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Outline of project:

Modelling the orbits of supermassive black holes in hydrodynamical simulation of galaxy formation.

Motivation: Supermassive black holes are ubiquitously observed in the centers of galaxies, and they play a critical role in current theories for galaxy formation, where they are supposed to suppress star formation in large galaxies by injecting energy into the gas. We hence want to simulate the growth of these black holes and the associated coevolution with the galaxy when studying numerical models of galaxy formation. In reality, it is believed that the black holes experience dynamical friction against the background of dark matter and stars, and possibly also through gas-dynamical processes, making them spiral in to the centers of galaxies on a reasonably short timescale. Only when they are positioned there, they can efficiently influence the whole galaxy and grow rapidly.

After galaxy mergers, the remnant (merged) black hole will return to the centre through these friction processes, after being kicked out through asymmetric emission of gravitational waves. To properly estimate how long the growth/feedback may be weak/interrupted after a merger (because the centre is not yet found again), and how many free floating massive black holes there may be, the orbit of the black holes needs to be followed reasonably accurately.

Problem: This is however not readily possible, because the mass of dark matter and star particles in N-body simulation is very much larger than in reality and similar to the mass of the supermassive black hole. As a result, two-body scattering effects will try to "heat-up" the central black hole particle and prevent it from experiencing proper dynamical friction, or in other words, the black hole will not return to the centre of the potential by itself under these conditions. Current simulation models therefore usually employ ugly tricks to "glue" the black hole particle to the potential minimum, for example by searching for the smallest black hole potential value among neighbors around the black hole, and then simply positioning the black hole particle to this minimum. This is for example done by the Illustris and Gigagalaxy projects. While this prevents that the central black hole particle is lost, it also prevents that the above questions can be studied, and potentially one also introduces severe inaccuracies in the efficiency with which black holes can grow.

Possible solutions: One approach for improving the modelling would be to augment the equation of motion for the black hole particle by an explicit dynamical friction force. This can be done in different ways.

- (1) One can try to add a suitably modified version of Chandrasekhar's dynamical friction formula, similar to how this is attempted in Tremmel et al. (2015, MNRAS, 451, 1868, arxiv:1501.07609). This involves some assumptions and technical approximations. In the formulation of Tremmel, they arranged it such that the force becomes ever weaker in the limit of infinite resolution, so that one argue it is a correction for the finite resolution of real-work simulations. However, the tests presented in the paper are not fully convincing and it is still unclear (certainly to me), whether this method works sufficiently well in practice (e.g. for galaxy merger simulations, and for cosmological simulations of galaxy formation).
- (2) Because it not really clear whether Chandrasekhar's formula applies well when the BH

is already close to the centre of the galaxy (where stars need to be ejected from the "losscone" and interactions with gas play a very important role), one may instead also conjecture different models. For example, if we assume that the BH is brought back efficiently to the potential minimum if it is displaced from the centre, we can construct an "optimum" friction force as follows: At the centre, the gradient of the potential is zero by definition, so that it can be approximated as quadratic to first order. The BH is hence expected to carry out harmonic oscillations around the centre. We can now try to define an optimum damping force that eliminates the oscillation on the shortest possible timescale (if the friction is too large, the motion will be "overdamped", and one takes longer to the centre than for a suitably smaller force). We can estimate the potential around the centre as $g(x) \sim phi_0 + 1/2 * (d^2phi/dx^2) * x^2$, and the second derivative along one direction can be estimated from Poisson's equation as (d^2phi/dx^2) ~ (1/3) (4 Pi G rho), where rho is the local total mass density. The harmonic oscillator's equation of motion will be $x'' = -om^2 * x = -dg/dx * x$, meaning that the oscillation frequency is om = sqrt[(1/3) (4 Pi G rho)]. For a critical damping we would then need to add a friction force as $x'' = -om^2 x - k x'$, where k = 2 om. In this case, we expect the orbit to decay on the local free fall timescale.

Work steps:

The above idea corresponding to (2) is implemented in AREPO in a first version, through the switch BH_FRICTION. This implementation tries to determine a reference velocity of the potential minimum by measuring its position in the environment of the BH, and then taking finite differences to estimate this velocity (this is needed because for applying the friction force antiparallel to the velocity one needs to know the local reference frame). The oscillation frequency is estimated from the total matter density, and from this a friction force is estimated antiparallel to the velocity agains the local frame of rest, with a tunable coefficient in front of order unity.

- As a first step, the implemented equations need to verified. This also requires that I explain you the many (in part quite complicated) technical aspects of the code that play a role here
- We should then test the effects this implementation has, and how well it is working, in a number of situations:
- (a) Simulations of Milky Way sized galaxies. Here, an important diagnostic would be to develop a plot script that shows the distance of BHs relative to the potential minima of the
- corresponding halos. This can be done based on the group catalogues, I think. First, one could look at the current state of affairs in some of the finished production simulations of Rob and Ruediger. Next, one should repeat one or two of these runs by disabling the BH centering that had been used (to see how bad the displacements get if one doesn't try to correct them). Finally, one should try one or several variants of the friction treatments above, starting with method (2) and later also with method (1).
- (b) Toy simulations of an isolated halo where one injects a BH with some velocity and impact parameter to study the decay of the orbit. Again, one should compare different friction treatments (ideally also at different resolution) with the results where no such thing is applied.

(c) Finally, one repeats the test of (a) for a cosmological simulation, where one can compile statistical results, e.g. about the distribution function of the average distances of BHs to their halo centers.

Project extension (phase two):

Adding BH merger kicks

Motivation: Whenever supermassive black holes merge during galaxy mergers, they emit a burst of gravitational wave radiation. This happens in an symmetric way such that there is a quite large recoil, kicking the BH out of the centre. It could even happen that the BH leaves the remnant halo entirely, but usually, it probably returns after some time. During this period, the galaxy may then grow unimpeded by the BH. We would like to find out how strongly predictions by galaxy formation are modified when the BH recoil kicks are accounted for. To this end, one can apply fitting functions produced by numerical relativity simulations to kick BH merger remnants depending on their mass ratio, whenever this happens in a cosmological simulation (see for example Sijacki et al., MNRAS, 2011, 414, 3656, arxiv:1008.3313). Combined with a treatment of BH friction, the BHs are then expected to return to the centers after a finite time, so that these effects can be studied in modern simulations of galaxy formation.

Work steps:

- One first needs to compile recent results from numerical relativity simulations to get the most up-to-date fitting function for the strength of the recoil kicks as a function of BH mass ratio.
- One then needs to implement an option in AREPO that adds a kick with the corresponding strength after the merger of two black holes.
- Then, one repeats a cosmological simulation of galaxy formation with the kicks enabled, and compares to the equivalent one without kicks.
- The quantities to focus on are, in particular, the galaxy luminosity function at the bright end, the BH mass function, and the color distribution of galaxies. Also interesting would be histograms that quantify the distance of BHs from the centre, the time of return to the centre, the number of free floating BHs, etc.
- Implications for the merger rate of BHs, and the distribution of the mass ratios, would also be particularly interesting as this is directly related to the statistics of gravitational wave emission events.