Physics 265 Problem Set 4

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1 Problem 4.1

> Derive explicitly the expression for the power P (Equation 26, p 172) of each thick lens considering the possible signs of the two radii of curvature for the first (front) and second surfaces.

The power of a thick lens is given by the sum of powers from each surface, accounting for thickness:

$$P = P_1 + P_2 - \frac{t}{n_2} P_1 P_2 \tag{1.1}$$

where t is the lens thickness, and n_2 is the lens's index of refraction. The powers P_1 and P_2 for each surface are expressed as:

$$P_1 = \frac{n_2 - n_1}{r_1} \tag{1.2}$$

and

$$P_2 = \frac{n_3 - n_2}{r_2} \tag{1.3}$$

where n_1 (n_3) is the index of refraction to the left (right) of the lens, and r_1 (r_2) is the radius of curvature of the left (right) surface.

- For the double-convex lens, considering $r_1 > 0$ and $r_2 < 0$, the power is obtained as:

$$\begin{aligned} P_{double-convex} &= \frac{n_2 - n_1}{+|r_1|} + \frac{n_3 - n_2}{-|r_2|} - \frac{t}{n_2}\left(\frac{n_2 - n_1}{+|r_1|}\right)\left(\frac{n_3 - n_2}{-|r_2|}\right) \\ P_{double-convex} &= \frac{n_2 - n_1}{+|r_1|} - \frac{n_3 - n_2}{|r_2|} + \frac{t}{n_2}\left(\frac{n_2 - n_1}{+|r_1|}\right)\left(\frac{n_3 - n_2}{|r_2|}\right). \end{aligned} \tag{1.4}$$

- For the plano-convex lens with $r_1 > 0$ and $r_2 = \infty$, the power simplifies to:

$$\begin{aligned} P_{plano-convex} &= \frac{n_2 - n_1}{|r_1|} + \cancelto{0}{\frac{n_3 - n_2}{|r_2 \to \infty|}} - \frac{t}{n_2}\left(\frac{n_2 - n_1}{|r_1|}\right)\cancelto{0}{\left(\frac{n_3 - n_2}{|r_2 \to \infty|}\right)} \\ P_{plano-convex} &= \frac{n_2 - n_1}{|r_1|}. \end{aligned} \tag{1.5}$$

- Similarly, for the convergent meniscus lens ($r_2 > r_1 > 0$), the power becomes:

$$P_{convergent\ miniscus} = \frac{n_2 - n_1}{|r_1|} + \frac{n_3 - n_2}{|r_2|} - \frac{t}{n_2}\left(\frac{n_2 - n_1}{|r_1|}\right)\left(\frac{n_3 - n_2}{|r_2|}\right). \tag{1.6}$$

- For the double-concave lens with $r_1 < 0$ and $r_2 > 0$, the power is given by:

$$\begin{aligned} P_{double-concave} &= \frac{n_2 - n_1}{-|r_1|} + \frac{n_3 - n_2}{+|r_2|} - \frac{t}{n_2}\left(\frac{n_2 - n_1}{-|r_1|}\right)\left(\frac{n_3 - n_2}{+|r_2|}\right) \\ P_{double-concave} &= -\frac{n_2 - n_1}{|r_1|} + \frac{n_3 - n_2}{|r_2|} + \frac{t}{n_2}\left(\frac{n_2 - n_1}{|r_1|}\right)\left(\frac{n_3 - n_2}{|r_2|}\right). \end{aligned} \tag{1.7}$$

- For the plano-concave lens with $r_1 = \infty$ and $r_2 > 0$, the power simplifies to:

$$
\begin{aligned}
P_{plano-concave} &= \cancelto{0}{\frac{n_2 - n_1}{|r_1 \to \infty|}} + \frac{n_3 - n_2}{|r_2|} - \frac{t}{n_2}\left(\cancelto{0}{\frac{n_2 - n_1}{|r_1 \to \infty|}}\right)\left(\frac{n_3 - n_2}{|r_2|}\right) \\
P_{plano-concave} &= +\frac{n_3 - n_2}{|r_2|}.
\end{aligned} \tag{1.8}
$$

- Finally, for the divergent meniscus lens ($r_1 > r_2 > 0$), the power is given by:

$$
P_{divergent\ miniscus} = \frac{n_2 - n_1}{|r_1|} + \frac{n_3 - n_2}{|r_2|} - \frac{t}{n_2}\left(\frac{n_2 - n_1}{|r_1|}\right)\left(\frac{n_3 - n_2}{|r_2|}\right). \tag{1.9}
$$

2 Problem 4.2

For the same focal lengths, refractive indices (n_2, $n_1 = n_3$), and radii of curvature (absolute values), rank the six lenses according to their light-gathering power P.

