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A survey on fully homomorphic encryption with statistical applications

Rener Oliveira
Rodrigo Targino

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Author/Advisor

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- Applied Math BSc, EMAp/FGV (2022)
- Quantitative Research Analyst, BOCOM BBM Bank (2021-)
- OpenFHE Developer (2023-)

Rodrigo Targino

- PhD in Statistics, UCL (2017)
- Assistant Professor EMAp/FGV (2017-)
- Bayesian Statistics
- Financial and Actuarial Risk Management

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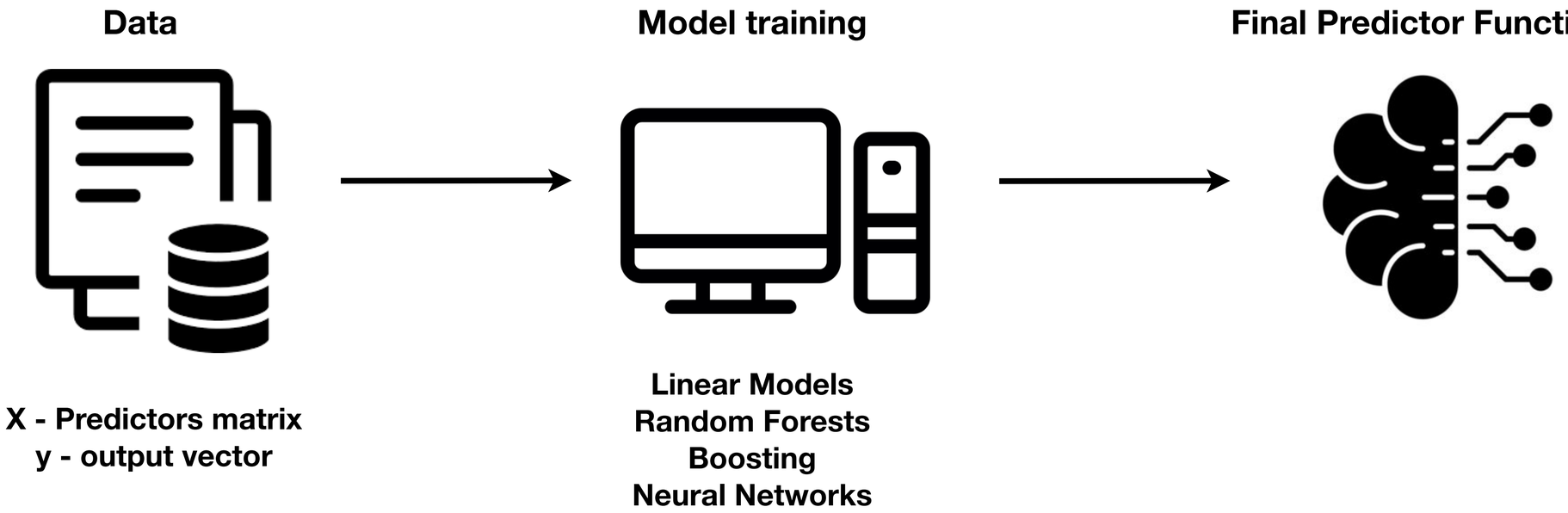
Introduction

Supervised machine learning setup:



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Introduction

Supervised machine learning setup:

Data

X - Predictors matrix
y - output vector

What if Data is sensitive?

- Medical records
- Financial data
- Genetic data

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Supervised machine learning setup:

Encrypted Data

%$*#!7E% - Encrypted predictors
$&h%$8 - Encrypted target vector

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Just encrypt it ;-)

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Maybe :)

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Privacy Homomorphisms, 1978

ON DATA BANKS AND PRIVACY HOMOMORPHISMS

Ronald L. Rivest
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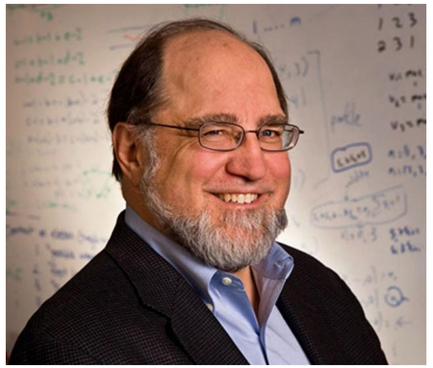

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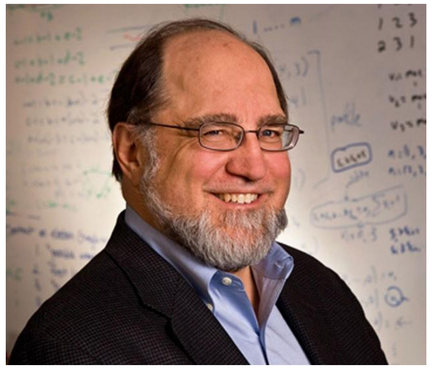

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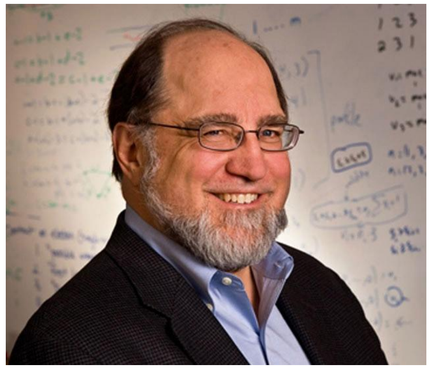

limitations on what can be accomplished, we shall see that it appears likely that there exist encryption functions which permit encrypted data to be operated on without preliminary decryption of the operands, for many sets of interesting operations. These special encryption functions we call "privacy homomorphisms"; they form an interesting subset of arbitrary encryption schemes

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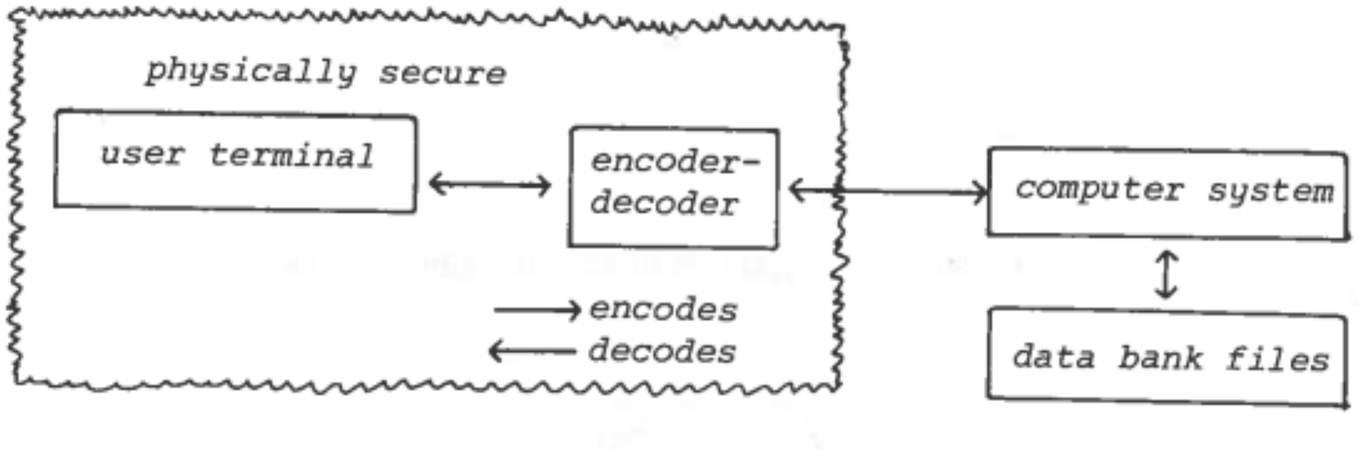

Figure 1

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Fully Homomorphic Encryption

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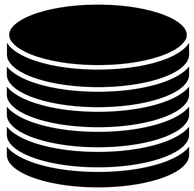
Fully Homomorphic Encryption

How to <u>securely</u> compute f(m1,m2,...) ?

Message space:
(m1, m2, ...)



