Final Review Questions

MECS 4602 Final Review

Q1. What are the conditions for a 3x3 matrix to represent a valid rotation in 3D space?

1. It needs to be orthonormal.
2. $|R|$ = 1 (determinant equal to 1)
3. $R^{-1} = R^T $

Q2. What is one advantage of representing a 3D rotation as a quaternion as opposed to a rotation matrix?

When the number of the variables that a rotation matrix contains is big (e.g. a total of 9 variables), the matrix itself will be complicated and such much extra information is a drawback particularly for applications where storage space is a premium. But quaternion only has 4 variables.

Q3. How is the rank of a matrix defined?

The dimensionality of $im(A)$ is equal to the rank of A.

the **column rank** of a matrix is equal to the maximum number of linearly independent column vectors.

the **row rank** of a matrix is equal to the maximum number of linearly independent row vectors.

column rank = row rank = **rank(A)**

Q4. What is the rank-nullity theorem?

$dim(im(A)) + dim(null(A)) = n$ $rk(A) + dim(null(A)) = n$

n is the number of columns of A.

When m>n or m=n , if matrix is full-rank , dim(null) = 0, if matrix is not full-rank, then dim(null)>0.

When m < n, matrix always has a non-0 nullspace , dimensionality is at least n-m.

Q5. When performing Singular Value Decomposition $A=U|Sigma V^{T}$,what is the structure of the $|Sigma$ matrix and what does it tell us about the rank of A ?

1. $\Sigma \in R^{m\times n}$ is a diagonal matrix, with diagonal entries in descending order.The entries on the diagonal of Σ are called **singular values**. It has exactly r non-zero entries, where $r = rk(A)$.

2. By counting the number of non-zero singular values, we will know the rank of the matrix.

Q6. For a full-rank matrix $A |in R^{m |times n}$ how many exact solutions does the linear system $Ax=b$ have for any $ b |in R^m $ in each of the following cases: m>n,m=n and m < n?

I DO NOT UNDERSTAND THE CONTENT IN THE HANDOUT!!

Q7. If a robot arm is operating in 3D space, what is the smallest number of joints it must have in order to have a redundancy in any configuration? Explain why that is the case.

7

Q8. How can we tell that a robot arm is in a singular configuration?

??? Jacobian is rank-defficient

det(J) = 0

Q9. What are the practical implications of a robot arm being in a singular configuration?

Practical implication: we change a joint value, the end-effector does not move..

Q10. How many dimensions does the configuration space of the Kuka LWR arm have?

It has seven dimensions, for there are seven revoluted joints.

Q11. What is the main advantage of the $A^{}$ **algorithm compared to Dijkstra's algorithm?***

A^{*} algorithm uses a heuristic to estimate how long the path from a cell to the goal could be . This makes this algorithm much faster in practise.(Though in worst case, it performs as bad as Dijkstra's algorithm.)

Q12. Which of the following algorithms guarantees that it will return the shortest possible path between the start and the goal:Dijkstra's,A,RRT,PRM?

Dijkstra's and A star algorithm.

Q13. If a joint trajectory as a function of time is composed exclusively of straight line segments, why is the trajectory impossible to execute on real robots?

Linear interpolation provides continuous position , but discontinuous velocity and acceleration.

Specifically, we want the robot to immediately stop right at the goal point. If the joint trajectory is composed exclusively of straight line segments, it means the acceleration is infinite, which is impossible to execute on real robots.

Q14. What is the advantage of computing a timing function for a trajectory segment using a bang-coast-bang profile, rather than using a polynomial segment?

With bang-coast-bang profile, we can make the path to be executed as fast as possible(or at least close to that), comparing to a polynomial segment(The velocity and acceleration might be below than what the joint can actually do.)

Q15. What are the advantages of a J-transpose method for Cartesian control as opposed to a J-inverse method?

1. J-transpose does not require matrix inversion.
2. It does not require special care around singularities.
3. It control explicitly regulates end-effector forces that the robot applies.

Q16. What is the role of the "prediction" and "update" steps during recursive state estimation?

Q17. What are the parameters that define a multivariate normal distribution?

$$ f_x(x) = \frac{1}{\sqrt{2 \pi |P|}}e^{- \frac{1}{2} (x - \bar{x})P^{-1}(x - \bar{x})^T} $$

where $\bar{x}$ is the mean vector and P is the covariance matrix.

Q18. Under what conditions is the Kalman Filter the optimal method for estimating the state of a system?

A linear system with white noise. ???

Q19. How does a Kalman Filter express uncertainty in its estimate of the state?

It is based on merging the uncertainty of our prediction with the uncertainty of our sensor information.

Q20. What is the main difference between a Kalman Filter and an Extended Kalman Filter for state estimation?

The Kalman Filter applies to linear systems.

Extended Kalman Filter applies to non-linear system.

Q21. What are the advantages of a particle filter over an Extended Kalman Filter for state estimation?