Electrical Power and Energy Systems 44 (2013) 798–809

Contents lists available at SciVerse ScienceDirect

Electrical Power and Energy Systems

journal homepage: www.elsevier.com/locate/ijepes

Power consumption analyzes with mathematical models into the overall electrical network

Snezana Tilovska *, Aristotel Tentov

University "St. Kiril and Metodij", Skopje, The Former Yugolav Republic of Macedonia

ARTICLE INFO

Article history:
Received 27 March 2012
Received in revised form 10 July 2012
Accepted 19 August 2012
Available online 26 September 2012

Keywords:
Electrical network
Power consumption
Measurements
Analytical model
Real-time measurements
Prediction

ABSTRACT

This paper performs analyzes using mathematical model of Fractional Brownian Distribution, M/G/∞ Model Described by Pareto Distribution, 'Heavy Tail' Distribution, and Gamma Distribution with Intergroup Characteristic.

For performed analyses into this research are used measurements for power consumption into overall electrical network in Macedonia in time period of 16 years.

The results enclosed with mathematical analytical model are compared with results from real measurements.

The calculated discrepancies confirms that enclosed analytical model with Intergroup Characteristic is optimal estimation model for modeling of the power consumption of electrical network.

The real-time analyses for consumption of electrical energy into the overall network or for the part of electrical network are very important for everyday real operations into the electrical networks in order to achieve the services into the electrical network with best performance and the smaller cost as it is possible.

In this research are prepared and predictions for power consumption quantities into the network, so the evaluated model is used as the best practice model for direct planning of the power consumption values into the electrical network (indirectly can be planned costs and operations into the electrical network related with the values of power consumption).

1. Introduction

Each country develop electrical network with defined topology, configuration and strength regarding the requirements of the population. Into the network can be found different demand requirement on random demand intervals in different regions or parts of the electrical network. In all distributed points of the electrical network should be provided uninterrupted power supply to the end customers with or without using hybrid electrical systems.

1.1. Electric power distribution

The modern distribution system [1–6] begins as the primary circuit leaves the sub-station and ends as the secondary service enters the customer's meter socket. A variety of methods, materials, and equipment are used among the various utility companies, but the end result is similar. First, the energy leaves the sub-station in a primary circuit, usually with all three phases, Fig. 1.

1.2. Electrical distribution system

There are a great many different standards for electrical distribution systems [7,8], but can be summarized the constraints and conventions regarding equipment powering devices as follows: mains voltage, neutral distribution and system earthing, wiring practices, product standards and clearance distances, types of fuses for fuse-holders or fused switches.

1.3. Cost analyses on demand interval into the electrical network

Due to the worldwide energy consumption into the electrical network become important issue [9], from one side and because of the requirements for improving of the operations and efficiency into the electrical networks from the other side, calculation of the power consumption into the electrical network as parameter who defines cost for electrical network operability, today became the most important and critical task.

Calculating the cost for energy consumption [10] is straight forward, and is the sum of power requirements accumulated over time. Energy consumption usually is described with power consumption over some time period defined on demand intervals. Calculating the costs associated with demand intervals can be much

* Corresponding author.
E-mail addresses: snezana.tilovska@telekom.mk (S. Tilovska), toto@feit.ukim.edu.mk (A. Tentov).

0142-0615/$ - see front matter © 2012 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.ijepes.2012.08.042

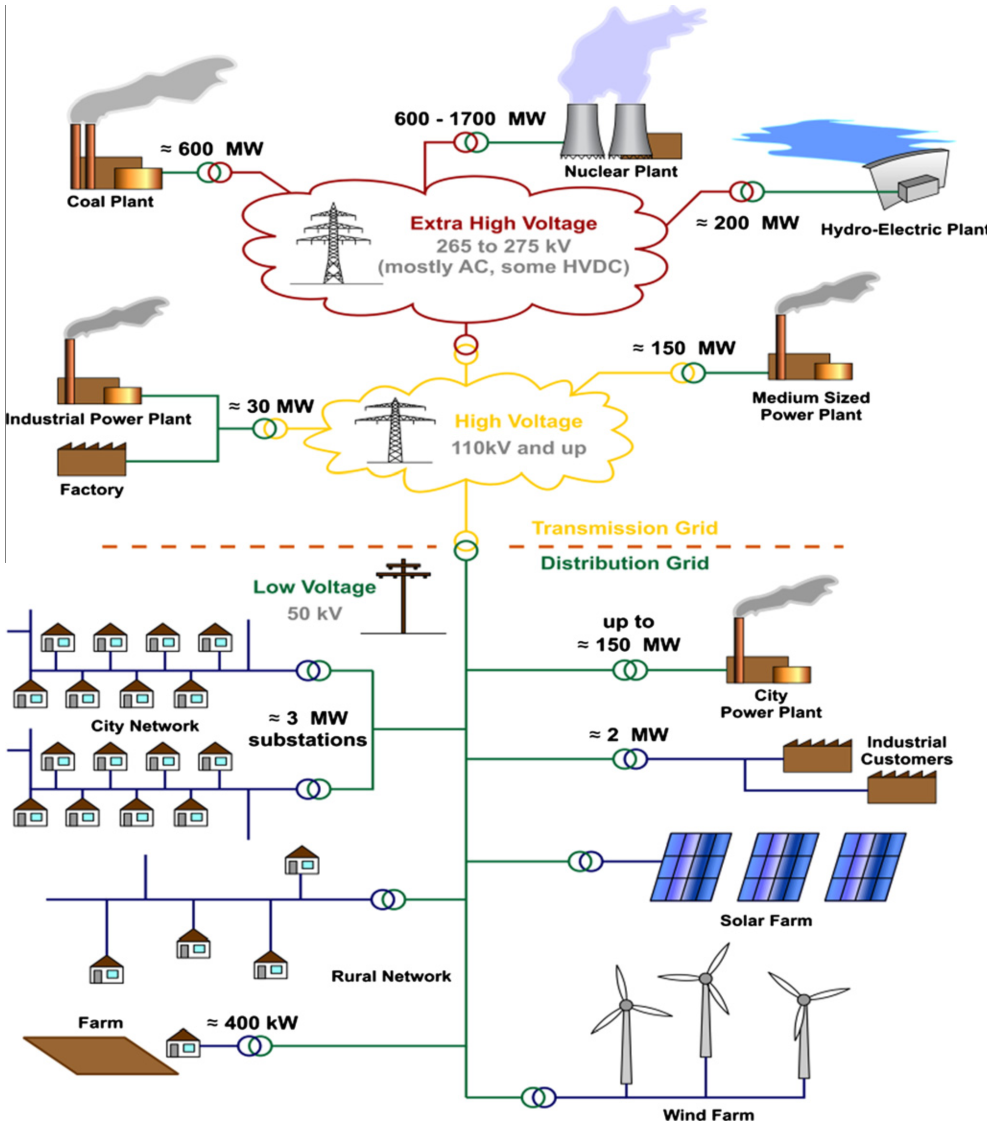

Fig. 1. General layout of electrical networks.

more complex. A demand charge is levied yearly, monthly or daily based on the maximum average rate of energy consumption over a time period of measurement (typically 15, 30 or 60 min). This time period of measurement is generally referred to as the demand interval. Additionally, many utility rates include provisions for assessing Peak Demand charges based on the single highest demand interval occurring during summer or winter months, that can substantially affect electrical capacities.

1.4. Power consumption analyses into the electrical network

This paper research analyses electrical power consumption measurement figures over the period January 1991–December 2006. The analyzed data includes information about the power consumption into the overall electrical network defined in hours per each day in a year for defined time period.

Into this research we collect measurements for electrical power consumption on the country level of Macedonia and we define them as power consumption measurements into the overall electrical network.

