Phase Shift Profilometry

Objectives

1. Derive phase from images projected with phase shifted sinusoids
2. Unwrap phase image using quality-guided phase unwrapping algorithm (optional)
3. Reconstruct 3D from phase difference of reference and object image phase.

Shape from Structured Light

When we say "structured light illumination", it means we are illuminating an object with a pattern. Suppose you have an image of vertical lines with sinusoidally varying intensity and you are projecting this image on a 3D object just like in Figure 1. These are halves of pingpong balls embedded in styrofoam.

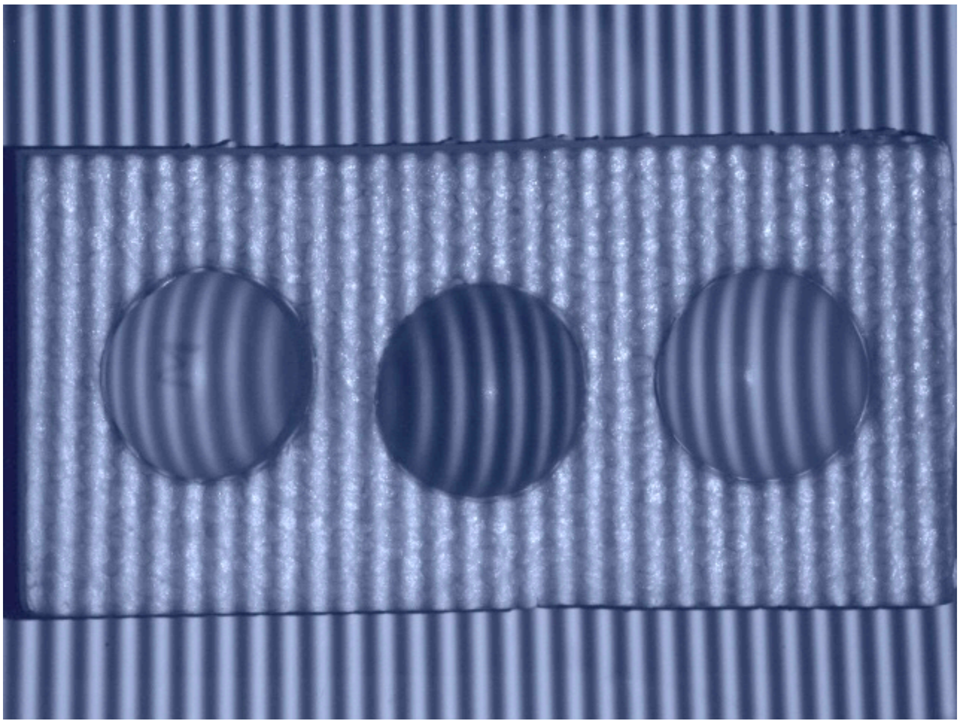

Figure 1. Pingpong ball halves in styrofoam illuminated by a vertical sinusoidal intensity pattern.

Notice that the reflected lines follow the curvature of the balls. The middle one is concave while the rightmost and leftmost are the convex halves. The shapes of the objects have caused a displacement in the reflected sinusoidal pattern.

Technically, we can already estimate the shape of the objects just from a single image like Figure 1 using the same concept in stereometry where disparity is inversely proportional to depth. The disparity here is the displacement in horizontal position of the reference line and the bulged line. But using the Phase Shift Profilometry technique we can instead estimate the phases of the intensity patterns for all pixels on the image. Getting the phase difference between that of flat reference object and a that of a 3D object we can then reconstruct the shape of the 3D object.

Phase Shift Profilometry (PSP)

Consider N intensity patterns of vertical sinusoids each phase shifted by k 2pi/N where k = 0 to N-1, and N is an integer. Suppose N = 4, then we have 4 phase shifted sinusoidal patterns. We will project each of these patterns to our object and capture an image each time. The four images can be expressed as intensity functions given by

$$\begin{aligned} I_1(x,y) &= I_o(x,y) + I_{mod}(x,y)cos(\phi(x,y)) \\ I_2(x,y) &= I_o(x,y) + I_{mod}(x,y)cos(\phi(x,y) + \tfrac{\pi}{2}) \\ I_3(x,y) &= I_o(x,y) + I_{mod}(x,y)cos(\phi(x,y) + \pi) \\ I_4(x,y) &= I_o(x,y) + I_{mod}(x,y)cos(\phi(x,y) + \tfrac{3\pi}{2}) \end{aligned} \tag{1}$$

where $I_o(x,y)$is the average background intensity, $I_{mod}(x,y)$is the intensity value of the fringe pattern and is the phase value.
We can express the equations in (1) as

$$\begin{aligned} I_1(x,y) &= I_o(x,y) + I_{mod}(x,y)cos(\phi(x,y)) \\ I_2(x,y) &= I_o(x,y) - I_{mod}(x,y)sin(\phi(x,y)) \\ I_3(x,y) &= I_o(x,y) - I_{mod}(x,y)cos(\phi(x,y)) \\ I_4(x,y) &= I_o(x,y) + I_{mod}(x,y)sin(\phi(x,y)) \end{aligned} \tag{2}$$

Subtracting like terms we get

$$\begin{aligned} I_4(x,y) - I_2(x,y) &= 2I_{mod}(x,y)sin(\phi(x,y)) \\ I_1(x,y) - I_3(x,y) &= 2I_{mod}(x,y)cos(\phi(x,y)) \end{aligned} \tag{3}$$

Solving for phase by we find

$$\phi(x,y) = \tan^{-1}\left(\frac{I_4(x,y) - I_2(x,y)}{I_1(x,y) - I_3(x,y)}\right). \tag{4}$$

Inverse tangent values will range from -π to π and so the phase values in $\phi(x, y)$ in (4) will wrap around to -π when the phase exceeds π, kind of like how interferogram patterns look like. Therefore the phase image will need to be unwrapped.

