7 Background to pull request: direct radiation perpendicular to the cell's surface

Let r be instantaneous direct radiation perpendicular to Earth's surface. dni denotes instantaneous surface radiation perpendicular to the beam and is calculated from r and the solar zenith angle α as

$$dni = \frac{r}{cos(\alpha)} \tag{21}$$

If the panel points towards the sun, it receives 100% of dni. If it doesn't, correction factors are needed to account for the difference. The correction can be obtained by evaluating the projections on a horizontal and vertical plane, respectively. Looking at the beam from the side (as opposed to from the top), one can decompose dni into a component parallel to the ground

$$dni_{||} = dni \cdot cos(h), \tag{22}$$

where h is solar altitude and one perpendicular to the ground

$$dni_{\perp} = dni \cdot sin(h). \tag{23}$$

The component perpendicular to the ground is independent of the azimuth angle (because the effective area doesn't change when rotating the panel along a rotational axis in parallel to the beam).

However, the component parallel to the ground is affected by changes of the azimuth angles:

$$I_{||} = cos(a_p - a_s) dni_{||}, \tag{24}$$

where a_p (a_s) is the panel (sun) azimuth angle.

In vector form, we can now write the radiation perpendicular to the plane as

$$\boldsymbol{I} = dni \left[cos(h) cos(a_p - a_s) \boldsymbol{e}_{||} + sin(h) \boldsymbol{e}_{\perp} \right] \tag{25}$$

Projecting onto the beam ($\boldsymbol{e}_{||} = sin(t) \boldsymbol{e}_{\boldsymbol{I}}$ and $\boldsymbol{e}_{\perp} = cos(t) \boldsymbol{e}_{\boldsymbol{I}}$) yields

$$\boldsymbol{I} = dni \left[cos(h) sin(t) cos(a_p - a_s) + sin(h) cos(t) \right] \boldsymbol{e}_{\boldsymbol{I}} \tag{26}$$

Only looking at the absolute values yields

$$I = r \frac{1}{cos(\alpha)} \underbrace{\left[cos(h) sin(t) cos(a_p - a_s) + sin(h) cos(t) \right]}_{g}, \tag{27}$$

where solar zenith $\alpha = \pi/2 - h$ and a_s vary in time while tilt and panel azimuth are kept constant unless tracking panels are assumed.

We are interested in temporal means of I rather than its instantaneous values. The mean of I over a timestep Δt is

$$\langle I \rangle = \frac{1}{\Delta t} \int I(t') dt', \tag{28}$$

where the bounds of the integral are typically centered around the timestep $[t - \Delta t/2, t + \Delta t/2]$, forward $[t, t + \Delta t]$ or backward $[t - \Delta t, t]$

7.1 The initial GSEE way to solve eq. 28

A helper variable is defined as

$$t_{\text{help}} = \begin{cases} t + 30Min & \text{, if t.hour is not sunset or sunrise hour} \\ t + N/2Min & \text{,otherwise, where } N \text{ is minutes of sunshine.} \end{cases} \tag{29}$$

Note that this assumes (1) hourly input and (2) considers values to be representative for the following hour (i.e., forward running means).

GSEE evaluates the angles α and a_s at t_{help}. It then makes three assumptions:

1. $\langle r\frac{1}{cos(\alpha)}g\rangle = \langle r\rangle\langle\frac{1}{cos(\alpha)}\rangle\langle g\rangle$
2. $\langle r\rangle = r\frac{N[Mins]}{60}$
3. $\langle\frac{1}{cos(\alpha)}\rangle = \frac{1}{cos(\alpha(t_{\text{help}})}$

Assumption 1 would be correct if two of the three terms were reasonably constant during a timestep. This is certainly not correct close to sunrise & sunset where $\frac{1}{cos(\alpha)} \rightarrow \infty$ and $r \rightarrow 0$ such that the entire term is well defined even though the zolar zenith contributions blows up.

Assumption 2 might be justified for hourly instantaneous values (i.e. observations) but not for model/reanalysis values. The models account for sub-resolution changes of the solar zenith angle in the calculation of r such that applying 2 means to account for this effect twice which ultimately implies a systematic underestimation of radiation around sunrise and sunset in GSEE.

Assumption 3 can be very wrong around $\alpha = \pi/2$.

7.2 Suggested alternative

One can solve eq. 28 by focusing on the highly variable part:

- $r \in [0, 1000W/m^2]$ ($1300W/m^2$ is ToA flux) $\Rightarrow$ 4 orders of magnitude
- $\frac{1}{cos(\alpha)} \in [-\infty, \infty]$ (numerically $[-10^{15}, 10^{15}]$ $\Rightarrow$ at least 30 orders of magnitude
- $g \in [-1, 1]$, $\Rightarrow$ less than 1 order of magnitude

which is the $\frac{1}{cos(\alpha)}$ contribution.

Using the following assumptions

1. g is reasonably constant per timestep and the angles for g can be calculated at the center of the timestep.
2. r is constant until the zenith angle α reaches a threshold at $t = t_{\text{thr}}$ and drops to zero afterwards,

we can write

$$\langle I\rangle = \langle r\frac{1}{cos(\alpha)}g\rangle = g(t_{\text{cen}})\langle r\frac{1}{cos(\alpha)}\rangle. \tag{30}$$

Exploiting assumption 2, it follows that

$$\langle r\frac{1}{cos(\alpha)}\rangle = \frac{1}{\Delta t}\left(r\int\limits_{t-\Delta t/2}^{t_{\text{thr}}} \frac{1}{cos(\alpha)}dt' + 0\int\limits_{t_{\text{thr}}}^{t+\Delta t/2} \frac{1}{cos(\alpha)}dt'\right), \tag{31}$$

where r is constant and taken directly from the reanalysis. (Rescale?)

For convenience we define a function

$$s = \begin{cases} \frac{1}{cos(\alpha)} & \text{, if } t < t_{thr} \\ 0 & \text{, otherwise,} \end{cases} \tag{32}$$

which finally yields

$$\langle I\rangle = g(t_{\text{cen}})r\langle s\rangle \tag{33}$$