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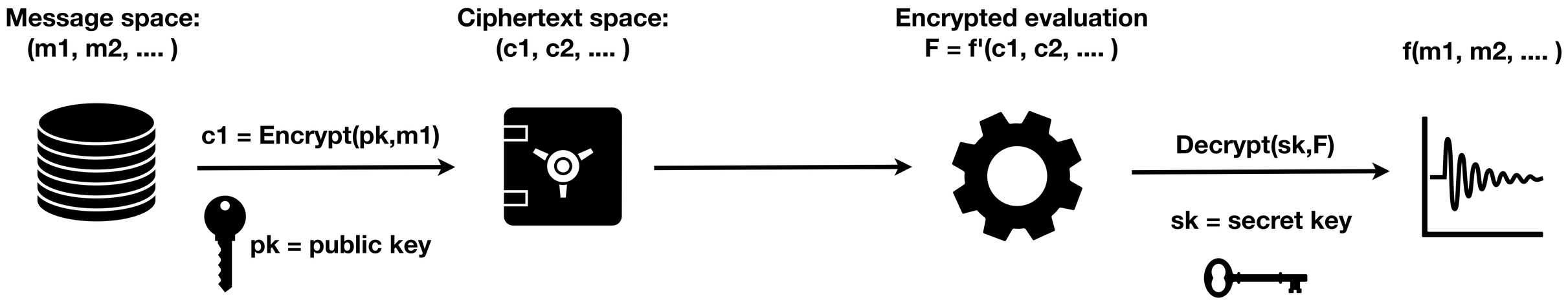

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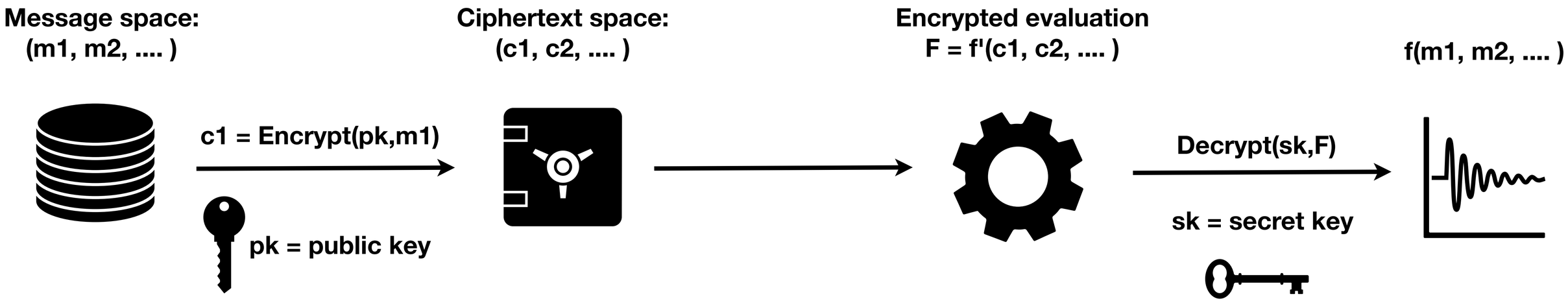

**An encryption scheme is called Fully Homomorphic,
if it can handle <u>any desired function f !</u>**

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Fully Homomorphic Encryption

Craig Gentry built the first FHE sheme, on his Ph.D. Thesis in 2009

Message space: {0,1}
Permitted functions:
additions and multiplications

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Semantically secure encryption

- Encrypt the same message 2+ times must generate different ciphertexts
- Encryption must be random

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Encrypt(pk,m) = c

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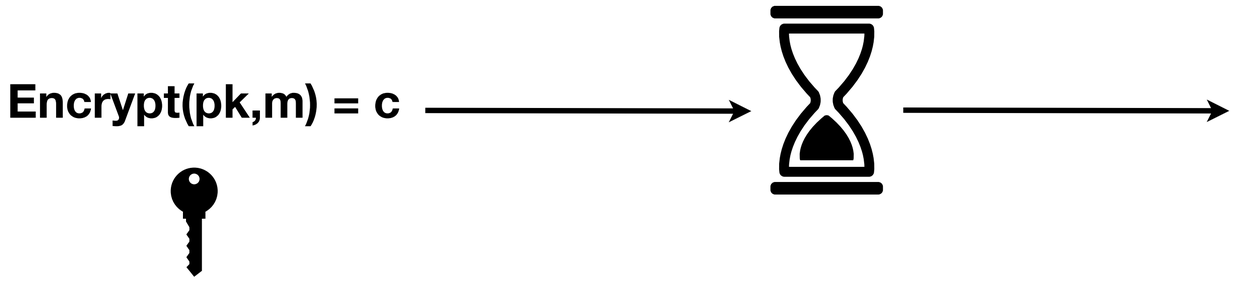

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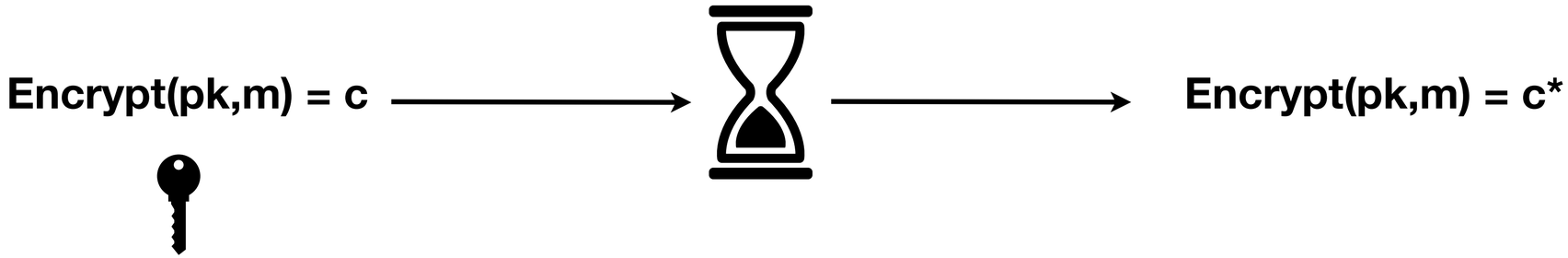

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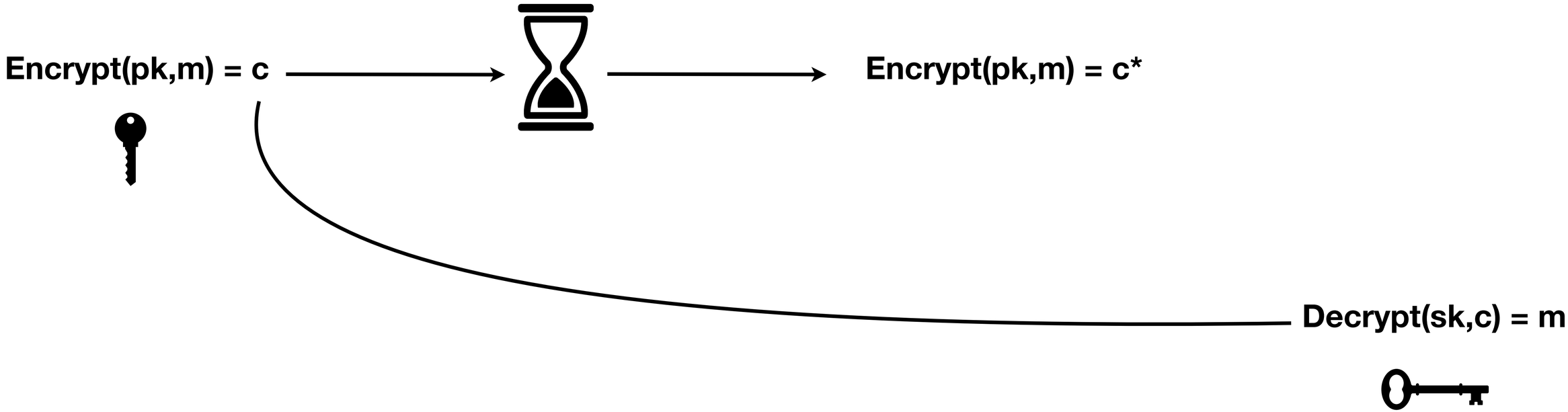

