

# Principe - Physics 265 PS6

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## 1 Physics 265 Problem Set 6

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```
[1]: import numpy as np
import matplotlib.pyplot as plt
from matplotlib import cm
```

## 2 Problem 6.1

### 2.1 Interference Signal Produced by Mercury Light Source

The visible light that is produced by an Hg lamp consists of the following spectral lines (nm) and their relative intensities (I): 312 (I = 70), 334 (I = 46), 365 (I = 96.7), 405 (I = 73), 436 (I = 93.3), 546 (I = 80), and 579 (I = 53):

Plot the total interference signal (Equation 15, Section 7.2, Born & Wolf) that is produced by an Hg lamp within the optical path difference range:  $-1.5 \leq \Delta S \leq 1.5$  (micron).

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Consideration of the optical path difference yields the total intensity ( $I$ ) for two monochromatic waves, expressed as:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \delta. \quad (1)$$

Expressing in terms of the wavelength of light, we have

$$= I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \left( \frac{2\pi}{\lambda_0} \Delta S \right). \quad (2)$$

For a single source, where  $I_1 = I_2$ , the total intensity ( $I$ ) reduces to:

$$I = 2I_1 + 2I_1 \cos \left( \frac{2\pi}{\lambda_0} \Delta S \right) \quad (3)$$

This further simplifies to:

$$I = 2I_1 \left( 1 + \cos \left( \frac{2\pi}{\lambda_0} \Delta S \right) \right) \quad (4)$$

$$I = 2I_1 \left( 2 \cos^2 \left( \frac{\pi}{\lambda_0} \Delta S \right) \right) \quad (5)$$

$$I = 4I_1 \cos^2 \left( \frac{\pi}{\lambda_0} \Delta S \right) \quad (6)$$

```
[2]: def total_intensity(I, delta_S , lambda_):
      return 4 * I * np.cos(np.pi*delta_S/lambda_)**2
```

```
[3]: peaks_mercury = np.array([312, 334, 365, 405, 436, 546, 579])
      intensities_mercury = np.array([70, 46, 96.7, 73, 93.3, 80, 53])/100

      opd = np.linspace(-1.5, 1.5, 1000)*1e3
```

```
[4]: n_y = len(peaks_mercury)

      fig, ax = plt.subplots(nrows=n_y,ncols=1, sharex='col',figsize=(9,6), dpi = 200)
      fig.patch.set_facecolor('None')

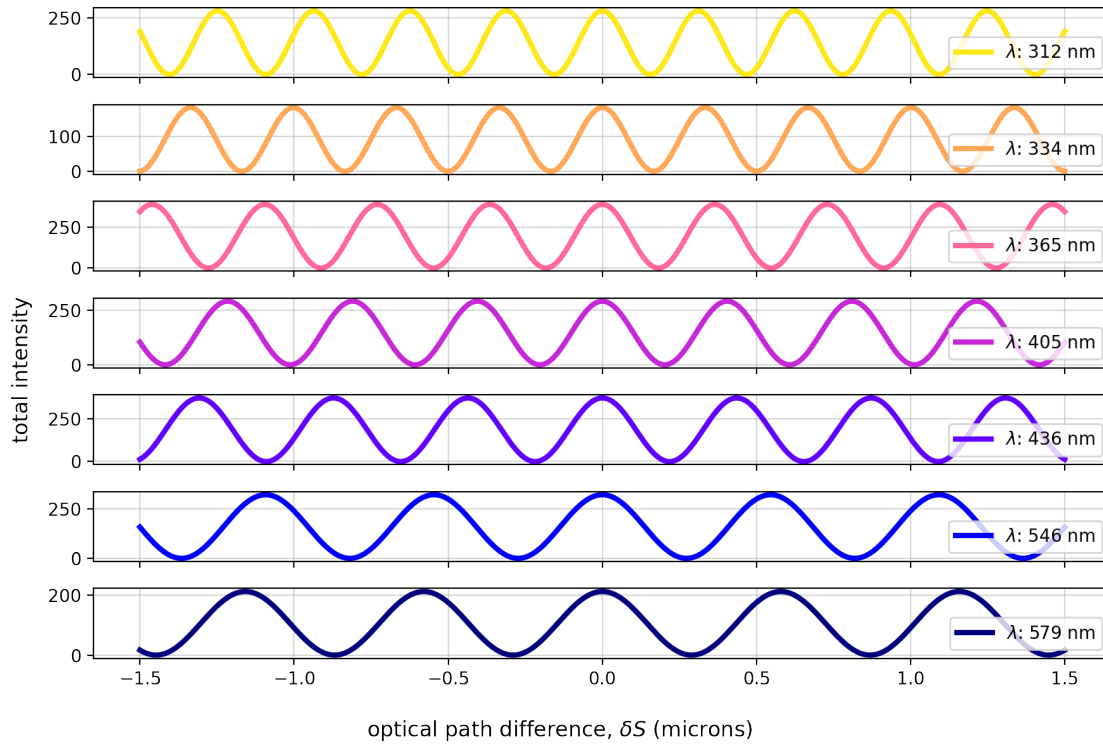
      total_I_mercury = np.zeros(len(opd))

