Principe - Physics 265 PS3

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1 Physics 265 Problem Set 3

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[25]:
```
import numpy as np
import matplotlib.pyplot as plt
```

2 Problem 3.1

2.1 Homogeneous dielectric film 1

`(see Born & Wolf Section 1.6.4 and Fig1.17 for schematic).`

`Generate the reflectivity plots (similar to Figure 1.18, normal incidence, 512 data points) of the film within the optical thickness range: 0 nh   o for incident wavelength o = 400 nm, 550 nm, 750 nm, and 1100 nm if applicable for your assigned materials. Additional information: index of layer 1 = 1.0, index of layer 2 = n1, index of layer 3 = n2.`

`At what  o-value would the film best function as a beam splitter. Explain.`

From Born & Wolf Section 1.6.4, Eq. 59 gives states that the reflectivity of a film is given by

$$R = \frac{r_{12}^2 + r_{23}^2 + 2r_{12}r_{23}\cos(2\beta)}{1 + r_{12}^2 r_{23}^2 + 2r_{12}r_{23}\cos(2\beta)} \tag{1}$$

where

$$\beta = \frac{2\pi}{\lambda_0} n_2 h \cos\theta_2 \tag{2}$$

For a dielectric film, the reflectivity at the interface is given by

$$r_{12} = \frac{n_1\cos\theta_1 - n_2\cos\theta_2}{n_1\cos\theta_1 + n_2\cos\theta_2} \tag{3}$$

$$r_{23} = \frac{n_2\cos\theta_2 - n_3\cos\theta_3}{n_2\cos\theta_2 + n_3\cos\theta_3} \tag{4}$$

where three layers have different indices of refraction.

For normal incidence, all the angular arguments $\theta = 0$ which would simplify all aforementioned equations into

$$r_{12} = \frac{n_1 - n_2}{n_1 + n_2} \tag{5}$$

and

$$r_{23} = \frac{n_2 - n_3}{n_2 + n_3}, \tag{6}$$

and β is reduced as well into

$$\beta = \frac{2\pi}{\lambda_0} n_2 h. \tag{7}$$

[26]:
```
def reflectivity(r_12, r_23, beta_):
    num = r_12**2 + r_23**2 + 2*r_12*r_23*np.cos(2*beta_)
    denom = 1 + num
    return num/denom

def reflectance(n_left, n_right):
    num = n_left - n_right
    denom = n_left + n_right
    return num/denom

def beta(lambda_0, n_2h):
    return 2*np.pi*n_2h/lambda_0
```

2.2 $n_1(\lambda)$ of BaSF10

$$n_1^2(\lambda) = 2.6531250 - 8.1388553 \times 10^{-3}\lambda^2 + 2.2995643 \times 10^{-2}\lambda^{-2} + 7.3535957 \times 10^{-4}\lambda^{-4} \tag{8}$$

$$-1.3407390 \times 10^{-5}\lambda^{-6} + 3.6962325 \times 10^{-6}\lambda^{-8} \tag{9}$$

[27]:
```
def BaSF10_n(lambda_):
    term1 = 8.1388553*10**-3 * lambda_**2
    term2 =  2.2995643*10**-2 * lambda_**-2
    term3 =  7.3535957*10**-4 * lambda_**-4
    term4 =  1.3407390*10**-5 * lambda_**-6
    term5 =  3.6962325*10**-6 * lambda_**-8
    return np.sqrt(2.6531250 - term1 + term2 + term3 - term4 + term5)
```

2.3 $n_2(\lambda)$ of BaK1

$$n_2^2(\lambda) = 2.4333007 - 8.4931353 \times 10^{-3}\lambda^2 + 1.3893512 \times 10^{-2}\lambda^{-2} + 2.6798268 \times 10^{-4}\lambda^{-4} \tag{10}$$

$$-6.1946101 \times 10^{-6}\lambda^{-6} + 6.2209005 \times 10^{-7}\lambda^{-8} \tag{11}$$

