A Framework for Contractual Graphs

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Abstract

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This paper studies contractual graphs, where the formation of edges between nodes is analyzed as a contractual agreement that is implemented upon fulfilment of underlying conditions. A contractual framework for graphs is especially useful in applications where AI-enabled software is employed to create or automate smart contracts between nodes. While some smart contracts may be easily created and executed, others may contain a higher level of ambiguity which may prevent their efficient implementation. Ambiguity in contractual elements is especially difficult to implement, since nodes have to efficiently sense the ambiguity and allocate appropriate amounts of computational resources to the ambiguous contractual task. This paper develops a two-node contractual model of graphs, with varying levels of ambiguity in the contracts and examines its consequences for a market where tasks of differing ambiguity are available to be completed by nodes.

The central theme of this paper is that as ambiguity increases, it is difficult for nodes to model the contract. Thus, while linguistic ambiguity or situational ambiguity might not be cognitively burdensome for humans, it might become expensive for nodes involved in the smart contract. The paper also shows that timing matters – the order in which nodes enter the contract is important as they proceed to sense the ambiguity in a task and then allocate appropriate resources. We propose a game-theoretic formulation to scrutinize how nodes that move first to complete a task are differently impacted than those that move second. We discuss the applications of such a contractual framework for graphs and obtain conditions under which two-node contracts can achieve a successful coalition.

Contribution to the field

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The central theme of this paper is that as ambiguity increases, it is difficult for nodes to model the contract. Thus, while linguistic ambiguity or situational ambiguity might not be cognitively burdensome for humans, it might become expensive for nodes involved in the smart contract. The paper also shows that timing matters – the order in which nodes enter the contract is important as they proceed to sense the ambiguity in a task and then allocate appropriate resources. We propose a game-theoretic formulation to scrutinize how nodes that move first to complete a task are differently impacted than those that move second. We discuss the applications of such a contractual framework for graphs and obtain conditions under which two-node contracts can achieve a successful coalition.

1. Introduction

Connections between entities are conveniently represented using graphs. The building blocks of graphs – nodes and edges – have long been used to represent various kinds of networks. The nodes represent entities involved in a transaction, and the edge represents the transaction itself. However, the transaction may not occur until underlying conditions for the implementation of that transaction have been satisfied. In this context, the transaction (represented by an edge in a graph) is not definitive, but is rather an outcome of a set of processes that have to be completed a priori. This paper studies such graphs and their applications, where the edges between nodes are created only upon fulfilment of underlying clauses. We call such graphs as *contractual graphs*, since the formation of an edge is similar to the execution of a contract upon fulfilment of underlying conditions.

Contractual graphs are best exemplified by the application of smart contracts, where AI-enabled nodes are tasked with the creation or execution of contracts. For example, upon transfer of funds from one account to another, the sale of an item is executed or a document is released. In this case, the nodes have to verify if the accounts have sufficient funds, if the transfer has taken place and if the entity has actual control over the item to be sold or the document to be released. Once these have been verified and the operations have taken place, the transaction is completed and can be represented as an edge in the contractual graph. For example, the work in (Bigi et al, 2015) describes smart contracts for use in applications such as barter, insurance, escrow, derivatives and general business contracts.

However, not all contracts can be clearly automated. For example, if the contract is executed only upon satisfactory completion of a project, the contract will have to clearly define the threshold for “satisfactory” performance. Any deviation from predetermined values for a task that varies in complexity will require a new smart contract to be created. Examples such as these abound in various domains, where the fuzziness of the contractual language can be parsed by humans, but poses tremendous difficulties while being converted into software. For the aforementioned example of what constitutes *satisfactory* performance, AI-enabled software could help by parsing through large datasets of similar projects that have been labeled as “*satisfactory*”, “*good*” or “*excellent*” and assign the appropriate label. Still, the ambiguity cannot always be resolved using code. Ambiguity, while helping humans achieve leeway in contractual relations, is not a welcome condition in software. Ordinary qualifiers such as “*few*”, “*some*”, or “*smart*” cannot be programmed effectively without assigning values to these qualifiers. As AI increasingly makes its way into various facets of our lives, it is important to understand that ambiguity in the code can lead to life-altering consequences. For example, when facial recognition software identifies an individual with some probability, the inherent ambiguity in the goal of the task can create adverse outcomes with far-reaching impact. While humans can navigate ambiguity based on situational cues, it is difficult to do so in software. Contractual relationships effected by AI, therefore, have to be cognizant of ambiguity as a key element of the relationship and a driver of resulting outcomes.

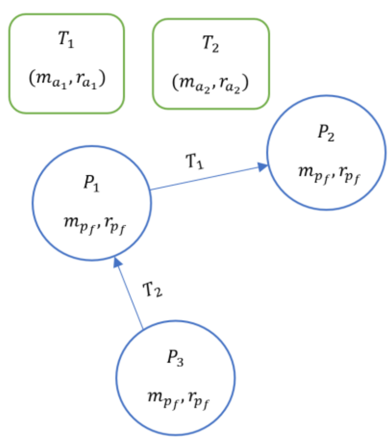

Figure 1: A marketplace with two ambiguous tasks T_1 and T_2, each with their levels of ambiguity (m_a) and resource consumption (r_a). Three players P_1, P_2 and P_3 assess the tasks to develop their individual perceptions of the ambiguity of a task (m_p) and the allocate resources accordingly (r_p).

