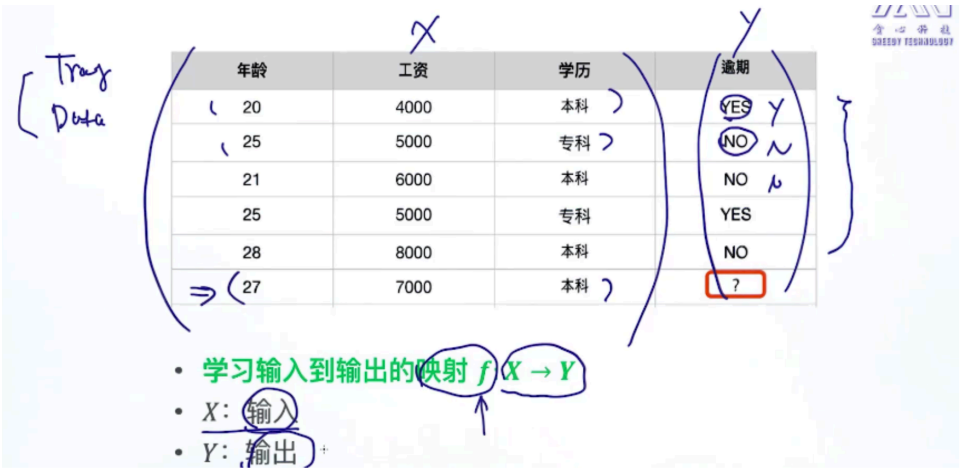
1. Logistic Regression

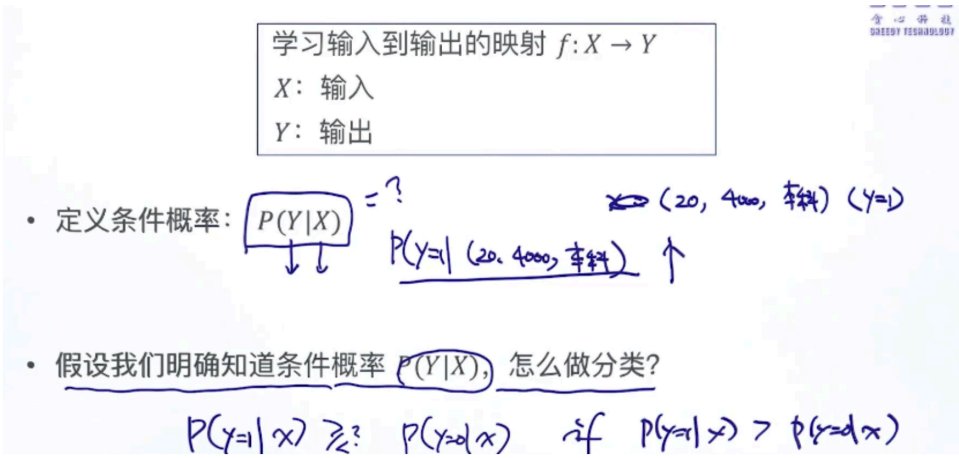
1.1 Classification tasks:

贷款违约、广告点击、商品推荐、情感分析、疾病诊断，etc

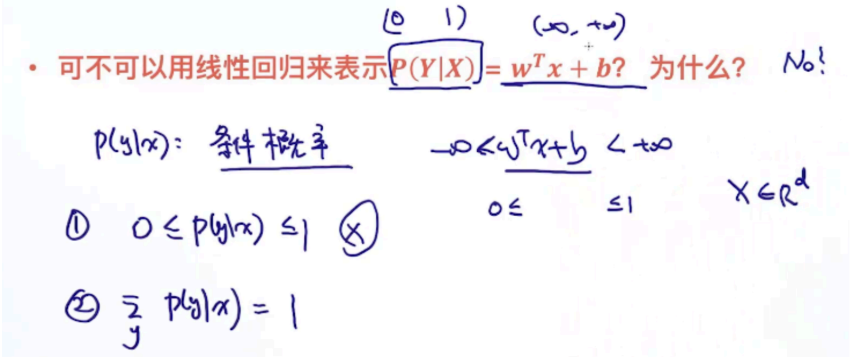
逻辑回归是一个很好的 baseline



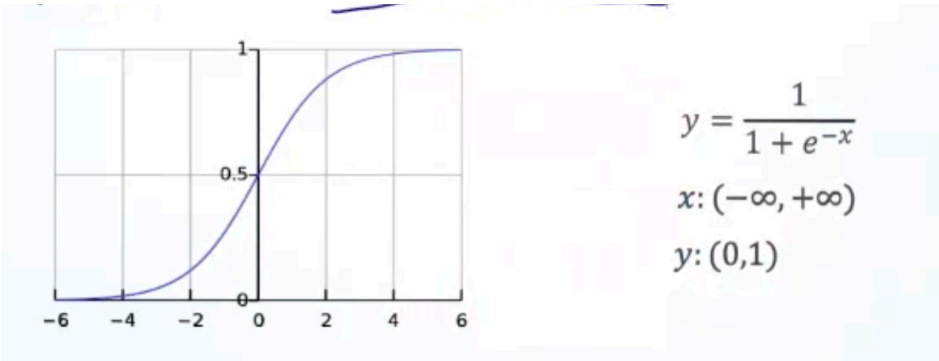
1.2 problem

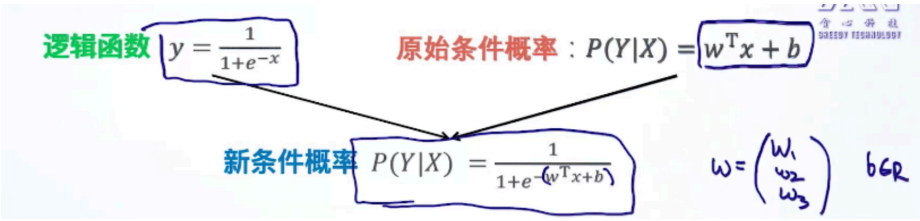


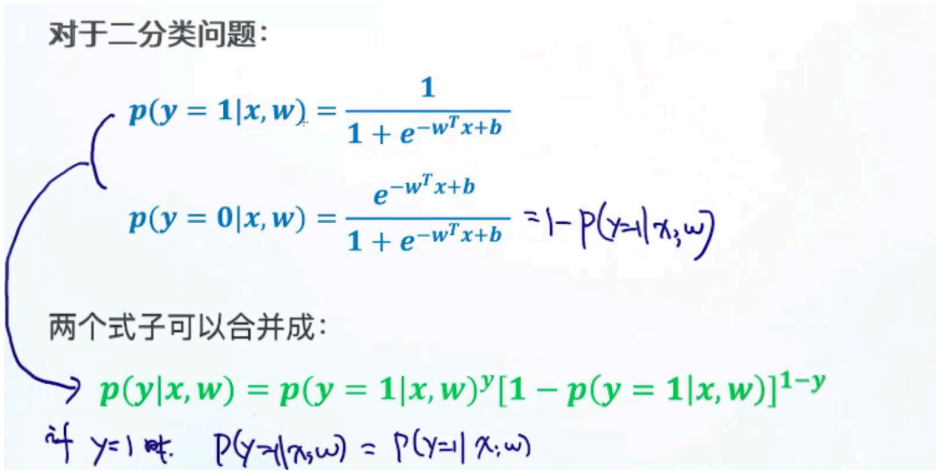
核心：怎么去设计条件概率



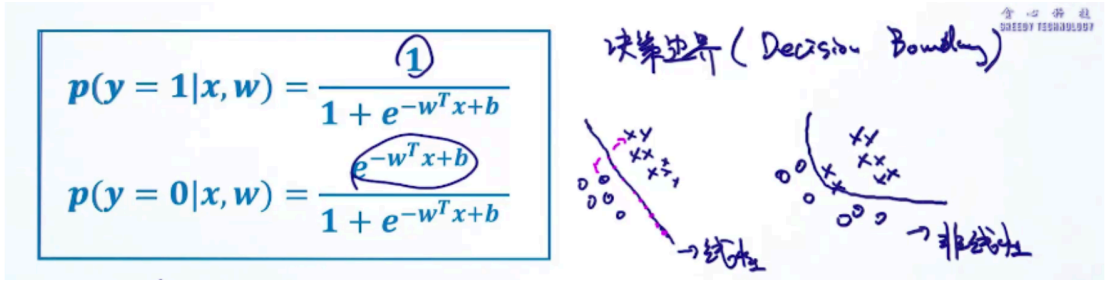
1.3 logistic function







1.4 linear classifier



1.5 objective function

假设我们拥有数据集 $D = \{(x_i, y_i)\}_{i=1}^{n} \quad x_i \in R^d, \quad y_i \in \{0, 1\}$

而且我们已经定义了：

$$p(y|x,w) = p(y=1|x,w)^y [1 - p(y=1|x,w)]^{1-y}$$

我们需要最大化目标函数：

$$\hat{w}_{MLE}, \hat{b}_{MLE} = argmax_w \prod_{i=1}^{n} p(y_i|x_i,w,b)$$