Phase Unwrapping

Phase unwrapping means adding integer amounts of 2π to pixels that have jumped from -π to π. Phase unwrapping techniques are built-in in MATLAB and other scientific programming software but often there can be unwrapping failure such as when the phase image is noisy, under-sampled, or discontinuous. Thus, there are phase unwrapping techniques that try to find the pixels where unwrapping failure might occur and unwrap them last. These are called "quality-guided" phase unwrapping methods [1] and they work by creating quality maps based on phase values of neighboring pixels and those pixels having high quality are unwrapped first following an unwrapping path. Contributed code to perform quality-guided unwrapping is also available for example in [2].

Phase-to-Height Conversion

Shown in Figure 2 is the setup for phase shift profilometry and the parameters for phase-to-height conversion. The range or object height Zw is expressed as

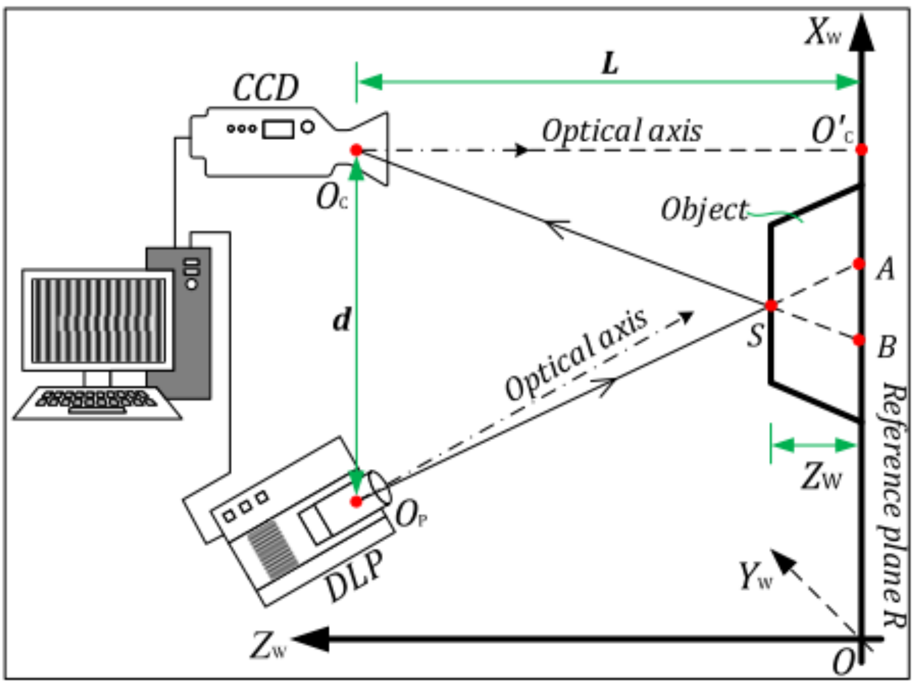

Figure 2. Setup for Phase Shift Profilometry and Phase-to-Height Conversion from [3].

$$Z_w(x,y) = \frac{Lp\Delta\phi(x,y)}{2\pi d + p\Delta\ phi(x,y)} \quad (4)$$

where L is the distance from the camera plane to the reference flat, p is the sinusoid or ("fringe") frequency, d is the distance between the projector and the camera, and $\Delta\phi(x,y)$ is the phase difference between the object phase image and reference flat phase image.

Procedure

1. You will be given images of objects and reference flats from PSP experiments. Create a code to compute the phase of the object and the reference flat. (Alternatively, you can estimate the reference flat by interpolation of the background.
2. Get the phase difference of the object and reference flat. Solve for Zw and display the height map as a mesh.
3. BONUS - Use quality-guide phase unwrapping contributed code instead of MATLAB's built in unwrap function.

Reference

[1] Zhao, M., Huang, L., Zhang, Q., Su, X., Asundi, A., & Kemao, Q. (2011). Quality-guided phase unwrapping technique: comparison of quality maps and guiding strategies. *Applied optics*, *50*(33), 6214-6224.
[2] Carey Smith (2022). QualityGuidedUnwrap2D_r1 (https://www.mathworks.com/matlabcentral/fileexchange/29499-qualityguidedunwrap2d_r1), MATLAB Central File Exchange. Retrieved May 20, 2022.
[3] Lu, J., Mo, R., Sun, H., & Chang, Z. (2016). Flexible calibration of phase-to-height conversion in fringe projection profilometry. *Applied Optics*, *55*(23), 6381-6388.

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