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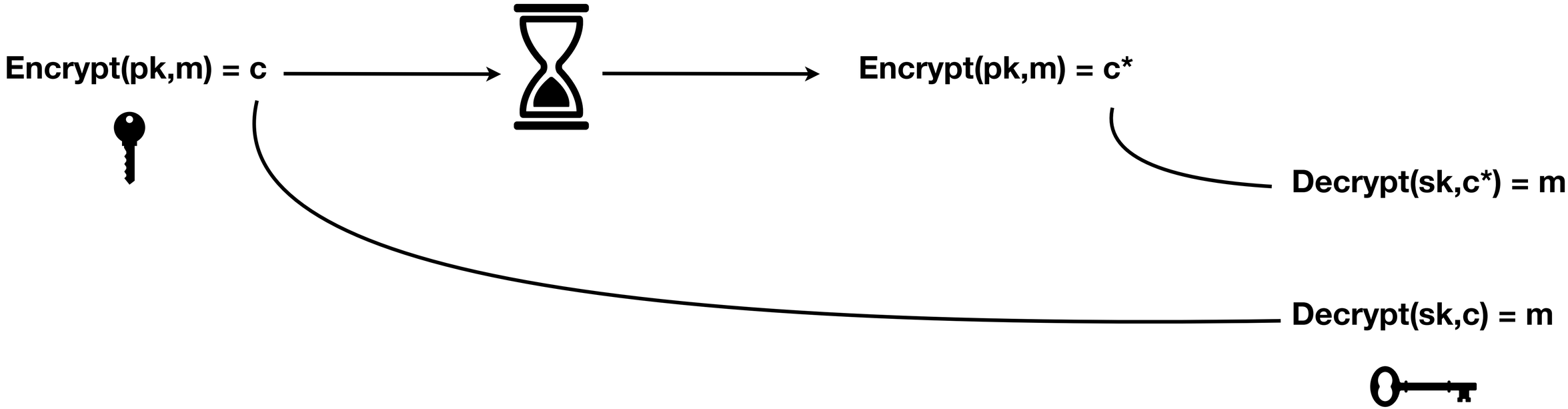

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Somewhat Homomorphic Encryption (SHE)

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A SHE scheme can homomorphically evaluate functions f with a limited depth

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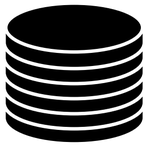

Message space:
{0,1}

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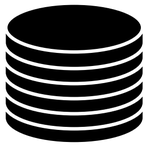

Message space:
{0,1}

Key:
p, odd integer

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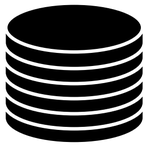

Message space:
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Key:
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Ciphertext space:
integers

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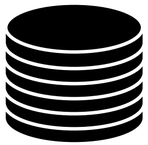

Message space:
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Encryption function:
c = pq + 2r + m

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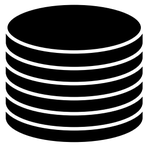

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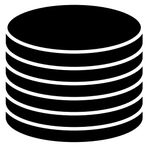

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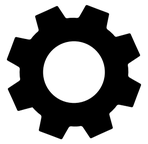

Decryption function:
(c mod p) mod 2

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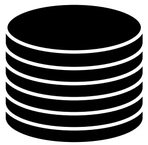

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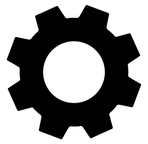

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If $2r < p$, the decryption is correct:

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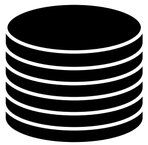

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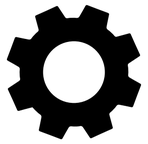

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If 2r < p, the decryption is correct:

$$
\begin{aligned}
&(c \bmod p) \bmod 2 \\
&= ((pq + 2r + m) \bmod p) \bmod 2 \\
&= ((2r + m) \bmod p) \bmod 2 \\
&= (2r + m) \bmod 2 \\
&= m
\end{aligned}
$$

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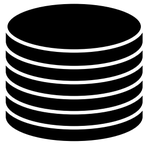

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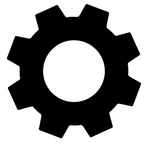

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$$

If we add/multiply two ciphertexts c1 and c2, the noise will be added/multiplied too

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Bootstrapping

Let's suppose we have a ciphertext c
of a message m, and want to reduce its noise

c

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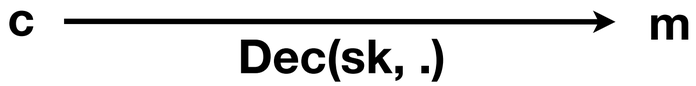

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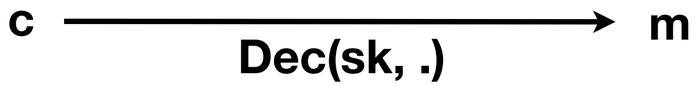
The Decryption function completely removes any noise !!
But reveals the message and uses the private key :(



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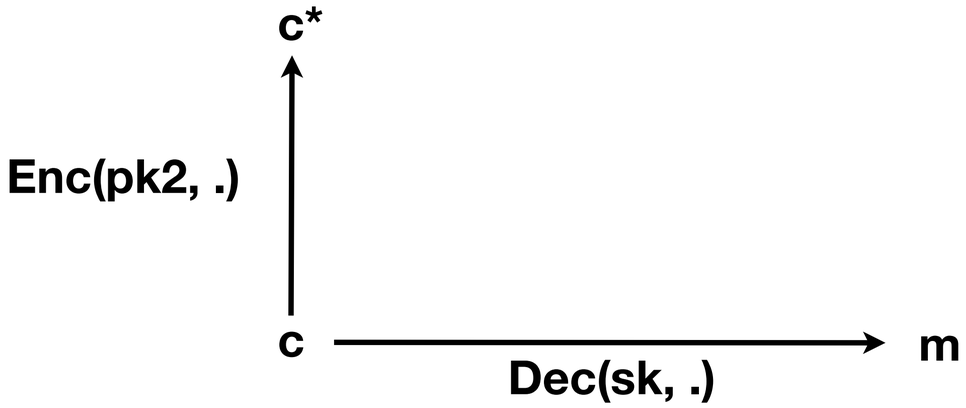

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c* is a double encryption of m

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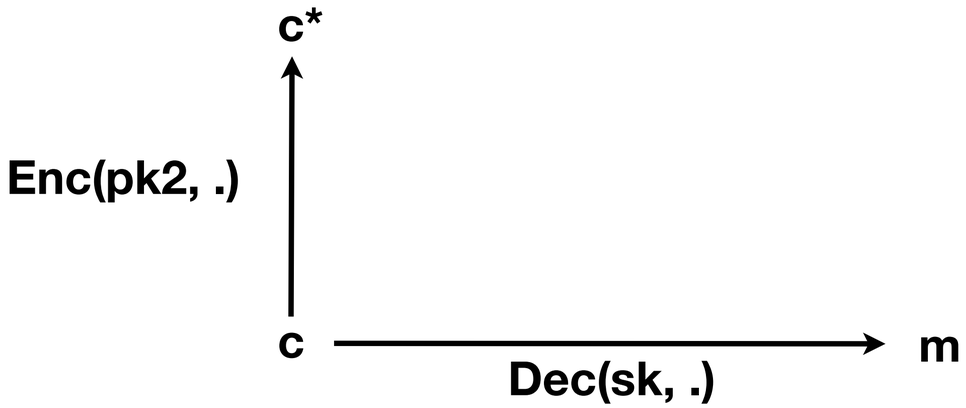

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We can homomorphically apply permitted functions f to c*

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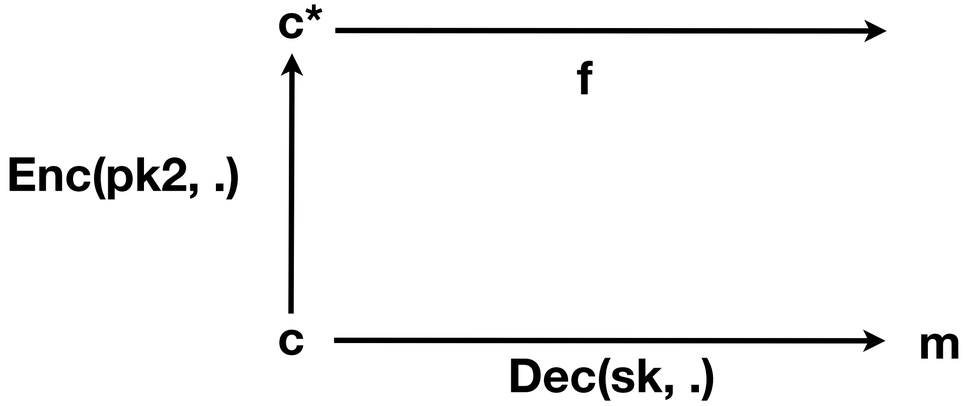