The overall electrical network in Macedonia covers about 180 km of distribution network at a voltage level of 110 kV, 1.161 km at 35 kV, 1.541 km at 20 kV, 8.124 km at 10 kV and 13.000 km at 0.4 kV and supplies with electricity 671.987 consumers from which 570.287 households (defined in Table 1) [11].

The population density in Macedonia is 81.47 people per square kilometer (in 2011) and about 62% of the population lives in urban areas [12].

The measurements of the power consumption into the overall electrical network were defined into the resulting database. The database consist records for about 140.160 power consumption values over this period.

Table 1
Number of the connections into overall electrical network.

Type of connections	Number of connections
5 kV customers	16
35 kV customers directly connected	3
10 (20) kV customers	1.156
0.4 kV customers	670.812
– Households	570.287
– Others (I and II tariff degree)	94.741
– Public lightening	5.784

From the enclosed database, as example, the plot of the power consumption on annual level for the year 2001 is defined in Fig. 2.

Into this research the first step was to analyze measured data for power consumption of electrical energy. We explore measured data for power consumption in configuration of 1 year, 1 month or 1 day and present them on the plot diagrams.

As the second step into the research we perform the deep analyzes on the structure and shape of the curve of the presented data on the plot diagrams.

In the third step we recognize that structure of the flow of the curve on the plot diagram with presented measured data for the power consumption are similar with the structure of the curve of self similar processes.

Therefore, we start with searching of the model formula of self similar type that will describe the power consumption into the overall electrical network.

Further, we use different mathematical models into the area of self similar processes: model of Fractional Brownian Distribution, M/G/∞ Model Described by Pareto Distribution, 'Heavy Tail' Distribution and Gamma Distribution with Intergroup Characteristics. Per each used mathematical model are defined variables who describe power consumption into the electrical network and parameter values that describe the characteristic nature of the electrical network.

The model of Fractional Brownian Distribution, M/G/∞ Model Described by Pareto Distribution, 'Heavy Tail' Distribution and Gamma Distribution with Intergroup Characteristics, into this paper, are described, discussed and compared with the real measurements for power consumption into the electrical network. The derivated data from the performed analyses and found discrepancies are reported into this paper and described on comparison diagrams.

The resulted discrepancies per each used model on daily basis, as most detail example analyse segment, defines that further researches should be performed with model of Gamma Distribution with Intergroup Characteristics.

So, additionally are performed analyses on daily, monthly and yearly level with Gamma Distribution with Intergroup Characteristic. From the comparison analyses of the enclosed mathematical model with Gamma Distribution with Intergroup Characteristics and values of power consumption with the real measurements over time are found a 0.07% annual discrepancies, 0.29% mount discrepancies and 3.35% day discrepancies. The level of these discrepancies confirms that the model developed with Gamma Distribution with Intergroup Characteristics can be used for modeling of power consumption into the electrical network.

Describing the power consumption values into the electrical network with model of Gamma Distribution with Intergroup Characteristic is used into the research area for the first time.

Into the research area can be found modeling of the stream outage distribution, delay factor and stream loss parameters into the key points of the IPTV network with self similar processes described with the model of Gamma Distribution with Intergroup Characteristics [15] .

The enclosed mathematical model defined with Gamma Distribution with Intergroup Characteristic is used for preparation of prediction for power consumption of electrical network for time period of a few months.

The model described in this paper can be used as a part of other already developed learning algorithms, robots or machines for predicting of the power consumption of the electrical energy.

1.5. Table with terminology and notation

In order to facilitate the reading and understanding of the paper in table bellow is defined short description of terminology and notation that is used within this study (Table 2).

2. Existing models for analyzes of power consumption of electrical energy

Into research area can be found other approaches for performing analyzes for power consumption of electrical energy: Analysis, modeling and design of energy management and multisource power systems, Predicting future hourly residential electrical consumption: A machine learning case study, Multiple regression models to predict the annual energy consumption in the Spanish banking sector, using pattern recognition to identify habitual behavior in residential electricity consumption, accurate quantitative estimation of energy performance of residential buildings using statistical machine learning tools, Performance analyses for DC power supplies, energy performance analyses for a Photovoltaic/Diesel/Battery hybrid power supply system, analyzes of the performance degradation caused by noise in power supply lines, models of battery, power electronic converter, and electric motor losses related to a typical 48 s track driving schedule and a data-driven approach for minimization of the energy to air condition a typical office-type facility [16–27].

Performance analyses of power consumption into overall electrical network with self-similar process with Gamma Distribution can be found into the research area in the world for the first time.

The proposed Gamma Distribution Model with Intergroup Characteristic is very efficient model for describing the real processes of power consumption into the overall electrical network with low level of calculated discrepancies. Model consist definition of set of parameters with values that refers calculated formula to enrich real measurements into the overall electrical network.

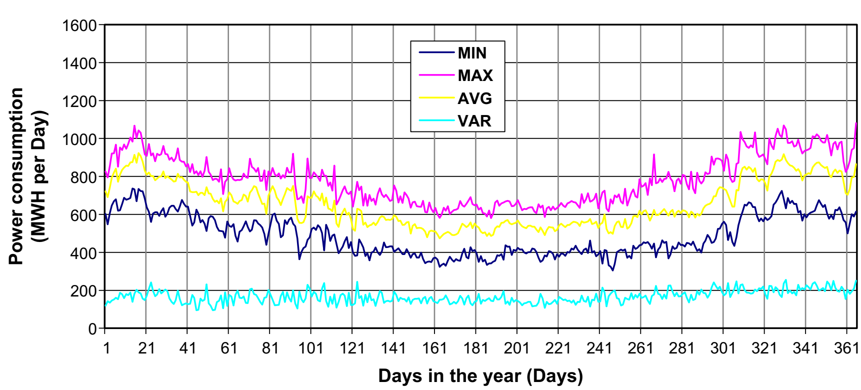

Fig. 2. Real measurements for power consumption per day for 1 year (example: year 2001) [13,14].

Table 2
Terminology and notation.

Terminology	Explanation
Power consumption	Power consumption refers to the electrical energy over time that must be supplied to an electrical device to maintain its operation
Electrical network	Electrical network is an interconnection of electrical elements such as transmission lines, voltage sources and switches
Self similar process	Stochastic process that exhibit the phenomenon of self-similarity
Gamma Distribution	Gamma Distribution is continuous probability distribution based on two parameters: shape and rate parameter
Intergroup Characteristic	Characteristic described with first order and second order statistical characteristic of I frame processes
Fractional Brownian Motion	Model based on FIFO queuing model with infinite buffer and constant service intensity
M/G/∞ Model with Pareto Distribution	M/G/∞ Model with service times corresponding to the Pareto Distribution
'Heavy Tail' Distribution	Distribution based on Pareto model used for advanced analyses
IPTV	Internet protocol television is system through which television services are delivered using the Internet protocol

Into this research are examined analyses of power consumption with mathematical model of Fractional Brownian Distribution, M/G/∞ Model Described by Pareto Distribution and 'Heavy Tail' Distribution. Because Fractional Brownian Distribution, M/G/∞ Model Described by Pareto Distribution and 'Heavy Tail' Distribution have not flow of the curve of the mathematical model that is similar with the flow of the curve of the real measurements for power consumption these distributions generate high level of calculated discrepancy and cannot be used as models for modeling of the power consumption into the electrical network.

Mathematical model defined with Gamma Distribution Model with Intergroup Characteristic is characterized with required speed of the changes from one point to another with predefined tendency factor, possibility to change the values upper or lower in the next calculated point after the last calculated value, possibility to have overall changing factor and therefore is excellent model for power consumption into overall electrical network.

The Gamma Distribution is fully characterized by the mean and the variance, and with deep analyses of the statistical characteristics of the processes reflecting distributions for the given *I* frame sizes, the result analyses shows that only Gamma Distribution with Intergroup Characteristic can be used as model for performance analyses of power consumption into the overall electrical network. The evaluated results confirms that the Gamma Distribution with Intergroup Characteristic is very flexible distribution that can reach values within time changes in real as it is power consumption into overall electrical network.

