Mathematical Logic_Assignment_3

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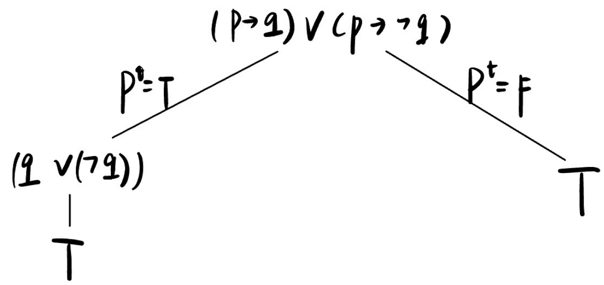
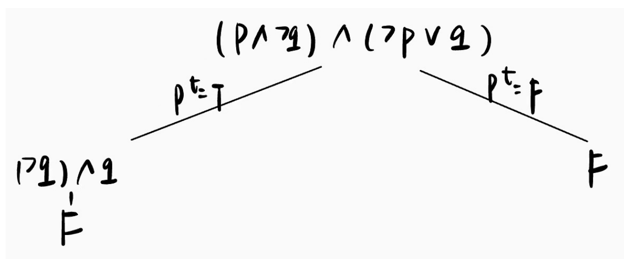
Contents

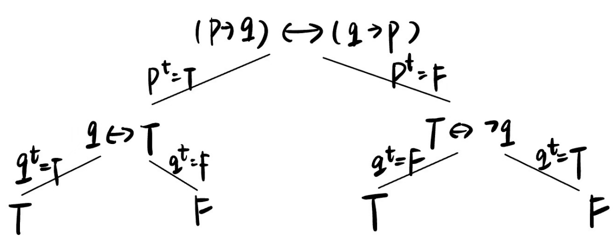
1 For each formula, whether it is tautology, contradiction, or neither?



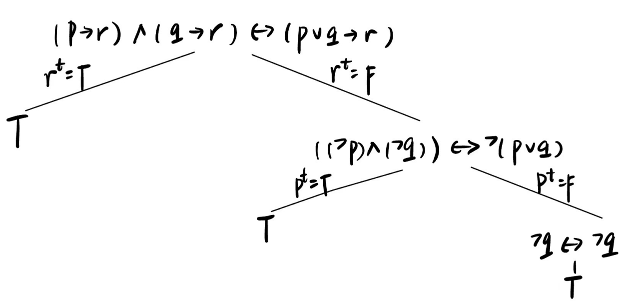
- **It is tautology.**



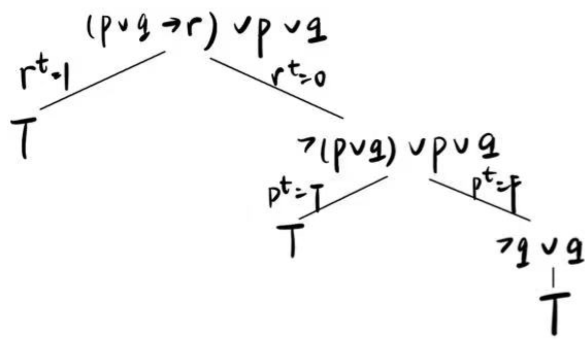
- **It is contradiction.**



- **It is neither tautology nor contradiction.**



- **It is tautology.**



- **It is tautology.**

2 Whether the following logical equivalences are correct?

- $(p \to (q \wedge (\neg q))) \equiv (p \to F) \equiv (\neg p)$
  The original formula is correct

- $(p \vee q) \wedge (\neg p \to \neg q) \equiv (p \vee q) \wedge (q \to p) \equiv (p \vee q) \wedge (\neg q \vee p) \equiv (p \vee q) \wedge (p \vee \neg q) \equiv p \vee (q \wedge \neg q) \equiv p \vee F \equiv p$
  The original formula is not correct.