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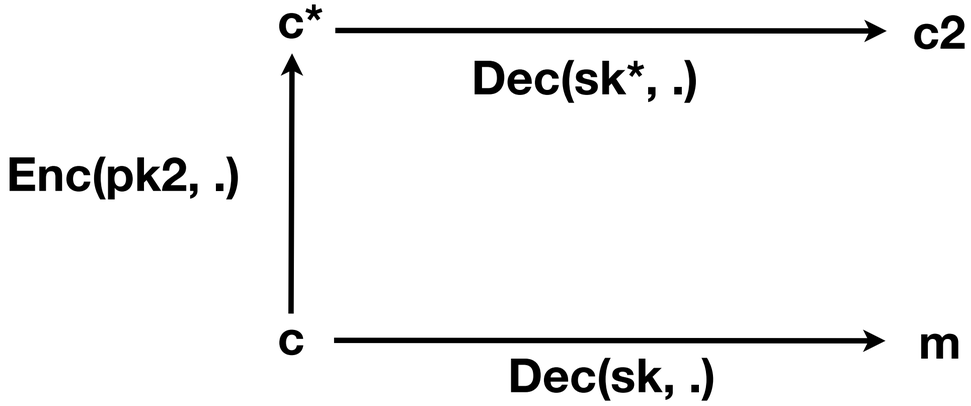

pk2 = second public key
sk* = Enc(pk2,sk)

The Decryption function completely removes any noise !!
But reveals the message and uses the private key :(

c* is a double encryption of m

We can homomorphically apply permitted functions f to c*

So, apply f(.) = Dec(sk*, .)

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Bootstrapping

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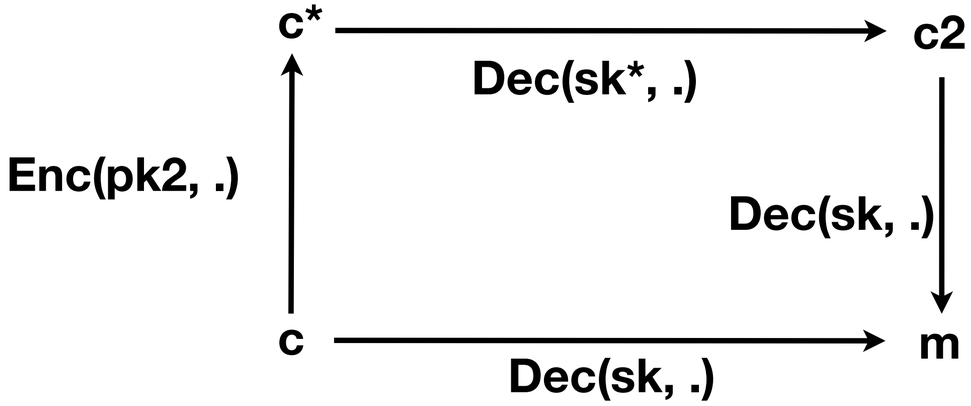

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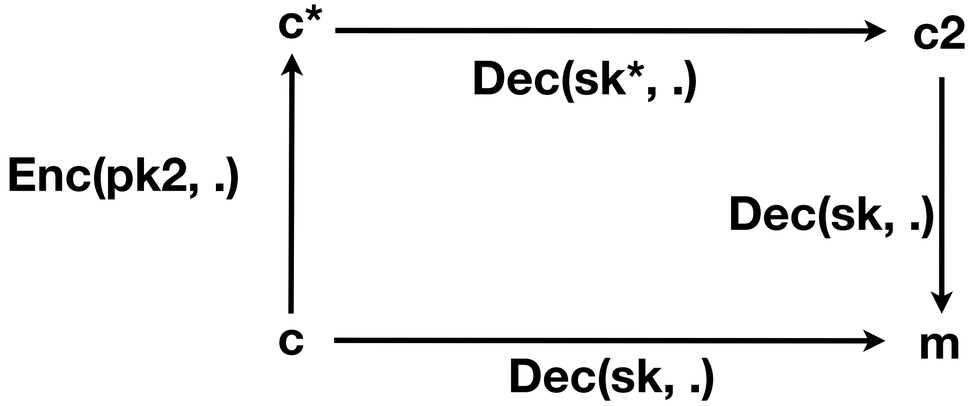

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A SHE scheme is called bootstrappable,
if the Decryption function is a permitted function

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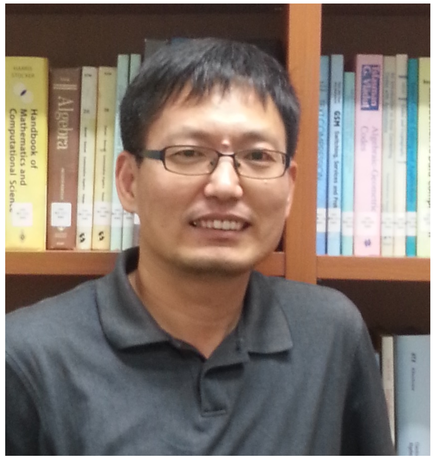
CKKS scheme

Cheon, Kim, Kim and Song (CKKS, 2017) - Homomorphic Encryption for arithmetic of approximate numbers



Jun Hee Cheon
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Hanyang University

Yongsoo Song
Seoul National University

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CKKS scheme

Plaintext Space: Polynomials

$$\mathcal{R}_q = \mathbb{Z}_q[X]/(X^N + 1)$$

Ciphertext Space: Polynomials tuples

$$\mathcal{R}_q \times \mathcal{R}_q.$$

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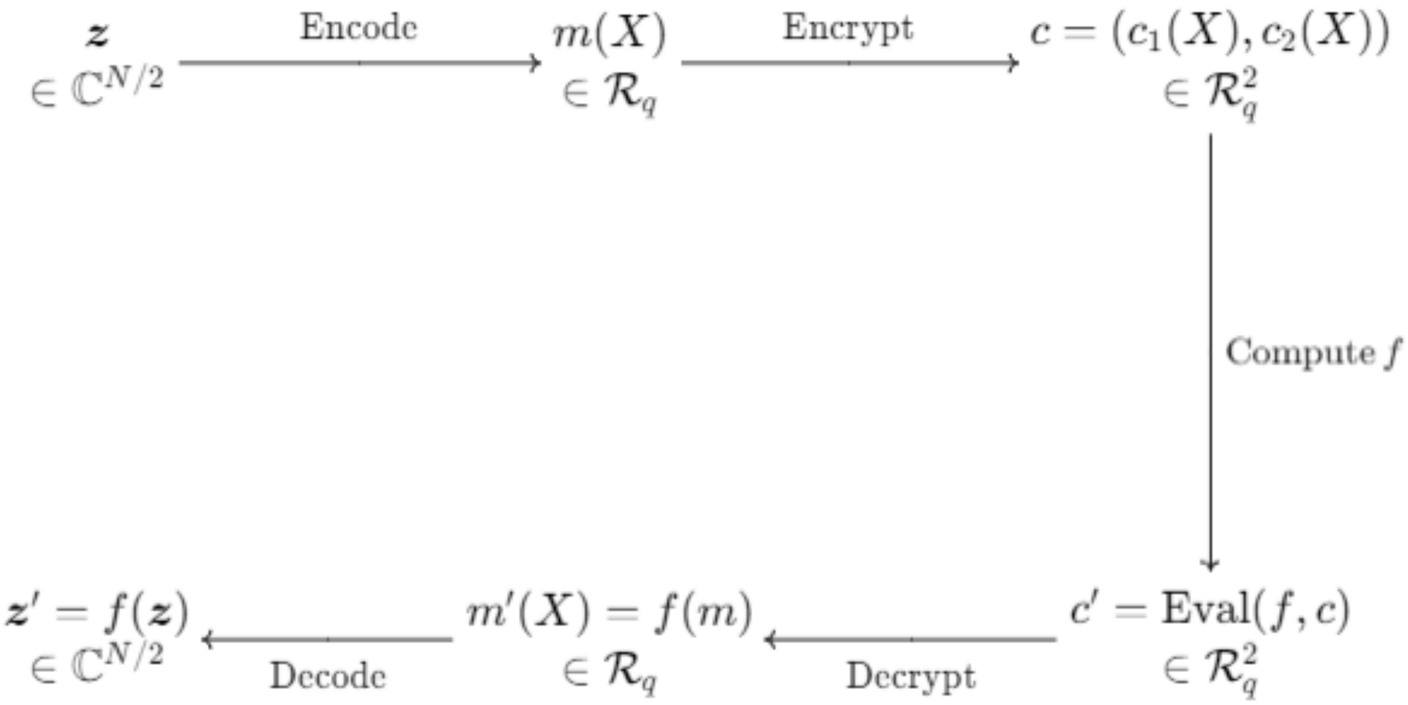
$$\mathcal{R}_q \times \mathcal{R}_q.$$

Complex Numbers are supported through encoding.