Transformative impacts on our energy security rely on creative approaches for consumption and generation of electricity. Technological contributions can impact both areas if they focus on problems of scale. For example, occupancy-based electrical loads (HVAC and lighting) accounted for roughly 50% of the total consumed electricity in the US in 2008. Meanwhile, roughly 50% of consumed oil in the US is imported. The US Department of Energy [24], has appropriately identified "sensing and measurement" as one of the "five fundamental technologies" essential for achieving energy security. Complementing reductions in consumption with increases in deployment of fossil-fuel-independent generation (solar and wind) and energy storage (batteries, capacitors and fuel cells) will yield a twofold impact. Lofty energy security goals can be made realizable by aggressive application of inexpensive technologies for minimizing waste and by maximizing energy availability from desirable sources. Long-standing problems in energy consumption and generation can be addressed by adding degrees of freedom to sensing and power conversion systems using multiple electrical sources. This principal drove the invention of the hybrid electric vehicle, which achieves efficiency increases by combining the energy capacity of gasoline with the flexible storage capability of batteries. Similarly, fresh strategies for electrical circuit design, control, and estimation in systems with multiple electrical sources can minimize consumption, extend the useful life of storage, and improve the efficiency of generation. A solar array constitutes a grid or network of panels or cells that may best be modeled and treated as independent sources needing careful control to maximize overall power generation. A fuel cell stack, an array of sources in its own right, is best used in a hybrid arrangement with batteries or capacitors to mitigate the impact of electrical transients. Meanwhile, room lighting constitutes a network of multiple electrostatic field sources that can be particularly useful for occupancy detection. Exploiting performance benefits of multi-source electrical networks requires an increased flexibility in the analysis required to make informed design choices. This thesis addresses the added complexity with linear analytical and modeling approaches that reveal the salient features of complicated multisource systems. Examples and prototypes are presented in capacitive sensing occupancy detectors, hybrid power systems and multi-panel solar arrays.

Traditional whole building energy modeling suffers from several factors, including the large number of inputs required for building characterization, simplifying assumptions, and the gap between the as-designed and as-built building. Residential modeling research regarding the monthly electrical consumption data is prepared by department of Electrical Engineering and Computer Science, University of Tennessee, TN, USA [23]. In this research is performed evaluation of seven different machine learning algorithms applied to a new residential data set that contains sensor measurements collected every 15 min, with the objective of determining which techniques are most successful for predicting next hour residential building consumption. In the research is performed validation of each learner's correctness on the ASHRAE Great Energy Prediction Shootout and confirmation of existing conclusions that Neural Network-based methods perform best on commercial buildings. Additional results show that these methods perform poorly on residential data, and that Least Squares Support Vector Machines perform best – a technique not previously applied to this domain.

A regression analysis of energy consumption in the banking sector in Spain are prepared by CIRCE red by prepared by energy consumption in the banking sector University of Zaragoza [25]. In this case study, the target area is the Spanish banking sector, for which are divided the available data into a prediction and a validation subset. Power models were developed using test data from 55 banks. From the analysis, three models were obtained; where the first proposed model can be used to predict the energy consumption of the whole banking sector, while the rest of the models estimate the energy consumption for branches with low winter climate severity (Model 2) and high winter climate severity (Model 3). Models 2 and 3 differ from the first model in that they need independent variables measured in situ. As a result, the uncertainty of the response variable in the function of the independent variables is reduced by 56.8% for the first model and by 65.2% and 68.5% for the second and third proposed models, respectively. The validation of the first model, which is the model with the lowest determination coefficient, shows that this model is appropriate for predicting the energy consumption of bank branches with good energy consumption performance and detecting inefficiencies in bank branches with poor energy consumption performance.

Recognizing habitual behavior and providing feedback in context are key to empower individuals to take control over residential electricity consumption is performed into the research prepared by Technical University of Lisbon and Faculty of Sciences and Technology, Coimbra University, Singapore [26]. This paper intends to discover whether habitual behavior can be identified by pattern recognition techniques. The data source is an experiment similar to a utility led advanced metering infrastructure implementation. The analysis discovers: (1) persistent daily outines and (2) patterns of consumption or baselines typical of specific weather and daily conditions. Approximately 80% of household electricity use can be explained within these two patterns, with several applicable "profiles" for this population, including: unoccupied baseline, hot working days, temperate working days, cold working days, and cold weekend days. The proposed methodology demonstrates that it is possible to use pattern recognition methodologies to recognize habitual electricity consumption behavior given the intrinsic characteristics of the family. This approach could be useful to improve small scale forecast, and as a mechanism to enable the provision of tailor-made information to the families.

A statistical machine learning framework to study the effect of eight input variables (relative compactness, surface area, wall area, roof area, overall height, orientation, glazing area, glazing area distribution) on two output variables, namely heating load (HL) and cooling load (CL), of residential buildings is developed by Oxford Centre for Industrial and Applied Mathematics (OCIAM), Mathematical Institute, University of Oxford, UK and Architectural Science Group, Welsh School of Architecture, Cardiff University, UK [27]. In the study is systematically investigated the association strength of each input variable with each of the output variables using a variety of classical and non-parametric statistical analysis tools, in order to identify the most strongly related input variables. Then, is compared a classical linear regression approach against a powerful state of the art nonlinear non-parametric method, random forests, to estimate HL and CL. Extensive simulations on 768 diverse residential buildings show that we can predict HL and CL with low mean absolute error deviations from the ground truth which is established using Ecotect (0.51 and 1.42, respectively). The results of this study support the feasibility of using machine learning tools to estimate building parameters as a convenient and accurate approach, as long as the requested query bears resemblance to the data actually used to train the mathematical model in the first place.

Magnet power supplies are critical components of a storage ring. The performance of power supplies directly impacts the stability and reliability of the storage ring operation. There are several type of DC power supplies in Duke FEL storage ring. The performance data of power supplies can be collected in a non-interruptive manner by an EPICS archive or by a MATLAB program regarding the research prepared by Duke University, Durham, USA [17]. MATLAB based tools have been developed to analyze the power supply data collected during the operation. Careful evaluation of data allows us to identify a power supply with degraded performance and provide a reference to perform preventive maintenance.

Energy performance analysis for a Photovoltaic/Diesel/Battery hybrid power supply system is described into the paper developed by Cape Peninsula University of Technology, Cape Town, South Africa [18]. The procedure starts by the identification of the hourly load requirements for a typical target consumer and the concept of load matrix was used to calculate an hourly load demand. Two competing energy dispatch strategies were considered. The first strategy is to switch the diesel generator only at night times or when the solar radiation is less than 80 W/m^2, herein referred to as the Night Strategy. The second one is a load following strategy where the diesel generator set is switched on if the load is greater than 80% of the average daily load and the photovoltaic (PV) hourly output is lower than the hourly load, herein called the load following strategy. A model that could simulate various operating conditions and produce corresponding energy flows was used. The model makes an hourly audit of energy flows and after inputting parameters like solar radiation data, temperature data, average day of the month, tilt angle, normalized PV area and storage capacity among other inputs, can output solar fraction; diesel fraction; load satisfied; and battery life among other outputs.

First statistical modeling technique for the power supply noise including inductive DI noise and power net IR voltage drop is prepared by University of California, Santa Barbara, USA [19]. The model is then integrated with a statistical timing analysis framework to estimate the performance degradation caused by the power supply noise. Experimental results of our analysis framework, validated by HSPICE, for benchmark circuits implemented on both 0.25 m, 2.5 V and 0.55 m, 3.3 V technologies are presented and discussed. The results show that on average, with the consideration of this noise effect, the circuit critical path delays increase by 33% and 18%, respectively for circuits implemented on these two technologies.

