Time Series. HW1. Roujie Liu.

1. (a) $Var(X+y) = Var(x) + Var(y) + 2Cov(X,y) = 9+4+2\times Cov(X,y)$

$Cov(X,y) = Corr(X,Y) \times \sqrt{Var(X) Var(Y)}$

$= 0.25 \times \sqrt{4\times 9} = 1.5.$

$Var(X+Y) = 4+9+3 = 16.$

(b) $Cov(X, X+Y) = Cov(X,X) + Cov(X,Y)$

$= Var(X) + Cov(X,Y)$

$= 9 + 1.5 = 10.5.$

(c) $Corr(X+Y, X-Y) = \frac{\cancel{Cov(X+Y, X-Y)}}{\sqrt{Var(X+Y) Var(X-Y)}}$

$Cov(X+Y, X-Y) = Cov(X, X-Y) + Cov(Y, X-Y)$

$= Cov(X,X) - Cov(X,Y) + Cov(Y,X) - Cov(Y,Y)$

$= Var X - 1.5 + 1.5 - Var Y$

$= 9-4 = 5.$

$Var(X-Y) = Var X + Var Y - 2Cov(X,Y)$

$= 9+4 - 2\times 1.5 = 10.$

$Corr(X+Y, X-Y) = \dfrac{5}{\sqrt{16\times 10}}$

$= \dfrac{5}{4\sqrt{10}}$

2. $Cov(X+Y, X-Y) = Cov(X, X-Y)$

$+ Cov(Y, X-Y) = Cov(X,X) -$
$Cov(X,Y) + Cov(Y,X) - Cov(Y,Y)$

$= Cov(X,X) - Cov(Y,Y) =$
$Var(X) - Var(Y) = 0.$

3. $E(Y_t) = E(5+2t+X_t)$

$= 5+2t+E(X_t)$

$E(X_t)=0$ because 0 mean stationary series.

$E(Y_t) = 5+2t$

b. autocovariance function $\gamma_t = Cov(Y_t, Y_{t-k})$

$= Cov(5+2t+X_t, 5+2(t-k)+X_{t-k})$

$= sign(bd) \, Cov(X_t, X_{t-k})$

$= Cov(X_t, X_{t-k})$

$\gamma_t = \gamma_k$

c. Y_t is not Stationary $E(Y_t) =$

$5+2t$ and t is time. The function is dependent on time, so it is not stationary.

4. ACF-A shows that the series is seasonal, and strong peak at lag 6, 12 shows seasonality of 12 months, and this match Series 1.
ACF - B shows the series is seasonal, and strong peak at 5, 10 shows seasonality of 10 lags/years and this match Plot 3.

ACF - C shows the slowly decaying ACF, and this means trend, and it matches the Plot 2.