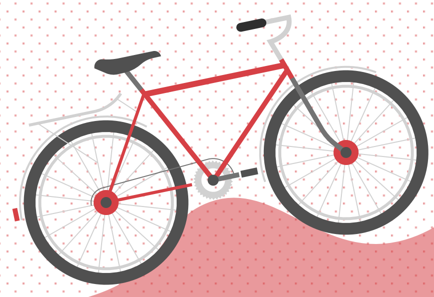
PHYSICS 301

Advanced Signal and Image Processing

ACTIVITY 04
RGB-to-Spectra using PCA



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2015-04622

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objectives

Compute the eigenspectral of paint pigments from the Munsell Color Chips database

Convert a color camera into a spectral imager using principal components analysis

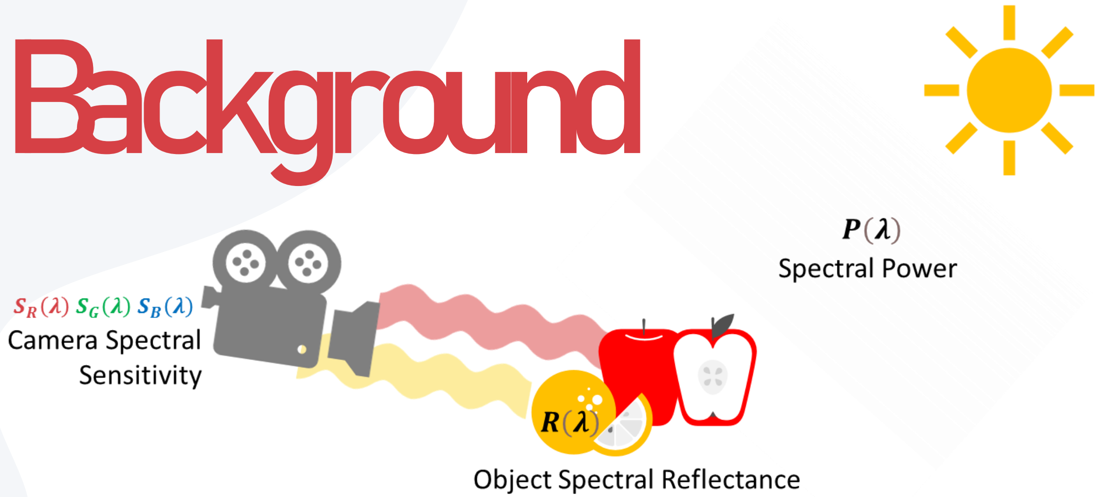
key take-aways

- We can accurately depict large datasets with only a few representative values through Principal Components Analysis.
- RGB digital counts were found to be sufficient in reconstructing the high-dimensional spectral information even with a novel set.
- Spectral (RMSE, SAM) and color error metrics (ΔE_{76}) should all be used in evaluating reconstruction as one supplements the other.

SOURCE CODE

- [Physics-301/Activity - 4 RGB-to-Spectra using PCA.ipynb at main · reneprincipejr/Physics-301 (github.com)]
- https://drive.google.com/file/d/11YbzA_dpg4L-_9xB1E44XnfDiXXtFVsc/view?usp=sharing

Background



$$C(\lambda) = \sum P(\lambda)R(\lambda)$$

$$q_n = \sum C_n S_n(\lambda)$$

Recalling trinity of color, the RGB values or q_n are obtained by multiplying the color signal $C_n(\lambda)$ with spectral sensitivities $S_n(\lambda)$ [1].

Suppose we have a spectral reflectance database and a default source, then we can represent the color signal as a linear superposition of the eigenvectors e_i and weights a_i by PCA's dimensional reduction given by

$$C(\lambda) \approx \sum_{i=1}^{m} a_i e_i(\lambda).$$

Background

The RGB $[q_R, q_G, q_B]$ values can then be obtained using:

$$q_R = a_1 \sum e_1(\lambda) S_R(\lambda) + a_2 \sum e_2(\lambda) S_R(\lambda) t \ldots + a_m \sum e_m(\lambda) S_R(\lambda)$$
$$q_G = a_1 \sum e_1(\lambda) S_G(\lambda) + a_2 \sum e_2(\lambda) S_G(\lambda) t \ldots + a_m \sum e_m(\lambda) S_G(\lambda)$$
$$q_B = a_1 \sum e_1(\lambda) S_B(\lambda) + a_2 \sum e_2(\lambda) S_B(\lambda) t \ldots + a_m \sum e_m(\lambda) S_B(\lambda)$$

$$\begin{bmatrix} q_R \\ q_G \\ q_B \end{bmatrix} = \begin{bmatrix} e_1 \cdot S_R & e_2 \cdot S_R & \cdots & e_m \cdot S_R \\ e_1 \cdot S_G & e_2 \cdot S_G & \cdots & e_m \cdot S_G \\ e_1 \cdot S_B & e_2 \cdot S_B & \cdots & e_m \cdot S_B \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix}.$$

