5 Assignment 5: FOL Basics (100 points)

5.1 Syntax (48 points)

Define the following symbols:

- Constants: a, b
- Functions: f^1, g^2
- Predicates: P^1, R^2, Q^3

Which of the following expressions are well-formed FOL formulas? In this question, a well-formed formula with parenthesis omitted by our convention can also be considered as a well-formed formula.

1. $Q(a)$
2. $P(y)$
3. $P(g(b))$
4. $\neg R(x, a)$
5. $Q(x, P(a), b)$
6. $P(g(f(a), g(x, f(x))))$
7. $Q(f(a), f(f(x)), f(g(f(z), g(a, b))))$
8. $R(a, R(a, a))$
9. $R(a, g(a, a))$
10. $g(a, R(a, a))$
11. $\forall x(\neg P(x))$
12. $\neg R(P(a), x)$
13. $\exists a R(a, a)$
14. $\exists x Q(x, f(x), b) \rightarrow \forall x R(a, x)$
15. $\exists x P(R(a, x))$
16. $\forall R(x, a)$

5.2 Formalization I (45 points)

Formalize the following sentences using FOL.

1. All Students are smart.
2. There exists a student.
3. There exists a smart student.
4. Every student loves some student.
5. Every student loves some other student.
6. There is a student who is loved by every other student.
7. Bill is a student.
8. Bill takes either Analysis or Geometry (but not both).
9. Bill takes Analysis and Geometry.
10. Bill doesn't take Analysis.
11. No students love Bill.
12. Every student takes at least one course.
13. Only one student failed Geometry.
14. No student failed Geometry but at least one student failed Analysis.
15. Every student who takes Analysis also takes Geometry.

5.3 Formalization II (7 points)

Consider a minimum language that doesn't have any non-logical symbols (constants, predicates, and functions). This language only has logical symbols.

Use this minimum language to formalize the following sentences:

- "There are more than 2 elements."
- "There are at most 2 elements."