METHODS AND APPLICATIONS OF WHITE NOISE ANALYSIS

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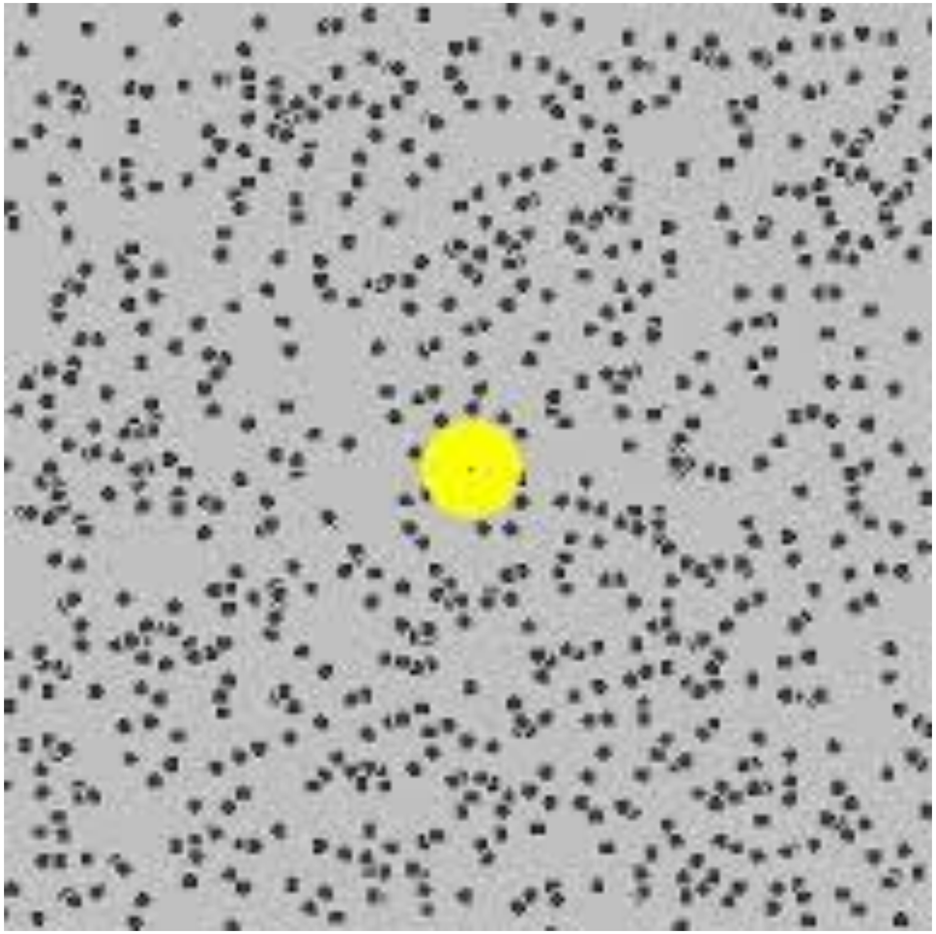
