PHYSICS 305: SPECIAL TOPICS IN THEORETICAL PHYSICS
(METHODS AND APPLICATIONS OF WHITE NOISE ANALYSIS)

1st Semester AY 2023-2024

PROGRAMMING ASSIGNMENT #1

October 3, 2023

Instructions: Submit a consolidated PDF file of your Python notebook(s) (with discussion, code, and plots) through Google Classroom. Properly label all your plots (include axis labels and legends, as appropriate). Briefly discuss your code, plots, and results (in formatted text cells).

Due Date: October 24, 2023 (Tues)

Problems:

1. (50 pts.) Perform stochastic process calculations on fractional Brownian motion.

    a. Generate fractional Brownian motion samples (at least 5 each) for $H = 0.1$, $H = 0.5$, and $H = 0.9$. (10 pts.)

    b. Calculate the PDF from the samples for at least 3 different lag times τ. Plot the PDFs overlaid with the analytical result (see next page).
        i. Generate plot showing PDFs for different H (small τ) (10 pts.)
        ii. Generate plots showing PDFs for different τ ($H=0.1$ and 0.9) (10 pts.)

    c. Calculate the MSD(τ) from the samples. Plot the individual and average MSD overlaid with the analytics result (see next page).
        i. Generate plot showing MSDs for different H (20 pts.)

2. (50 pts.) Perform stochastic process calculations on your selected dataset.

    a. Generate x(t) plot(s) of your dataset (10 pts.)
    b. Generate PDF plot(s) (20 pts.)
    c. Generate MSD plot(s) (20 pts.)

Reference Formulas:

The probability density function (PDF) has the form,

$$P(x_1, t; x_0, 0) = \frac{1}{\sqrt{2\pi(MSD)}} \exp\left[\frac{-(x_1 - x_0)^2}{2(MSD)}\right] , \tag{1}$$

where the mean square deviation (MSD) is given by:

$$MSD = g(t)^2 \int_0^t [f(t-\tau)\, h(\tau)]^2 d\tau \ , \tag{2}$$

with t a constant final time in Eq. (2). Functions $g(t), f(t-\tau)$, and $h(\tau)$ determine the type or behavior of the stochastic process.

The following MSD's can be plugged-in to Eq. (1):

1) **Ordinary Brownian motion (Wiener process):**

$$MSD = 2Dt \quad (D \text{ is a constant diffusion coefficient}) \tag{3}$$

2) **Fractional Brownian Motion:**

$$MSD = \frac{t^{2H}}{2H\left[\Gamma\left(H+\frac{1}{2}\right)\right]^2} \tag{4}$$

The H is Hurst exponent, $0 \le H \le 1$, and $\Gamma(\alpha)$ is the Gamma function.

Derivation of the MSD for fractional Brownian motion (Dr. Bernido's lecture): https://youtu.be/hlG7YLgelQ4