Here we see an association between the weights (eigenvalues) and the RGB value through the transformation matrix $\mathbf{T}$. Weiner estimation method then allows us the obtain the weights $\mathbf{a}$:

$$\mathbf{q} = \mathbf{T}\mathbf{a}$$

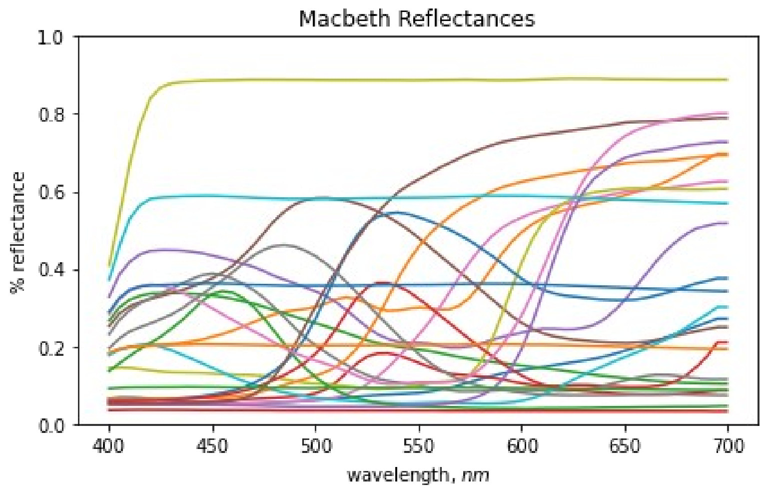
$$\mathbf{a} = (\mathbf{T}^T\mathbf{T})^{-1}\mathbf{T}^T\mathbf{q}.$$

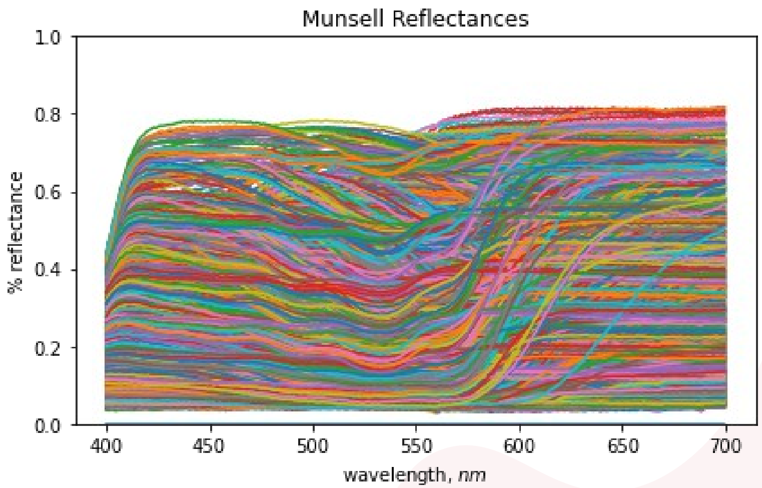
Since the eigenvectors are already established and the weights can be determined using RGB, then the spectrum of the color signal can be recreated [1].

Background

Like the compression demonstrated in facial reconstruction using Principal Components Analysis (PCA) [2], here we attempt to reduce the reflectance database into a few representative eigenspectra which yields the transformation matrix $\mathbf{T}$ necessary to represent the spectral reflectance using only the input digital color [1].

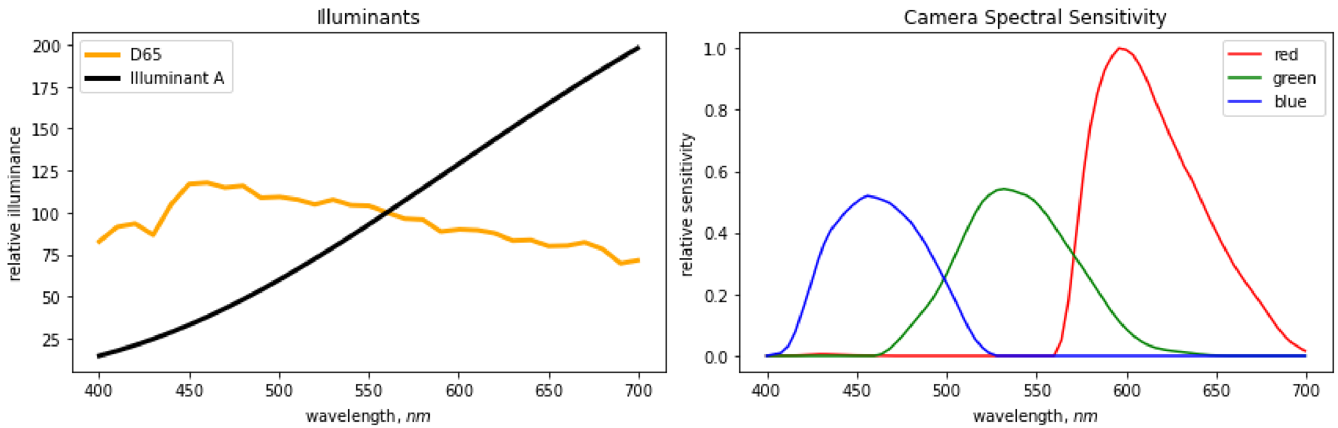
To facilitate this process called spectral super-resolution, we use the 1269 Munsell color chips ensemble and then, we attempt to recover the spectra of 24 Macbeth color patches using the rendered RGB values [3].





Transformation Matrix

Show below are the illuminants to be tested and the spectral sensitivity to be used in obtaining the transformation matrix $\mathbf{T}$.