Applying $n_1 = n_3$ and using the same magnitudes for the radii, $|r_1| = |r_2| = r$, the expressions for the powers are modified as follows:

- For the double-convex lens:

$$
\begin{aligned}
P_{double-convex} &= \frac{n_2 - n_1}{+|r_1|} - \frac{n_3 - n_2}{|r_2|} + \frac{t}{n_2}\left(\frac{n_2 - n_1}{+|r_1|}\right)\left(\frac{n_3 - n_2}{|r_2|}\right) \\
P_{double-convex} &= 2\left(\frac{n_2 - n_1}{r}\right) - \frac{t}{n_2}\frac{(n_2 - n_1)^2}{r^2}.
\end{aligned} \tag{2.1}
$$

- For the plano-convex lens:

$$
\begin{aligned}
P_{plano-convex} &= \frac{n_2 - n_1}{|r_1|} \\
P_{plano-convex} &= \frac{n_2 - n_1}{r}.
\end{aligned} \tag{2.2}
$$

- For the convergent meniscus lens:

$$
\begin{aligned}
P_{convergent\ miniscus} &= \frac{n_2 - n_1}{|r_1|} + \frac{n_3 - n_2}{|r_2|} - \frac{t}{n_2}\left(\frac{n_2 - n_1}{|r_1|}\right)\left(\frac{n_3 - n_2}{|r_2|}\right) \\
P_{convergent\ miniscus} &= \cancelto{0}{\frac{n_2 - n_1}{r} + \frac{n_1 - n_2}{r}} + \frac{t}{n_2}\frac{(n_2 - n_1)^2}{r^2} \\
P_{convergent\ miniscus} &= \frac{t}{n_2}\frac{(n_2 - n_1)^2}{r^2}.
\end{aligned} \tag{2.3}
$$

- For the double-concave lens:

$$
\begin{aligned}
P_{double-concave} &= -\frac{n_2 - n_1}{|r_1|} + \frac{n_3 - n_2}{|r_2|} + \frac{t}{n_2}\left(\frac{n_2 - n_1}{|r_1|}\right)\left(\frac{n_3 - n_2}{|r_2|}\right) \\
P_{double-concave} &= -2\left(\frac{n_2 - n_1}{r}\right) - \frac{t}{n_2}\frac{(n_2 - n_1)^2}{r^2}.
\end{aligned} \tag{2.4}
$$

- For the plano-concave lens:

$$
\begin{aligned}
P_{plano-concave} &= +\frac{n_3 - n_2}{|r_2|} \\
P_{plano-concave} &= -\frac{n_2 - n_1}{r}. \qquad (2.5)
\end{aligned}
$$

- For the divergent meniscus lens:

$$
\begin{aligned}
P_{divergent\ miniscus} &= \frac{n_2 - n_1}{|r_1|} + \frac{n_3 - n_2}{|r_2|} - \frac{t}{n_2}\left(\frac{n_2 - n_1}{|r_1|}\right)\left(\frac{n_3 - n_2}{|r_2|}\right) \\
P_{divergent\ miniscus} &= \cancelto{0}{\frac{n_2 - n_1}{r} + \frac{n_1 - n_2}{r}} + \frac{t}{n_2}\frac{(n_2 - n_1)^2}{r^2} \\
P_{divergent\ miniscus} &= \frac{t}{n_2}\frac{(n_2 - n_1)^2}{r^2}. \qquad (2.6)
\end{aligned}
$$

Letting $x = \frac{n_2 - n_1}{r}$, $y = \frac{t}{n_2}$, and ranking the powers in decreasing order, we have

$$
\begin{aligned}
P_{double-convex} &= 2x - x^2 && (2.7) \\
P_{convergent\ miniscus} &= yx^2 && (2.8) \\
P_{divergent\ miniscus} &= yx^2 && (2.9) \\
P_{plano-convex} &= x && (2.10) \\
P_{plano-concave} &= -x && (2.11) \\
P_{double-concave} &= -2x - yx^2. && (2.12)
\end{aligned}
$$

3 Problem 4.3

> Derive the expression for the resulting power of the plano-convex (4.15b), convergent meniscus (4.15c), plano-convex (4.15e) and the divergent meniscus (4.15f) lens when the order of their two surfaces is reversed to face the incident light coming from the left side. For the plano-convex lens, the said situation would correspond to having the plane surface as the first one to meet the incident light.