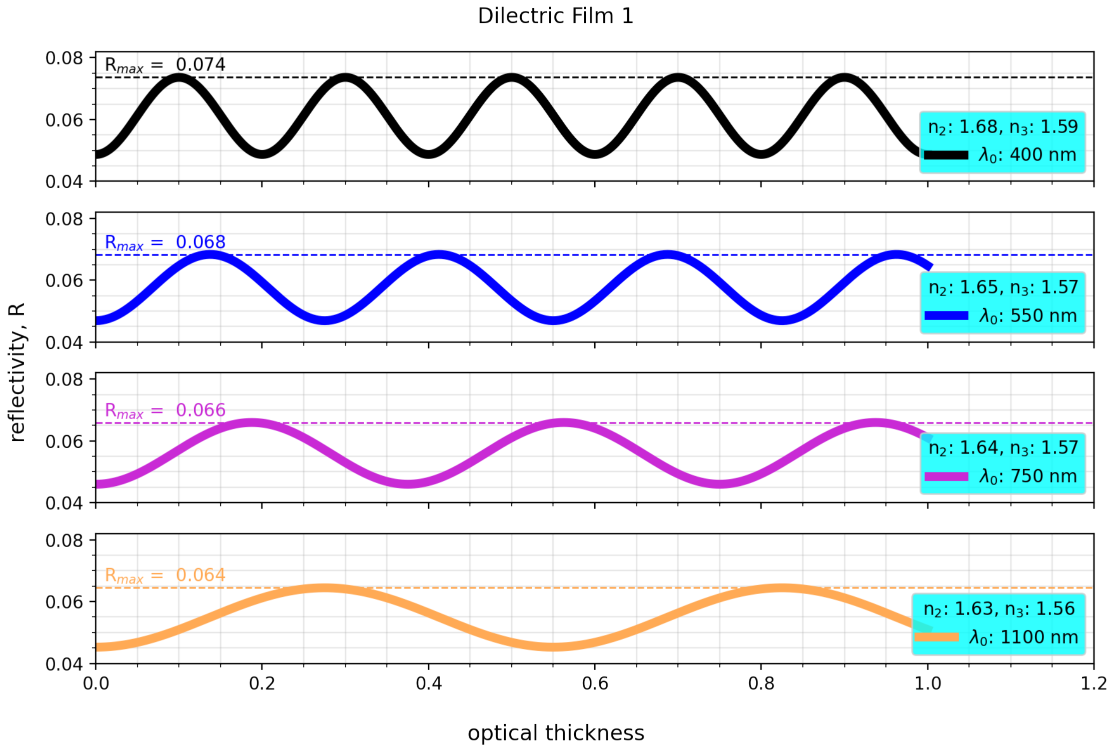
[28]:
```python
def BaK1_n(lambda_):
    term1 = 8.4931353*10**-3 * lambda_**2
    term2 =  1.3893512*10**-2 * lambda_**-2
    term3 =  2.6798268*10**-4 * lambda_**-4
    term4 =  6.1946101*10**-6 * lambda_**-6
    term5 =  6.2209005*10**-7 * lambda_**-8
    return np.sqrt(2.4333007 - term1 + term2 + term3 - term4 + term5)
```

[29]:
```python
n = 512 #points
lambda_0_list = np.array([400, 550, 750, 1100])/1e3
lambda_0_list
```

[29]:
```
array([0.4 , 0.55, 0.75, 1.1 ])
```

[30]:
```python
n_y = len(lambda_0_list)
fig, ax = plt.subplots(nrows=n_y,ncols=1, sharex='col',figsize=(9,6), dpi = 200)
fig.patch.set_facecolor('None')

for i, ax in enumerate(ax):
    lambda_0 = lambda_0_list[i]
    n_2h = np.linspace(0,1,n+1)
    n1 = 1.0
    n2 = BaSF10_n(lambda_0)
    n3 = BaK1_n(lambda_0)

    r_12 = reflectance(n1, n2)
    r_23 = reflectance(n2, n3)
    beta_list = beta(lambda_0, n_2h)

    R = reflectivity(r_12, r_23, beta_list)
    rmax = np.max(R)

    ax.grid(alpha = 0.3, which = 'minor')
    ax.plot(n_2h, R, lw = 5, color = plt.cm.gnuplot2(i/n_y), label =␣
↪'$\lambda_0$: %.0f nm' % (lambda_0*1e3))
    ax.legend(loc = 'lower right', title = "n$_2$: %.2f, n$_3$: %.2f" % (n2,␣
↪n3),
              facecolor = 'cyan')
    ax.axhline(rmax, lw = 1, ls = '--', color = plt.cm.gnuplot2(i/n_y))
    ax.text(0.009, rmax + 0.0019, 'R$_{max}$ = % .3f' % (rmax), color = 'w')
    ax.text(0.01, rmax + 0.002, 'R$_{max}$ = % .3f' % (rmax), color = plt.cm.
↪gnuplot2(i/n_y))
    ax.set_xlim(0, 1.2)
    ax.set_ylim(0.04,0.082)
    ax.minorticks_on()

fig.supxlabel('optical thickness')
```

```python
fig.supylabel('reflectivity, R')
fig.suptitle('Dilectric Film 1')
plt.tight_layout()
```