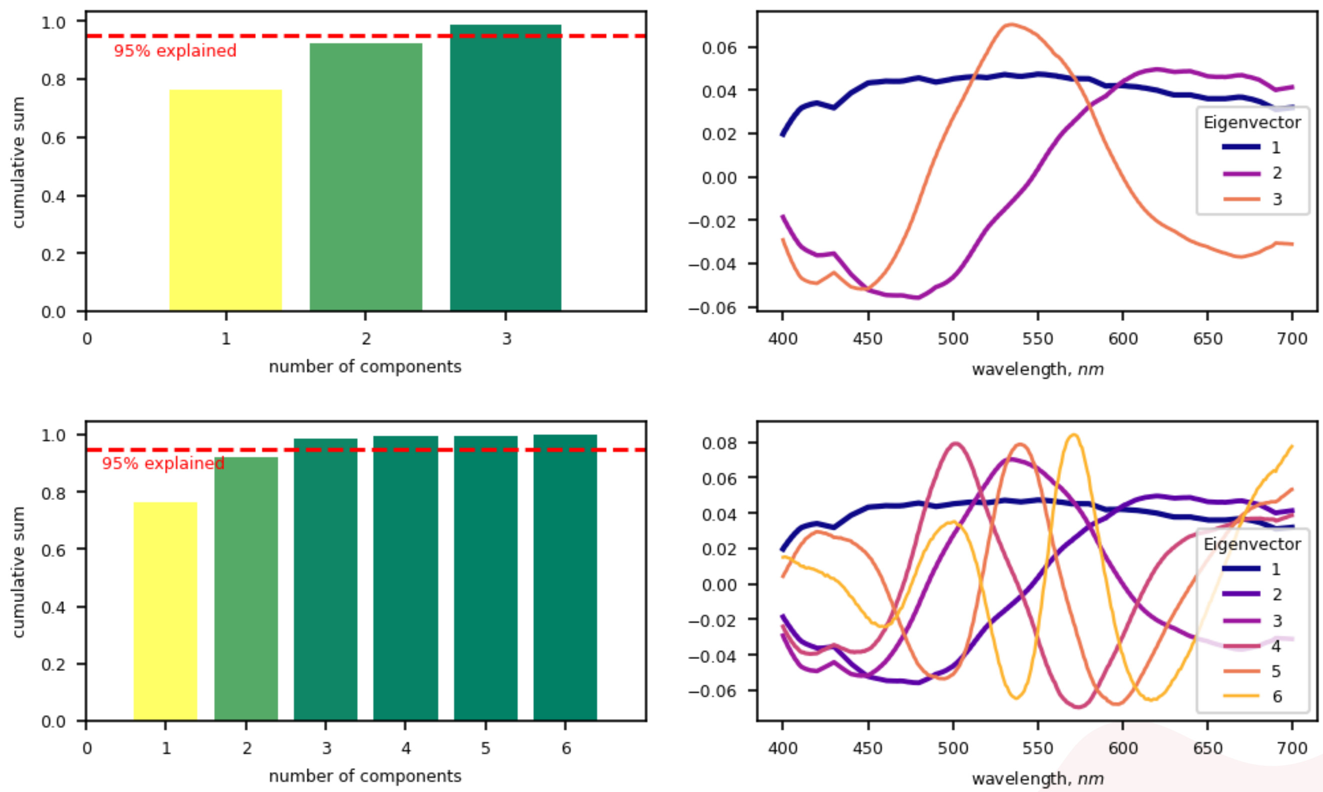


To quantify the reconstruction accuracies, we employed Root-Mean-Square-Error (RMSE) and Spectral-Angle-Mapper (SAM) metrics which measures the residual error and structure shape similarity respectively between the actual $\hat{r}$ and reconstructed r spectra [4]:

$$RMSE = \sqrt{\frac{\sum_N (r(N) - \hat{r}(N))}{n}} \qquad SAM = \cos^{-1}\left(\frac{\sum_N r(N) * \hat{r}(N)}{\sqrt{\sum_N r(N)}\sqrt{\sum_N \hat{r}(N)}}\right).$$