In this paper, we study how ambiguity impacts contractual graphs. Specifically, we use a game-theoretic approach to study how tasks of varying ambiguity are perceived and completed by nodes. To do this, we assume a marketplace with ambiguous tasks (Figure 1). We model the contractual graph as a two-player game, where edges are formed only when two nodes cooperate to complete a task. We show that timing matters, i.e. there are specific advantages to the first mover and the second mover in cooperating to complete the task. Here, two contracts are formed: P_3P_1 commit to task T_1 and P_1P_2 commit to task T_2. The direction of the arrow determines the first mover. Here P_3 is the first mover for task T_2. P_1 is the second mover for task T_2 and is the first mover for task T_1. Similarly, P_2 is the second mover for task T_2. In contract P_3P_1, P_3 is the first mover and P_1 is the second mover and are denoted accordingly in the notation.

Next, using the Shapley value, we quantify the value that the first mover and the second mover bring to a coalition. These results are then used to create contractual graphs, whose edges reflect the outcomes of ambiguity and timing in task execution. Further, we provide a representation of such contractual graphs that can be used in learning important information about the community of nodes and the nature of the contracts that are formed in the marketplace.

The rest of this paper is organized as follows. Section 2 presents an overview of related work in contract theory and the application of the Shapley value in diverse settings. Section 3 presents our model for contractual graphs, and Section 4 derives the Shapley value for 2-player coalitions in contractual graphs. Section 5 presents our findings of simulations. A kernel representation of contractual graphs is presented in Section 6. Finally, Sections 7 and 8 present directions for future work and conclude the paper, respectively.

2. Related Work in Contract Theory and Coalition Value

The use of game theory to study the mechanism of contract formation has been studied in (Katz, 1990). Here, the author explored contract formation by addressing two questions. First, what are the actions or intentions that are required for a contractual obligation? Second, how do these actions or intentions affect the content of the contract itself? At the heart of this analysis, lies the issue of ambiguity in determining the set of actions or intentions that signal the formation and content of a contract. While legal rules provide a robust framework for determining the outcomes of contracts, the author emphasized the role of social norms and ethical precepts in the outcomes. To this end, the author identified different types of costs involved in contract formation and contract execution and proposes a game-theoretic economic analysis of the steps involved in contract offer, acceptance and rejection.

Another case for the incompleteness of contracts from an economics viewpoint has been made in (Tirole, 1999). Here, the author explains that most contracts are in fact, incomplete. Citing an example of mission and vision statements at higher levels of administration in various domains, the author explains how vague statements such as “increase security” or “provide robust frameworks” are often used to issue directives. The vagueness is inevitable especially since the costs of specifying every contingency and breach can be prohibitively expensive, even if such specifications can be explicitly specified. Such incomplete contract models are characterized by three types of costs: unforeseen contingencies, costs of writing contracts and costs of enforcing contracts. This theme has been continued in (Sklaroff, 2017), where the specific case of inflexibility in smart contracts was studied. The author presented the three features of smart contracts – automation, decentralization and anonymity – that require the formation of fully specified terms for entities to verify the terms of the contract without ambiguity. The author argued that contractual language offers two important attributes – linguistic ambiguity and enforcement discretion that provide powerful efficiencies in the contracting process. This is because smart contracts create transaction costs that are often inflexible. Smart contracts require future stages and terms to be clearly defined beforehand, which is difficult in volatile or unknown environments. Further, generic smart contracts lead to unpredictable and expensive litigation, and create challenging outcomes in cases of breach. Further work in the challenges of smart contracts and the law has been studied in (Verstraete, 2018) where the author presents challenges concerning enforcement and governance in smart contracts, which explicitly eschew central entities in transactions and their outcomes. Another example of developing contracts with predetermined outcomes is in (Asgaonkar and Krishnamachari, 2018) where the authors describe double-sided payment functions in smart contracts to ensure trustworthy transactions.

The aspect of cognitive burden has been addressed in (Tirole, 2014) where the author studies scenarios where having additional information is costly and therefore, players choose scenarios with less information. The paper describes such scenarios as cognitive traps, where the additional cognitive burden imposed by choosing options with higher information is the less desirable option for players. Thus, a player is hurt by choosing the cognitively burdensome option that requires the player to process larger amount of information. The paper further considers cognition-intensive contracting, where the parties to a contract attempt to understand the likely implications of the contract.

The value that players bring to a coalition can be measured in several ways, one of which is the Shapley value. In (Chalkiadis et al, 2012), the authors present an overview of the applications of the Shapley value in cooperative game theory for AI and computer science. The Shapley value has been used extensively in feature selection in machine learning. Originally derived as a game-theoretic mechanism to characterize the value of each player in a coalition, the derivation of the Shapley value is contingent upon the fulfilment of the following conditions: symmetry (a player's contribution and not the label assigned to the player is the factor that determines the player's Shapley value), linearity (utility functions are linear), and carrier (dummy players are assigned a value of zero, and similarly players who make a contribution receive a value that divides the worth of the coalition among players.