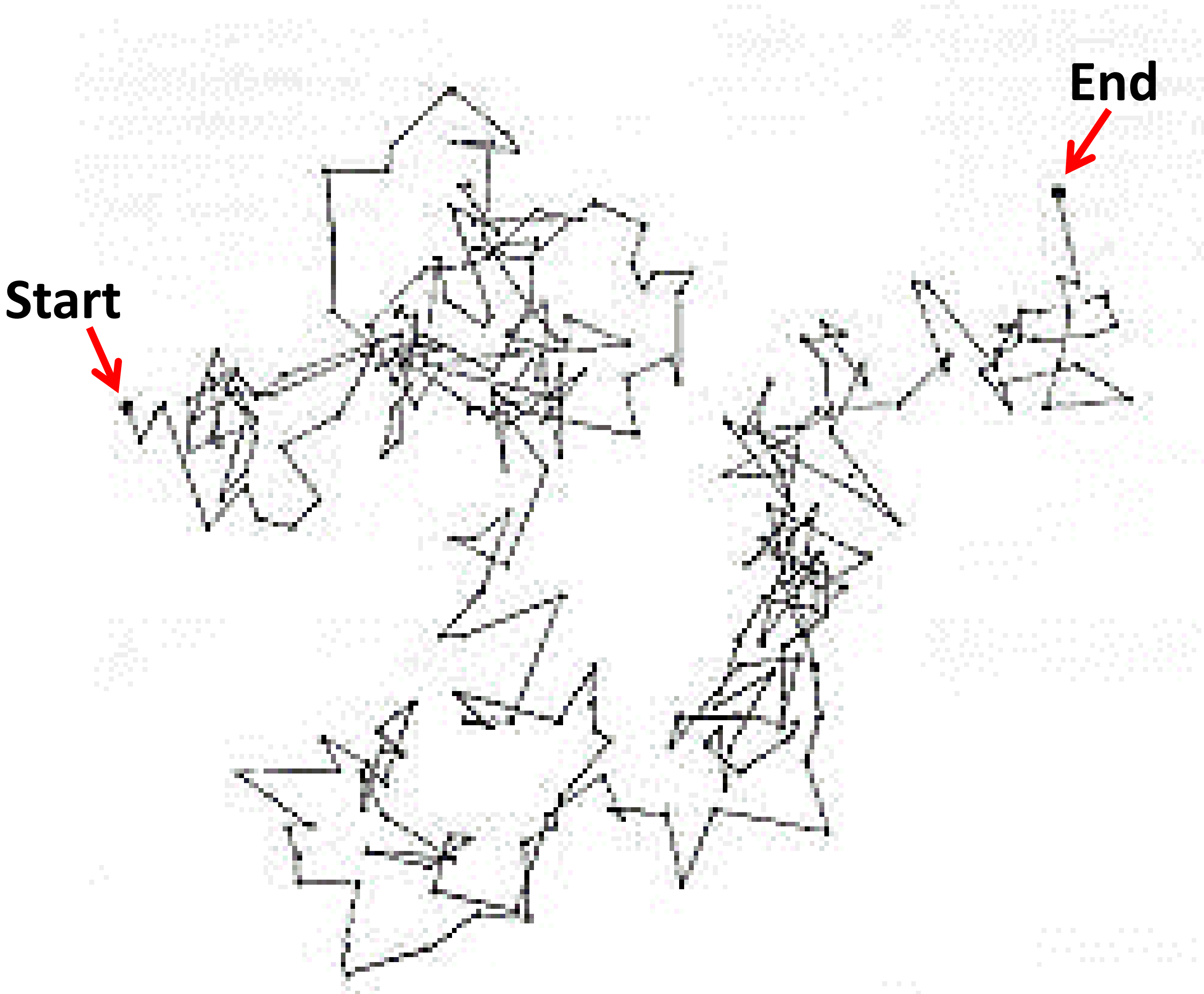
1827: The botanist Robert Brown observed a jittery motion of a particle in a fluid.



