3 =  7.3535957*10**-4 * lambda_**-4
    term4 =  1.3407390*10**-5 * lambda_**-6
    term5 =  3.6962325*10**-6 * lambda_**-8
    return np.sqrt(2.6531250 - term1 + term2 + term3 - term4 + term5)
```

1.2 $\frac{dn_1(\lambda)}{d\lambda}$ of BaSF10

$$\frac{dn_1(\lambda)}{d\lambda} = \frac{1}{2}n_1(\lambda)\left[2 \cdot 8.1388553 \times 10^{-3}\lambda - 2 \cdot 2.2995643 \times 10^{-2}\lambda^{-3} - 4 \cdot 7.3535957 \times 10^{-4}\lambda^{-5} - 6 \cdot 1.3407390 \times 10\right. \tag{3}$$

[18]:
```
def d_BaSF10_n(lambda_):
    term1 = 2 * 8.1388553*10**-3 * lambda_
    term2 = -2 * 2.2995643*10**-2 * lambda_**-3
    term3 = -4 * 7.3535957*10**-4 * lambda_**-5
    term4 = -6 * 1.3407390*10**-6 * lambda_**-7
    term5 = -8 * 3.6962325*10**-7 * lambda_**-9

    TERM1 = 1/2*BaSF10_n(lambda_)
    TERM2 = -term1 + term2 + term3 - term4 + term5
```

```
    return TERM1*TERM2
```

1.3 $n_2(\lambda)$ of BaK1

$$n_2^2(\lambda) = 2.4333007 - 8.4931353 \times 10^{-3}\lambda^2 + 1.3893512 \times 10^{-2}\lambda^{-2} + 2.6798268 \times 10^{-4}\lambda^{-4} \tag{4}$$

$$-6.1946101 \times 10^{-6}\lambda^{-6} + 6.2209005 \times 10^{-7}\lambda^{-8} \tag{5}$$

[19]:
```
def BaK1_n(lambda_):
    term1 = 8.4931353*10**-3 * lambda_**2
    term2 =  1.3893512*10**-2 * lambda_**-2
    term3 =  2.6798268*10**-4 * lambda_**-4
    term4 =  6.1946101*10**-6 * lambda_**-6
    term5 =  6.2209005*10**-7 * lambda_**-8
    return np.sqrt(2.4333007 - term1 + term2 + term3 - term4 + term5)
```

1.4 $\frac{dn_2(\lambda)}{d\lambda}$ of BaK1

$$\frac{dn_2(\lambda)}{d\lambda} = \frac{1}{2}n_2(\lambda)\,[2 \cdot 8.4931353 \times 10^{-3}\lambda - 2 \cdot 1.3893512 \times 10^{-2}\lambda^{-3} - 4 \cdot 2.6798268 \times 10^{-4}\lambda^{-5} - 6 \cdot 6.1946101 \times 10 \tag{6}$$

[20]:
```
def d_BaK1_n(lambda_):
    term1 = 2 * 8.4931353*10**-3 * lambda_
    term2 = -2 * 1.3893512*10**-2 * lambda_**-3
    term3 = -4 * 2.6798268*10**-4 * lambda_**-5
    term4 = -6 * 6.1946101*10**-6 * lambda_**-7
    term5 = -8 * 6.2209005*10**-7 * lambda_**-9

