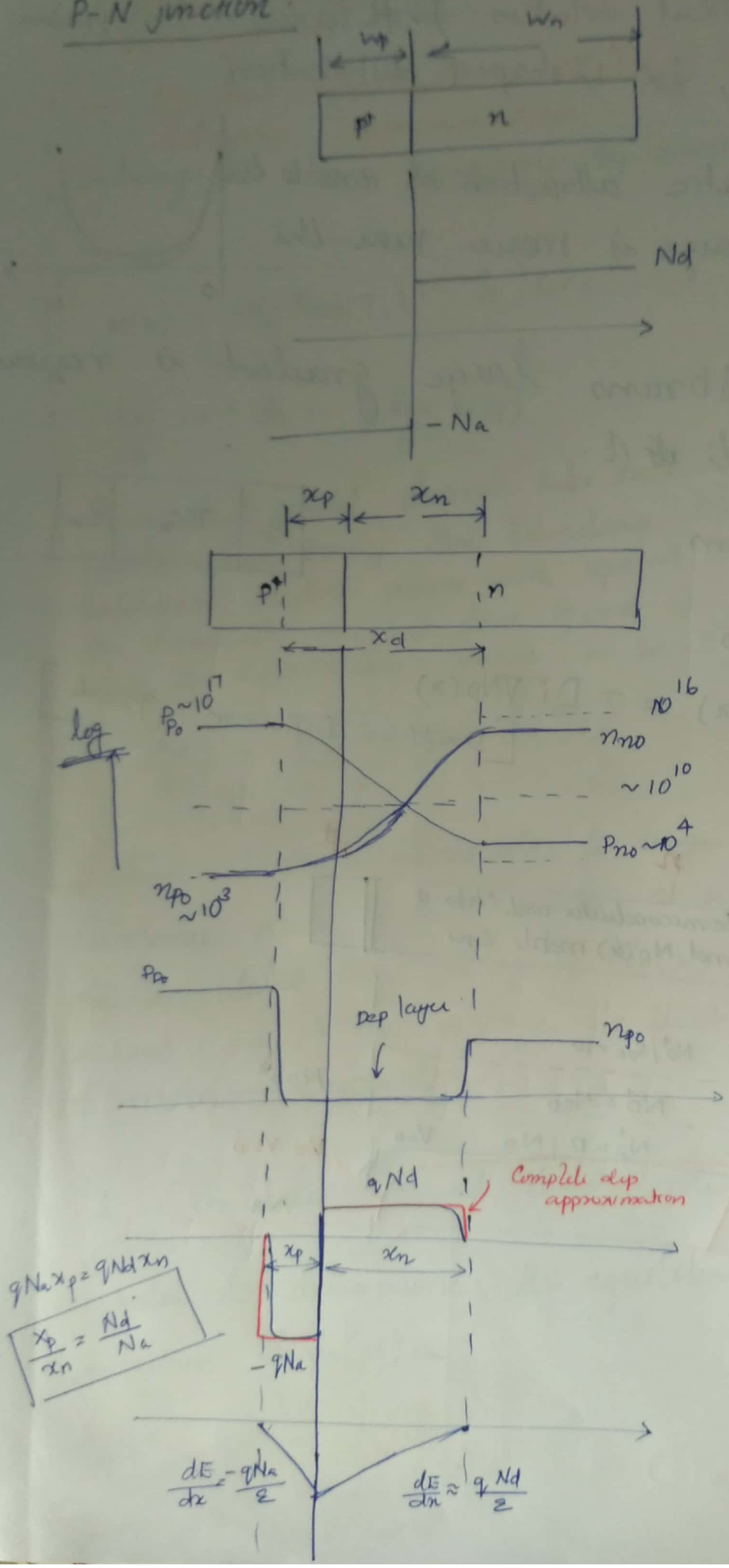
P-N junction



n, p, J_n, J_p, and E as fn of 'x'.