Models of battery, power electronic converter, and electric motor losses related to a typical 48 s track driving schedule are developed by Department of Energy and Environment, Goteborg, Sweden [20]. The important losses within a typical electric motor such as stator copper, rotor copper, and core losses were modeled and simulated over the entire speed range. A power electronic converter was modeled; including the switching and conduction losses for both MOSFETs and the anti parallel power diodes. The energy storage was modeled as a generic model capable of representing losses and the state of charge (SOC) of the battery over the driving cycles. The energy captured during regenerative braking was also considered in the simulation. Finally, the overall electric traction motor drive system efficiency was estimated based on the individual model based efficiency analysis in that research. The battery and induction motor parameters, which were used in the simulation, were calculated using the measurement data obtained through laboratory tests. The complete electric traction drive system was simulated and observed using the drive cycle of the ICE carting at the race day for 48 s (one lap). The total average efficiency of the electric drive system was 66.7%. The average power of the electric motor was 5.4 kW and the total energy consumed by this electric traction drive system was 920 W h for one whole race. The battery can supply the electric traction drive system for 22 min. The regenerative braking energy can be used to charge the battery and reduce the energy usage in the system, but has only a small effect due to the short time of the regenerative braking period.

A data-driven approach for minimization of the energy to air condition a typical office-type facility is presented in the research prepared by Department of Mechanical and Industrial Engineering, The University of Iowa, Iowa, USA [9]. Eight data-mining algorithms are applied to model the nonlinear relationship among energy consumption, control settings (supply air temperature and supply air static pressure), and a set of uncontrollable parameters. The multiple-linear perceptron (MLP) ensemble outperforms other models tested in this research, and therefore it is selected to model a chiller, a pump, a fan, and a reheat device. These four models are integrated into an energy optimization model with two decision variables, the set point of the supply air temperature and the static pressure in the air handling unit. The model is solved with a particle swarm optimization algorithm. The optimization results have demonstrated the total energy consumed by the heating, ventilation, and air-conditioning system is reduced by over 7%.

The demand for electricity and provides out-of-sample forecasting at the sectored level using a panel co integration approach is defined into the research prepared by The Egyptian Cabinet – IDSC [22]. The econometric model permits cross-sectional heterogeneity within a dynamic framework that incorporates information on relevant income and prices of domestic and foreign goods. Both the short-run dynamics and the long-run slope coefficients are allowed to vary across cross-sections. Also, the testing for unit roots and co integration in panels allow for heterogeneous fixed effects and deterministic trends. Using Egyptian data, it is shown that the empirical model produces reliable ex-post forecasts near the end of the full sample period. These pseudo forecasts represent what one would expect, if the forecasting relationship is stationary. It is then possible to conduct long-term ex-ante forecasting under plausible assumptions for policy purposes.

3. A few concepts for modeling of self-similar processes

3.1. Concept of self similar processes

Self-similar processes are types of stochastic processes that exhibit the phenomenon of self-similarity [28]. A self-similar phenomenon behaves the same when viewed at different degrees of magnification, or different scales on a dimension (space or time). Self-similar processes can sometimes be described using heavy-tailed distributions, also known as long-tailed distributions. Example of such processes includes traffic processes such as packet inter-arrival times and burst lengths. Self-similar processes can exhibit long-range dependency.

The design of robust and reliable networks and network services has become an increasingly challenging task in today's world. To achieve this goal, understanding the characteristics of traffic plays, performance models, bottlenecks into the networks, throughput, specific functions of the packets that are transmitted over the network, key points into the network is more and more critical role. Empirical studies of measured traffic traces have led to the wide recognition of self-similarity in network traffic.

3.1.1. Fractals in self similar processes

B. Mandelbrot introduced the term 'fractal' for geometrical objects: lines, surfaces and spatial bodies having a strongly irregular form [29]. These objects can possess the property of self similarity. The term 'fractal' comes from the Latin word fractals and can be translated as fractional or broken. The fractional object has an infinite length, which essentially singles it out on the traditional Euclidean geometry background. As the fractal has the self-similar property it is more or less uniformly arranged in a wide scale range; i.e. there is a characteristic similarity of the fractal when considered for different resolutions. In the ideal case self-similarity leads to the fractional object being invariant when the scale is changed. When self-similar traffic is mentioned, it will be assumed that its time realizations are fractals.

3.1.2. Definition of self similar process

A random process in discrete time or the time series $\{X(t), t \in R\}$, is considered where $X(t)$ is interpreted as the traffic volume (measured in packets, bytes or bits) up to moment t [30].

Definition 1. Consider that a real-valued process $\{X(t), t \in R\}$ has stationary increments if $\{X(t + \Delta t) - X(\Delta t), t \in R\} = \{X(t) - X(0), t \in R\}$ for all $\Delta t \in R$.

The sequence of increments for $\{X(t), t \in R\}$ at discrete time is defined as $Yk = X(k + 1) - X(k)$, $k \in Z$. For the purpose of traffic simulation the process $X(t)$ is considered as stationary in the wide sense, applying the restriction that the covariance function $R(t_1, t_2) = M[(X(t_1) - m)(X(t_2) - m)]$ is invariant with regard to the shift, i.e. $R(t_1, t_2) = R(t_1 + k, t_2 + k)$ for any t_1, t_2, $k \in Z$. It is assumed that two first moments $m = [X(t)]$, $\sigma^2 = [M(X(t) - m^2]$ exist and are finite for any $t \in Z$. Since at the stationary condition, the covariance function will be designated as $R(k)$ and the correlation factor as $r(k) = R(k)/R(0) = R(k)/\sigma^2$.

3.2. Model described by Fractional Brownian Motion

Let consider a simple FIFO queuing model, with an infinite buffer and constant service intensity r [29]. The total traffic arriving in the queue will be described with the function $A(t)$. The function $Q(t)$ describes the total traffic that is processed in the queue in time period t.

The buffer length of the queue in moment t is defined with the function $Q(t, r)$

$$Q(t, r) = \sup_{0 \leqslant s \leqslant t} [A(t) - A(s) - r(t - s)]$$

where $A(t) - A(s)$ is the traffic value that arrives for processing during the time interval $[s, t]$ and $r(t - s)$ is the traffic value which was processed during the same time interval.

If the process $A(t)$ is fractal process defined with Fractional Brownian Motion defined with the equation in log–log scale with the equation $\log(P[Q > L]) = f(L) = -\log[L^{2(1-H)} r^{2H} (1 - H)^{-2(1-H)}$- $> L]) = f(L) = -\log[L^{2(1-H)} r^{2H} (1 - H)^{-2(1-H)} H^{-2H}/2]$.

3.3. M/G/∞ Model Described by Pareto Distibution

The concept of M/G/∞ Model is used for modeling of approximately self-similar traffic [29]. In this research we will use the M/G/∞ Model defined with Pareto Distribution function

$$R(\tau) = \lambda \int_{\tau}^{\infty} (k/x)^{\alpha} dx = k^{\alpha} / (\alpha - 1)^{(1-\alpha)}$$

It is known that the $\{X_t\}_{t=0,1,2,\dots}$ process is asymptotically self-similar if $R(\tau) \sim \tau - DL(\tau)$, $\tau \to \infty$ for $0 < D < 1$, and L is the slowly varying function at infinity. Therefore, for $\kappa \geqslant 0$ and $1 < \alpha < 2$ the denumerable process for the M/G/1 Model with service times corresponding to the Pareto Distribution is asymptotically self-similar and, hence, long-range dependent.

The process is exactly self-similar if $R(\tau) = [(\tau + 1)^{2H} - 2\tau^{2H} + (\tau - 1)^{2H}]/2$ for $0.5 < H < 1$.