    TERM1 = 1/2*BaK1_n(lambda_)
    TERM2 = -term1 + term2 + term3 - term4 + term5
    return TERM1*TERM2
```

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2 Problem 5.1

2.1 Dispersion by a Prism

Plot the angular dispersion OE as a function differential wavelength D [see Eqn 37, p 193, Born & Wolf) for a prism (base lengths b = 2.5, 5 and 7.5 cm, l_1 = 5 cm) that is made of your assigned dielectric material for the range: 400 nm 800 nm (D = 0.1 nm).

Which of the two materials is more suitable for a prism spectrometer? Explain succinctly.

The angular dispersion by a prism is given by

$$\Delta\varepsilon = \frac{b}{l_1}\frac{dn}{d\lambda}\Delta\lambda \tag{7}$$

[21]:
```python
def dE(b, l1 , dn, d_lambda):
    return (b/l1) * dn * d_lambda
```

[22]:
```python
d_lambda = 0.1
lambda_list = np.arange(400, 800, d_lambda)/1e3
lambda_list
```

[22]:
```
array([0.4   , 0.4001, 0.4002, …, 0.7997, 0.7998, 0.7999])
```

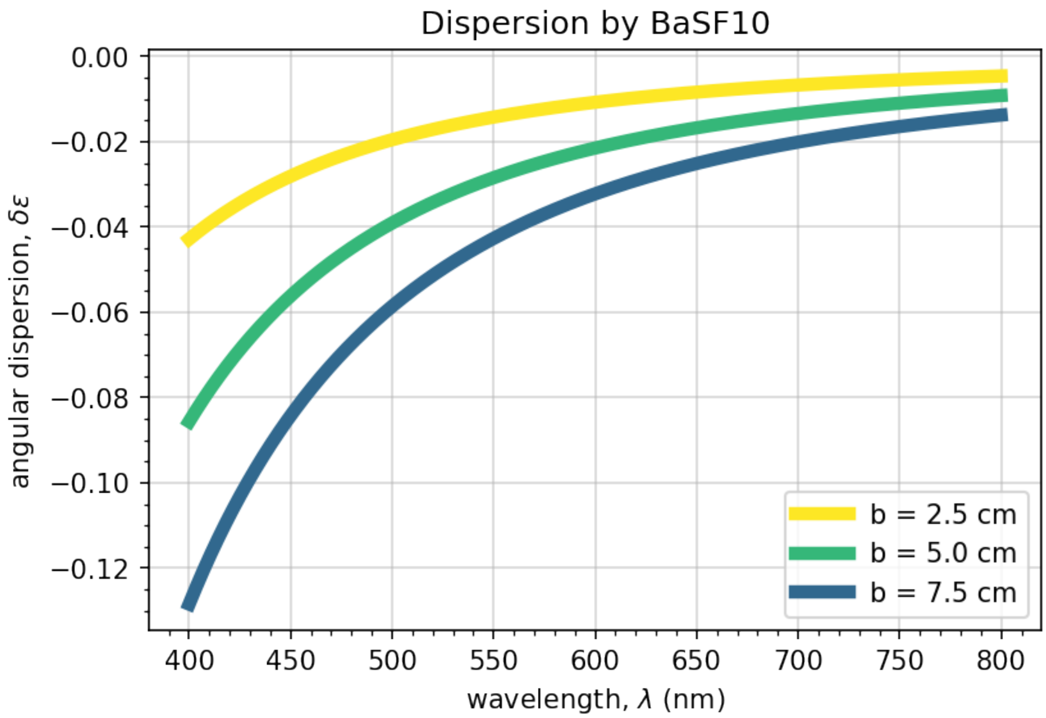
[23]:
```python
b_vals = np.array([2.5, 5, 7.5])
```

[24]:
```python
plt.figure(dpi = 150)
plt.grid(alpha = 0.5)

for i, b in enumerate(b_vals):
    d_epsilon = dE(b, 5, d_BaSF10_n(lambda_list), d_lambda = d_lambda)
    plt.plot(lambda_list*1e3 , d_epsilon , '-', lw =5,
            color = plt.cm.viridis_r(i/len(b_vals)), label = 'b =% .1f cm' % b)