A beam splitter divides an incoming beam of light into two or more separate beams. The effectiveness of a beam splitter is often evaluated based on its reflectivity and absorption characteristics. In practical terms, a high reflectivity and low absorption are desirable qualities for an efficient beam splitter.

From the results above, **the optimal wavelength (λ_0) for an effective beam splitter operation is identified at λ = 400 nm.** This specific wavelength value demonstrates relatively high reflectivity at R$_{max}$ = 0.074, making it the most favorable choice for optimal beam splitting performance.

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3 Problem 3.2

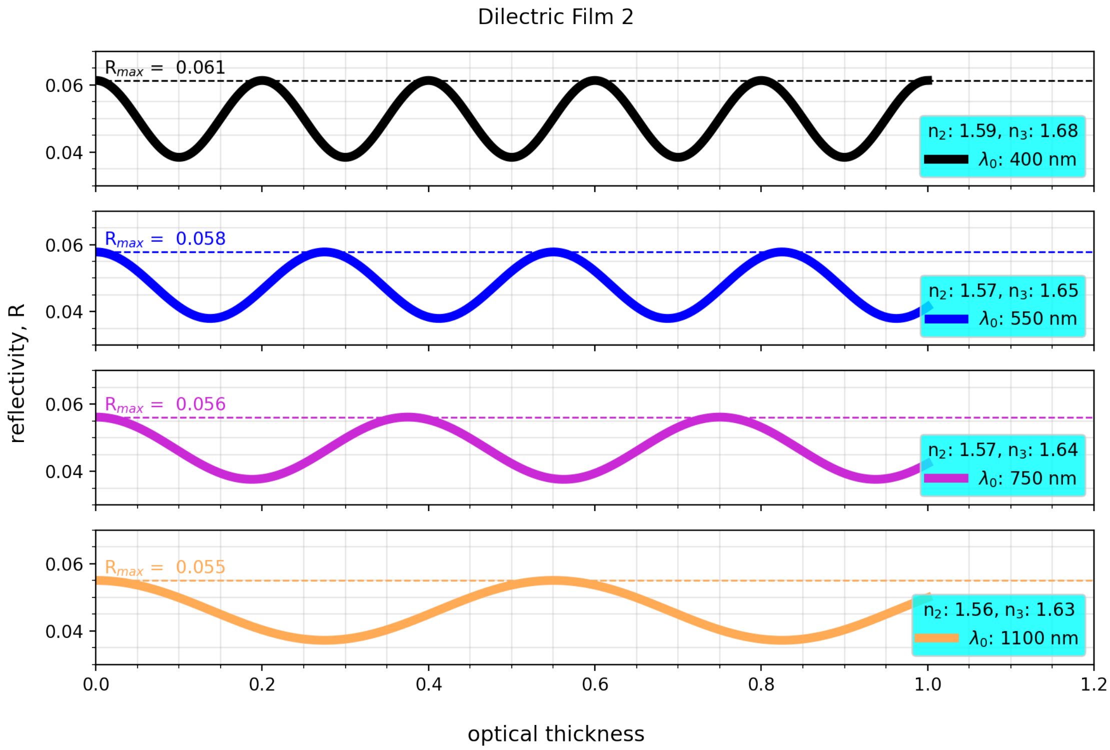
3.1 Homogeneous dielectric film 2.

Generate the reflectivity plots (normal incidence) of the film within the optical thickness range: 0   nh   o for incident wavelength o = 400 nm, 550 nm, 750 nm, and 1100 nm. Additional information: index of layer 1 = 1.0, index of layer 2 = n2, index of layer 3 = n1.

At what o-value would the film best function as a beam splitter. Which performs better as a beam splitter, film 1 or film 2? Explain.