$$\hat{w}_{MLE}, \hat{b}_{MLE} = argmax_{w,b} \prod_{i=1}^{n} p(y_i|x_i,w,b) \quad ①$$

$$= \arg\max_{w,b} \log\left(\prod_{i=1}^{n} p(y_i|x_i,w,b)\right)$$

$$= \arg\max_{w,b} \sum_{i=1}^{n} \log p(y_i|x_i,w,b)$$

$\log(xyz) = \log x + \log y + \log z$

Objective Function

$$\arg\max_{w,b} \sum_{i=1}^{n} \log p(y_i|x_i,w,b)$$

$$argmin_{(w,b)} - \sum_{i=1}^{n} \log p(y_i|x_i,w,b)$$

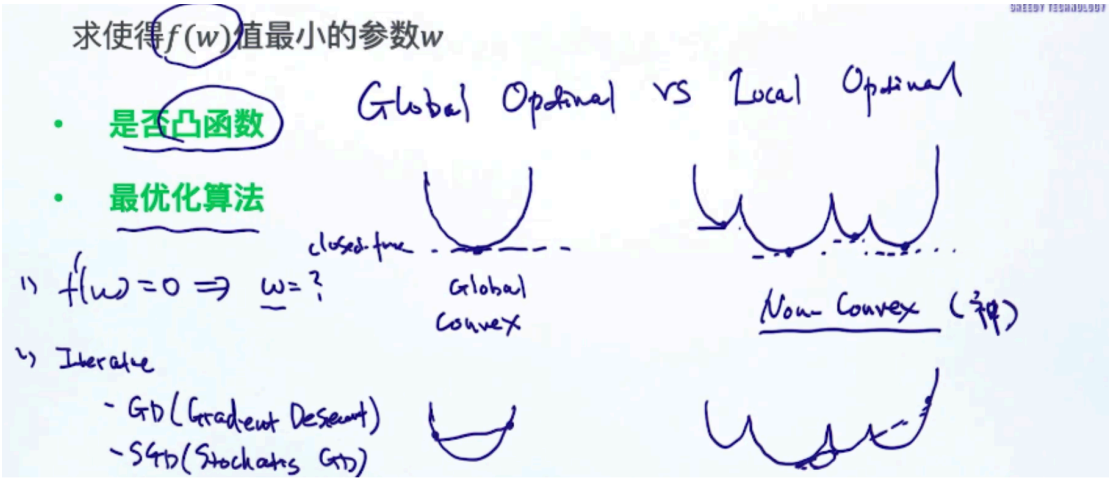
$$= \arg\min_{w,b} - \sum_{i=1}^{n} \log\left[p(y=1|x,w,b)^y \cdot [1 - p(y=1|x,w,b)]^{1-y}\right]$$

$$= \arg\min_{w,b} - \sum_{i=1}^{n} y \cdot \log p(y=1|x,w,b) + (1-y) \log[1 - p(y=1|x,w,b)]$$