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CKKS scheme

Plaintext Space: Polynomials

$$\mathcal{R}_q = \mathbb{Z}_q[X]/(X^N + 1)$$

Ciphertext Space: Polynomials tuples

$$\mathcal{R}_q \times \mathcal{R}_q.$$

Secret key: (1,s)
s - random polynomial from Rq

Public Key:

$$\mathrm{pk} = (b, a), \text{ with } b = [-a \cdot s + e]_{q_L}$$

$$a \xleftarrow{\mathcal{U}} \mathcal{R}_{q_L} \text{ and } e \xleftarrow{\mathcal{DG}(\sigma^2)} \mathcal{R}.$$

Encryption:

$$\mathrm{Enc}(\mathrm{pk}, m) = [v \cdot \mathrm{pk} + (m + e_0, e_1)]_{q_L}$$

Decryption:

$$\mathrm{Dec}(\mathrm{sk}, \boldsymbol{c}) = [\langle \boldsymbol{c}, \mathrm{sk} \rangle]_{q_\ell} = [c_1 + c_2 \cdot s]_{q_\ell}, \text{ for } \boldsymbol{c} = (c_1, c_2)$$

Allowed operations:
Additions, Multiplications, Rotations

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Security - Learning with Errors (LWE)

$$\boldsymbol{a}_i \xleftarrow{u} \mathbb{Z}_q^n$$

$$b_i = \langle \boldsymbol{a}_i, \boldsymbol{s} \rangle + e_i \pmod{q}$$

$$e_i \xleftarrow{\chi} \mathbb{Z}_q$$

LWE Problem: Guess s, given (ai,bi)

If ei = 0, we could easily find s, solving As = b

With ei > 0, this is a NP-hard problem

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Ring Learning with Errors (RLWE)

$$a_i \xleftarrow{\mathcal{U}} \mathcal{R}_q$$

$$b_i = a_i \cdot s + e_i \pmod{q}$$

$$e_i \xleftarrow{\chi} \mathcal{R}$$

RLWE Problem: Guess s, given (ai,bi)

CKKS public key security is based on RLWE:

$$\mathrm{pk} = (b, a), \text{ with } b = [-a \cdot s + e]_{q_L}$$

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Logistic Regression

$$\boldsymbol{X} \in \mathbb{R}^{n \times d}$$

$$\boldsymbol{y} \in \{0, 1\}^n$$

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Logistic Regression

$$\boldsymbol{X} \in \mathbb{R}^{n \times d}$$

$$\boldsymbol{y} \in \{0, 1\}^n$$

$$\ln\left(\frac{p(\boldsymbol{x})}{1 - p(\boldsymbol{x})}\right) = \boldsymbol{w}^T \boldsymbol{x}'$$

$$\boldsymbol{w} = (w_0, w_1, \ldots, w_d)^T \in \mathbb{R}^{d+1}$$

$$\boldsymbol{x}' = (1; \boldsymbol{x})$$

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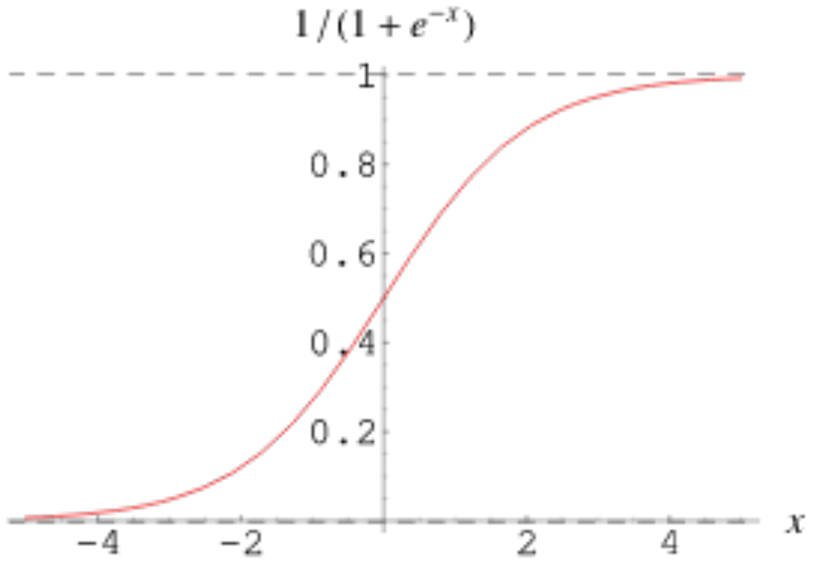
$$\boldsymbol{x}' = (1; \boldsymbol{x})$$

$$p(\boldsymbol{x}) = \sigma(\boldsymbol{w}^T \boldsymbol{x}')$$

$$\sigma(x) = 1/(1 + e^{-x})$$



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Logistic Regression - Training

Training algorithm: Find w that maximizes the likelihood ...

$$\mathcal{L}(\boldsymbol{w}) = \prod_{i=1}^{n} \sigma(\boldsymbol{w}^T \boldsymbol{x}_i')^{y_i} (1 - \sigma(\boldsymbol{w}^T \boldsymbol{x}_i'))^{1-y_i}$$

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... or minimizes the negative log-likelihood

$$\ell(\boldsymbol{w}) = -\frac{1}{n} \ln(\mathcal{L}(\boldsymbol{w}))$$

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$$\ell(\boldsymbol{w}) = -\frac{1}{n} \ln(\mathcal{L}(\boldsymbol{w}))$$

Gradient Descent Method:

$$\boldsymbol{w}_{i+1} = \boldsymbol{w}_i - \alpha \nabla \ell(\boldsymbol{w}_i),$$

$$\text{where } \nabla \ell(\boldsymbol{w}_i) = -\frac{1}{n} \sum_{j=1}^{n} \sigma(-\boldsymbol{w}_i^T \boldsymbol{z}_j) \boldsymbol{z}_j$$

Two inner products involved!
exponential function in sigmoid!

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Logistic Regression - Private Training