3.4. Model with 'Heavy Tail' Distribution

For advanced analyzes of network performance, usually can be used the Pareto Distribution as DHT [29]. It can be said that X has a distribution with a 'Heavy Tail' if

$$P[Z > x] \sim cx^{-\alpha} \text{ when } x \to \infty$$

where $0 < \alpha < 2$ is called the 'tail' index or form parameter and c is a positive constant.

The no exponential distributions of network performance parameters can be described with Pareto Distribution defined with the equation

$$W(x) = ab^a / x^{(a+1)}$$

where a is the shape parameter of Pareto Distribution and b is a scaling parameter.

3.5. Model with Intergroup Characteristics

The model of Intergroup Characteristic is analyzed as the fourth approach in this research.

The intergroup traffic character can be described by the first-order and second-order statistical characteristics of I frame processes [29]. Gamma Distribution is a good approximation of the I process:

$$F_{X_I}(r) = r^{m_I-1}e^{-r/l_I}/(\Gamma(m_I)l_I^{m_I}), \quad \forall r > 0$$

where m_I is the shape parameter and I is the scaling coefficient. They are related by the m value and the variance σ_I^2 of the I frame trace using the following relations:

$$m_I = \sigma_I^2/\mu_I^2 \quad \text{and} \quad l_I = \sigma_I^2/\mu_I$$

The I frame trace has self-similar properties and can be characterized by the SRD parameter l_I, the LRD parameter H_I and the 'boundary parameter' K_I. Thus the autocorrelation function calculated as has the form

$$R_{X_I X_I}(m) = \begin{bmatrix} e^{-\lambda_I m}, & m \leqslant K_I \\ Lm^{-\beta_I}, & m > K_I \end{bmatrix}$$

where $\beta_I = 2 - 2H_I$. The same procedure can be used to describe P and B frame distributions.

Can be shown that the ACF of I processes for analyzed sequences has two different characteristics: the self-similar character (long-range dependence), described by the Hurst exponent HI, and exponential decay similar to the function $e^{-\lambda_I x}$ over short time intervals.

Two regions are divided by coefficient K_I characterizing the boundary. For example, in the case of the cartoon the exponent $H_{Ij} = 0.873, \lambda_j = 0.891$ and coefficient $K_{Ij} = 30$ frames. The same character is typical for the correlation functions of B and P frames.

Taking into account that the Gamma Distribution is fully characterized by the mean and the variance, it is necessary to analyze the statistical characteristics of the processes reflecting the B and P frame size distributions for the given I frame sizes.

This model reflects two main statistical characteristics of the real video sequence: the quotient distribution with the Heavy Tail and the long-range part of the autocorrelation function.

The mathematical model of intergroup traffic described by the first-order and second-order statistical characteristics of I frame processes defined with Gamma Distribution is used for modeling of outage distribution parameter, stream loss parameters and delay factor parameter into the key points at the IPTV networks [15]. The simplicity of the covered model and the accuracy of the enclosed results confirm that the described research can be used as the base in further analyses, studies and researches in the field.

4. Self-similarity for modeling of power consumption

4.1. Measurement configuration

This research analyses electrical network consumption over the period January 1991–December 2006. The database includes information about the power consumption into the overall electrical network defined in hours per each day in a year for defined time period of 16 years.

The measurement unit for value of electrical network consumption is defined in MW for defined figures.

The resulting database included information about 140.160 power consumption figures. The range value per each figure comes from about 250 up to 1350 value metric.

4.2. Scenario practice

The following scenario was configured:

- *Step 1.* The parameter power consumption of electrical energy is measured per each hour during the time period of 16 years.
- *Step 2.* The measurement results were recorded into Documents.
- *Step 3.* The received data are prepared for further analyzes with mathematical analytical model.
- *Step 4.* The mathematical model with Fractional Brownian Motion with specific parameters is enclosed for mathematical modeling of real measurements.
- *Step 5.* The mathematical model with M/G/∞ Model Described by Pareto Distribution with specific parameters is used for modeling the real measurements.
- *Step 6.* The mathematical model with Model with 'Heavy Tail' Distribution with specific parameters is practiced for modeling the real measurements.
- *Step 7.* The mathematical model with Intergroup Characteristic with specific parameters is used for modeling the real measurements.
- *Step 8.* The values of the real measurements are related with results from the mathematical models.
- *Step 9.* Comparison analyzes are performed with calculation of the discrepancies between the results from the analytical model and results from the real measurements.
- *Step 10.* Confirmation of acceptable mathematical model regarding the figures from comparison analyses on daily, monthly and yearly level.
- *Step 11.* Preparation the figures of power consumption into the electrical network with mathematical calculation prediction for time period of few months with model of Gamma Distribution.

4.3. Method for calculating of the discrepancies

Into this research we use estimator arithmetic mean for calculation of the discrepancies. Suppose we have a random sample space $\{a_1, \ldots, a_n\}$. The estimator for calculating the discrepancies is arithmetic mean as the sample average is defined via equation

$$A := \frac{1}{n}\sum_{i=1}^{n} a_i$$

If the list is a statistical population, then the mean of that population is called a population mean [31].

4.4. Comparison analyses with assumed model – Fractional Brownian Motion

We assume that Fractional Brownian Function have values for parameters $H = 0.7$, $r = 5$ and L takes values from 100 up to 23000. The result mathematical modeling curve is compared with the curve that represents the real measurements. Into this research as example measurement is analyzed 1 day measurement of power consumption for Sixth of March 1991.

The predefined parameters for Fractional Brownian Function are scaled in order to have as much as possible closer results to real measurements.

Regarding the value of calculated discrepancy approximately is equal to 9.06%.

Because of the high value of calculated discrepancies and shape of the mathematical model curve, modeling of the power consumption into electrical network with Fractional Brownian Function is not acceptable. The Fractional Brownian Function cannot describe the real processes of power consumption into the electrical network.

4.5. Comparison analyses with assumed model – M/G/∞ Model Described by Pareto Distribution

Into this research is analyzed M/G/∞ Model Described by Pareto Distribution with parameter values $H = 0.8$ and values for τ parameter from 1 up to 24 with step 1 as predefined scaling parameters.

The values enclosed with the mathematical model are compared with values from real measurements.

Discrepancy calculation value 7.35% and mathematical model curve shape defines that M/G/∞ Model with Distribution Pareto Function cannot describe real power consumption values into the electrical network.

4.6. Comparison analyses with assumed model – 'Heavy Tail' Distribution

The third evaluated model into this research defined with self-similar 'Heavy Tail' Distribution takes parameters $a = 1.4$, $b = 5$ and values for x parameters starts from 70 up to 93 with step 1.

The values that derivate from mathematical model and real measurements for power consumption on the same date, Sixth of March 1991 defines discrepancy with value 23.58% and not adequate shape curve for analyzed mathematical model. So, the model with 'Heavy Tail' Distribution cannot be used for modeling of the power consumption into the electrical network.

4.7. Comparison analyses with assumed model – Intergroup Characteristic

In this research is assumed that the self-similar process, described with the Intergroup Characteristic with first order and second order statistical characteristic of I frame processes as mathematical model can explain the behavior of power consumption of electrical network.

The Gamma Distribution described with Intergroup Characteristic is defined with the equation:

$$R_{X_I X_I}(m) = \begin{cases} e^{-\lambda_I m}, & m \leqslant K_I \\ L m^{-\beta_I}, & m > K_I \end{cases}$$

where m_I is the shape parameter, I is the scaling coefficient, μ is mean and σ_I^2 is variance of the I frame trace, K is boundary parameter, H is LRD parameter, λ is SRD parameter. The relational are defined with the following relations:

$$m_I = \sigma_I^2/\mu_I^2,$$
$$l_I = \sigma_I^2/\mu_I$$
and $\beta_I = 2 - 2H_I$.

In Table 3 are defined an example estimation of Hurst exponent.

