CPSC 4040/6040
Computer Graphics Images

Assigned: Oct. 24, 2015
Due: Nov. 3, 2015 (start of class)

Quiz — Extra Credit *(Grading: 0–10 points)*

This quiz is an opportunity for to practice working with matrices and seeing the relationship between linear algebra and image warps. This quiz will be graded and valued as a normal quiz, and added to your final quiz grade for the class.

For each question, you are encouraged to think about the implications of each question that you're given. When possible, even if not explicitly stated, consider what would happen to an image as a result of each of these warps. Try to be comfortable both with the algebra as well as the conceptual issues.

1. Compute the (3×3) matrix for applying a translation of -50 pixels horizontally and 20 pixels vertically (call this matrix A). Next, compute the matrix for a rotation of 60 degrees *clockwise* (call this matrix B). Finally, compute the matrix for first applying the translation A and then applying the rotation B (call this C). Compare this with the matrix (called D) of first applying B and then translating by A. Does $C = D$? Show your work for the matrix multiplications.

2. Compute the inverse matrices for A and B. Show your work. Use Figure 10.2 from House's book. Next, compute the inverse of C. Finally, multiply $A^{-1}B^{-1}$. Compare this with C^{-1}.

3. Consider the following sequence of operations. First, take the matrix for shearing by a factor of -0.5 in the x direction (matrix E). Next, take the matrix for shearing by a factor of 0.8 in the y direction (matrix F). Compute the matrix $G = EFE$. What type of matrix is this?

4. Suppose that the matrix

$$H = \begin{bmatrix} 1 & 0 & 100 \\ 0 & 1 & -50 \\ 0 & 0 & 1 \end{bmatrix}$$

   determines a forward warp from the input image (u, v) space to the output image (x, y) space. What kind of warp is this? If the input image is 400 pixels wide $\times$ 300 pixels high, what will be the minimum required size for the output pixmap? Where does output pixel $(121, 76)$ come from in the input image? What about output pixel $(0, 0)$?

5. Consider the two matrices:

$$J = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \quad K = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1000 \\ 0 & 1 & 1 \end{bmatrix}$$

   that determine forward warps from the input image (u, v) space to the output image (x, y) space. What kind of warps are these? If an input image is 400 pixels wide $\times$ 200 pixels high, what will be the minimum required size for the output pixmap for each warp? Be careful. Compare your results with Fig. 9.11 on page 130 of House's book.