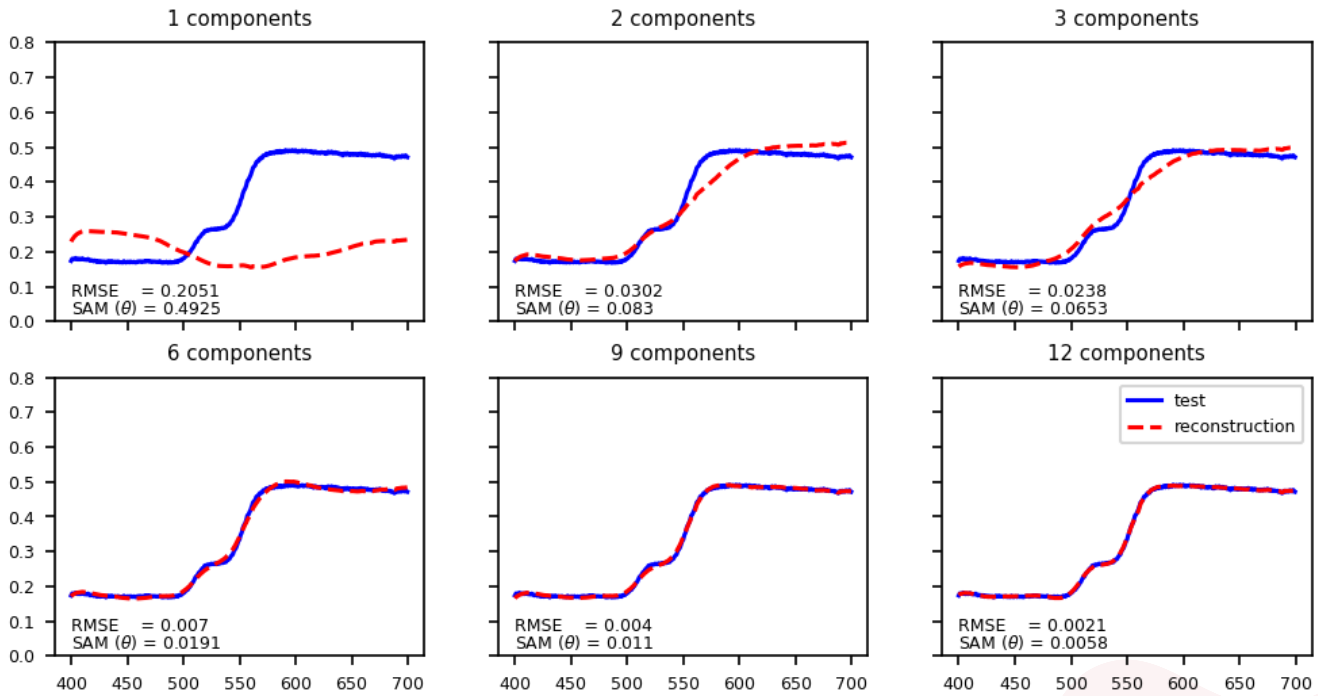
Percent Explained

Taking the cumulative sum of the normalized eigenvalues, we can see that three (3) principal components can represent 95% of the variance in the Munsell reflectance database. Therefore, we expect a spectrum to be properly represented using only three input values.



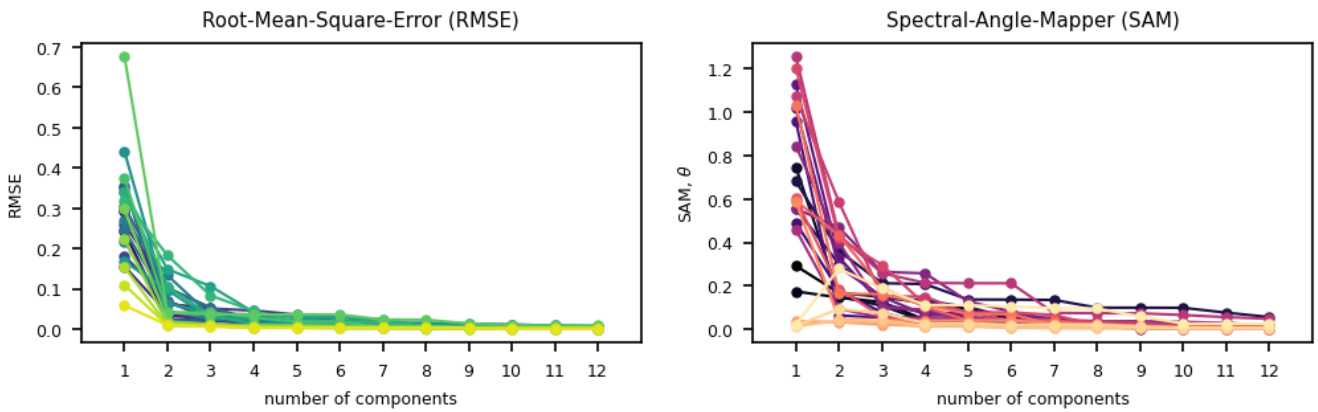
Reconstruction

Shown below is the sample reconstruction of a random Munsell reflectance spectrum using increasing number of principal components. At N = 3, the residual error of the reconstruction was reduced by almost ten-folds and the shape similarity has improved by seven-folds. At N = 6 onwards, the reconstruction is almost perfect with imperceivable errors.



