Super-Resolution Motion Deblurring

Suppose a scene is represented as f(x,y). This is traditionally known as the “object”. An imaging device captures the scene and produces an output g(x,y), that is, the “image”. The image will never be as pristine as the object since the imaging device, as well as the capture conditions, can introduce distortion. A model of the image formation can be represented by

$$g(x,y) = h(x,y) * f(x,y) + n(x,y) \qquad (1)$$

where h(x,y) is the point-spread function of the imaging device, “*” is the convolution operator, and n(x,y) is noise.

The Fourier Transform of Eq. (1) is given by

$$G(u,v) = H(u,v)F(u,v) + N(u,v) \qquad (2)$$

where we note that a convolution in (x,y) space is a multiplication in (u,v) space, (u,v) being spatial frequency coordinates after Fourier Transformation. H(u,v) which is the FT of h(x,y) is also known as the Optical Transfer Function (OTF) of the imaging device.

If the OTF is known and there is no noise, Eq. (2) suggests that we can recover f(x,y) by getting the inverse of F(u,v) given by

$$F(u,v) = \frac{G(u,v)}{H(u,v)} \text{ and then } f(x,y) = F^{-1}\left\{\frac{G(u,v)}{H(u,v)}\right\}. \qquad (3)$$

Equation 3 is only valid under two unrealistic conditions : (1) that the image has no noise, and (2) H(u,v) has no zero values . The division in Eq (3) is an element-per-element division.

Wiener Filtering

One way of recovering f(x,y) is through Minimum Mean Square Error (Wiener) Filtering. The objective is to find an estimate $\hat{f}$ that will minimize the mean square error between it and the scene f. If we define a square error function as

$$e^2 = E\left\{\left(f - \hat{f}\right)^2\right\} (4)$$

where E{} is the expectation. In the frequency domain the solution which minimizes Eq (4) is

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2 + S_n(u,v)/S_f(u,v)}\right]G(u,v) \qquad (5)$$

where Sn and Sf are the power spectrum of the noise and the object, respectively. Eqtn (5) nearly looks like Eq(3) , in fact it becomes Eq (3) if there is no noise.

For practical applications, the ratio of Sn and Sf can be replaced by a single number known as the noise to signal ratio (NSR) . Although Sn can be estimated or even measured, Sf is not known.

Thus Eq (5) can be applied as

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2+K}\right]G(u,v) \qquad (6)$$

where K is the NSR.

Removing Motion Blur

Motion blur can be removed in two scenarios:

1. If there's a single image.
2. If there is a low resolution video of the moving object.

If a single blurred image is all you have, you may use Eq. (6) with an estimate of the point spread function h(u,v) which represents the direction and extent of motion.

If you have a low resolution video then you may register (realign) each frame as in our first activity and deblur using Eq (6) and with h(u,v) this time as that of the point spread function of the camera.

Activity

Go through the example in

[Deblur Images Using a Wiener Filter - MATLAB & Simulink Example (mathworks.com)]

After running the example, capture your own actual, motion blurred image. A remote control car will be provided. You can use other moving scenes.

Reference

Gonzales and Woods, "Digital Image Processing" 3rd Edition, Chapter 5.7-5.8, (2008) Pearson Prentice Hall,