plt.legend()
plt.xlabel('wavelength, $\lambda$ (nm)')
plt.ylabel('angular dispersion, $\delta \epsilon$')
plt.minorticks_on()
plt.title('Dispersion by BaSF10')
```

[24]:
```
Text(0.5, 1.0, 'Dispersion by BaSF10')
```

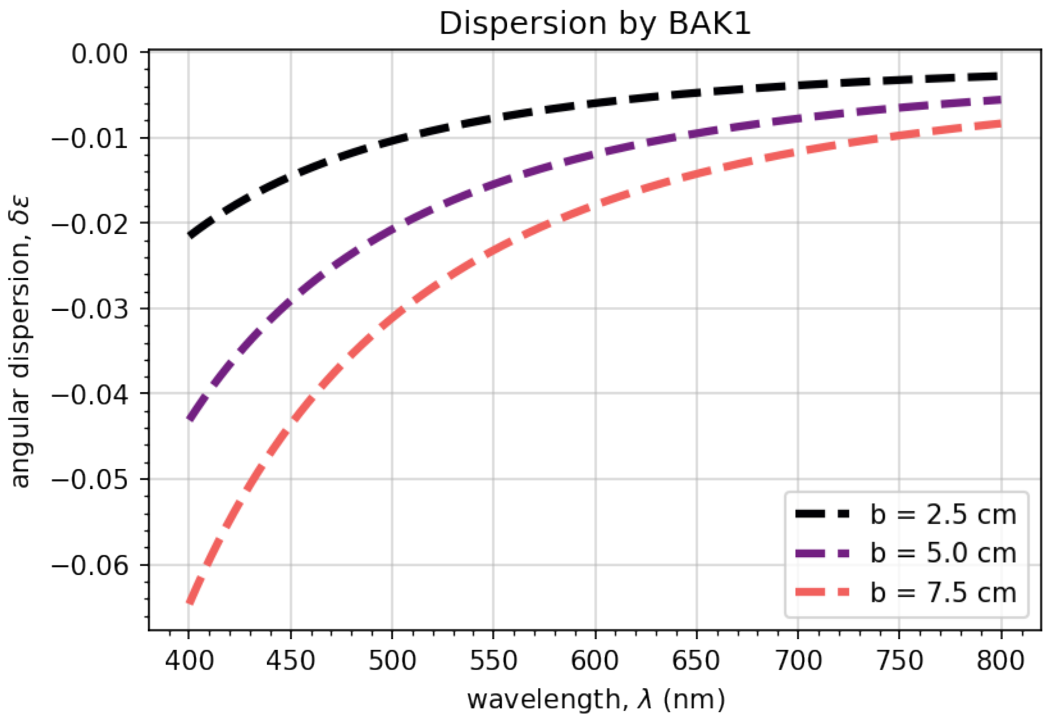


[25]:
```
plt.figure(dpi = 150)
plt.grid(alpha = 0.5)

for i, b in enumerate(b_vals):
    d_epsilon = dE(b, 5, d_BaK1_n(lambda_list), d_lambda = d_lambda)
    plt.plot(lambda_list*1e3 , d_epsilon , '--', lw = 3,
            color = plt.cm.magma(i/len(b_vals)), label = 'b =% .1f cm' % b)