For the assumed model in this research it is defined that $H = 0.9$, $\lambda = 0.9$, $L = 100$ and $K = 100$.

The parameter β is calculated with formula $\beta = 2–2 * H$, and receive value $\beta = 0.20$.

Table 3
Hurst exponent estimate (example).

Shape parameter	Log–log correlation	Variance time	*R/S* statistics
$\alpha = 2.0$	$H = 0.592$	$H = 0.544$	$H = 0.672$
$\alpha = 1.7$	$H = 0.758$	$H = 0.794$	$H = 0.684$
$\alpha = 1.4$	$H = 0.7996$	$H = 0.8198$	$H = 0.811$
$\alpha = 1.1$	$H = 0.828$	$H = 0.745$	$H = 0.884$

The shape parameter and the scaling coefficient are related by the μ value and σ^2 of the I frame process receive values in range from 0.19 to 0.44, with step 0.05.

The values of the parameters we define by reheseal.

The comparison analyzes can be found with diagram on which can be found the percentage of discrepancies between the values for power consumption from mathematical model and values for power consumption from real measurements.

4.7.1. 1st class of measurements

The measurements are performed during time period of 1 day with measurement interval – 1 h. With performed measurements are received data values for power consumption of electrical energy per 1 day. The measurement result values are uploaded and matched with the calculated figures from closed analytical mathematical model for self similar process. The total discrepancies value is 3.35% (Fig. 3).

With this research is developed mathematical model with acceptable value of discrepancy factor and acceptable shape and flow of the curve.

Therefore, the mathematical model described with the Intergroup Characteristic, can describe in detail real processes into the electrical network, as power consumption into the electrical network.

4.7.2. 2nd class of measurements

The measurements are performed during time period of 1 month with measurement interval by 1 h per each day in month.

With performed measurements are recorded data for power consumption of electrical energy per 1 month. The measurement result values are uploaded and matched with the calculated data values from mathematical model for self similar process. The total discrepancies value is 0.29% (Fig. 4).

4.7.3. 3rd class of measurements

The measurements are performed during time period of 1 year with measurement intervals of 1 h per each month per each day in month and per each hour per day. With performed measurements are received data for power consumption of electrical energy per 1 year. The measurement result values are uploaded and matched with the calculated figures from closed analytical model for self similar process. The total discrepancies value is 0.07% (Fig. 5).

4.7.4. Summary for 1st, 2nd and 3rd class of measurements

The level of discrepancies calculated with estimator arithmetic mean for 1st, 2nd and 3rd class of measurements confirms that power consumption on daily, monthly and yearly basis into the electrical network can be described with Self similar process defined with Gamma Distribution with Intergroup Characteristic.

This mathematical model formula receives typical values per defined parameters that allow current description of the real process.

Plot of real measurements and plot of mathematical model defined in Fig. 3 show that the curve of the mathematical model goes a little bit faster from one point to another one than the curve of the real measurements. This faster change of the values and received level per each point define the level of discrepancy for calculated model.

With further changes of the predefined parameters into the used mathematical formula can be reached smaller or bigger level of calculated discrepancy.

The monthly level of discrepancy defined into this study is smaller than the daily level of discrepancy, because of the bigger number of calculated – modeled values per month than the num-

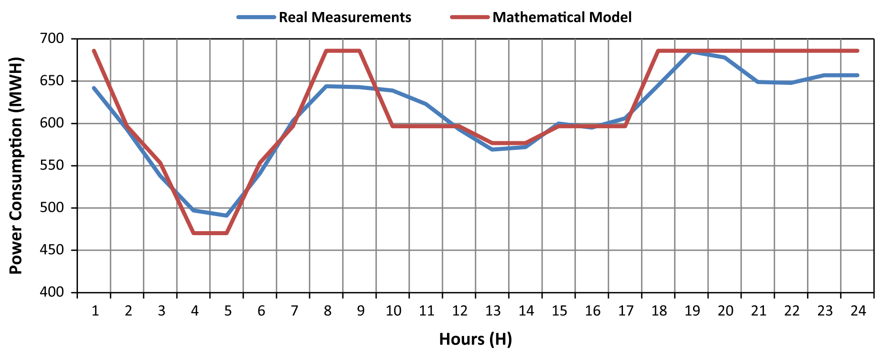

Fig. 3. Comparison analyzes for power consumption per 1 day (example measurement data for day: 6th March 1991).

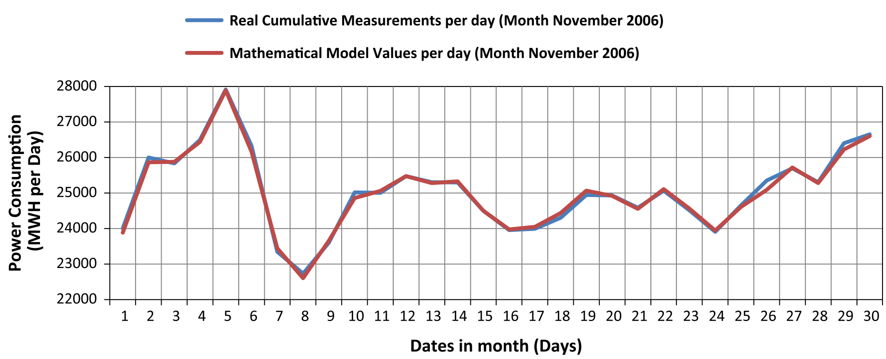

Fig. 4. Comparison analyzes for power consumption per 1 month (example measurement data for month: November 2006).

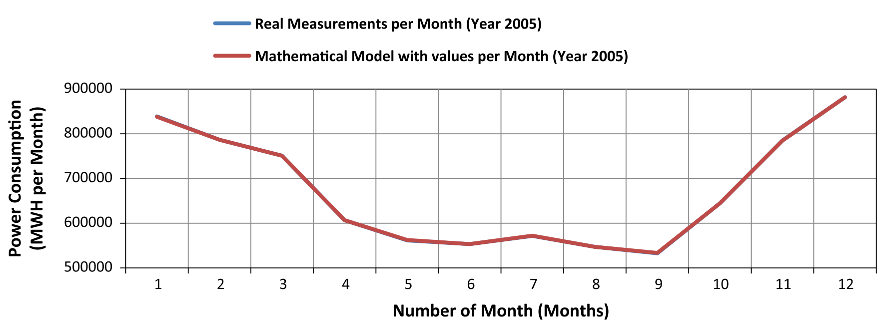

Fig. 5. Comparison analyzes for power consumption per 1 year (example measurement data for year: 2005).

ber of values per day. The same reason is for the bigger accuracy on yearly level than on the monthly level.

4.7.5. 4th class of measurements – using different K parameter

The measurements that are performed during time period of 1 day, for example day 6th of March 1991, with measurement interval by 1 h defined as 1st class of measurements in this document will be analyzed with different value for parameter K than that defined previously in this research. We take that K parameter receives value equal to 2. The measurement result values are uploaded and matched with the calculated figures from closed analytical from the same model, but only with different value for K. Into this configuration we enclose discrepancy equal to 6.13% (Fig. 6).

4.7.6. 5th class of measurements – using different K and H parameter

In the 5th class of measurements we perform comparison analyses of the measurements that are performed during time period of 1 day (6th of March 1991) defined as 1st class of measurements and the same model with different H and K parameter. Into this class of measurements we take that K parameter receives value equal to 1 and parameter H receives value equal to 0.6. The calculated delta measurement result values enclose discrepancy equal to 5.62% (Fig. 7).

4.7.7. 6th class of measurements – using different Lambda parameter

Into this modeling configuration we use all the predefined parameters from the model configuration that is defined within this paper, but we change the value of Lambda parameter. We use the same measurements per 1 day, the measurements that are performed during time period of 1 day on 6th of March 1991 with measurement interval by 1 h. Then we compare the results defined as 1st class of measurements in this document and the results from the configuration with different Lambda parameter. We take that Lambda parameter receives value equal to 0.5. Into this configuration we enclose discrepancy equal to 6.83% (Fig. 8).