The versatility of the Shapley value to diverse applications has led to development in multiple interpretations of the Shapley value. In Sundarajan and Najmi (2019), the authors study several Shapley values for application to a dataset with ten features, and show that the Shapley value with Conditional Expectations was the most sensitive for their application. In Cohen et al (2005), the authors propose a Contribution-Selection Algorithm (CSA) that ranks each feature according to its contribution value. This algorithm is able to iteratively select n top features with highest contribution values, and is able to remove features with lowest contribution values. Another way in which the Shapley value has been studied is in Bilbao et al (2008) for the case of bicooperative games. In addition to applications in game-theoretic coalitions, the Shapley value has been used in determining centrality, i.e. the set of most influential nodes in a network (Michalak et al, 2013). Here, the authors describe the application of Shapley value in determining the group of nodes that have the largest influence on the network. Additional work on the role of the Shapley value in social networks is in Narayanam et al (2014) with applications to community detection and information spread. Other interpretations of the Shapley value include the computation of a bounded rationality Shapley value that ensures that the share of each agent reflects its contribution to the difficulty of computing the coalition values.

While the Shapley value offers a mechanism for determining the value of every player to a coalition, egalitarianism still remains another solution for determining the value of a player. In egalitarian solutions, the worth of each player is determined by equal division. This has led to research that combines the equal division of egalitarian solution versus the marginalism offered by Shapley value solutions. Work in van den Brink et al (2013) offers such a concept in the form of egalitarian Shapley values in both cooperative and non-cooperative scenarios.

3. Model for Contractual Graphs

We now introduce the model to study how ambiguity affects contract modeling by nodes/players in a marketplace. Assume two players P_1 and P_2, who both seek a high reputation and are equally abled. The first mover assesses the ambiguity in the contract. Assume that once a player enters a contract, she will complete the task. That is, the model does not study instances where nodes back off after entering the contract. Consequently, the first mover in the contract always commits. We assume a two-player coalition for task completion. Thus, although the first mover has modeled the task in the form of a contract, she requires the help of another player to complete the task. This other player, whom we label as the second mover, has the option to accept/reject offer of coalition with the first mover. For this paper, we study only the case where second mover accepts.

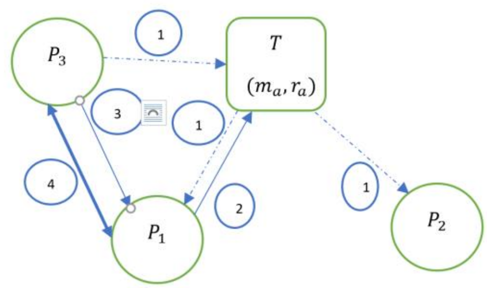

Figure 2: Steps leading to a contract. Assume that a task T appears in the marketplace, with its ambiguity and resource consumption requirements denoted by m_a and r_a respectively.

Figure 2 shows the steps in the process leading to a contract in a marketplace with one ambiguous task T and three players P_1, P_2, and P_3. The ambiguity and resource consumption of T are given by (m_a, r_a) in order.

Step 1: Players P_1, P_2, and P_3 assess the ambiguity of task T. Each player develops her individual perception of the ambiguity of task T as m_{p_i}, and accordingly allocates resources r_{p_i} for task completion, where $i \epsilon \{1,2,3\}$.

Step 2: P_1 decides to create a contract and thereby commits to task T. Since we assume two-player coalitions, P_1 needs another player to commit to the contract to complete T. Information about this contract is made available to the marketplace as a hard information signal similar to the work in Rajan, et al (2010).

Step 3: The remaining players in the marketplace (P_2 and P_3) receive the signal. Of these two players, P_3 decides to collaborate with P_1.

Step 4: The contract is finalized with P_1 as the first mover and P_3 as the second mover in the contract.

We assume that players are rational, intelligent and have common knowledge. A player assesses a task's ambiguity and determines its payoff as the difference between its perception of the work it might be required to do for the task and the actual work that is required for the task. Thus, the initial payoff awareness (IPA) for a node is given by:

Initial payoff awareness (IPA) = Perceived work – Actual work (1)

We define the amount of work to be performed as a product of the ambiguity and the resource commitment. Let the perceived ambiguity for the first mover node be m_{p_f}. The first mover accordingly allocates r_{p_f} resources for the task. Let the actual ambiguity of the task be m_a, and the resources required for the task be r_a. Thus, the initial payoff awareness for a first-mover node is given by

$$IPA_f = m_{p_f} r_{p_f} - m_a r_a \tag{2}$$

Similarly, let the perceived ambiguity for the second mover node be m_{p_s}. The second mover accordingly allocates r_{p_s} resources for the task. Thus, the initial payoff awareness for a second mover node is given by

$$IPA_s = m_{p_s} r_{p_s} - m_a r_a \tag{3}$$

We are interested in the difference between the perception of ambiguity and the actual ambiguity. We denote the difference in the perception and actual ambiguity for the first mover as $m_{p_f} - m_a = \lambda$. Similarly, we denote the difference in the perception of ambiguity and the actual ambiguity as $m_{p_s} - m_a = \mu$. Further, we assume that the difference in perception of resource consumption for both players is the same, i.e. $r_{p_f} - r_a = \delta = r_{p_s} - r_a$. Without loss of generality, as we show in the next section of our paper, the constant δ is transformed into a player-specific values δ_1 and δ_2 for the two players. This transformation drives the derivation of player-specific payoff calculations, and ultimately the derivation of the Shapley value-based representation of the contractual graphs.