      for i, ax in enumerate(ax):
          I_i = intensities_mercury[i]
          lambda_ = peaks_mercury[i]
          I = total_intensity(I_i, opd , lambda_)

          total_I_mercury += I

          ax.grid(alpha = 0.5)
          ax.plot(opd*1e-3, I*100, lw = 3, color = plt.cm.gnuplot2_r((i+1)/(n_y+1)),
                  label = '$\lambda$: %.0f nm' % (lambda_))
          ax.legend(loc = 'lower right', facecolor = 'white')

      fig.supxlabel('optical path difference, $\Delta S$ (microns)')
      fig.supylabel('total intensity')
      plt.tight_layout()
```



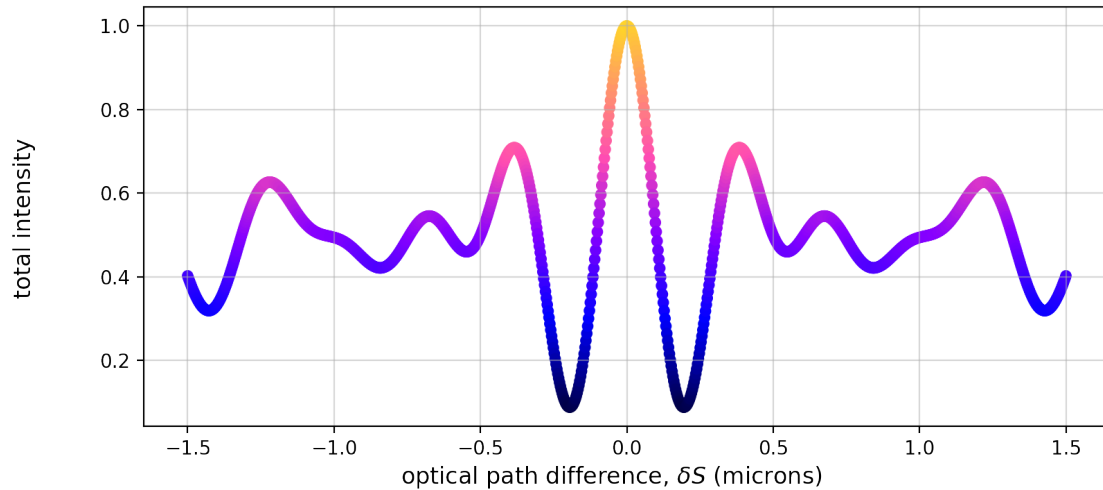
```
[5]: fig, ax = plt.subplots(nrows=1,ncols=1, sharex='col',figsize=(9,4), dpi = 200)
fig.patch.set_facecolor('None')

x = opd*1e-3
y = total_I_mercury*100

plt.grid(alpha = 0.5)
plt.scatter(x, y/max(y), c=cm.gnuplot2(y/max(y*1.2)), edgecolor='none')

fig.supxlabel('optical path difference,  $\delta S$  (microns)')
fig.supylabel('total intensity')
```

```
[5]: Text(0.02, 0.5, 'total intensity')
```



### 3 Problem 6.2

#### 3.1 Interference Signal Produced by Sodium Light Source.

The visible light that is produced by a Sodium lamp consists of two (doublet) lines: 589.6 nm ( $I = 70$ ), and 589 nm ( $I = 70$ ).

Plot the total interference signal that is produced by Na lamp within the optical path difference range:  $-1.5 \leq \Delta S \text{ (micron)} \leq 1.5$ .

```
[6]: peaks_sodium = np.array([589, 589.6])
      intensities_sodium = np.array([70, 70])/100

      opd = np.linspace(-1.5, 1.5, 1000)*1e3
```

```
[7]: n_z = len(peaks_sodium)

      fig, ax = plt.subplots(nrows=n_z,ncols=1, sharex='col',figsize=(9,3), dpi = 200)
      fig.patch.set_facecolor('None')

      total_I_sodium = np.zeros(len(opd))

      for i, ax in enumerate(ax):
          I_i = intensities_sodium[i]
          lambda_ = peaks_sodium[i]
          I = total_intensity(I_i, opd, lambda_)