---

[31]:
```
n_y = len(lambda_0_list)
fig, ax = plt.subplots(nrows=n_y,ncols=1, sharex='col',figsize=(9,6), dpi = 200)
fig.patch.set_facecolor('None')

for i, ax in enumerate(ax):
    lambda_0 = lambda_0_list[i]
    n_2h = np.linspace(0,1,n+1)
    n1 = 1.0
    n3 = BaSF10_n(lambda_0)
    n2 = BaK1_n(lambda_0)

    r_12 = reflectance(n1, n2)
    r_23 = reflectance(n2, n3)
    beta_list = beta(lambda_0, n_2h)

    R = reflectivity(r_12, r_23, beta_list)
    rmax = np.max(R)

    ax.grid(alpha = 0.3, which = 'minor')
    ax.plot(n_2h, R, lw = 5, ls = '-',
            color = plt.cm.gnuplot2(i/n_y), label = '$\lambda_0$: %.0f nm' % (lambda_0*1e3))
    ax.legend(loc = 'lower right', title = "n$_2$: %.2f, n$_3$: %.2f" % (n2, n3),
              facecolor = 'cyan')
    ax.axhline(rmax, lw = 1, ls = '--', color = plt.cm.gnuplot2(i/n_y))
    ax.text(0.009, rmax + 0.0019, 'R$_{max}$ = % .3f' % (rmax), color = 'w')
    ax.text(0.01, rmax + 0.002, 'R$_{max}$ = % .3f' % (rmax), color = plt.cm.gnuplot2(i/n_y))
    ax.set_xlim(0, 1.2)
    ax.set_ylim(0.03,0.07)
    ax.minorticks_on()

fig.supxlabel('optical thickness')
fig.supylabel('reflectivity, R')
fig.suptitle('Dilectric Film 2')
plt.tight_layout()
```



For the fielectric film 2, **the optimal wavelength (λ_0) is at λ = 400 nm** as well.

However, compared to film 1, it only attained a reflectivity of $R_{max} = 0.061$. Therefore, film 1 is a better choice as an electric film beam splitter which has an $R_{max} = 0.074$.

4 Problem 3.3

4.1 Quarter-wave film 1 (optical thickness = o/4).

Generate the reflectivity plot (see Figure 1.19, normal incidence, 512 data points) of the film versus n(o) within the range: 400 nm o 800 nm. Additional information: index of layer 1 = 1.3, index of layer 2 = n1, index of layer 3 = n2.

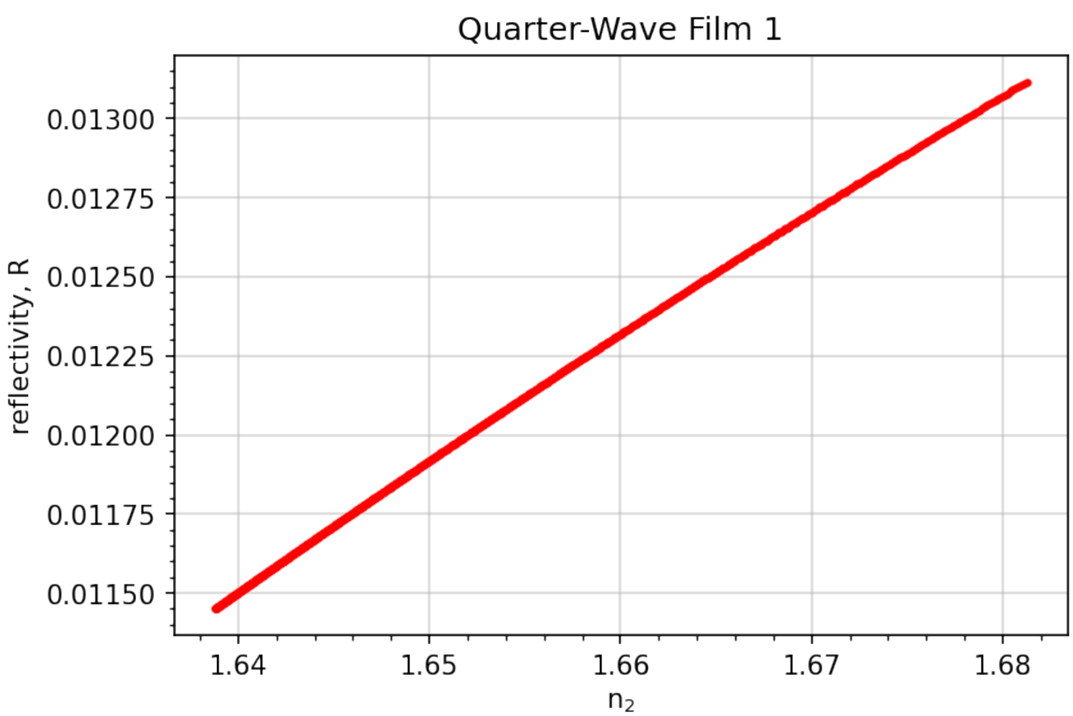
[32]:
```
def beta_2(lambda_, n_2, h):
    return 2*np.pi*n_2*h/lambda_
```