We now develop a matrix for the initial payoff awareness for the first mover (Table 1) and second mover (Table 2) based on the difference in ambiguity and resource consumption perception and reality. The table contains four options corresponding to the possible difference in perceptions of ambiguity (high/low), the actual levels of ambiguity (high/low), and the difference in perceptions and actual values of resource commitment (high/low).

Perceived	Actual	λ	δ
High	Low	> 0	> 0
High	High	$= 0$	$= 0$
Low	Low	$= 0$	$= 0$
Low	High	< 0	< 0

Table 1: Initial payoff awareness matrix for first mover

Perceived	Actual	μ	δ
High	Low	> 0	> 0
High	High	$= 0$	$= 0$
Low	Low	$= 0$	$= 0$
Low	High	< 0	< 0

Table 2: Initial payoff awareness matrix for second mover

In Tables 1 and 2, we assume that each of the four cases are equally probable. Thus, the initial payoff awareness calculations can be obtained from equations (1) – (2) and Table 1 as follows. For the first mover,

$$IPA_f = m_{p_f} r_{p_f} - m_a r_a$$

$$= \frac{1}{4}[(m_a + \lambda)(r_a + \delta) - m_a r_a] + \frac{1}{4}(0) + \frac{1}{4}(0) + \frac{1}{4}\left(m_{p_f} r_p - m_a r_a\right)$$

$$IPA_f = \frac{\delta\lambda}{2} \tag{4}$$

Similarly, IPA calculations for the second mover are given by equations (1) and (3) and Table 2 as

$$IPA_s = \frac{\delta\mu}{2} \tag{5}$$

Next, we assume that nodes are motivated by reputation, which in turn is a proportional to the number of tasks completed. Reputational incentives or motivations have been extensively studied in game-theoretic modeling for applications such as cooperation in wireless networks (Jaramillo and Srikant, 2010) and human computation systems (Ghosh, 2013). Assume that reputation is function of work ω for each contract which in turn is a function of the IPA. Thus, the higher the perception of an increased payoff, the higher is the probability that the task will be modeled as a contract and a two-player coalition will be formed to complete the task. Let ω_1 and ω_2 be the work performed by first mover and second mover respectively.

$$\omega = \omega_1 + \omega_2 \tag{6}$$

Assume q_1 and q_2 are number of coalitions that the first and second mover are a part of respectively. Thus, $q = q_1 + q_2$ is the total number of coalitions. We assume an inverse resource function, where number of coalitions is inversely proportional to the work required for all coalitions. A detailed treatment of inverse resource functions in (Agaltsov et al, 2018). Thus, for some constant N, we have

Thus, $q = N - \omega$, (7)

We now have the framework to obtain the collective payoff from a coalition. The collective payoff function (p_{coll}) is modeled by assuming that the more one player expends resources and works harder, the less valuable it is to the other player.

$$p_{coll} = q\omega_i - a\omega_i^2 \tag{8}$$

Here, $i \epsilon \{1, 2\}$ for players 1 and 2 and a denotes variable costs. Thus, substituting (8) in (7) and (6), we get

$$p_{coll} = (N - \omega_1 - \omega_2)\omega_1 - a\omega_1^2 \quad (9)$$

For best response, we set

$$\frac{dp_{coll}}{d\omega_1} = 0 \quad (10)$$

We get

$$\omega_1 = \frac{N-\omega_2}{2(1-a)} \quad (11)$$

Since work ω is a function of initial payoff awareness, from equation (4), we have

$$\omega_1 = f\left(\frac{\delta\lambda}{2}\right) \quad (12)$$

$$\omega_2 = f\left(\frac{\delta\mu}{2}\right) \quad (13)$$

Without loss of generality, we assume constants δ_1 and δ_2 such that the above equations are expressed as

$$\omega_1 = \frac{\delta\lambda}{2} \quad (14)$$

$$\omega_2 = \frac{\delta_2\mu}{2} \quad (15)$$

Substituting in equation (11), we get an equation for the relationship between the payoff factors at best response and obtain the ambiguity ratio (AR) of the first and second movers, λ/μ, as follows.

$$\frac{\lambda}{\mu} = \frac{2N-\mu\delta_2}{2\delta_1(1-a)} \quad (16)$$

Thus, $\lambda \propto -\mu$. This shows that the higher the difference in perception and actual values of ambiguity (λ) for the first mover, the lower is the difference in perception and actual values of ambiguity for the second mover (μ).

Substituting this value of λ in the equation for collective payoff, we observe the performance of the collective payoff at best response as follows

$$p_{coll} = \frac{\lambda\delta_1}{4}(2N - 2a - \mu\delta_2 - \delta_1\lambda^2) \quad (17)$$

4. Derivation of the Shapley Value for 2-player Contracts

In this section, we analyze the two-player coalition to examine the contribution of each player to the coalition. To do this, we use the results from equations 4 and 5 from the above section. These results show the work of the first mover (P_1) and second mover (P_2), which is proportional to their IPA. Further assume that when the two players work together in a coalition, the players each can reduce their work load. These reductions are denoted as τ_1 and τ_2 for the first mover and second mover respectively. Thus, the contribution matrix of the individual players and the coalition are summarized in Table 3.