4.7.8. Summary for 6th, 7th and 8th class of measurements

The curves defined with the 6th, 7th and 8th class of measurements shows which changes on the flow of the curve defined with

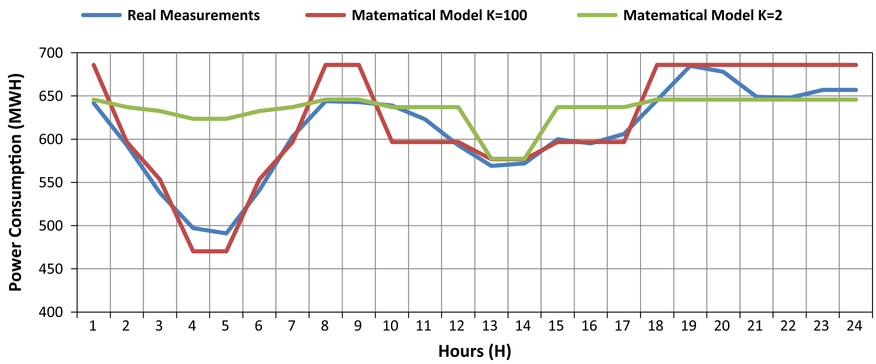

Fig. 6. Comparison analyzes for power consumption per 1 day with $K = 2$ (example measurement data for day: 6th March 1991).

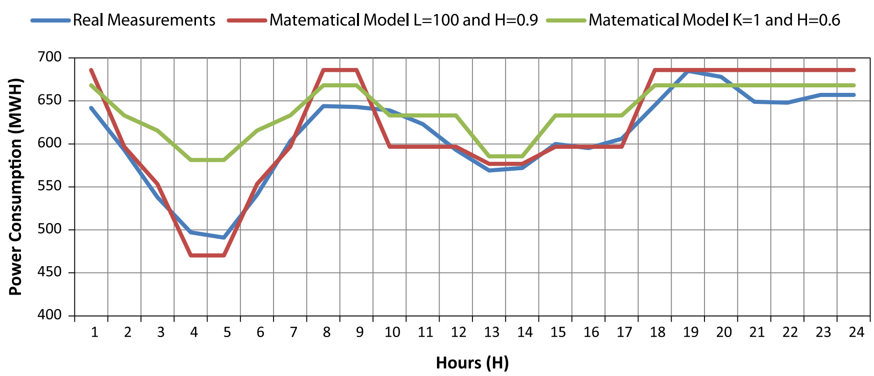

Fig. 7. Comparison analyzes for power consumption per 1 day with $K = 1$ and $H = 0.6$ (example measurement data for day: 6th March 1991).

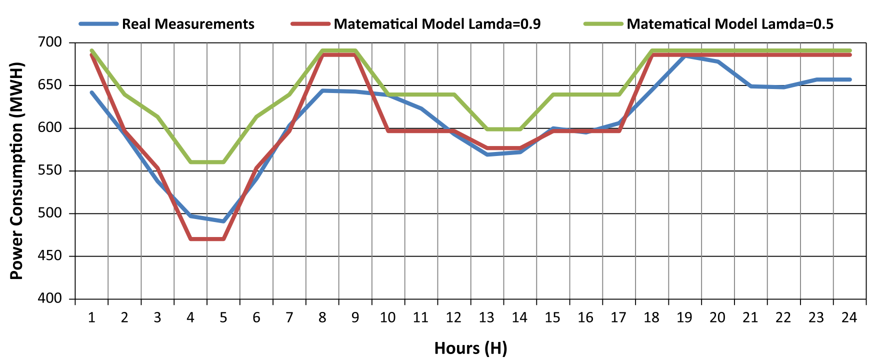

Fig. 8. Comparison analyzes for power consumption per 1 day with Lambda = 0.5 (example measurement data for day: 6th March 1991).

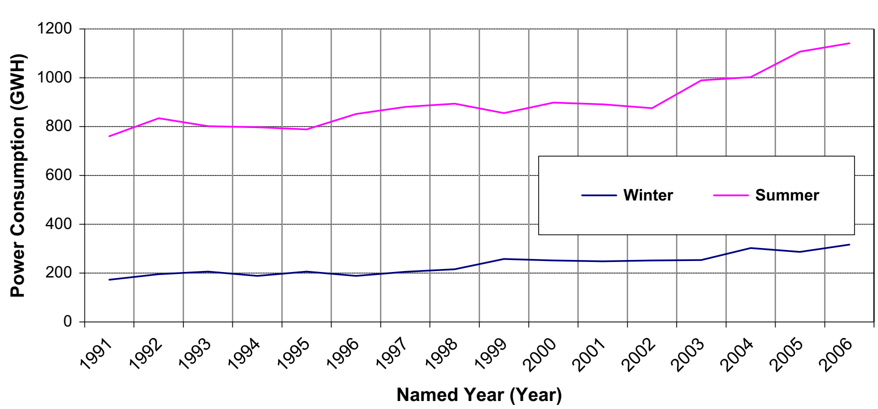

Fig. 9. Consumption of power per year for time period 1991–2006, [13,14].

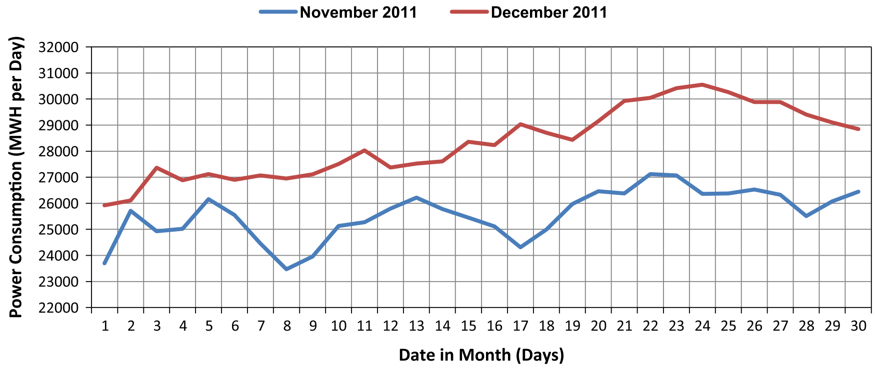

Fig. 10. Predictions for November and December 2011 with Gamma Distribution.

the mathematical model can be reached with changing the value of *K*, *H* or Lambda parameter.

The graph defined in Fig. 6 shows that if parameter *K* receives smaller value, for example, equal to 2 then the flow of the curve goes much slower then when this parameter receives value equal to 100. This slower changing of the curve of the mathematical model defines bigger level of discrepancy and therefore used mathematical model of self similar process defined with Gamma Distribution with Intergroup Characteristic cannot be used as mathematical model for description of power consumption into the electrical network.

Further, after changing of the value of parameter *K* we change the level of parameter *H* and we assume that $H = 0.6$ (in the enclosed model *H* is equal to 0.9). From the flow of the graph defined in Fig. 7 can be shown that with this changing of the parameter *H* is received faster flow of the curve of the mathematical model then the flow defined in Fig. 6. But this dynamic of changing of the flow of the curve is not enough to describe the real process – power consumption into the electrical network and therefore the received discrepancy is with higher level. So, this configuration for mathematical model cannot be used for modeling the power consumption into the electrical network.

Fig. 8 shows the changes that are received on the plot with changing of the parameter Lambda equal to 0.5. Again this change of the parameter defines slower changes of the flow of the graph and bigger discrepancy calculation.

The changes of these three parameters shows that the values that are predefined in the mathematical model for 1st, 2nd and 3rd class of measurements are stabile parameter values and they define the model for power consumption into the electrical network.

