1 Assignment 1: Preliminaries (100 points)

1.1 Set operations (12 points)

Find the union, intersection, and difference $(A - B)$ of the following pairs of sets.

a) $A = \{x \mid x \in \mathbb{N}, \text{x is a factor of } 12\}$

$B = \{x \mid x \in \mathbb{N}, \text{x is a multiple of } 2, x < 12\}$

b) A = The set of all letters of the word FEAST

B = The set of all letters of the word TASTE

1.2 Find the equivalence classes (8 points)

Let S be the power set of $\{1, 2, 3\}$. R is an equivalence relation on S, defined as "set A and set B have the same number of elements". Please find the equivalence classes of R.

1.3 Find the equivalence classes II (20 points)

Find the equivalence classes for each of the following equivalence relations R on $\mathbb{Z}$.

a) $mRn \Leftrightarrow |m - 3| = |n - 3|$

b) $mRn \Leftrightarrow m + n$ is even

1.4 Partial order and total order relation (20 points)

Determine whether or not each of the following relations is a partial order and state whether or not each partial order is a total order. Explain why.

a) $(N \times N, \preccurlyeq)$ where $(a, b) \preccurlyeq (c, d)$ if and only if $a \leq c$.

b) $(N \times N, \preccurlyeq)$ where $(a, b) \preccurlyeq (c, d)$ if and only if $a \leq c$ and $b \geq d$.

1.5 One-to-one and onto functions (20 points)

Determine whether the following function is one-to-one/onto? Explain why.

a) The function $f : \mathbb{Z} \to \mathbb{Z}$ is defined by

$$f(x) = \begin{cases} 2x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

b) The function $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R} \times \mathbb{R}$ is defined by:

$$f(x, y) = (x + y, 3y)$$

1.6 Proof by induction (20 points)

Prove by induction that $1 + 2 + ... + n = \frac{n(n+1)}{2}$ for every positive integer n.