**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans) we have *μ* = 45, *σ* = 8

To find that the service manager cannot meet his commitment the time taken to complete the work should be more than 50 minutes so we need to find P(X>50) which means

= 1 - P(X<=50) = 1 - P(Z<=(x-*μ*)/*σ*)

= 1 - P(Z<=(50-45)/8)

= 1 - P(Z<=0.625)

= 1 - 0.73237 (probability of a Z score)

= 0.2676

The answer is **B. 0.2676**

2- The current age (in years) of 400 clerical employees at an insurance claims processing

center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each

statement below, please specify True/False. If false, briefly explain why.

1. More employees at the processing center are older than 44 than between 38 and 44.

Ans) we have *μ* = 38, *σ* = 6

To find More employees at the processing center are older than 44 we need to find probability P(X>44) which means

= 1 - P(X<=44) = 1 - P(Z<=(x-*μ*)/*σ*)

= 1 - P(Z<=(44-38)/6)

= 1 - P(Z<=1)

= 1 – 0.8414 (probability of a Z score)

= **0.1586**

To find More employees at the processing center are between 38 and 44 we need to find probability P(X<=44) - P(X>=38) which means

= P(X<=44) - P(Z>=(x-*μ*)/*σ*)

= P(X<=44) - P(Z>=(38-38)/6)

= P(X<=44) - P(Z>=0)

= 0.8414 - 0.5 (probability of a Z score)

= **0.3414**

**True,** More employees at the processing center are older than 44 than between 38 and 44.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans) we have *μ* = 38, *σ* = 6

To find no of employees to be attracted to the training proram for employess under age 30 we need to find probability P(x<30) which means

= P(Z<=30) = P(Z<(x-*μ*)/*σ*)

= P(Z<(30-38)/6)

= P(Z<-1.333)

= 0.912 (probability of a Z score)

We can get the no employees by N \* P(x<30)

= 400 \* 0.912

= **36.484**

**True,** training program for employees under the age of 30 at the center can be expected to attract about 36 employees.

3- If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the

difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans) Given *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2)

2 *X1* ~ *N*(2μ, 4σ2) ………(i)

And *X1 + X*2 ~ *N*(μ + μ, σ2 + σ2 )

*X1 + X*2 ~ *N*(2μ, 2σ2) ………..(ii)

Now 2 *X*1 – (*X*1 + *X*2) ~ *N*(2μ - 2μ, 4σ2 + 2σ2) …(substituting values from i and ii)

2 *X*1 – (*X*1 + *X*2) ~ *N*(0, 6σ2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans) The probability of ettin value between a and b should be 0.99.

Here we have P(a<x<b) = 0.99, *μ* = 100, *σ* = 20

Directly from formula stats.norm.interval(0.99,100,20)

= **(48.48341392902199, 151.516586070978)**

Option **D.** is the correct answer.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

we have combined mean of both profits = 5 + 7 = 12

Mean profit in Rs = 12\* 45 = Rs. **540** million

combined variance of both profits = √ (32 + 42 ) = 5

std.dev in Rs = 5 \* 45 = Rs. **225** million

1. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans) we have combined mean of both profits = 5 + 7 = 12

Mean profit in Rs = 12\* 45 = Rs. 540 million

combined variance of both profits = √ (32 + 42 ) = 5

std.dev in Rs = 5 \* 45 = Rs. 225 million

Range for the annual profit of the company with 95% probability

= stats.norm.interval(0.95,540,225)

= **(99.00810347848784, 980.9918965215122)**

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans) For the 5th percentile of profit x we can use Z= (x - *μ* )/ *σ*

Therefore x = Z*σ*  + *μ*

= (-1.645)\*(225) + 540 (here Z=-1.645 for 5th percentile)

= **170** million in Rupees.

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans) Probability of division 1 making a loss P(x<0)

= stats.norm.cdf(0,5,3)

= **0.047790352272814**

Probability of division 1 making a loss P(x<0)

= stats.norm.cdf(0,7,4)

= **0.040059156863817**