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Quiz 5

(Grading: 0–10 points)

1. Suppose that you are warping an 8-bit greyscale image of size  $256 \times 256$  into another greyscale image of size  $256 \times 256$ , using a warp whose inverse map is defined by:

$$u(x, y) = \frac{W}{2} \left( 1 + \cos\left(\frac{4\pi x}{W}\right) \right)$$

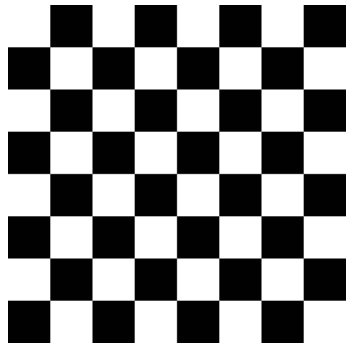
$$v(x, y) = \frac{y^2}{H}$$

Where  $W, H = 255$ .

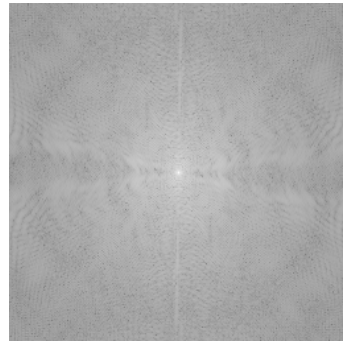
These equations are clearly nonlinear, but they are nothing a calculator cannot handle. The next four questions deal with this warp.

One way to better understand what this warp does is to see where pixels warp to and from. First, use the above equations to compute the  $(u, v)$  location of the pixel  $(120, 240)$ . Use a calculator / computer program, there is no need to show your work.

2. Next, estimate how much stretching is occurring at  $(120, 240)$ . This can be done by computing  $\partial u$  and  $\partial v$  at this pixel (see House 11.8.3). You do not need to differentiate the equations. Instead, approximate  $\partial u$  by estimating both  $\frac{\partial u}{\partial x}$  and  $\frac{\partial u}{\partial y}$  using finite differences. Specifically, estimate  $\frac{\partial u}{\partial x}$  as the difference in the  $u$  values for pixels  $(121, 240)$  and  $(119, 240)$  and divide by two. Similarly, estimate  $\frac{\partial u}{\partial y}$  using the  $u$  values for pixels  $(120, 241)$  and  $(120, 239)$ . You can then approximate  $\partial u$  as  $\max(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y})$ . Repeat this process for  $\partial v$ .
3.  $\partial u$  and  $\partial v$  indicate how much  $u$  and  $v$  change locally as you move around in the output image. A large change would indicate that pixels in the output image are taken from pixels that are far away in the input image. Similarly, small values for  $\partial u$  and  $\partial v$  indicate that nearby pixels are from closely grouped pixels in the input. What is the situation at  $(120, 240)$ ? Describe the types of artifacts you expect to occur at this location.
4. Consider implementing the above warp as an animated morph. What is a reasonable  $(u, v)$  value for the output pixel  $(120, 240)$  at time  $t = 0.3$ ? Assuming that  $t = 0$  refers to the original, unmorphed image and that  $t = 1.0$  refers to the output image completely morphed. There is more than one acceptable answer: be sure to explain how you arrived at this  $(u, v)$  and show your work.
5. Examine the four images on the next page using a pdf viewer (a printed version may be hard to see). Which image (a)-(d) matches which FFT (e)-(h)?



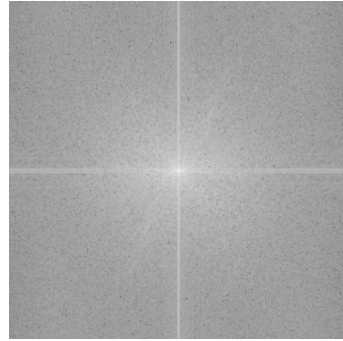
(a)



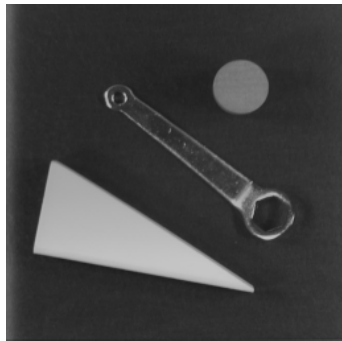
(e)



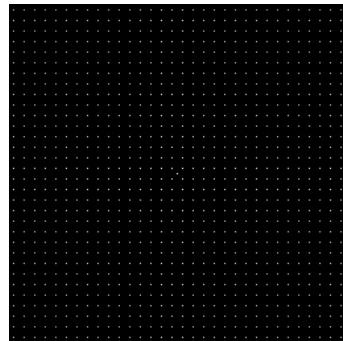
(b)



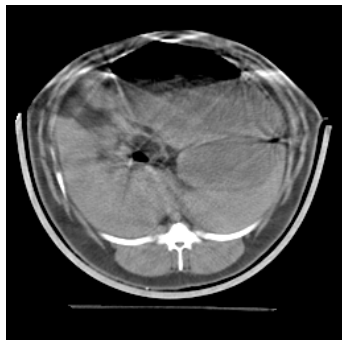
(f)



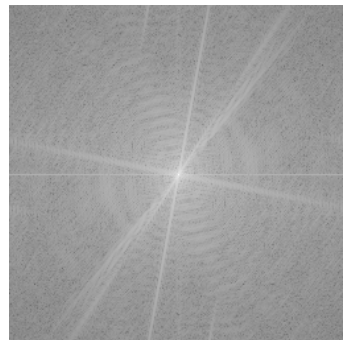
(c)



(g)



(d)



(h)

Figure 1: Input Images (left) and Input FFTs (right)