- $((p \to q) \to q) \to q \equiv (\neg(\neg p \vee q) \vee q) \to q \equiv (p \wedge \neg q) \vee q \to q \equiv (q \vee p) \wedge (q \vee \neg q) \to q \equiv (q \vee p) \to q \equiv \neg(q \vee p) \vee q \equiv (\neg q \wedge \neg p) \vee q \equiv (q \vee \neg q) \wedge (q \vee \neg p) \equiv q \vee \neg p \equiv p \to q$
  The original formula is correct.

- $(p \to \neg q) \to r \equiv (\neg p \vee \neg q) \to r \equiv \neg(p \wedge q) \to r \equiv p \wedge q \vee r$
  The original formula is correct.

3 Whether the following logical consequences are correct?

p	q	$\neg p$	$\neg q$	$p \wedge \neg q$	$p \wedge q$	$p \wedge \neg q \to p \wedge q$
1	1	0	0	0	1	1
0	0	1	1	0	0	1
1	0	0	1	1	0	0
0	1	1	0	0	0	1

  The logical consequence is correct.

p	q	$p \to q$	$\neg p$	$\neg q$	$\neg p \to \neg q$
1	0	0	0	1	1
1	1	1	0	0	1
0	0	1	1	1	1
0	1	1	1	0	0

  The logical consequence is not correct.

p	q	$p \to q$	$\neg q$	$(p \to q) \wedge \neg q$	$\neg p$
1	1	1	0	0	0
0	0	1	1	1	1
1	0	0	1	0	0
0	1	1	0	0	1

  The logical consequence is correct.

p	q	r	$q \wedge r$	$p \to q \wedge r$	$p \to q$	$(p \to q) \to r$
1	1	1	1	1	1	1
1	1	0	0	0	1	0
1	0	1	0	0	0	1
1	0	0	0	0	0	1
0	1	1	1	1	1	1
0	1	0	0	1	1	0
0	0	1	0	1	1	1
0	0	0	0	1	1	1

  The logical consequence is not correct.

4 Prove

we need to find a truth valuation v under which all of the premises are true and the conclusion A is false.

- we can find a counter-example: $A^v = 1$, $B^v = 1$, $C^v = 0$. In this case, the left side is 1 but the right side is 0.
- we can find a counter-example: $A^v = 1$, $B^v = 0$, $C^v = 1$. In this case, the left side is 1 but the right side is 0.
- we can find a counter-example: $A^v = 1$, $B^v = 0$, $C^v = 0$. In this case, the left side is 1 but the right side is 0.

5 Formalizing problems

Let's assume that $A^v = 1$ represents A contains a treasure and $A^v = 0$ represents A contains a fatal trap. The same rule satisfies for B.

- *At least one of these two trunks contains a treasure* can be written as $(A \vee B)$.
  In A there's a fatal trap can be written as $(\neg A)$.
  either both the inscriptions are true, or they are both false can be represented by the symbol $\leftrightarrow$
  In conclusion, we can formalize the puzzle in propositional logic as $(A \vee B) \leftrightarrow (\neg A)$.
A	B	$A \vee B$	$\neg A$	$(A \vee B) \leftrightarrow (\neg A)$
1	0	1	0	1
0	1	1	1	1
1	1	1	0	0
0	0	0	1	0

  In conclusion, Aladdin can choose a trunk being sure that he will find a treasure and the trunk B should be open.

6 Adequate Sets

We know that $(p \to q)$ and $((\neg p) \vee q)$ are logically equivalent.
Hence, $((\neg p) \to q)$ and $(p \vee q)$ are logically equivalent.
Therefore, $\vee$ can be definable in terms of $\neg$ and $\to$.
According to **DeMorgan's Law**, $(p \wedge q)$ and $(\neg((\neg p) \vee (\neg q)))$ are logically equivalent.
Therefore, $\wedge$ can also be definable in terms of $\neg$ and $\to$.
We know that $\{\neg, \wedge, \vee\}$ is an adequate set of connectives.
So in conclusion, $\{\neg, \to\}$ is an adequate set of connectives.