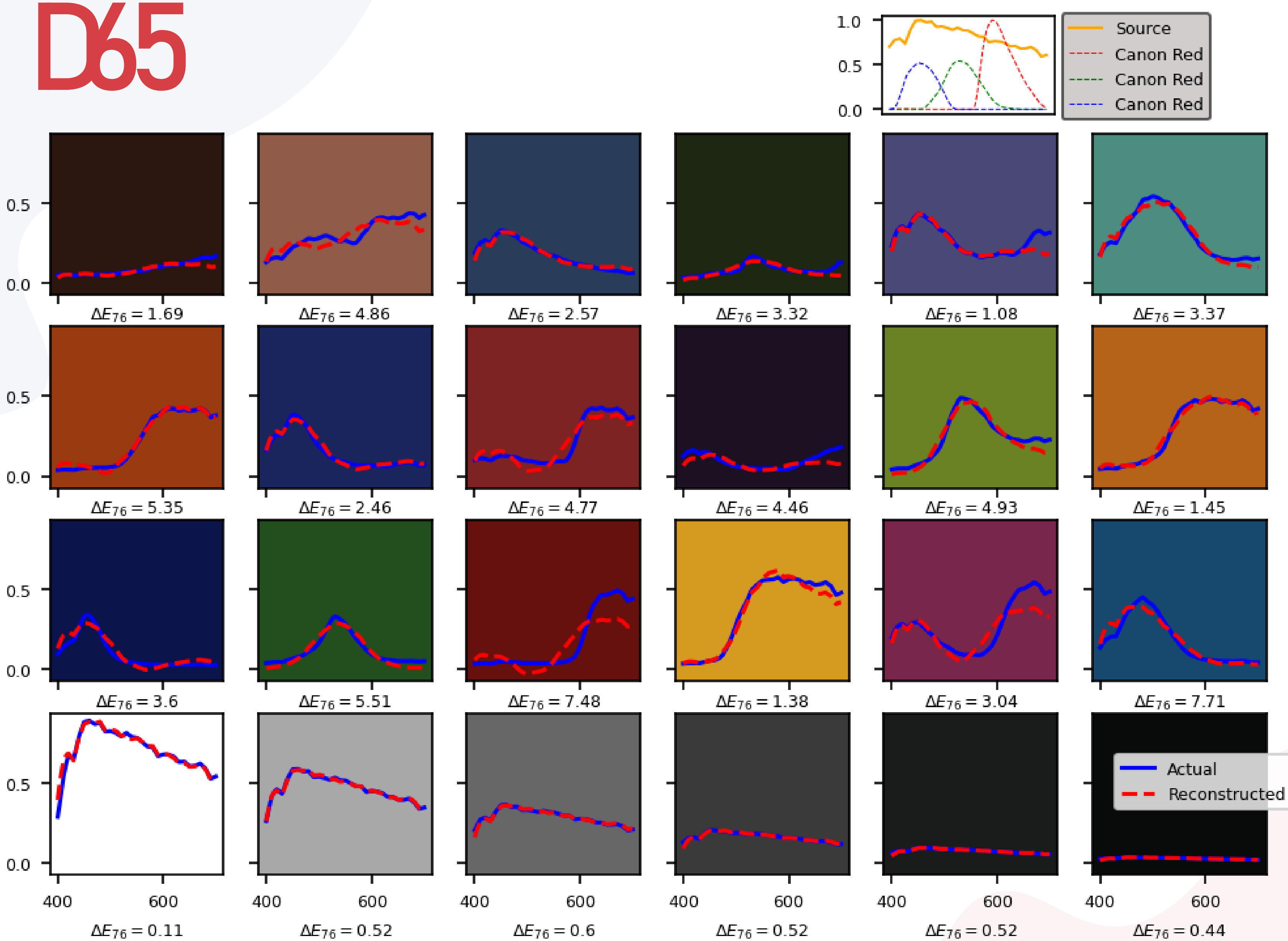
Reconstructing Macbeth CC

Shown below are the accuracies for the reconstruction of the 24 reflectance spectra from the Macbeth color chart. At N = 3, majority of the patches have RMSE < 0.1 and SAM < 0.2, quite low even though these were not part of the PCA ensemble where eigenvectors were obtained.



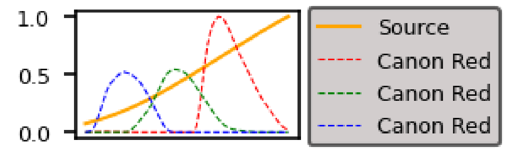
Aside from spectral accuracy, we employed color difference ΔE_{76} which measures the Euclidean distance between the two spectra transformed in the CIELAB uniform color space [5]. In short, ΔE_{76} quantifies how perceptually different the actual Macbeth color signals are from the PCA reconstructed spectra. From here onwards, we use (N = 3) three principal components.