$x_d = x_p + x_n$

Space charge

$$\rho = q(p - n + N_d)_{n\,side}$$

$$= q(p - n - N_a)_{p\,side}$$

In dep region $p \approx n \approx 0$

Now Gauss' Law

$$\frac{dE}{dx} = \rho/\varepsilon$$

on p $\frac{dE}{dx} = \frac{-qN_a}{\varepsilon}$

on n $\frac{dE}{dx} = \frac{+qN_a}{\varepsilon}$

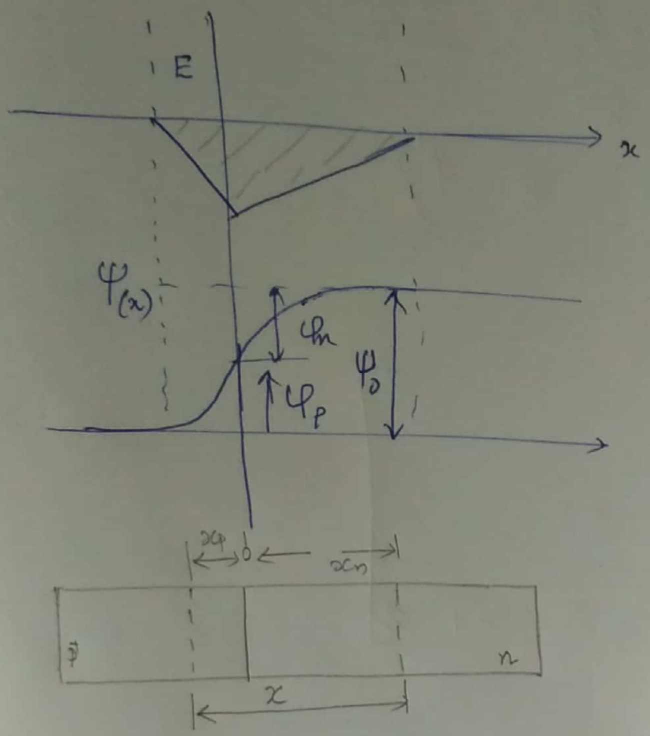
By Poisson eqn $\quad \nabla^2\psi(x) = \rho/\varepsilon$

substituting $E = \nabla\psi(x)$ in above

$$\frac{d^2\psi(x)}{dx^2} = \rho/\varepsilon$$

$\psi_0 = V_t \ln \frac{N_c N_a}{n_i^2}$

$X = \sqrt{\frac{2\varepsilon\psi_0}{q}\left(1/N_a + 1/N_D\right)}$



→ equations

→ Now how to find $E(x)$, $\psi(x)$, $n(x)$ and $p(x)$?

→ Analytical solutions by integration

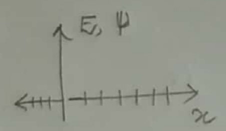
→ Numerical solutions by FDM

$E = cx$

1) $\frac{dE}{dx} = \frac{-qN_a}{\varepsilon} \qquad 0 < x < x_p$

2) $\frac{dE}{dx} = \frac{qN_d}{\varepsilon} \qquad 0 < x < x_n$

3) $\frac{d^2\psi}{dx^2} = \rho/\varepsilon$



Boundary conditions.

$E(x_n) = E(x_p) = 0$

$E(0) = E_{max}$

$\psi(x_n) = 0 \qquad \psi(x_p) = \psi_0$

$\psi_0 = V_t \ln \frac{N_a N_d}{n_i^2}$

Solution using FDM.

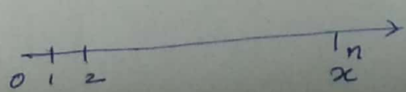
1, 2

$$\frac{dE}{dx} = \rho/\varepsilon \quad \text{for} \quad 0 < x < x_p$$
$$\cancel{0 < x} \; -x_n < x < 0$$

3

$$\frac{d^2\psi(x)}{dx^2} = \rho/\varepsilon \qquad -x_n < x < x_p$$

$x_j = j/n+1 \; ; \; j = 0, 1, \ldots n+1$



n nodes

x_j nodepoints

$$\left.\frac{dE}{dx}\right|_{j+1} \approx \frac{E(x_{j+1}) - E(x_j)}{h} \qquad \frac{dE}{dx} = f(x)$$

$$E_{j+1} = E_j + \Delta h f_{n+1}$$

$$-\frac{d^2\psi}{dx^2}(x_j) \approx \frac{-\psi(x_{j-1}) + 2\psi(x_j) - \psi(x_{j+1})}{h^2}$$

$$-u_{j-1} + 2u_j - u_{j-1} = h^2 f'_j$$

$$Tu = h^2 f'$$

$$T = \begin{bmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & \\ 0 & -1 & 2 & -1 & \\ & & & & \\ & & & -1 & 2 \end{bmatrix}$$

$$\boxed{\psi = T^{-1} h^2 f'}$$

Steps

→ Derive the corresponding poisson equation

→ Initialize charge distribution [From analytical & intuitive modeling]

→ Make T matrix $\quad y = 1 + \sqrt{1 \pm 4x^2}$

→ Make $h^2 f'$ matrix by applying boundary conditions

→ Find ψ.