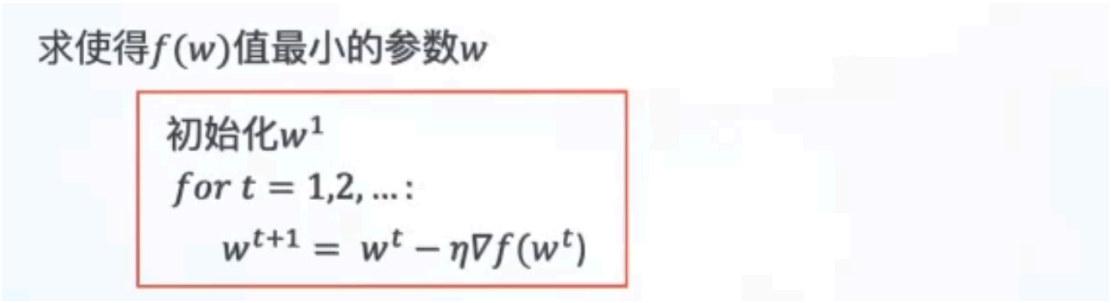
$\log(a^y b^x) = \log(a^y) + \log(b^x) = y \log a + x \log b$

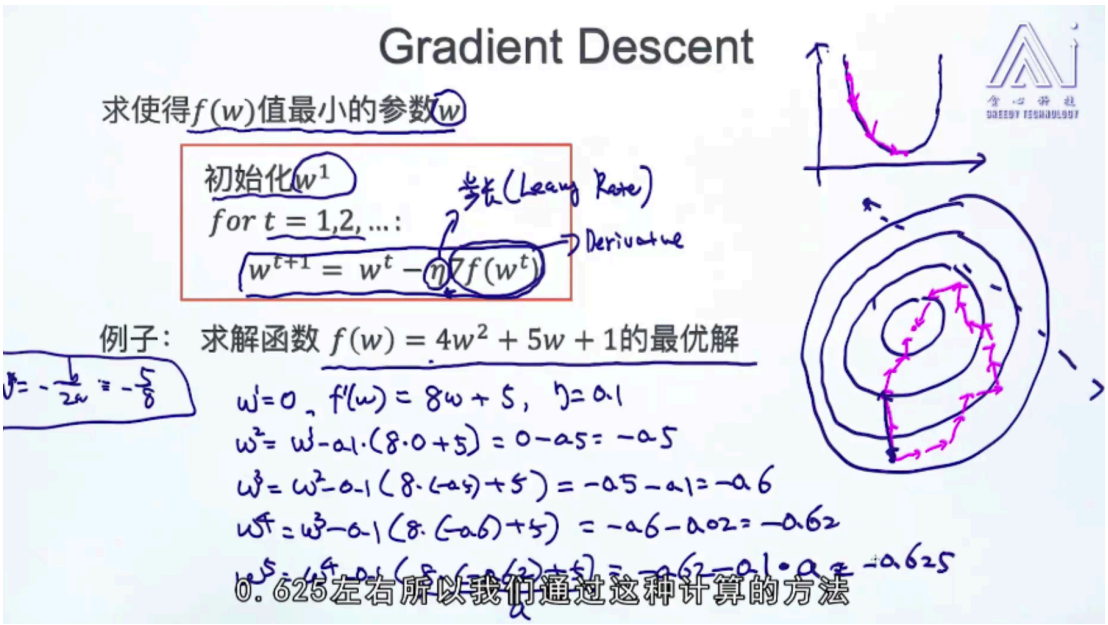
2. 梯度下降

2.1 求解函数的最小值



2.2 梯度下降





2.3 逻辑回归的梯度下降

$$p(y=1|x,w)=\frac{1}{1+e^{-w^Tx+b}}$$

$$argmin_{w,b}-\sum_{i=1}^{n} y_i \log p(y_i=1|x_i;w)+(1-y_i)\log(1-p(y_i=1|x_i;w))$$