Player(s)	Contribution
P_1	$\frac{\delta\lambda}{2}$
P_2	$\frac{\delta\mu}{2}$
P_1, P_2	$\left(\frac{\delta\lambda}{2} - \tau_1\right) + \left(\frac{\delta\mu}{2} - \tau_2\right)$

Table 3: Contribution matrix of coalition

Order of players	Marginal Contribution P_1	Marginal contribution P_2
P_1P_2	$\frac{\delta\lambda}{2}$	$\frac{\delta\mu}{2} - (\tau_1 + \tau_2)$
P_2P_1	$\frac{\delta\lambda}{2} - (\tau_1 + \tau_2)$	$\frac{\delta\mu}{2}$

Table 4: Marginal contribution matrix

The Shapley value offers a way to quantify the contribution of each player to the coalition. In machine-learning applications, the Shapley value has been widely used to determine the value of a particular set of features to the overall representation of the dataset. For example, in seeking to determine a predictor for longevity, which feature contributes most to longevity from a pool of features including education, genetics, diet, and others? The seminal paper by Shapley (Shapley, 1953) describes how to determine the value of the expected marginal contribution of a coalition by considering all possible orders in which coalitions can be formed between players and assigning each player her marginal contribution.

Applying this technique to the two-player coalition, we get the marginal contribution matrix (Table 4). Specifically, the Shapley value emphasizes the role of timing. If the coalition is formed with P_1 first and then P_2 as shown in the first row, the marginal contributions of P_1 and P_2 are given as follows:

Averaging the columns, we get P_1's contribution to the coalition as follows:

$$\frac{\delta\lambda-(\tau_1+\tau_2)}{2} \tag{18}$$

Similarly, we get P_2's contribution to the coalition as

$$\frac{\delta\mu-(\tau_1+\tau_2)}{2} \tag{19}$$

5. Results

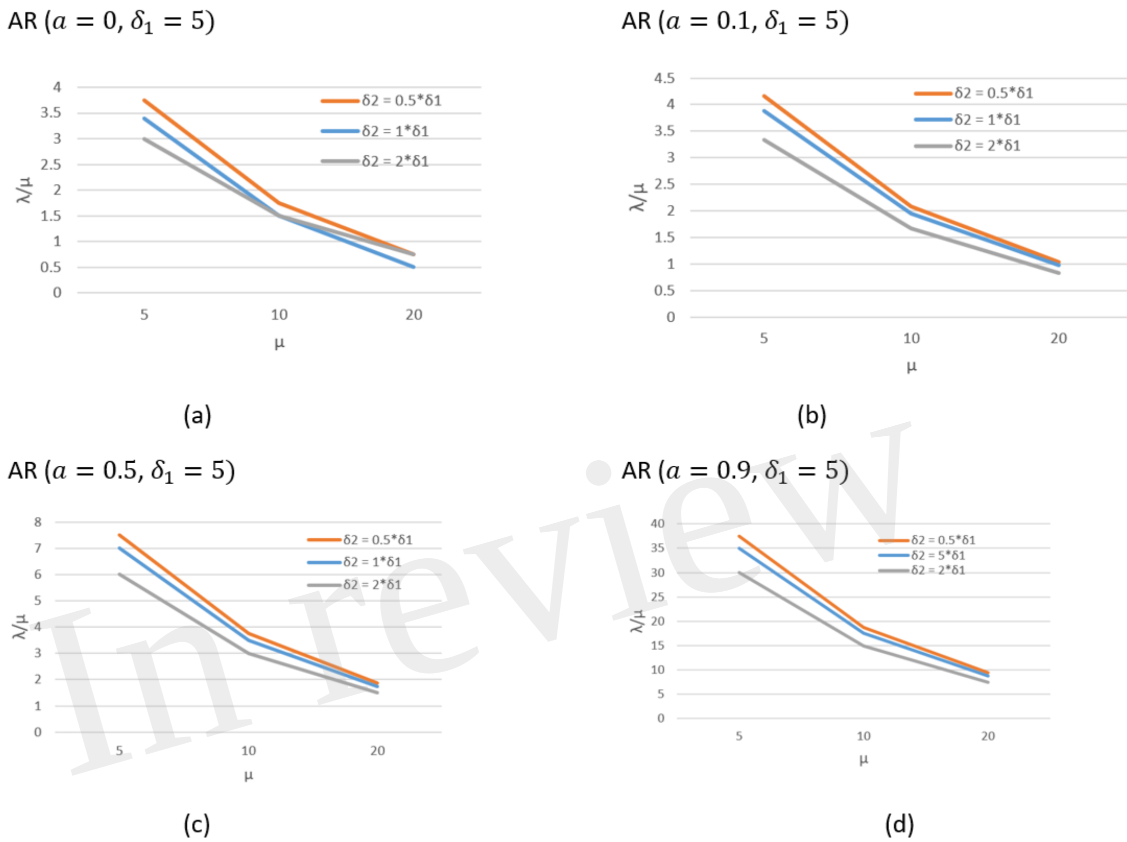

Figure 3. Relationship between the ambiguity ratio (AR) as a function of μ. As the variable costs (denoted by a) increase, the ambiguity ratio increases. Further, as the difference between actual and perceived resource commitment levels of the second mover increase, the value of λ decreases.