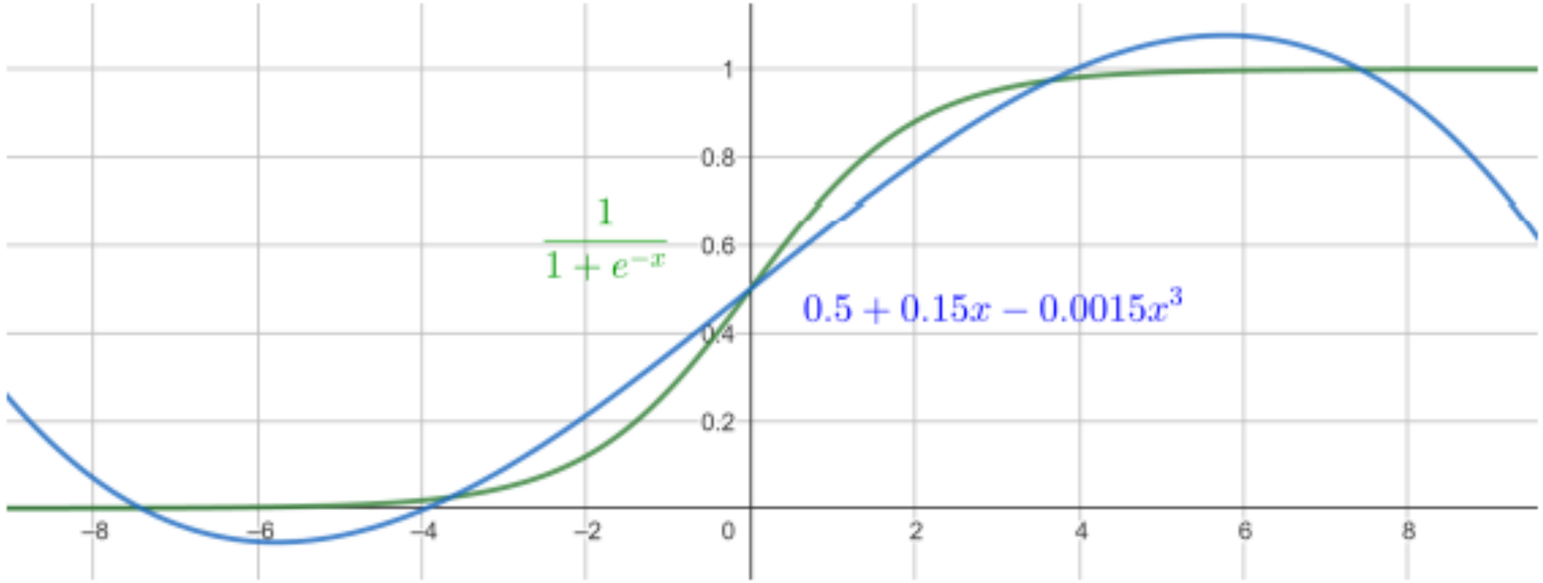

Figure 6 – Sigmoid polynomial approximation

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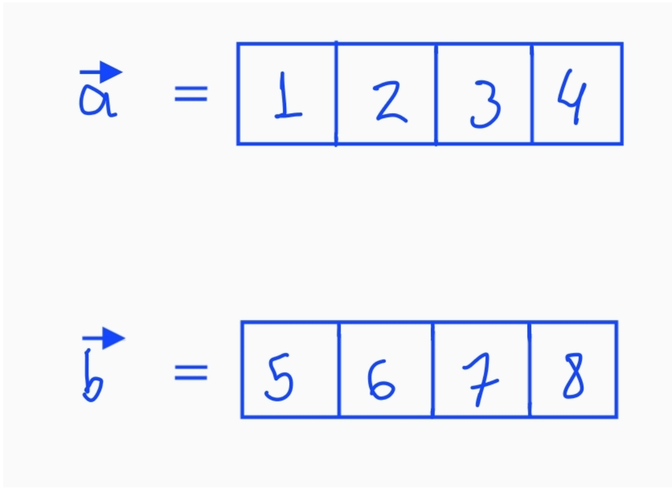
Logistic Regression - Private Training

Calculating Inner products
Naive approach: encrypt each entry as a ciphertext



a1.b1 + a2.b2 + a3.c3 + a4.c4 = 70

4 multiplications, 3 additions required

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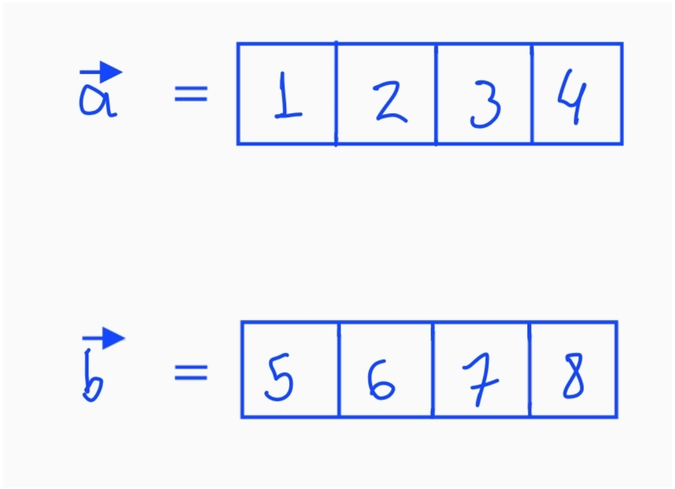
Logistic Regression - Private Training

Calculating Inner products
SIMD approach: encrypt each vector as a ciphertext



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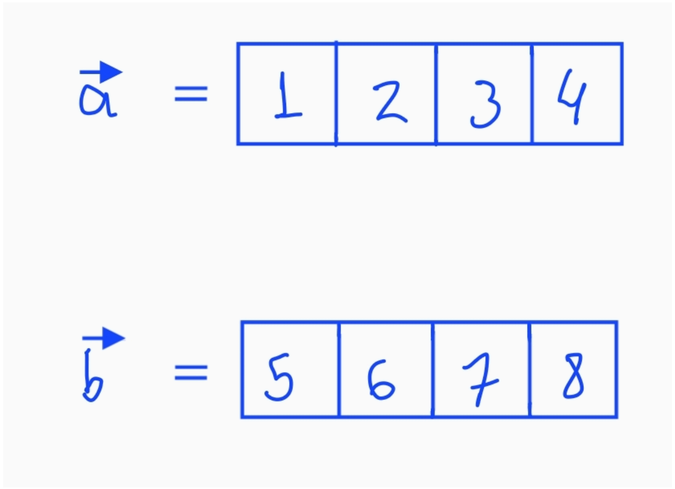
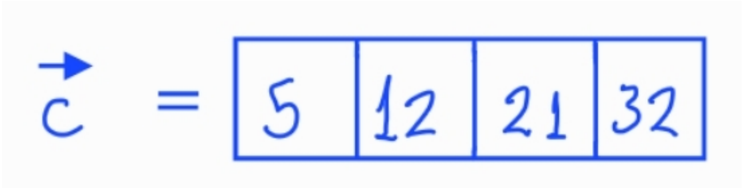
Logistic Regression - Private Training

1st Step: Do 1 SIMD multiplication



c = a.b



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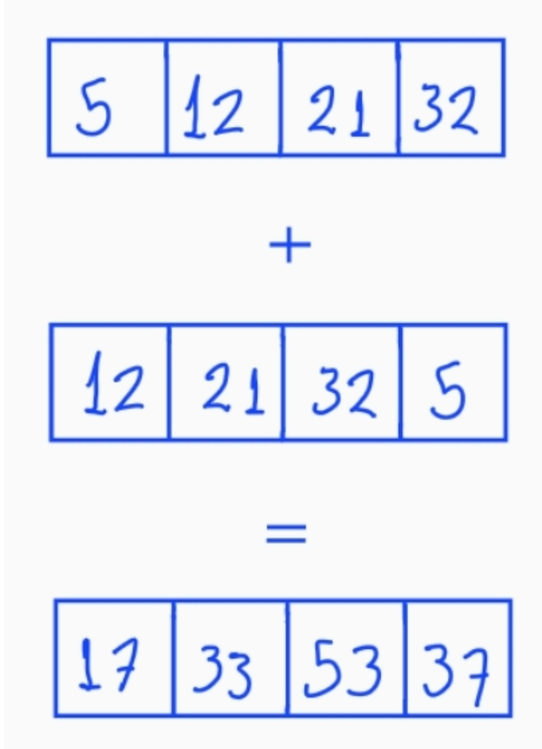
Logistic Regression - Private Training

2nd Step: c = c + left_rotate(c,1)



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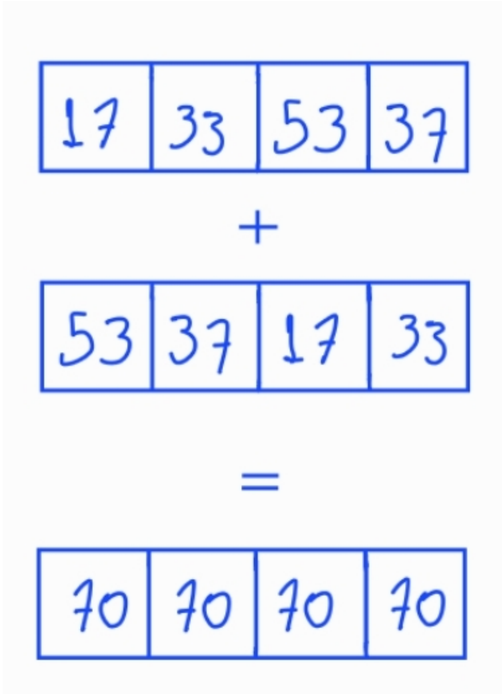
Logistic Regression - Private Training