[33]:
```
n = 512 #points
lambda_list = np.linspace(400, 800, 1000)/1e3
h_list = lambda_list/4
```

[34]:
```
n1 = 1.3
n2 = BaSF10_n(lambda_list)
n3 = BaK1_n(lambda_list)
```

```
r_12 = reflectance(n1, n2)
r_23 = reflectance(n2, n3)
beta_list = beta_2(lambda_list, n2, h_list)

R_qwf_1 = reflectivity(r_12, r_23, beta_list)

plt.figure(dpi = 150)
plt.grid(alpha = 0.5)
plt.plot(n2 , R_qwf_1 , 'ro-', markersize =2)
plt.xlabel('n$_2$')
plt.ylabel('reflectivity, R')
plt.minorticks_on()
plt.title('Quarter-Wave Film 1')
```

[34]: Text(0.5, 1.0, 'Quarter-Wave Film 1')



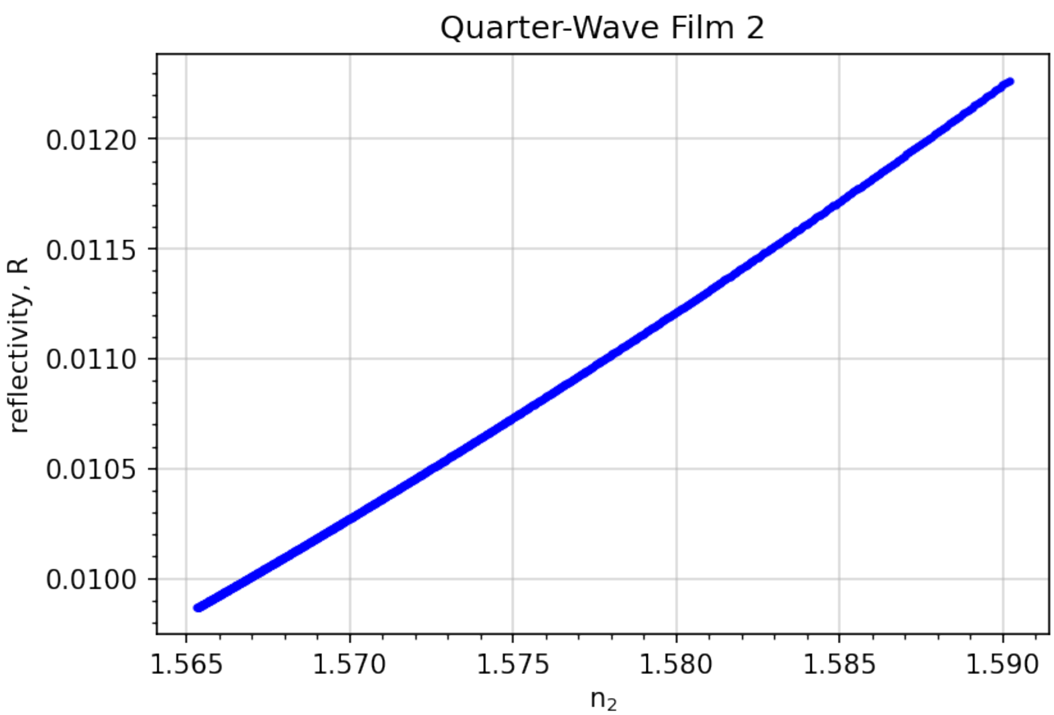
5 Problem 3.4

5.1 Quarter-wave film 2

Generate the reflectivity plot (normal incidence) of the film versus n(o) within: 400 nm o 800 nm. Additional information: index of layer 1 = 1.3, index of layer 2 = n2, index of layer 3 = n1. Which performs better as an anti-reflective coating, film 1 or film 2? Explain.