Figure 3 shows the relationship between the AR and the difference in perceived and actual ambiguity levels for the second mover (μ) at best response. From sub-figures (3a) – (3d), we see the inverse relationship between AR and μ. Thus, the difference in ambiguity levels for both parties in the contract are contrarian at the point of best response. Further, as the variable costs (a)increase, AR increases signifying a directly proportional relationship between the ambiguity and the variable costs. Additionally, AR is dependent on the difference in perceived and actual resource commitments of the first mover (δ_1) and the second mover (δ_2). Specifically, when the difference in perceived and actual resource commitments of the first mover (δ_1) is half that of the second mover (δ_2), the difference in ambiguity levels of the first mover is the highest. This difference in λ decreases as the value of δ_1 increases with respect to δ_2, thereby rendering λ inversely proportional to the ratio given by δ_1/δ_2. In summary, the difference in perceived and actual ambiguity levels for the first mover (λ) is directly proportional to the variable costs a.

The value of λ is inversely proportional to the difference in perceived and actual ambiguity levels for the second mover (μ) and to the ratio of difference in resource commitments of the first and second movers (δ_1/δ_2). These relationships at best response show that more resource-intensive contracts, denoted by increase in a and the ratio (δ_1/δ_2). , cause the first mover to have a larger gap in the perception of ambiguity. Thus, the larger the costs of executing the contract, the greater is the tendency for a mismatch between perceived and real ambiguity. Further, the difference in ambiguity perceptions for the first and second movers is inversely proportional signaling the conflicting role of contractual commitments for the players.

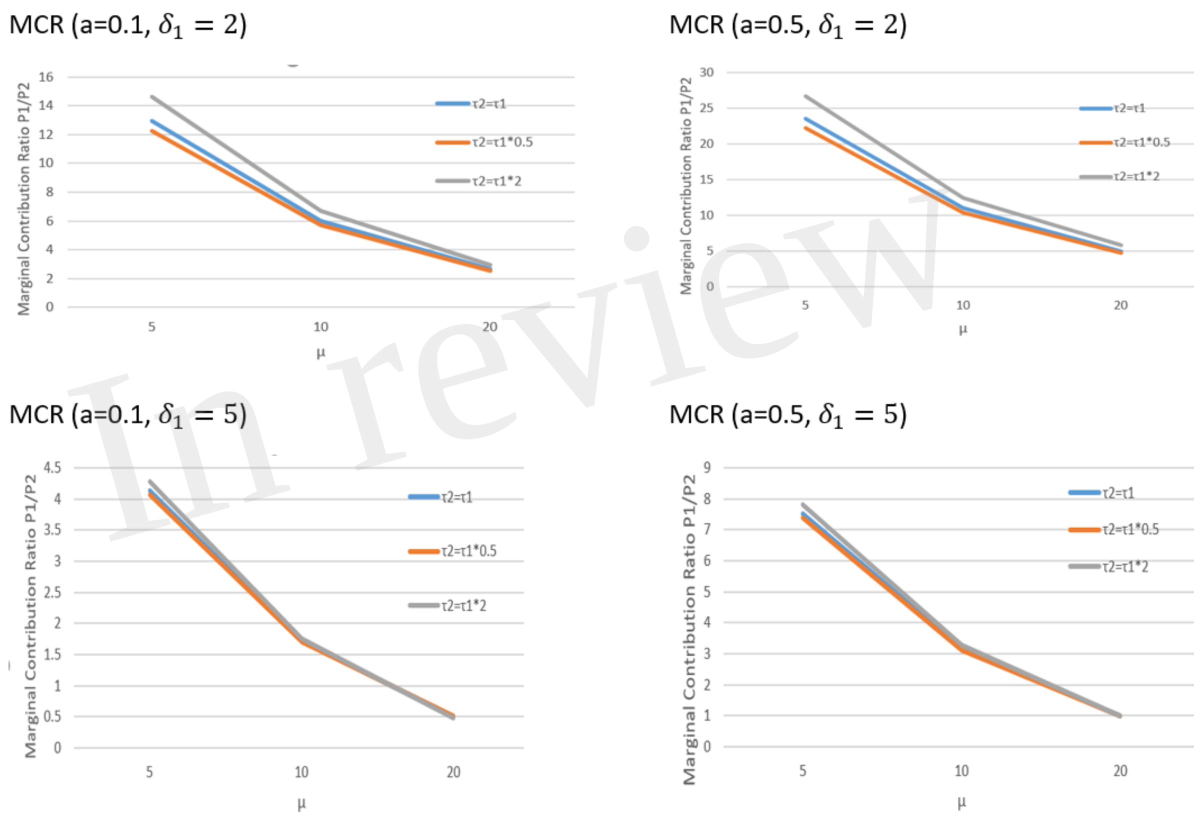

Figure 4: Shapley value findings. The variation of the marginal contribution ratio of the two players is explored as a function of the difference in perceived and actual ambiguity levels for the second mover $(\boldsymbol{\mu})$, variable costs $(\boldsymbol{a})$, and the difference in perceived and actual resource commitments of the first mover $(\boldsymbol{\delta_1})$.