$$= \underset{w,b}{\operatorname{argmin}} \underbrace{-\sum_{i=1}^{n} y_i \cdot \log \sigma(w^T x_i+b)+(1-y_i)\cdot \log[1-\sigma(w^T x_i+b)]}_{L(w,b)}$$

$$\frac{\partial L(w,b)}{\partial w} = -\sum_{i=1}^{n} y_i \cdot \frac{\sigma(w^T x_i+b)\cdot[1-\sigma(w^T x_i+b)]}{\sigma(w^T x_i+b)}\cdot x_i + (y_i-1)\cdot \frac{\sigma(w^T x_i+b)\cdot[1-\sigma(w^T x_i+b)]}{1-\sigma(w^T x_i+b)}\cdot x_i$$

$$= -\sum_{i=1}^{n} y_i(1-\sigma(w^T x_i+b))\cdot x_i + (y_i-1)\cdot \sigma(w^T x_i+b)\cdot x_i$$

$$= \sum_{i=1}^{n}[y_i - \sigma(w^T x_i+b)]\cdot x_i = \sum_{i=1}^{n}[\sigma(w^T x_i+b)-y_i]\cdot x_i$$

$$L(w,b) = -\sum_{i=1}^{n} y_i \log \sigma(w^T x_i+b)+(1-y_i)\log[1-\sigma(w^T x_i+b)]$$

$$\frac{\partial L(w,b)}{\partial b} = -\sum_{i=1}^{n} y_i \cdot \frac{\sigma(w^T x_i+b)[1-\sigma(w^T x_i+b)]}{\sigma(w^T x_i+b)} + (1-y_i)\cdot \frac{-\sigma(w^T x_i+b)[1-\sigma(w^T x_i+b)]}{1-\sigma(w^T x_i+b)}$$

$$= -\sum_{i=1}^{n} y_i\cdot(1-\sigma(w^T x_i+b)) + (1-y_i)\cdot(-\sigma(w^T x_i+b))$$

$$= \sum_{i=1}^{n}[\sigma(w^T x_i+b)-y_i]$$

初始化 w, b

for $t=1,2\cdots$

learning rate

$$w^{t+1} = w^t - \eta \cdot \sum_{i=1}^{n}[\sigma(w^T x_i+b)-y_i]\cdot x_i$$

$$b^{t+1} = b^t - \eta \cdot \sum_{i=1}^{n}[\sigma(w^T x_i+b)-y_i]$$

Gradient Descent

但是它的风险很大

Stochastic Gradient Descent for Logistic Regression

for $t=1,2\cdots$

$$w^{t+1} = w^t - \eta_t \sum_{i=1}^{n}[\sigma(w^T x_i+b)-y_i]\cdot x_i \quad O(n)$$

$$b^{t+1} = b^t - \eta_t \sum_{i=1}^{n}[\sigma(w^T x_i+b)-y_i] \quad O(n)$$

n: # of samples

① Gradient Descent

for $t=1,2\cdots$

mini-batch = sample($\{(x_i,y_i)\}_{i=1}^{n}$, m)

$$w^{t+1} = w^t - \eta_t \cdot \sum_{(x_i,y_i)\in \text{mini-batch}}[\sigma(w^T x_i+b)-y_i]\cdot x_i$$

$$b^{t+1} = b^t - \eta_t \sum_{(x_i,y_i)\in \text{mini-batch}}[\sigma(w^T x_i+b)-y_i]$$

① + ②

Mini-batch Gradient Descent

for $t=1,2\cdots T$

shuffle($\{(x_i,y_i)\}_{i=1}^{n}$)

for $i=1\cdots n$

$$w^{new} = w^{old} - \eta_t[\sigma(w^T x_i+b)-y_i]\cdot x_i$$

$$b^{new} = b^{old} - \eta_t[\sigma(w^T x_i+b)-y_i]$$

② Stochastic Gradient Descent