4.7.9. 7th class of measurements – prediction

The measurements are performed during time period 1991–2006. Those measurements are calculated and used for performing the predictions for time period November and December 2011. With performed analyses of power consumption per year in time period 1991–2006 can be found that the level of power consumption is bigger in last few years. Results from analyses are defined in Fig. 9.

For performing of the predictions for November and December 2011, are taken associate values for power consumption form the same months in 2006 and 2005, defined with Gamma Distribution curve that were analyzed previously.

In the prediction is defined *trend factor* for power consumption in the electrical network defined for the predefined time period in the years and the *weight factor* for power consumption in the electrical network that defines the weather condition prognoses into the predefined time period.

Performed analyses of value of average power consumption for November and December 2011 are defined in Fig. 10.

5. Conclusion

In this paper, we present the first characterized research for performed analyzes power consumption of electrical energy with closed empirical model based on fractal shapes represented with Fractional Brownian Distribution, $M/G/\infty$ Model Described by Pareto Distribution, 'Heavy Tail' Distribution and self similar processes with Intergroup Characteristic with Gamma Distribution.

The results obtained by using the proposed models are compared with the real power consumption measurements and achieved results confirm that covered model with Intergroup Characteristic with Gamma Distribution is model that can be used for performing analyses of power consumption of electrical networks.

Categories that included weather, equipment failure, human error, fires, and others are not directly analyzed into the research, but their impact is indirectly reflected. In addition, information about the total number of customers served by the affected utilities, as well as total population quantity and population density of the state affected in each incident, also was not analyzed into the research and they are indirectly calculated via analyses of electrical power consumption. Related with the simplicity of the covered model of Intergroup Characteristic with Gamma Distribution that is implemented for performing of mathematical analyzes of performance of power consumption of electrical network, on one side, and the accuracy in results achieved from the mathematical model compared with the real-time measurement result, the model is confirmed as the model with high level of quality that can be used in further analyses and researches.

The research can have impact on better automation of energy capacities, saving of energy cost into the electrical networks and planning of operation of production of electricity blocks in process of distribution of electrical energy to end consumers.

The covered model can be implemented into the all analyses for electrical power consumption of active blocks or subsystems into the electrical networks.

References

[1] Electric power distribution. <http://en.wikipedia.org/wiki/Electric_power_distribution.htm>.
[2] Understanding electricity. <http://inventors.about.com/library/inventors/blelectric1.htm>.
[3] Block Diagrams. <http://www.kpsec.freeuk.com/bdiags.htm>.
[4] Using the electrical wiring diagram. <http://www.autoshop101.com/forms/elec12.pdf>.
[5] Electricity. <http://en.wikipedia.org/wiki/Electricity>.
[6] Energy story. <http://www.energyquest.ca.gov/story/chapter02.html>.

[7] EWICS. Electric power systems cyber security: power substation case study. <http://www.nfpa.org/assets/files/PDF/Research/PowerConsumption.pdf.2006>.

[8] Schneider electric. Electrical power supply. <http://dk.schneider-electric.nu/nu_dk/Skole_CD/Automation%20Solution%20Guide/asg-2-electrical-power-supply.pdf>.

[9] Kusiak A, Li M, Tang F. Modeling and optimization of HVAC energy consumption. <http://www.icaen.uiowa.edu/~ankusiak/Journal-papers/HVAC_1_2010.pdf.Elsevier Ltd. May 2010>.

[10] Ripley. Power consumption. <http://www.RipleyLC.com/Tech_Power_Consumption.pdf>.

[11] ERC. Quality of electricity supply national report of FYR of Macedonia. <http://www.ecrb.eu/portal/page/portal/ECRB_HOME/ECRB_DOCUMENTS/ECRB%20WORKSHOP%20MATERIAL/EnC%20-%20QoSR%20Study%20-%20National%20Report%20-%20FYR%20of%20Macedonia.pdf>.

[12] IndexMundi. Population density (number of people per square kilometer). <http://www.indexmundi.com/g/g.aspx?v=21000&c=mk&l=en>.

[13] MANU. Strategy for development of energetic network in Republic of Macedonia in period 2008–2020 with vision until 2030. <http://www.gec.mk/EE%20vo%20Makedonija/Strategija%20MANU.pdf>.

[14] ELEM. Year report. <http://www.elem.com.mk/index.php?option=com_content&view=article&id=198&Itemid=166&lang=mk>.

[15] Tilovska S, Tentov A. Performance model of key points at the IPTV networks. <http://www.cscjournals.org/csc/manuscriptinfo.php?ManuscriptCode=71.72.65.76.43.47.50.48.100&JCode= IJCN&EJCode=69.70.63.74.102&Volume=43.106&Issue=44.104. June 2011>.

[16] Zhang T. RTOS performance and energy consumption analyses based on an embedded system tested. <http://www.ece.umd.edu/~blj/papers/thesis-MS-zhang–embedded.pdf.2001>.

[17] Li J, Wu YK, Hartman S. Power supply performance monitoring and analyses using operation data. <http://epaper.kek.jp/p03/PAPERS/WPAG005.PDF.2003>.

[18] Tazvinga H, Fore S. An Energy performance analyses for a photovoltaic/diesel/battery hybrid power supply system. <http://researchspace.csir.co.za/dspace/handle/10204/4153.March2010>.

[19] Jiang YM, Cheng KT. Analyses of performance impact caused by power supply noise in deep submission devices. <http://ieeexplore.ieee.org/xpl/freeabs_all.jsp?arnumber=782118&abstractAccess=no&userType=inst. August2002>.

[20] Istardi D. Modeling and energy consumption determination of an electric go-kart. <http://webfiles.portal.chalmers.se/et/MSc/DidiIstardi.MSc.pdf.2009>.

[21] A brief description of the electricity consumption time series. <http://www.math.univ-toulouse.fr/staph/npfda/npfda-electricity-des.pdf>.

[22] Alaa El-Shazly, IDSC. Electricity demand analysis and forecasting: a panel cointegration approach. <http://www.idsc.gov.eg/Publications/PublicationDetails.aspx?typeid=10&id=216.September2010>.

[23] Edwards ER, New J, Parker EL. Predicting future hourly residential electrical consumption: a machine learning case study. <http://dx.doi.org/10.1016/j.enbuild.2012.03.010.March2012>.

[24] Cooley, Jacob J. Analysis, modeling and design of energy management and multisource power systems. <http://dspace.mit.edu/handle/1721.1/65966.2011>.

[25] Aranda A, Ferreira G, Mainar-Toledo MD, Scarpellini S, Sastresa EL. Multiple regression models to predict the annual energy consumption in the Spanish banking sector. <http://www.sciencedirect.com/science/article/pii/S0378778812001326.March 2012>.

[26] Abreu MJ, Pereira FC, Ferrao P. Using pattern recognition to identify habitual behavior in residential electricity consumption. <http://www.sciencedirect.com/science/article/pii/S0378778812001363.March 2012>.

[27] Tsanas A, Xifara A. Accurate quantitative estimation of energy performance of residential buildings using statistical machine learning tools. <http://www.sciencedirect.com/science/article/pii/S037877881200151X.March2012>.

[28] Wikipedia. Self similar process. <http://www.enon.wikipedia.org/wiki/Self-similar_process.January2010>.

[29] Sheluhin OI, Smolskiy SM, Osin AV. Self-similar processes in telecommunications, 1st ed.; 2007. <http://www.wiley.com>.

[30] Simonoff J, Zimmerman J, Restrepo C, Dooscin N, HartWell N, Miller J, et al. Statistical analyzes of electric power outages. <http://icisnyu.org/assets/documents/NYUYr1ElecCase3Stats072905Final.pdf.July2005>.

[31] Wikipedia. Mean squared error. <http://en.wikipedia.org/wiki/Mean_squared_error>.