Next, we present the findings of the Shapley values of players involved in the coalition in terms of the marginal contribution ration (MCR). From Figure 4, we see that as μ increases, MCR decreases. The MCR is directly proportional to the ambiguity ratio, and affirms the inversely proportional relationship between the ambiguities experienced by the two players that was shown in Figure 3. This continues in the form of decreased contribution by each player to the coalition resulting in a decrease in the MCR. As τ_2/τ_1 increases, MCR increases. The variables τ_1 and τ_2 denote the benefits to the first mover P_1 and second mover P_2 respectively in the form of reduction in work loads due to the formation of a coalition. We see that an increase in τ_2/τ_1 denotes an increase in the benefits to each player from the coalition and so each player brings higher value to the coalition which is manifested in the form of an increase in the MCR. As variable costs denoted by α increase, MCR increases. This is because the MCR is directly proportional to the ambiguity ratio AR, and therefore, an increase in the costs signals greater ambiguity to the two players. As δ_1 increases, MCR decreases. An increase in the resource commitment decreases the MCR for the players since it denotes tasks of increasing complexity.

6. Kernel Representation for Contractual Graphs

In this section, we propose a simple kernel for contractual graphs that depict marketplaces of two-player coalitions formed to complete ambiguous tasks. Specifically, we address the question of how we can determine if two marketplaces of ambiguous tasks are similar. The implications of this question are several that range from evaluating marketplaces with similar resources, players and tasks. Several metrics can be used to study the effectiveness of a marketplace, such as number of coalitions, average size of coalitions and average task completion rate. An analysis of kernels of successful marketplaces can yield insights about the factors that significantly impact the efficiency of a marketplace of ambiguous tasks.

Since our model assumes two-player coalitions, we use the Shapley value of a two-player coalition developed in the previous section to build the kernel. Assume there are n players in a marketplace, which results in potential $n!/(n-2)!$ coalitions of two-players. A player can be a part of multiple coalitions. Each coalition represents an edge on the contractual graph, and the weight of the edge is denoted by the Shapley value of the coalition. As derived in the above section, the Shapley value is dependent on the order in which the players form the coalition.

Consider two contractual graphs in Figure 5. The Shapley value of each coalition is arked on the edge. Two graphs are considered similar if the sums of Shapley values of coalitions are equal.

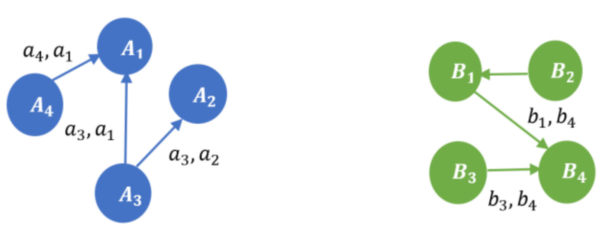

Figure 5. Contractual graphs G_1 (left, in blue) and G_2 (right, in green) are depicted. Each directional edge represents a contract in a 2-player coalition to complete an ambiguous task. The Shapley value of a coalition is the weight of each edge. For example, the edge with weight (a_4, a_1) represents the coalition A_4A_1.

We now present the conditions under which two contractual graphs are considered similar. Let graph $G_1 \equiv (V_1, E_1)$, where V_1 is the finite set of vertices in graph G_1. Let E_1 be the set of edges in graph G_1, such that $E_1 \subseteq \{\{u, v\} \subseteq V_1 | u \neq v\}$. An edge (u, v) denotes a coalition in which u is the first mover and v is the second mover in the coalition. Assume that there are N tasks of varying ambiguity in a marketplace. Consider another contractual graph G_2, such that $G_2 \equiv (V_2, E_2)$. Similar to G_1, E_2 and V_2 are the set of vertices and edges in G_2 respectively. Let the number of edges in graphs G_1 and G_2 be denoted by n_1 and n_2 respectively.

Two graphs G_1 and G_2 are similar if and only if they satisfy the following three properties.

1. Edge cardinality equivalence: The number of edges in graph G_1 is equal to the number of edges in graph G_2, i.e. $n_1 = n_2 = n$.
2. First mover equivalence: The sum of the Shapley values of the first movers in graph G_1 is equal to the sum of the Shapley values of the first movers in graph G_2. Thus,

$$\sum_{i=1}^{n} u_{i-G_1} = \sum_{i=1}^{n} u_{i-G_2}$$

where, u_{i-G_1} and u_{i-G_2} are the Shapley values of first movers in graphs G_1 and G_2 respectively.

3. Second mover equivalence: The sum of the Shapley values of the second movers in graph G_1 is equal to the sum of the Shapley values of the second movers in graph G_2. Thus,

$$\sum_{i=1}^{n} v_{i-G_1} = \sum_{i=1}^{n} v_{i-G_2}$$

where, v_{i-G_1} and v_{i-G_2} are the Shapley values of second movers in graphs G_1 and G_2 respectively.

This theorem can be generalized to k contractual graphs to establish an upper bound on the Shapley values of all coalitions.