Particle in a Fluid

https://www.google.com.ph/search?q=Images+Brownian+motion&tbm=isch&imgil=tPFFqxL-3jc8kM%253A%253BYSbYechQHNVs2M%253Bhttp%25253A%25252F%25252Fwww.tutorvista.com%25252Fcontent%25252Fphysics%25252Fphysics-i%25252Fmatter%25252Fbrownian-motion.php&source=iu&usg=__3NI8UiQQxifPTCMez8FfN6iS2Hc%3D&sa=X&ei=EifkU_PPE4bd8AWq24GIAw&ved=0CBwQ9QEwAA&biw=1093&bih=468#facrc=_&imgdii=_&imgrc=tPFFqxL-3jc8kM%253A%3BYSbYechQHNVs2M%3Bhttp%253A%252F%252Fimages.tutorvista.com%252Fcontent%252Fmatter%252Fbrownian-motion.jpeg%3Bhttp%253A%252F%252Fwww.tutorvista.com%252Fcontent%252Fphysics%252Fphysics-i%252Fmatter%252Fbrownian-motion.php%3B430%3B305

1827: R. Brown observed jittery motion of particle in fluid.



https://www.google.com.ph/search?q=Images+Brownian+motion&tbm=isch&imgil=tPFFqxL-3jc8kM%253A%253BYSbYechQHNVs2M%253Bhttp%25253A%25252F%25252Fwww.tutorvista.com%25252Fcontent%25252Fphysics%25252Fphysics-i%25252Fmatter%25252Fbrownian-motion.php&source=iu&usg=__3NI8UiQQxifPTCMez8FfN6iS2Hc%3D&sa=X&ei=EifkU_PPE4bd8AWq24GIAw&ved=0CBwQ9QEwAA&biw=1093&bih=468#facrc=_&imgdii=_&imgrc=tPFFqxL-3jc8kM%253A%3BYSbYechQHNVs2M%3Bhttp%253A%252F%252Fimages.tutorvista.com%252Fcontent%252Fmatter%252Fbrownian-motion.jpeg%3Bhttp%253A%252F%252Fwww.tutorvista.com%252Fcontent%252Fphysics%252Fphysics-i%252Fmatter%252Fbrownian-motion.php%3B430%3B305

Theory on Brownian Motion
(1905 Doctoral Thesis)

https://www.pinterest.com/pin/333688653613483844/

NO MEMORY OF THE PAST

http://www.riskencyclopedia.com/articles/brownian_motion/

Pictures of Louis Bachelier - MacTutor History of Mathematics (st-andrews.ac.uk)

Louis Bachelier

French Mathematician

1900: PhD Thesis: *The Theory of Speculation*

Stock Prices are subjected to a random movement

Birth of Mathematical Finance

Aged 15

http://en.wikipedia.org/wiki/Louis_Bachelier

He is credited as the first person to model the stochastic process now called **Brownian motion.**

Correspondence between Finite and Infinite Dimensions:

FINITE DIMENSIONS	INFINITE DIMENSION
Independent variable: x_j	***Independent random variable***: $\omega(t)$
Coordinate system: $(x_1, \dots, x_n)$	***Coordinate system***: $\{\omega(t); t \in \mathbf{R}\}$
Function: $f(x_1, \dots, x_n)$	***Functional***: $\Phi(\omega(t);\ t \in \mathbf{R})$
Space: R^n	***Space of Hida distributions***: S^*
Lebesgue measure: dx	***Gaussian measure***: $d\mu(\omega)$

White Noise Analysis works with the Gelfand triple: $S \subset L^2 \subset S^*$

Space of test functions: S

Hilbert space of square integrable functions: L^2

T. Hida, H. H. Kuo, J. Potthoff, and L. Streit, *White Noise. An Infinite Dimensional Calculus* (Kluwer, 1993)

Get the *T*-transform of the following:

(1) $\Phi = 1$

(2) $\Phi(\omega) = \exp(i\langle\omega, \eta\rangle)$

(3) $\Phi(\omega) = \exp\left(-i\langle\omega, \eta\rangle\sqrt{2}\, y\right)$

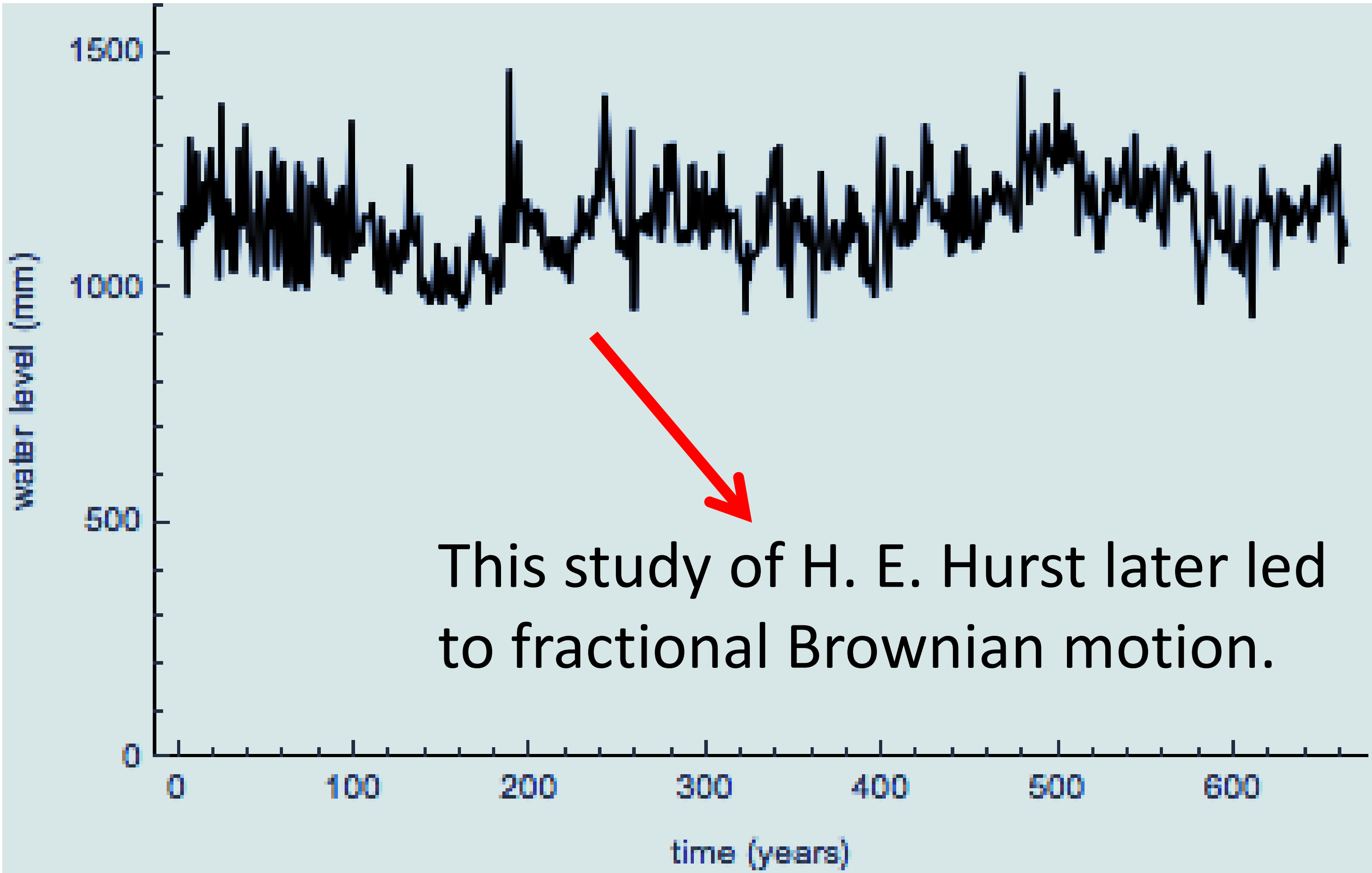
(4) Express your answer in number (3) in terms of the Hermite polynomials, $H_k(x)$. Take the norm, $\int \eta^2 d\tau = 1$.

Note: The generating function for Hermite polynomials is:

$$\exp\left[\sqrt{2}\, y\,\langle\xi, \eta\rangle - y^2\right] = \sum_{k=0}^{\infty} \frac{y^k}{k!} H_k\left(\langle\xi, \eta\rangle / \sqrt{2}\right)$$

TIME SERIES

Example of Big Data: **Nile River**



Graph courtesy of M. G. O. Escobido

Let $B(t)$ be the ordinary Brownian Motion.

Fractional Brownian Motion:

$$B^H(T) = \frac{1}{\Gamma\left(H + \frac{1}{2}\right)} \int_0^T (T-t)^{H-(1/2)} dB(t)$$