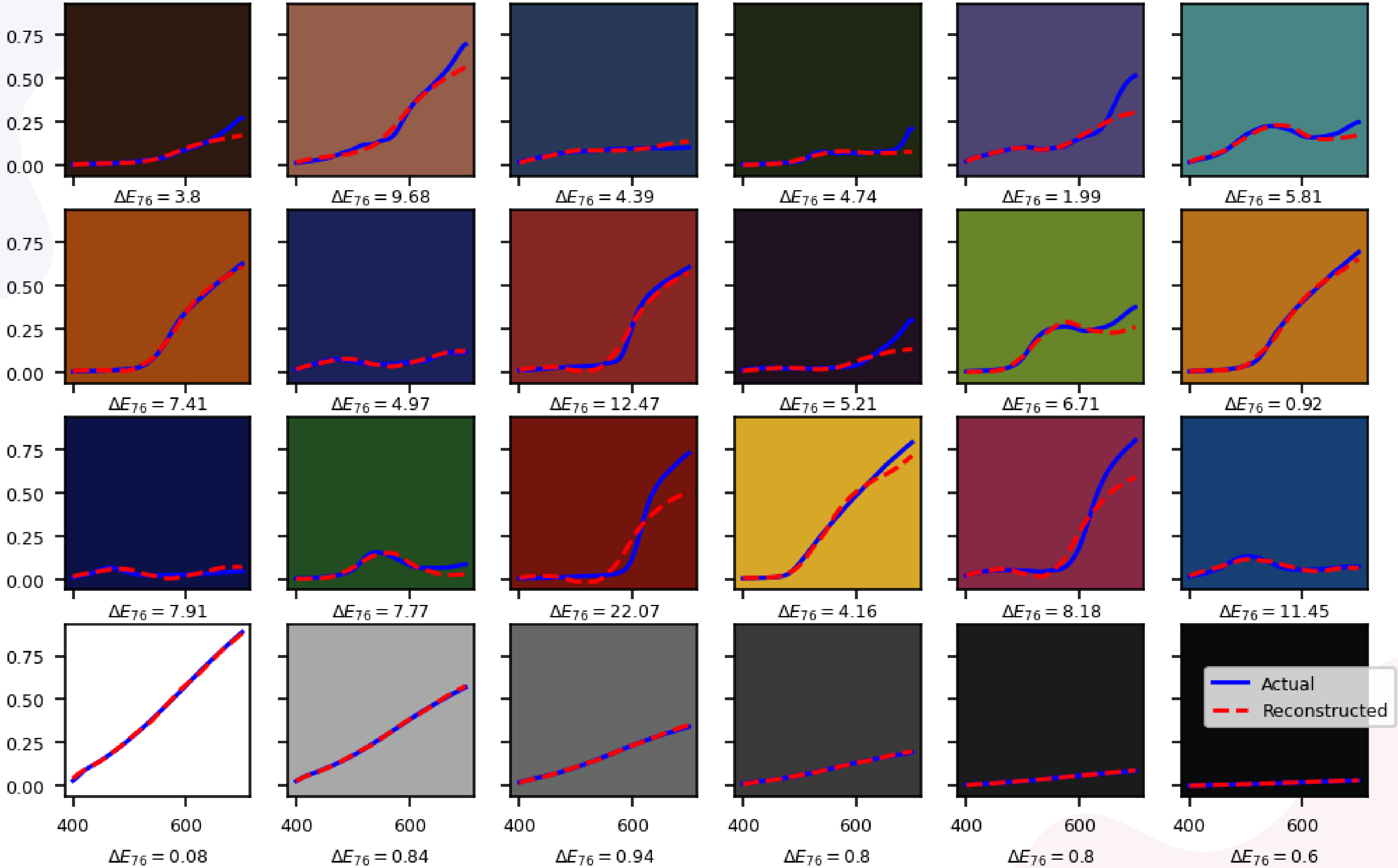
D65



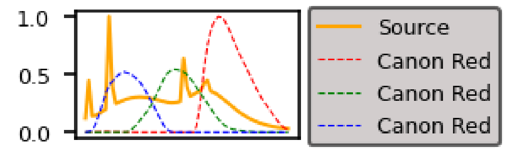
1.0
0.5
0.0
Source
Canon Red
Canon Red
Canon Red
0.5
0.0
400
600
$\Delta E_{76} = 1.69$
$\Delta E_{76} = 4.86$
$\Delta E_{76} = 2.57$
$\Delta E_{76} = 3.32$
$\Delta E_{76} = 1.08$
$\Delta E_{76} = 3.37$
$\Delta E_{76} = 5.35$
$\Delta E_{76} = 2.46$
$\Delta E_{76} = 4.77$
$\Delta E_{76} = 4.46$
$\Delta E_{76} = 4.93$
$\Delta E_{76} = 1.45$
$\Delta E_{76} = 3.6$
$\Delta E_{76} = 5.51$
$\Delta E_{76} = 7.48$
$\Delta E_{76} = 1.38$
$\Delta E_{76} = 3.04$
$\Delta E_{76} = 7.71$
$\Delta E_{76} = 0.11$
$\Delta E_{76} = 0.52$
$\Delta E_{76} = 0.6$
$\Delta E_{76} = 0.52$
$\Delta E_{76} = 0.52$
$\Delta E_{76} = 0.44$
Actual
Reconstructed