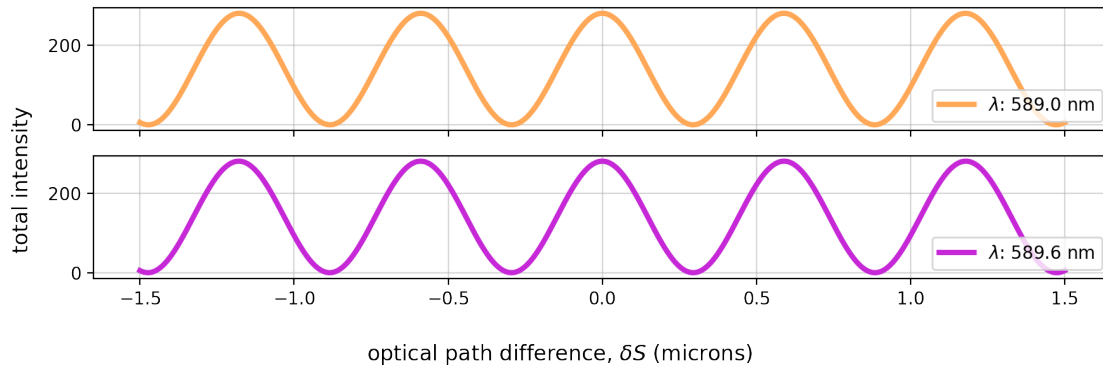
          total_I_sodium += I
```

```

ax.grid(alpha = 0.5)
ax.plot(opd*1e-3, I*100, lw = 3, color = plt.cm.gnuplot2_r((i+1)/(n_z+2)),
        label = '$\lambda$: %.1f nm' % (lambda_))
ax.legend(loc = 'lower right', facecolor = 'white')

fig.supxlabel('optical path difference, $\delta S$ (microns)')
fig.supylabel('total intensity')
plt.tight_layout()

```



```

[8]: fig, ax = plt.subplots(nrows=1,ncols=1, sharex='col',figsize=(9,4), dpi = 200)
fig.patch.set_facecolor('None')

x = opd*1e-3
y = total_I_sodium*100

plt.grid(alpha = 0.5)

plt.plot(opd*1e-3, I*100, lw = 1, color = plt.cm.gnuplot2_r(1/3), ls = '-')
plt.plot(opd*1e-3, I*100, lw = 1, color = plt.cm.gnuplot2_r(2/3), ls = '--')

plt.scatter(x, y, c=cm.gnuplot2(y/max(y*1.2)), edgecolor='none')

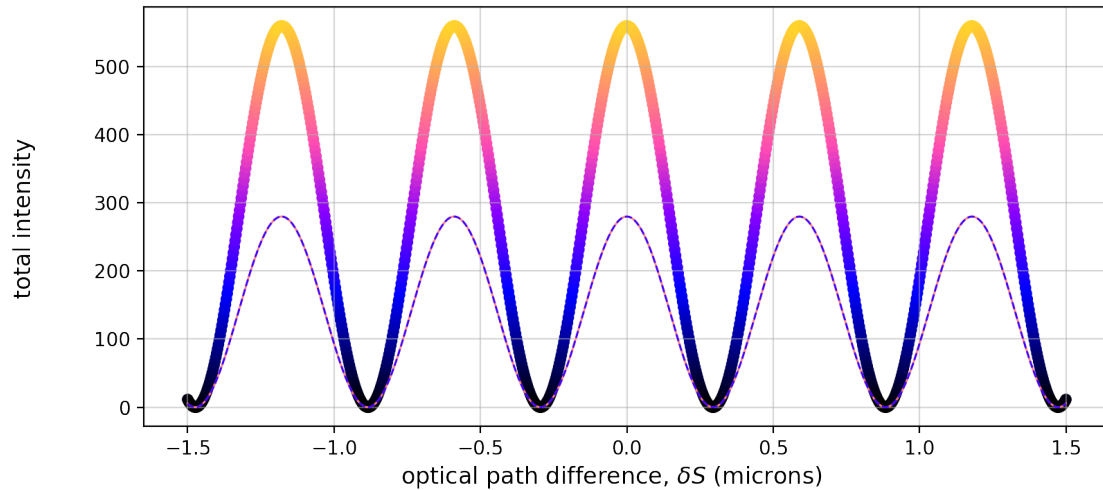
fig.supxlabel('optical path difference, $\delta S$ (microns)')
fig.supylabel('total intensity')

```

```

[8]: Text(0.02, 0.5, 'total intensity')

```



## 4 Problem 6.3

### 4.1 Visibility

Which light source (Hg or Na) produces an interference signal with higher visibility  $V$  where  $V = (I_{\max} - I_{\min}) / (I_{\max} + I_{\min})$ ?

```
[9]: def visibility(I):
    I_max = np.max(I)
    I_min = np.min(I)
    V = (I_max - I_min) / (I_max + I_min)
    print('Maximum Intensity: % .2f' % I_max)
    print('Minimum Intensity: % .2f' % I_min)
    print('Visibility: % .6f' % V)
    return I_max, I_min, V
```

#### 4.1.1 Mercury

```
[10]: visibility(total_I_mercury)
```

```
Maximum Intensity: 20.48
Minimum Intensity: 1.81
Visibility: 0.837981
```

```
[10]: (20.477134574247376, 1.8050759135570595, 0.8379805347817697)
```

### 4.1.2 Sodium

```
[11]: visibility(total_I_sodium)
```

```
Maximum Intensity: 5.60
```

```
Minimum Intensity: 0.00
```

```
Visibility: 0.999978
```

```
[11]: (5.599941704572872, 6.23219742121721e-05, 0.999977742168071)
```

From the calculations, **sodium light source produces interference signal with higher visibility compared to mercury.**