Riemann-Liouville representation

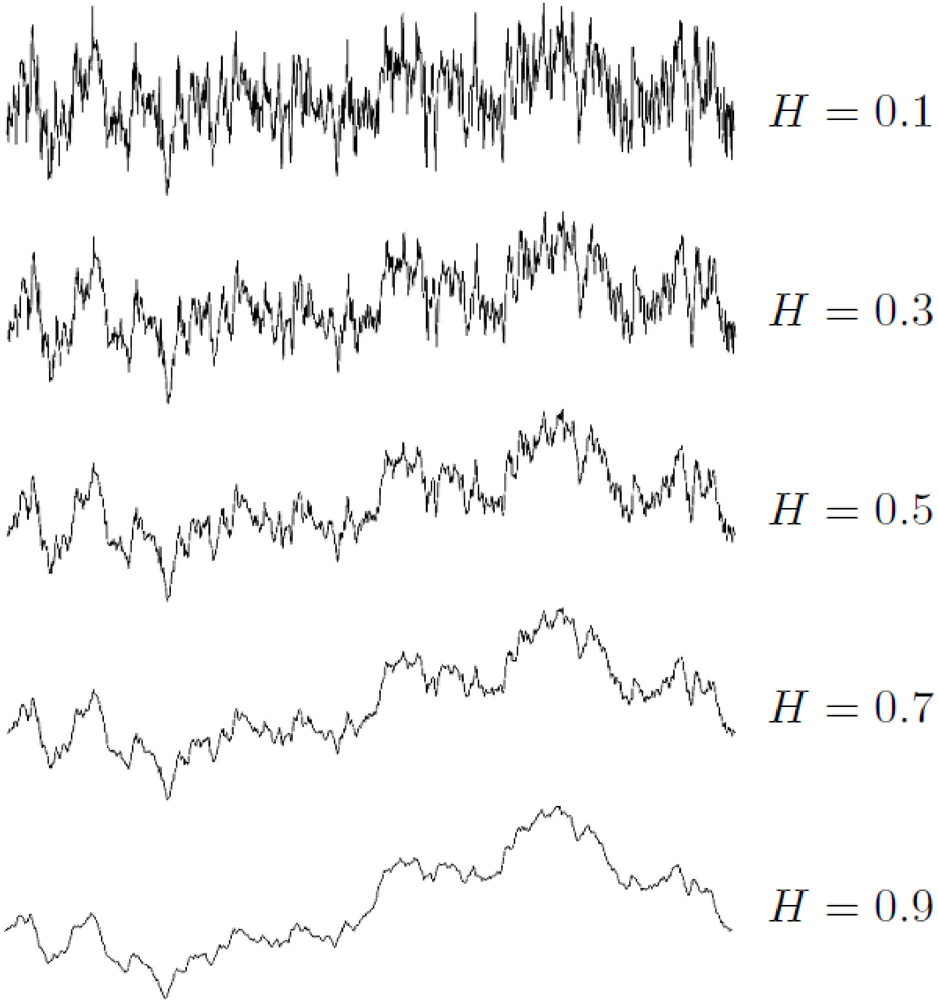
H **is the Hurst Exponent**

$0 < H < ½$: *Subdiffusion*

$½ < H < 1$: *Superdiffusion*

$H = ½$: *Normal Diffusion*

What is Memory?



Short Memory Domain
(negatively correlated)

No Memory ;
(*Brownian Motion*)

Long Memory Domain
(positively correlated)

Graphs by S. Shevchenko
Jagna Workshop (2014)

Fractional Brownian Motion:

$$x(T) = x_0 + \frac{1}{\Gamma(H+1/2)} \int_0^T (T-t)^{H-\frac{1}{2}}\, dB(t)$$

Biophysical Journal Vol. **103** (2012)1839– 1847

Universal Algorithm for Identification of Fractional Brownian Motion. A Case of Telomere Subdiffusion

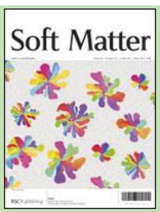
K. Burnecki, E. Kepten, J. Janczura, I. Bronshtein, Y. Garini, and A. Weron

MATHEMATICAL FINANCE

An International Journal of Mathematics, Statistics and Financial Economics

NO ARBITRAGE UNDER TRANSACTION COSTS, WITH FRACTIONAL BROWNIAN MOTION AND BEYOND

Paolo Guasoni, *Mathematical Finance* **16** (2006) 569-582.



