Mathematical Logic_Assignment_2

Hongli SHEN

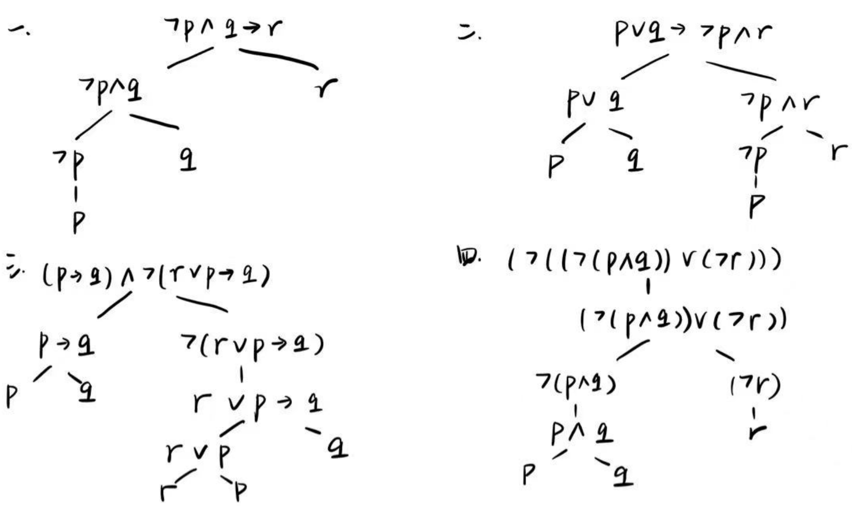
March 2024

Contents

1 Which of the following are well formed propositional formulas?

- It is not a well formed propositional formula.
- It is a well formed propositional formula.
- It is not a well formed propositional formula.
- It is a well formed propositional formula.
- It is not a well formed propositional formula.
- It is not a well formed propositional formula.
- It is not a well formed propositional formula.
- It is a well formed propositional formula.
- It is a well formed propositional formula.

2 Parse Tree



3 Propositional Formalization

- (D→(B∧C))
- ((A∧B)→(¬D))
- (D↔(¬(C∧A)))
- ((D∧(¬C))→A)
- (((¬D)→C)∨(D→(¬B)))

- (((¬(B∧C))∧D)→A)
- ((((A∧B)∧C)↔(¬D))∨(((¬A)∧(¬B))→((¬C)→(¬D))))

4 Properties of formulas I

① **Base Case**: For the simplest propositional language, which contains only one item. $m = 1$ and $n = 0$. Thus, $m = n + 1$. Satisfied.
② **Inductive Step**: Suppose that a propositional language A contains k atoms and satisfies $m = n + 1$.
Let us consider a propositional language that contains $(k + 1)$ atoms. we can write it as $B = A \star P$ ($\star$ includes $\wedge \vee \rightarrow \leftrightarrow$) and P stands for a new atom. The total number of logical connectives in B is obviously $(k - 1 + 1)$.
Therefore, for B, $(m_B = k + 1)$ and $(n_B = n_A + 1 = k - 1 + 1 = k)$.
Therefore, $(m_B = n_B + 1)$.
$(m = n + 1)$ stays true for all propositional language.

5 Properties of formulas II

① If **UV** is a formula in propositional language. Then **V** is the suffix of **UV**, which contains more closing brackets than opening brackets.
② If **VW** is a formula in propositional language. Then **V** is the prefix of **VW**, which contains more opening brackets than closing brackets.
③ **v** cannot satisfy the statements above at the same time.
In conclusion, **UV** and **VW** cannot both be formulas in propositional language.