[35]:
```python
n1 = 1.3
n2 = BaK1_n(lambda_list)
n3 = BaSF10_n(lambda_list)

r_12 = reflectance(n1, n2)
r_23 = reflectance(n2, n3)
beta_list = beta_2(lambda_list, n2, h_list)

R_qwf_2 = reflectivity(r_12, r_23, beta_list)

plt.figure(dpi = 150)
plt.grid(alpha = 0.5)
plt.plot(n2 , R_qwf_2 , 'bo-', markersize =2)
plt.xlabel('n$_2$')
plt.ylabel('reflectivity, R')
plt.minorticks_on()
plt.title('Quarter-Wave Film 2')
```

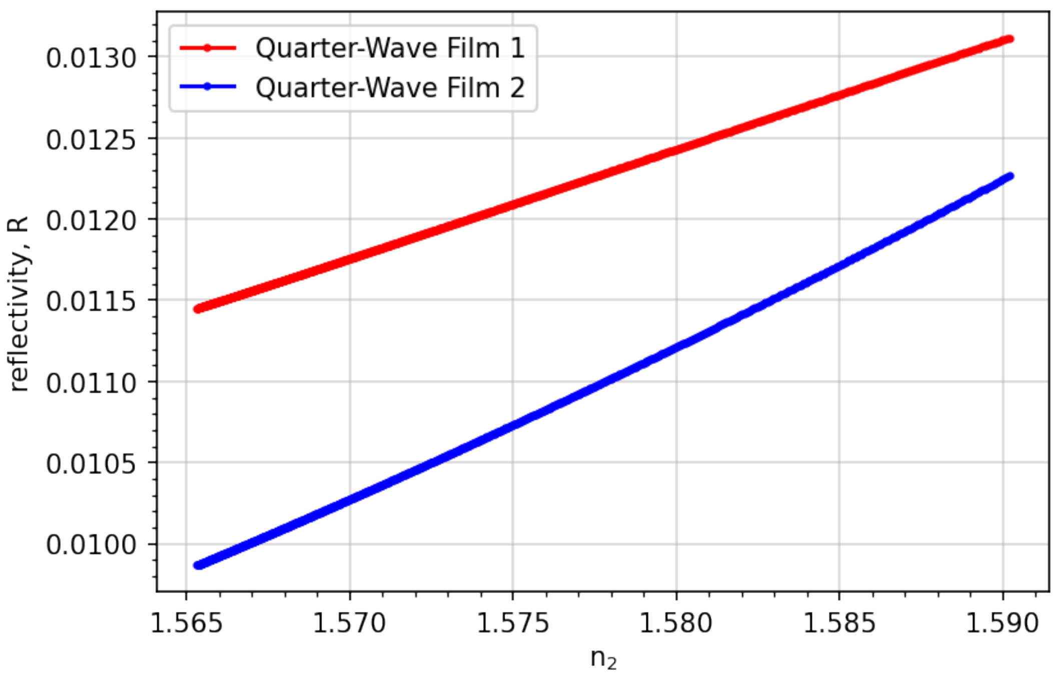
[35]:
```
Text(0.5, 1.0, 'Quarter-Wave Film 2')
```



5.2 Comparison of Quarter-Wave Film 1 and Film 2

[36]:
```
plt.figure(dpi = 150)
plt.grid(alpha = 0.5)
plt.plot(n2 , R_qwf_1 , 'ro-', markersize = 2, label = 'Quarter-Wave Film 1')
plt.plot(n2 , R_qwf_2 , 'bo-', markersize = 2, label = 'Quarter-Wave Film 2')
plt.xlabel('n$_2$')
plt.ylabel('reflectivity, R')
plt.minorticks_on()
plt.legend()
```

[36]:
```
<matplotlib.legend.Legend at 0x7fe2ed2261f0>
```



In the context of anti-reflective coatings, a desirable characteristic is the reduction of reflectivity, in contrast to the requirements for a beam splitter. Comparing two films, it is evident that **film 2 outperforms film 1 in terms of anti-reflective properties**. This conclusion is drawn from the observation that film 2 exhibits lower reflectivity, making it more effective as an anti-reflective coating.

[ ]:
```
```