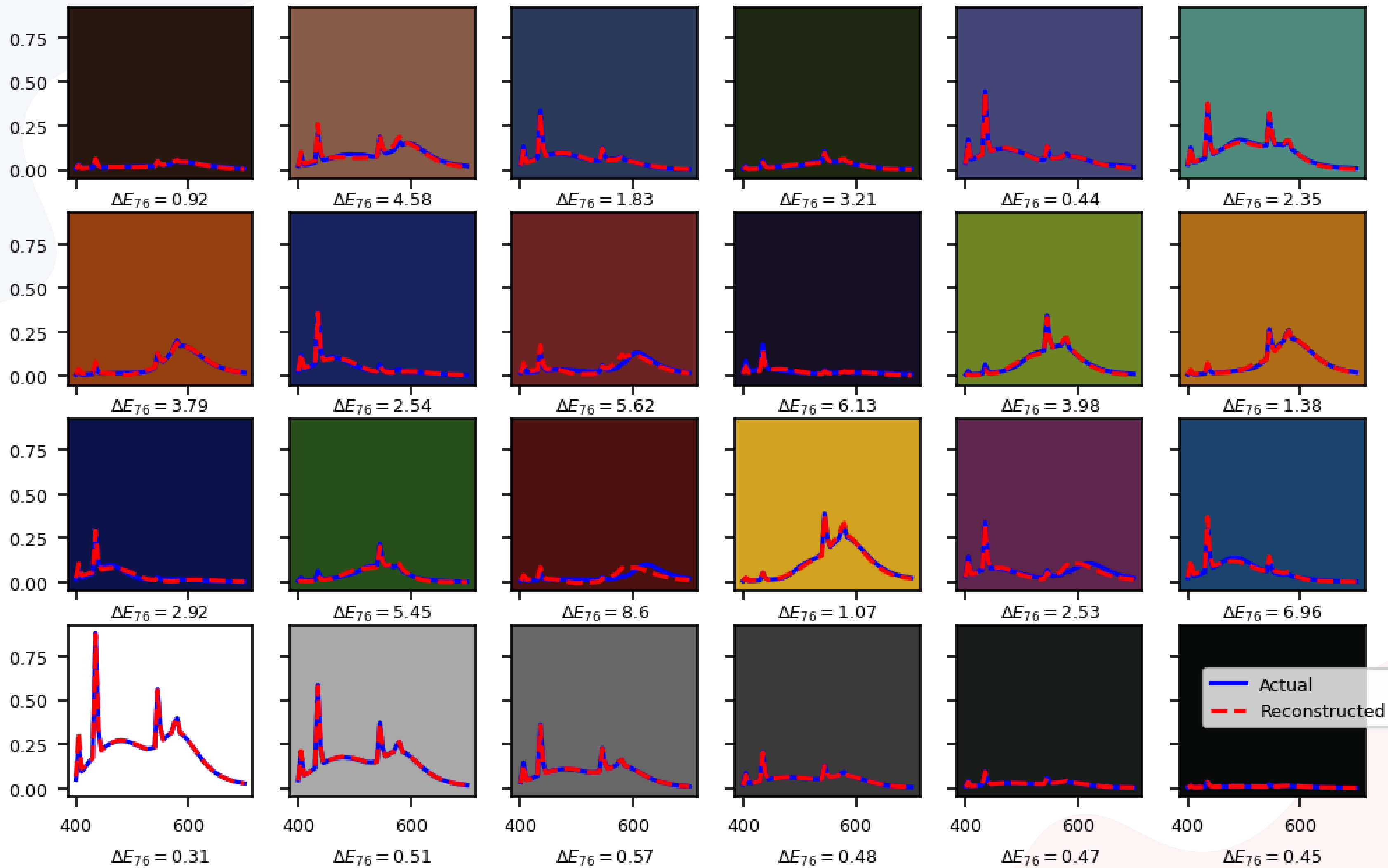
Illuminant A





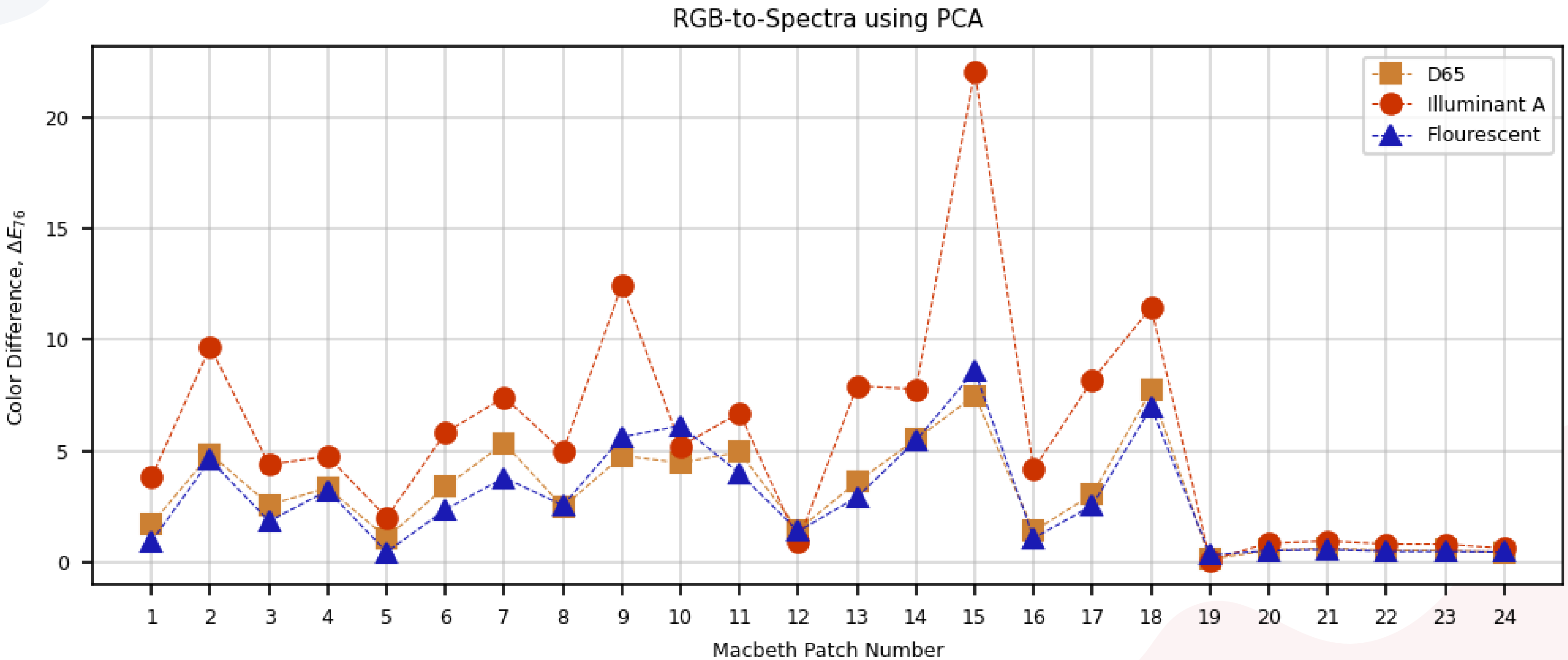
Flourescent





Discussion

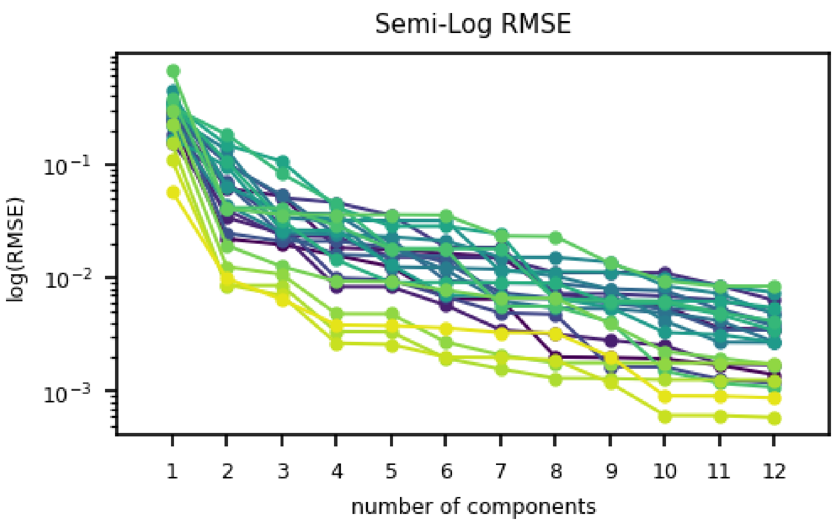
Overall, we successfully reconstructed the never-before-seen Macbeth color spectra using the Transformation Matrix derived from the Munsell ensemble's eigenvectors. Across different illuminants, majority of the reconstructions returned imperceivable color difference ($\Delta E_{76} < 10$) [5], with a few exceptions due to Illuminant A which has a skewed power spectrum.

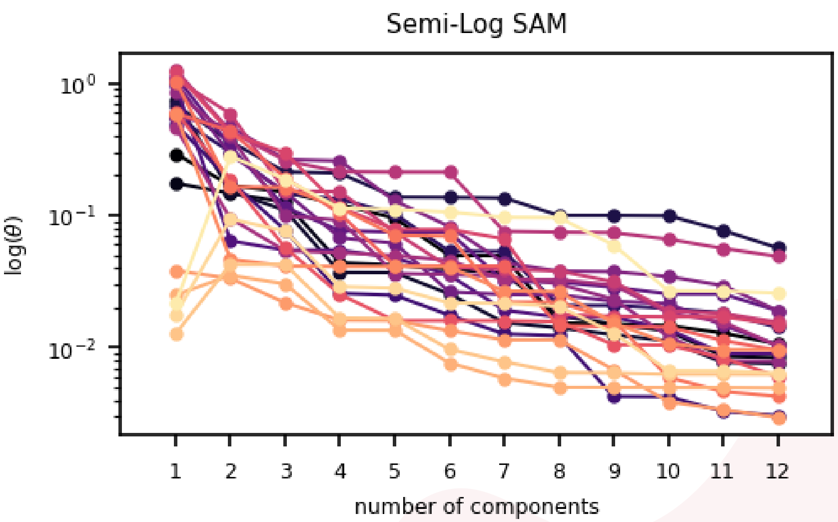


Conclusions

Both the spectral (RMSE, SAM) and colorimetric (ΔE_{76}) accuracy has shown how we can accurately depict large datasets with only a few representative values through Principal Components Analysis.

In this activity, we were able to convert a digital camera into a spectral imager where the RGB digital counts were found sufficient to reconstruct the high-dimensional spectral information using PCA.





reflection

As mentioned previously, I have used PCA before in spectral resolution but in this report, I was able to compare how the method fared across all different light sources. The visualizations were stand-alone and self-explanatory. One important finding is that it turns out, the skewed power distribution contributed to rendering spectra with large color difference. The take-away here is the same as what my undergraduate thesis proposed; we should implement both spectral and color error metrics to evaluate the spectral reconstruction.

In this activity, I'd give myself a score of **96/100.**

references

- [1] M. Soriano, Physics 301 – RGB-to-Spectra using PCA, (2022).
- [2] sklearn.decomposition.PCA — scikit-learn 1.1.1 documentation
- [3] J. P. S. Parkkinen, J. Hallikainen, and T. Jaaskelainen, Characteristic spectra of Munsell colors, J. Opt. Soc. Am. 6, 318 (1989).
- [4] P. E. Dennison, K. Q. Halligan, and D. A. Roberts, A comparison of error metrics and constraints for multiple endmember spectral mixture analysis and spectral angle mapper, Remote Sens. Environ. 93, 359 (2004).
- [5] W. Mokrzycki and M. Tatol, Color difference Delta E - A survey, Mach. Graph. Vis. 20, 383 (2011).