Fractional Brownian motion in crowded fluids

D. Ernst, M. Hellmann, J. Köhler, and M. Weiss

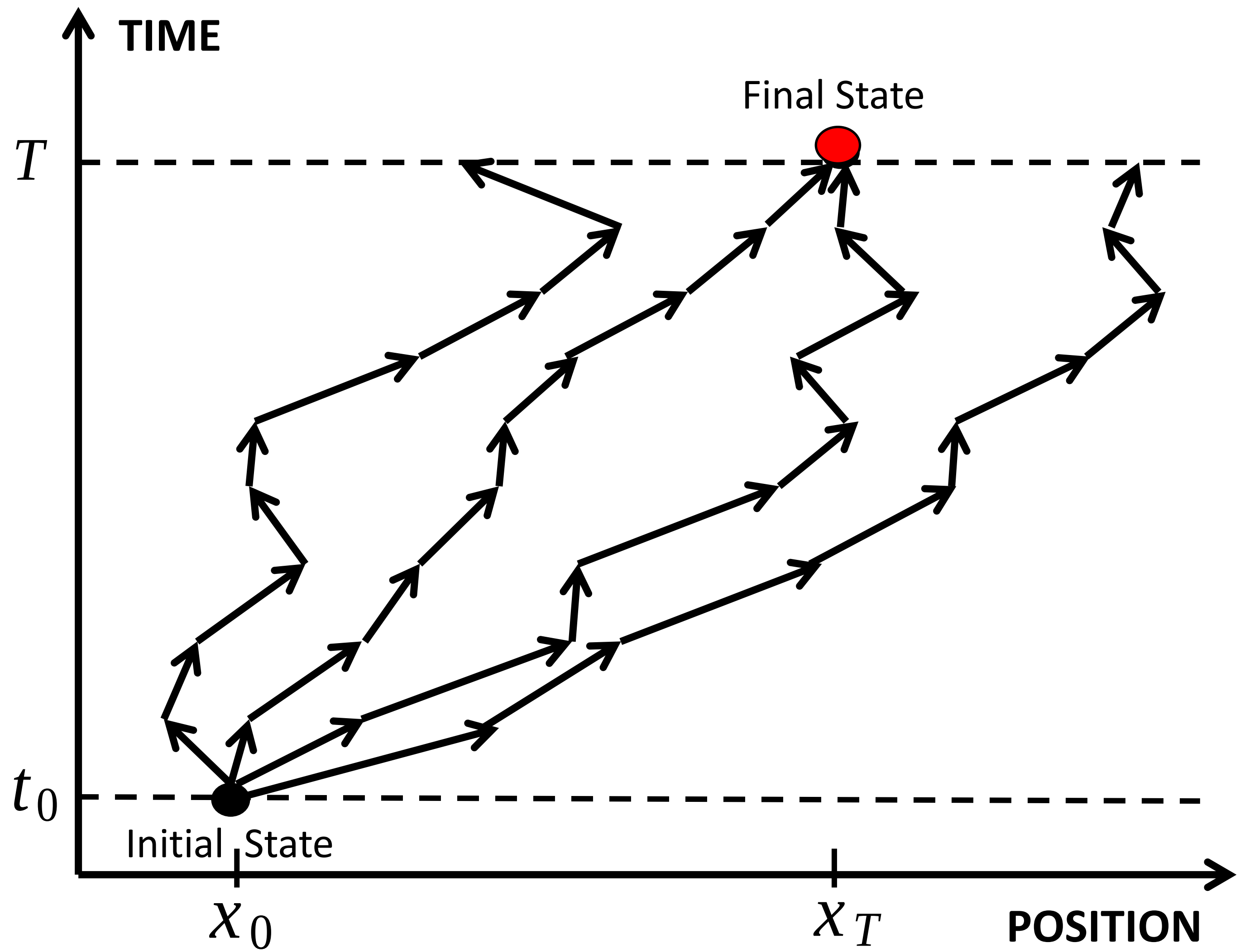
Soft Matter (2012)