- For the plano-convex lens with $r_1 = \infty$ and $r_2 < 0$ and , the power simplifies to:

$$
\begin{aligned}
P_{plano-convex} &= \cancelto{0}{\frac{n_2 - n_1}{|r_1 \to \infty|}} + \frac{n_3 - n_2}{-|r_2|} - \frac{t}{n_2}\left(\cancelto{0}{\frac{n_2 - n_1}{|r_1 \to \infty|}}\right)\left(\frac{n_3 - n_2}{|r_2|}\right) \\
P_{plano-convex} &= -\frac{n_3 - n_2}{|r_2|}. \qquad (3.1)
\end{aligned}
$$

- Similarly, for the convergent meniscus lens ($r_2 < r_1 < 0$), the power becomes:

$$
\begin{aligned}
P_{convergent\ miniscus} &= \frac{n_2 - n_1}{-|r_1|} + \frac{n_3 - n_2}{-|r_2|} - \frac{t}{n_2}\left(\frac{n_2 - n_1}{-|r_1|}\right)\left(\frac{n_3 - n_2}{-|r_2|}\right) \\
P_{convergent\ miniscus} &= -\frac{n_2 - n_1}{|r_1|} - \frac{n_3 - n_2}{|r_2|} - \frac{t}{n_2}\left(\frac{n_2 - n_1}{|r_1|}\right)\left(\frac{n_3 - n_2}{|r_2|}\right). \qquad (3.2)
\end{aligned}
$$

- For the plano-concave lens with $r_1 < 0$ and $r_2 = \infty$, the power simplifies to:

$$P_{plano-concave} = \frac{n_2 - n_1}{-|r_1|} + \cancelto{0}{\frac{n_3 - n_2}{|r_2 \to \infty|}} - \frac{t}{n_2}\left(\frac{n_2 - n_1}{-|r_1|}\right)\cancelto{0}{\left(\frac{n_3 - n_2}{|r_2 \to \infty|}\right)}$$

$$P_{plano-concave} = -\frac{n_2 - n_1}{|r_1|}. \tag{3.3}$$

- Finally, for the divergent meniscus lens ($r_1 < r_2 < 0$), the power is given by:

$$P_{divergent\ miniscus} = \frac{n_2 - n_1}{-|r_1|} + \frac{n_3 - n_2}{-|r_2|} - \frac{t}{n_2}\left(\frac{n_2 - n_1}{-|r_1|}\right)\left(\frac{n_3 - n_2}{-|r_2|}\right)$$

$$P_{divergent\ miniscus} = -\frac{n_2 - n_1}{|r_1|} - \frac{n_3 - n_2}{|r_2|} - \frac{t}{n_2}\left(\frac{n_2 - n_1}{|r_1|}\right)\left(\frac{n_3 - n_2}{|r_2|}\right). \tag{3.4}$$

4 Problem 4.4

In focusing an incident beam of parallel rays (plane waves) with a plano-convex lens, which first surface gives better result – the convex or the plane? Briefly explain the answer.

We have solved the power for the plano-convex lens for two possible orientations in Eq. (1.6) & (2.2). Given that their radius of curvature is equal and $n_1 = n_3$, the expressions for their respective powers is

$$\begin{aligned}\text{1st surface is convex: } P_{plano-convex} &= \frac{n_2 - n_1}{|r|}\\ \text{1st surface is plane: } P_{plano-convex} &= -\frac{n_3 - n_1}{|r|}\\ &= -\frac{n_1 - n_2}{|r|}\\ &= \frac{n_2 - n_1}{|r|}.\end{aligned}$$

Hence, regardless of the orientation of a plano-convex lens, it will generate the same power.