Mathematical Logic_Assignment_1

Hongli SHEN

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Contents

1 Set Operations

- $A = \{1, 2, 3, 4, 6, 12\} \qquad B = \{2, 4, 6, 8, 10\}$
  $A \cup B = \{1, 2, 3, 4, 6, 8, 10, 12\}$
  $A \cap B = \{2, 4, 6\}$
  $A - B = \{1, 3, 12\}$

- $A = \{F, E, A, S, T\} \qquad B = \{T, A, S, T, E\}$
  $A \cup B = \{F, E, A, S, T\}$
  $A \cap B = \{T, A, S, E\}$
  $A - B = \{F\}$

2 Find the Equivalence Classes

2.1

$S = \{\varnothing, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$
$[\varnothing]_R = \{\varnothing\}$
$[\{1\}]_R = \{\{1\}, \{2\}, \{3\}\}$
$[\{1, 2\}]_R = \{\{1, 2\}, \{1, 3\}, \{2, 3\}\}$
$[\{1, 2, 3\}]_R = \{\{1, 2, 3\}\}$
These four sets are disjoint

2.2

- $\because |m - 3| = |n - 3|$
  $\therefore the\ distance\ from\ m, n\ to\ 3\ are\ the\ same$
  $every\ m, n\ with\ the\ same\ distance\ are\ in\ the\ same\ equivalent\ class.$
  $[3]_R = \{3\}$
  $[2]_R = \{2, 4\}$
  $\ldots$
  $[3 - i]_R = \{3 - i, 3 + i\}$
  $\ldots$

- $\because m + n\ is\ an\ even\ number$
  $\therefore there\ are\ only\ two\ cases.$
  ① $m, n \in 2\mathbb{Z} + 1.$
  ② $m, n \in 2\mathbb{Z}.$
  $[1]_R = \{i \mid i \in 2\mathbb{Z} + 1\}.$
  $[0]_R = \{i \mid i \in 2\mathbb{Z}\}.$

3 Partial Order and Total Order Relation

- **the relation is not a partial order**
  we can guarantee that in the antisymmetric case, a must equal to c. However, the question does not require anything about b and d. Therefore, (a, b) *and* (c, d) may not equal.

e.g (a,b)=(1,2) (c,d)=(1,3). It obviously against the antisymmetric rule.
In conclusion, it is not a partial order relation and thus it is not a total order relation.

- $\because$ *for all* $(a,b) \in \mathbb{N} \times \mathbb{N}, (a,b) \preccurlyeq (a,b)$.
  $\therefore \preccurlyeq$ *is reflective.*
  $\because$ *for all* $(a,b),(c,d),(e,f) \in \mathbb{N} \times \mathbb{N}$, *if* $(a,b) \preccurlyeq (c,d)$, $(c,d) \preccurlyeq (e,f)$, *then* $(a,b) \preccurlyeq (e,f)$. $\therefore \preccurlyeq$ *is transitive*
  $\because$ *for all* $(a,b),(c,d) \in \mathbb{N} \times \mathbb{N}$, *if* $(a,b) \preccurlyeq (c,d)$ *and* $(c,d) \preccurlyeq (a,b)$, *then* $(a,b) = (c,d)$.
  $\therefore \preccurlyeq$ *is antisymmetric*
  In conclusion, it is a partial order relation. But it is not a total order relation.
  e.g if $(a,b) = (1,2)$ *and* $(c,d) = (3,4)$, *then neither* (a,b) *nor* (c,d) *is in* $\preccurlyeq$.

4 One-to-one and Onto Functions

- for every $\mathbb{Z}$ in the domain, there exists a corresponding $f(x)$ in $\mathbb{Z}$. But if $f(x) > 0$ and $f(x) \neq 2\mathbb{Z}$, then $x \notin \mathbb{Z}$
  In conclusion: the function is one-to-one.

- $\forall\ (x,y) \in \mathbb{R} \times \mathbb{R}$, there exists only one $(x+y, 3y) \in \mathbb{R} \times \mathbb{R}$
  $\forall\ (x+y, 3y) \in \mathbb{R} \times \mathbb{R}$, there exists only one $(x,y) \in \mathbb{R} \times \mathbb{R}$
  In conclusion: the function is both one-to-one and onto, which also called bijective.

5 Proof by Induction

① when n = 1, 1=$\dfrac{n(n+1)}{2}$, satisfied.

② for any $n \in \mathbb{N}$, suppose $1+2+\ldots+n \ = \ \dfrac{n(n+1)}{2}$

$f(n+1) \ = \ \dfrac{n(n+1)}{2} + (n+1) = \dfrac{n^2+3n+2}{2} \ = \ \dfrac{(n+1)(n+2)}{2}$, satisfied.
Then, for any $n \in \mathbb{N}$, satisfies the equation.