A note on the use of fractional Brownian motion for financial modeling

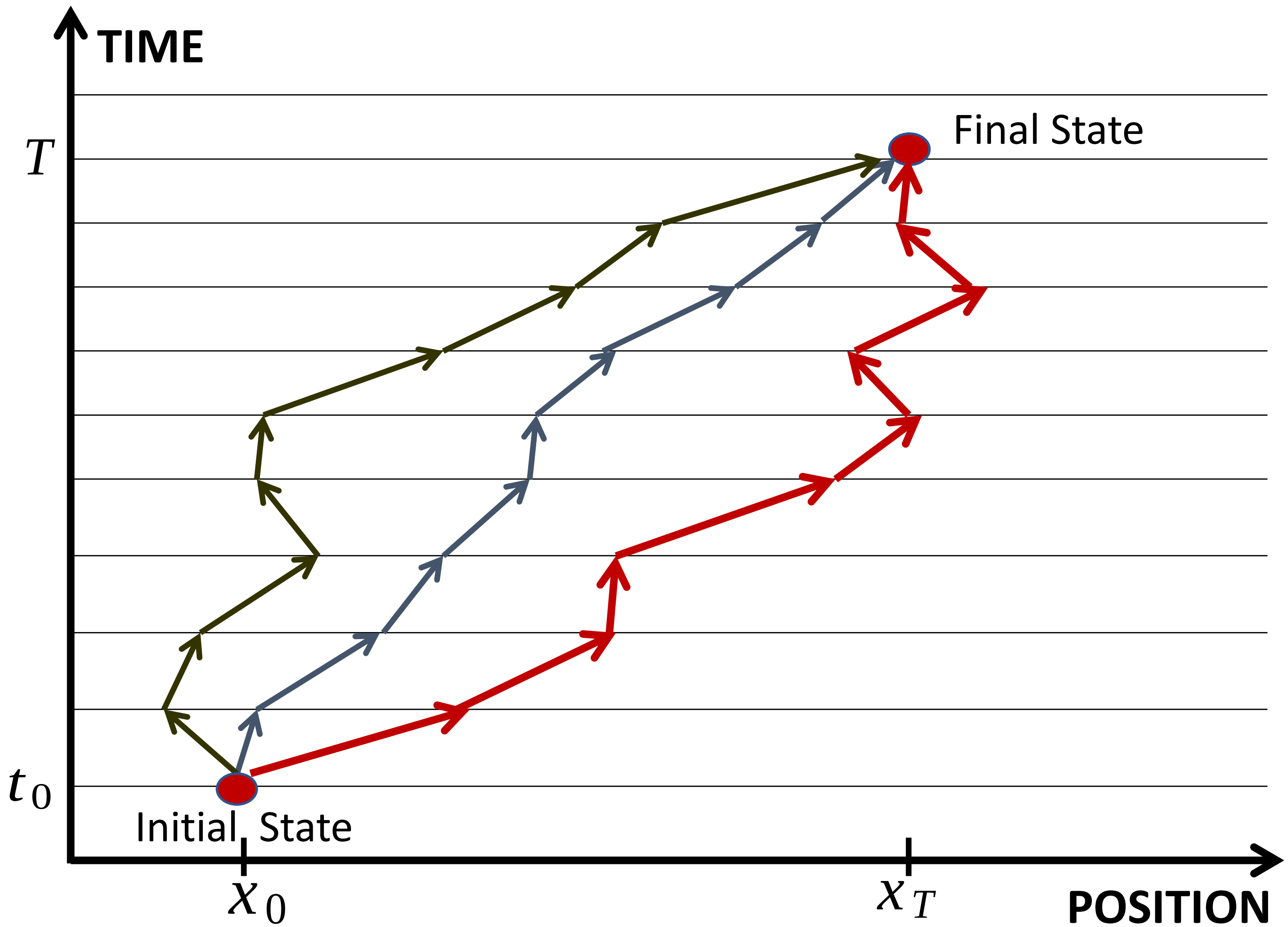
S. Rostek and R. Schöbel ,

Economic Modelling **30** (2013) 30-35.

$$x(T) = x_0 + \sqrt{2D}\ B(T)$$



Feynman‘s Sum-Over-All Paths



$$\delta(x(T) - x_T)$$