3nd Step: c = c + left_rotate(c,2)



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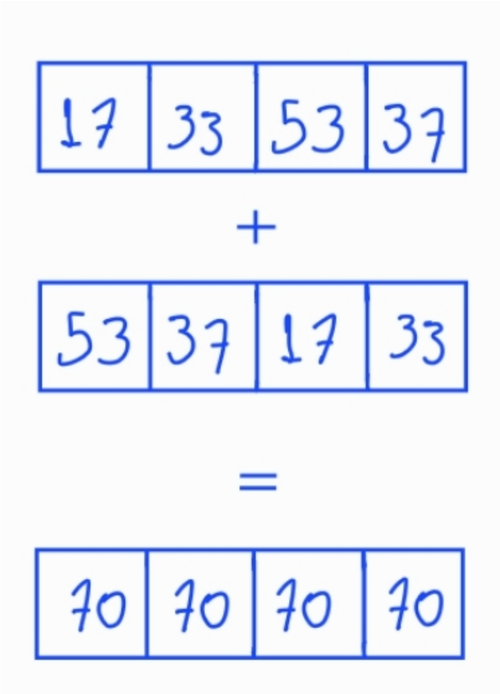
Logistic Regression - Private Training

3nd Step: c = c + left_rotate(c,2)



Total cost: 1 multiplications + 2 additions

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Logistic Regression - Private Training

In general a d-dimensional inner product requires:

- Naive approach: d multiplications, d-1 additions
- SIMD approach: 1 multiplication, log(d) additions

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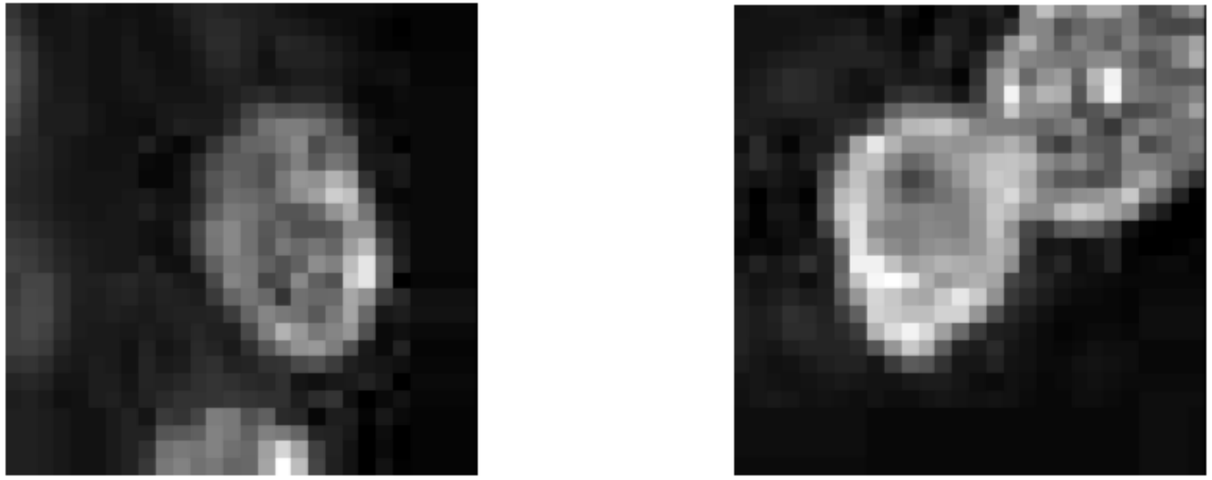
Simulation Results

- HEAAN + HELR C++ packages;
- Dataset: Subset of the TissueMNIST dataset; 92672 rows and 196 columns;
- Machine: 64-bit quad-core Intel Core i5-6200U 2.3GHz CPU, 16GB of RAM;



(a) Collecting Duct (b) Proximal Tubule Segments

Figure 7 – TissueMNIST

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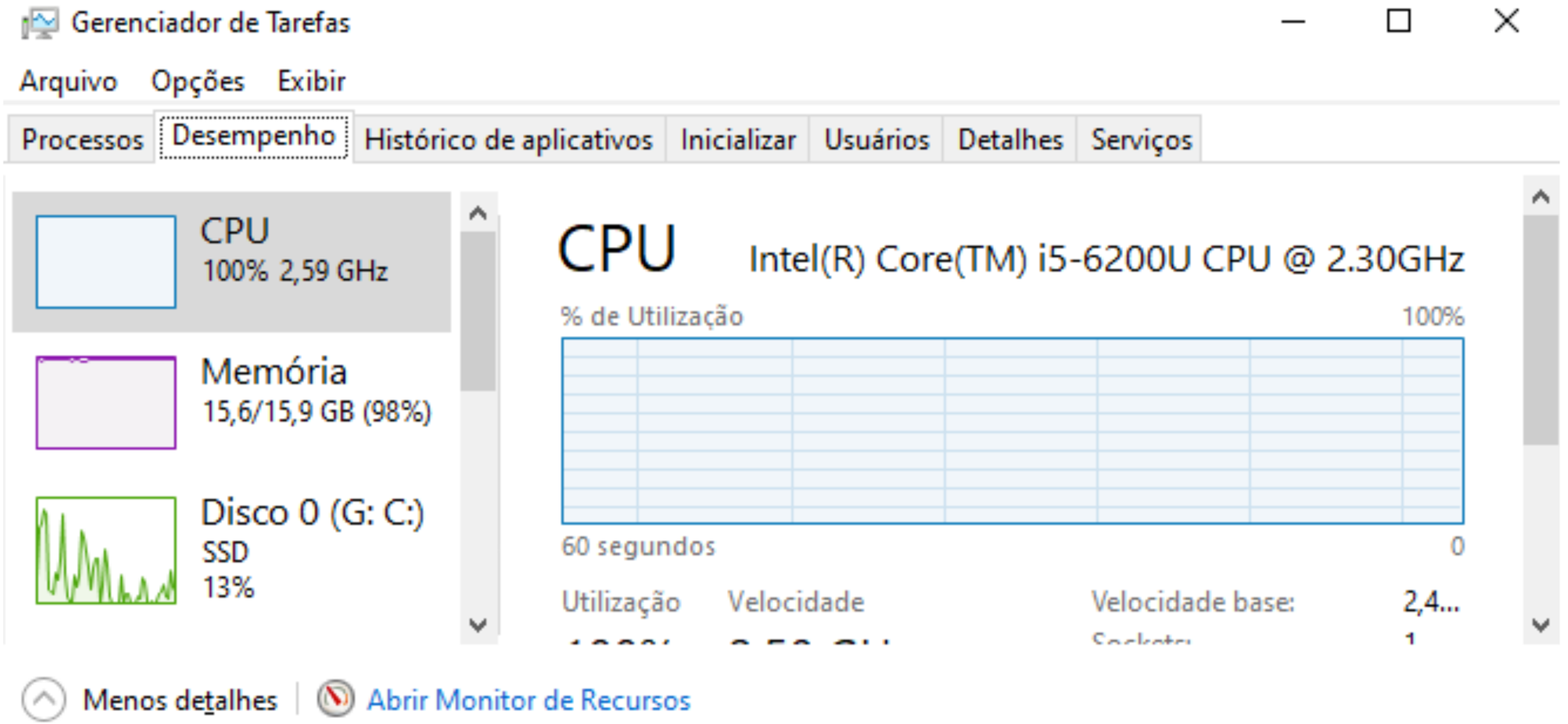
Simulation Results



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Simulation Results

KeyGen time	8.52 min
Encrypt time	19.17 min
Training time	6.19 hours
Public key size	2.62 GB

	Encrypted	Unencrypted
Accuracy	64.6211%	64.6363%
AUC	81.7039%	81.6996%

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Text organization

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References

Rivest, 1978 - ON DATA BANKS AND PRIVACY HOMOMORPHISMS;

Gentry, 2009 - Fully Homomorphic Encryption using Ideal Lattices;

Gentry, 2010 - Fully Homomorphic Encryption over the Integers;

CHEON, Jung Hee; KIM, Andrey, et al., 2017 - Homomorphic Encryption
for Arithmetic of Approximate Numbers;

HAN, Kyoohyung et al., 2018 Efficient Logistic Regression on Large Encrypted Data.