plt.legend()
plt.xlabel('wavelength, $\lambda$ (nm)')
plt.ylabel('angular dispersion, $\delta \epsilon$')
plt.minorticks_on()
plt.title('Dispersion by BAK1')
```

[25]:
```
Text(0.5, 1.0, 'Dispersion by BAK1')
```



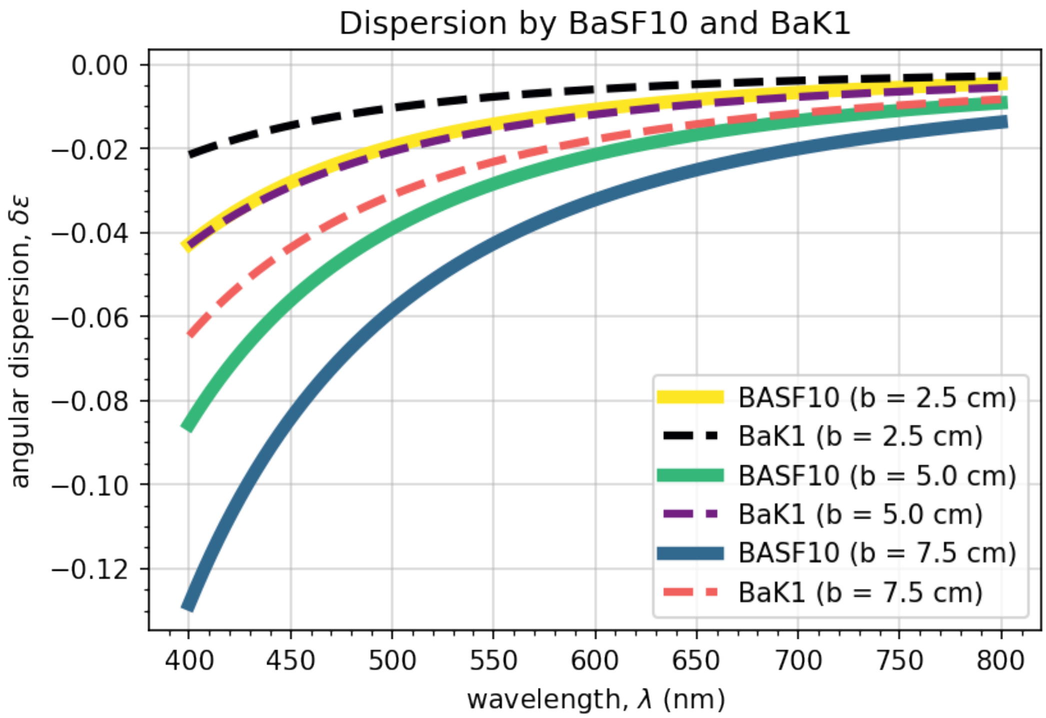
[26]:
```
plt.figure(dpi = 150)
plt.grid(alpha = 0.5)

for i, b in enumerate(b_vals):
    d_epsilon_1 = dE(b, 5, d_BaSF10_n(lambda_list), d_lambda = d_lambda)
    plt.plot(lambda_list*1e3 , d_epsilon_1 , '-', lw = 5,
             color = plt.cm.viridis_r(i/len(b_vals)), label = 'BASF10 (b =% .1f␣
 ↪cm)' % b)

    d_epsilon_2 = dE(b, 5, d_BaK1_n(lambda_list), d_lambda = d_lambda)
    plt.plot(lambda_list*1e3 , d_epsilon_2 , '--', lw = 3,
             color = plt.cm.magma(i/len(b_vals)), label = 'BaK1 (b =% .1f cm)' %␣
 ↪b)


plt.legend()
plt.xlabel('wavelength, $\lambda$ (nm)')
plt.ylabel('angular dispersion, $\delta \epsilon$')
plt.minorticks_on()
plt.title('Dispersion by BaSF10 and BaK1')
```

[26]:
```
Text(0.5, 1.0, 'Dispersion by BaSF10 and BaK1')
```



Across all prism base lenegths b, **BaSF10 consistently has greater magnitudes of angular dispersion compared to BaK1**. This characteristic positions fused **BaSF10 as a more suitable choice for a prism spectrometer**, as it exhibits significantly larger deviations in dispersion across all wavelengths.

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3 Problem 5.2

3.1 The Prism Spectrometer

```
Born & Wolf (Fig 4.28, p 192). Which one will yield a better spectral resolution
(separation) for the multi-wavelength image point P' - a spectrometer with a
longer or shorter focal length for lens L2? Is it essential that the focal
lengths of L_1 and L_2 be equal? Explain.
```

Better spectral resolution is achieved by maximizing the separation of light. Longer focal lengths contribute to a higher magnitude of angular dispersion. **Hence, a longer focal length in L__2 contributes to a broader separation of light compared to a lens with a shorter focal length.**

Based on 4.28, the focal length in L__1 and L__2 doesn't have to be equal since what enters and comes out of the prism are parallel rays after encountering the L__1 and before entering L__2. What matters in the prism dispersion is the shape and refractive index of the prism.

```
[ ]:
```