Theorem: The upper bound of the sum of the Shapley values of the coalitions in k contractual graphs is given by y, where y is given by

$$y \leq \sum_{s=1}^{k} \left(\sum_{i=1}^{n} u_{i-G_s} + v_{i-G_s} \right)$$

7. Future Work

This paper presented an introduction to the theory of contractual graphs, by modeling contracts are commitments between two players to complete an ambiguous task. The theory of contractual graphs can be extended in several ways, some of which are summarized below.

Role of incentives: While ambiguity of tasks can represent hurdles in task completion, incentives might be able to alleviate some of the costs involved in accepting tasks with high ambiguity. Additionally, incentives might be able to nudge the formation of coalitions thereby encouraging second movers to collaborate frequently with first movers on task completion. The role of incentives in economic literature has been widely studied (Benabou and Tirole, 2006), (DeMarzo and Sanikov, 2016). In (Benabou and Tirole, 2006), where the authors explore the basis of incentives in psychology. Specifically, the role of incentives in intrinsic, extrinsic and reputational motivations have been considered in the development of game-theoretic models to gain insights into individual contributions and interactions. Graph theoretic representation learning can be enriched with the incorporation of additional sociological constructs such as identity (Akerlof and Kranton, 2000) to explain outcomes of interactions between nodes.

Negotiations over time: Our model for studying the performance of two-player coalitions in solving ambiguous tasks uses a single-stage contract. That is, the first mover assesses the ambiguity and creates a contract that the second mover accepts. In practice, however, contract formation goes through multiple stages of bargaining and negotiations over the terms of the contract. Additionally, problems related to imperfect information such as moral hazards and adverse selection can significantly impact the outcome of the contract formation, interpretation and execution.

Spectrum of perceived ambiguity: In our model, we chose a binary system for modeling ambiguity (high/low). While easier to model binary choices, in practice, ambiguity lies along a spectrum. Understanding the impact of ambiguity for a range of values between 0 and 1 can help further illustrate its impact on the formation of a contract.

Backing off from contracts with penalties/prorated benefits: While our model assumes that players commit to the contract and stay committed until task completion, it would be worth investigating how players would behave if they had the option to exit the contract. Similar to real-world situations where premature contract termination results in penalties or prorated rewards, modeling a marketplace of ambiguous tasks with players who have the option of exiting the contract would provide insights into real-world situations. Examples of such situations include students who sign up for classes and withdraw, renters who terminate the lease before its due date and employees who leave prior to the end of their probationary period. In all of these cases, embedded penalties/prorated rewards exist in the contractual terms and it would be worth investigating those through the lens of ambiguity. The players who have exited the contract are now freelancers of some sort, and how they impact the dynamics of the marketplace would be an interesting direction for future research.

Coalitions of multiple players: While our model chose two-player coalitions, extending this model to n-player coalitions would be beneficial. In practice, two-player coalitions are rarer than n-player coalitions. Understanding how coalitions of n players react to ambiguous tasks and each other's perceptions of ambiguity will have a significant impact on the performance of the coalition.

8. Conclusions

Contractual graphs arise in multiple situations. The agents involved in contractual graphs form connections among themselves upon the fulfilment of underlying conditions. For example, sending a packet from one node to another only when the packet is received with minimal distortion can be viewed as contract between two nodes only to transmit high-fidelity data. Another example of a contractual graph could be the formation of a connection in a social network, where two individuals form a "friend" connection only if they are separated by

n friends. In this case, the contract is fulfilled only if there exist fewer than n degrees of separation. This paper looked at the case of smart contracts as an application of a contractual graphs, but as noted in the examples above, contractual graphs can be used to study a variety of graphs where the formation of edges is dependent upon the completion of specific conditions. However, the conditions underlying the formation of an edge are not always specific enough to be readily automated in software. This motivates the need for a study into how ambiguity impacts contract formation. Thus, we studied ambiguous marketplaces, defined as marketplaces containing tasks of varying ambiguity that players could seek to complete in two-player coalitions.

We studied how the perception of ambiguity impacts the resources allocated by a player to the task. Using a game-theoretic formulation, we developed a model that assigns payoffs to players for the tasks completed. We distinguished between how the order in which players proceed to enter the two-player coalition affects the value that each player brings to the coalition. This metric was quantified using the Shapley value, that showed how the ambiguity of the task, costs of performing the task and resource allocation all played a part in determining the performance of the coalition. The Shapley value of a two-player coalition was then used to assign a tuple of edge weights. The set of all edge weights then formed a graph kernel for the contractual graph. We derived the conditions under which two contractual graphs are similar using edge cardinality, first-mover and second-mover equivalence. Finally, we proposed an upper limit on the sum of Shapley values of coalitions in k contractual graphs.

Contractual graphs offer several avenues for exploration due to their importance to network science. For example, we have assumed two-player coalitions where players complete the task, once they enter into the contract. In practice, however, coalition sizes may vary and players may have the option to leave the coalition even after they have entered into a contract. Similarly, new players may join the coalition even after the task has begun execution. Contractual graphs can be studied broadly, as well as in several niche environments. A better understanding of the behavior of contractual graphs can lead to the development of effective representative kernels, which in turn would facilitate learning about graphs.

9. References

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