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FHE libraries - Overview

Library/ Scheme or Extension	BGV	BGV Bootstr.	BFV	CKKS	CKKS Bootstr.	DM	CGGI	Threshold FHE (MP)	PRE (MP)
Concrete							✓		
FHEW						✓			
HEAAN				✓	✓				
HELib	✓	✓		✓					
Lattigo			✓	✓	✓			✓	
OpenFHE	✓	*	✓	✓	✓	✓	✓	✓	✓
PALISADE	✓		✓	✓		✓	✓	✓	✓
SEAL	✓		✓	✓					
TFHE							✓		

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HELib	✓	✓		✓					
Lattigo			✓	✓	✓			✓	
OpenFHE	✓	*	✓	✓	✓	✓	✓	✓	✓
PALISADE	✓		✓	✓		✓	✓	✓	✓
SEAL	✓		✓	✓					
TFHE							✓		

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FHE libraries - OpenFHE

Library/ Scheme or Extension	BGV	BGV Bootstr.	BFV	CKKS	CKKS Bootstr.	DM	CGGI	Threshold FHE (MP)	PRE (MP)
Concrete							✓		
FHEW						✓			
HEAAN				✓	✓				
HELib	✓	✓		✓					
Lattigo			✓	✓	✓			✓	
OpenFHE	✓	*	✓	✓	✓	✓	✓	✓	✓
PALISADE	✓		✓	✓		✓	✓	✓	✓
SEAL	✓		✓	✓					
TFHE							✓		

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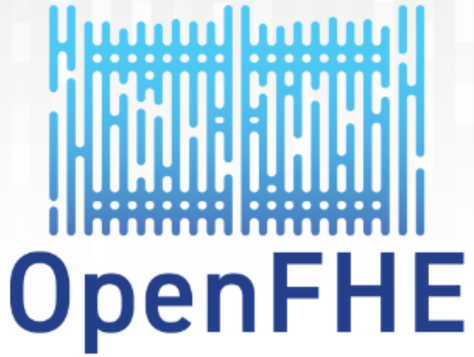
FHE libraries - OpenFHE



Community Growth:

OpenFHE is an open-source project that provides efficient extensible implementations of the leading post-quantum Fully Homomorphic Encryption (FHE) schemes.

Paper 2022/915

OpenFHE: Open-Source Fully Homomorphic Encryption Library

Ahmad Al Badawi, Duality Technologies
Jack Bates, Duality Technologies
Flavio Bergamaschi, Intel Corporation
David Bruce Cousins, Duality Technologies
Saroja Erabelli, Duality Technologies
Nicholas Genise, Duality Technologies
Shai Halevi, Algorand Foundation
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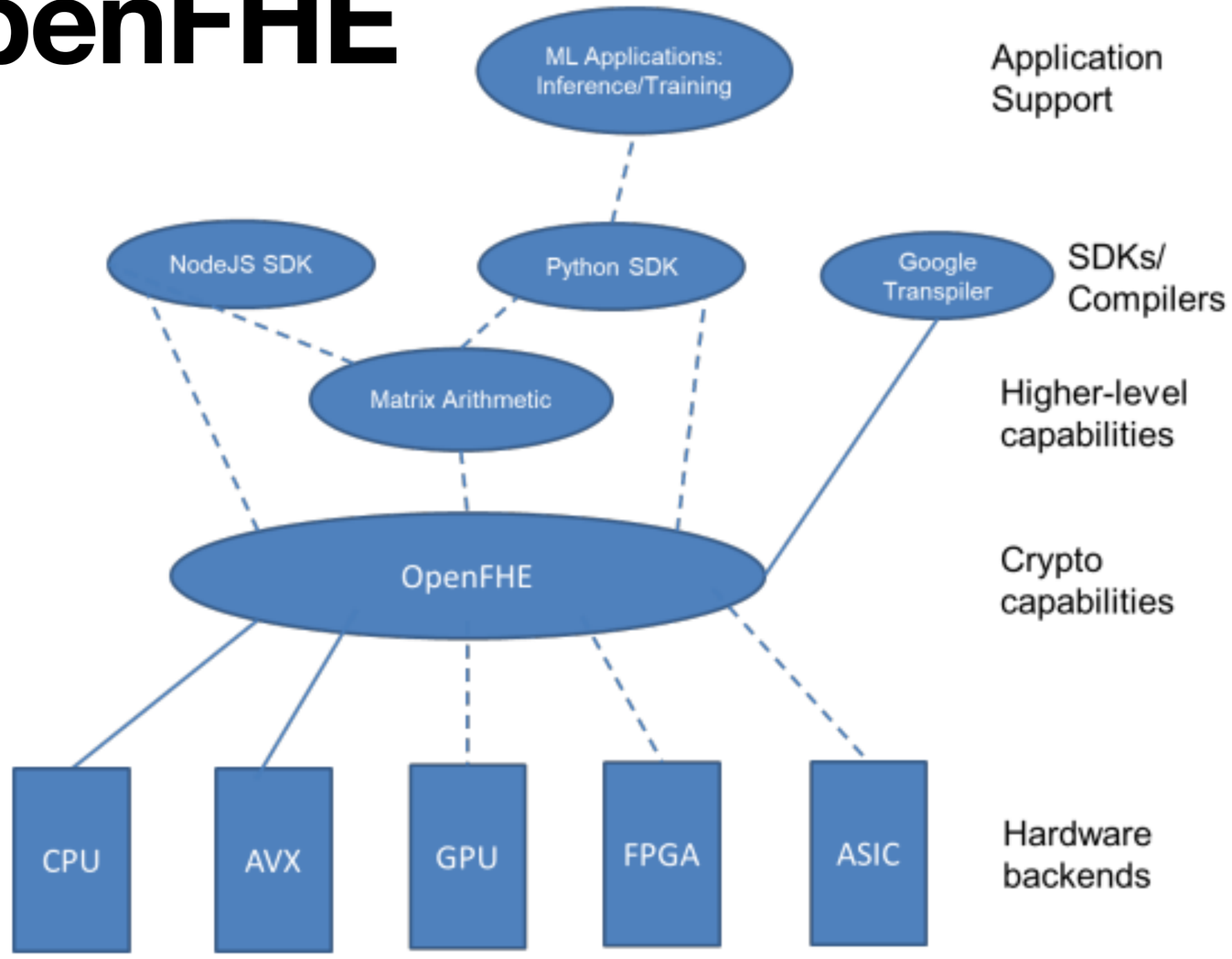
FHE libraries - OpenFHE



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FHE libraries - OpenFHE Python

Goals:

- **Make FHE more accessible for non-C++ users and ML community**
- **Keep C++ efficiency**

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- Matthew Tripplet
- Rener Oliveira

Project History
- Feb/2023 - July/2023: First pre-release v0.8.0
- Ago/2023 - Now: working on stable release

Sponsorship:
- NumFocus (also finances Numpy, scikit-learn, pandas, etc)

Paper 2022/915

OpenFHE: Open-Source Fully Homomorphic Encryption Library

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FHE libraries - OpenFHE Python

openfhe-python

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API REFERENCE:

- CryptoContext
- Ciphertext
- Plaintext
- Public Key
- Private Key
- KeyPair
- PKE enums
- BinFHEContext
- LWECiphertext
- LWEPrivateKey
- BinFHE enums

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OpenFHE - Python

Fully Homomorphic Encryption (FHE) is a powerful cryptographic primitive that enables performing computations over encrypted data without having access to the secret key. OpenFHE is an open-source FHE library that includes efficient implementations of all common FHE schemes: BFV, BGV, CKKS, DM and CGGI.

`openfhe-python` is a Python library built as a wrapper for the main capabilities of OpenFHE C++ library. It provides a more user-friendly interface for Python developers, while keeping the efficiency of C++ FHE operations.

API Reference:

- CryptoContext
- Ciphertext

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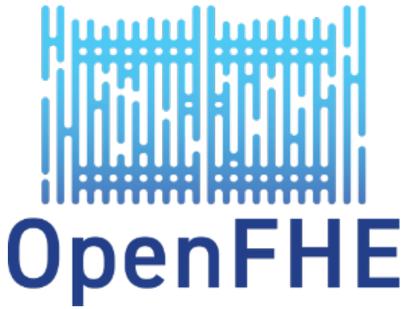
FHE libraries - OpenFHE Python



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