5.1 Syntax

② ④ ⑥ ⑦ ⑨ ⑪ ⑭ are well-formed FOL formulas.

5.2 Formalization

Def: $S(x)$: x is a student.

$P(x)$: x is smart.

$L(x,y)$: x loves y

$T(x,y)$: x takes y (courses)

$C(x)$: x is a course.

$F(x,y)$: x fails y

Constants:

Bill: a guy called Bill.

Analysis: a course named Analysis

Geometry: a course named Geometry.

1. $\forall x (S(x) \rightarrow P(x))$
2. $\exists x S(x)$
3. $\exists x (S(x) \wedge P(x))$
4. $\forall x (S(x) \rightarrow \exists y (S(y) \wedge L(x,y)))$
5. $\forall x (S(x) \rightarrow \exists y (S(y) \wedge L(x,y) \wedge \neg(x=y)))$
6. $\exists x (S(x) \wedge \forall y (S(y) \wedge \neg(y=x) \rightarrow L(y,x)))$
7. $S(Bill)$
8. $(T(Bill, Analysis) \vee T(Bill, Geometry)) \wedge \neg(T(Bill, Analysis) \wedge T(Bill, Geometry))$
9. $T(Bill, Analysis) \wedge T(Bill, Geometry)$
10. $\neg T(Bill, Analysis)$.
11. $\forall x (S(x) \rightarrow \neg L(x, Bill))$
12. $\forall x (S(x) \rightarrow \exists y (C(y) \wedge T(x,y)))$
13. $\exists x ((S(x) \wedge F(x, Geometry)) \rightarrow \forall y (S(y) \wedge \neg(y=x) \rightarrow \neg F(y, Geometry)))$
14. $\forall x (S(x) \rightarrow \neg F(x, Geometry)) \wedge \exists y (S(y) \wedge F(y, Geometry))$
15. $\forall x ((S(x) \wedge T(x, Analysis)) \rightarrow T(x, Geometry))$

5.3 Formalization.

- "There are more than 2 elements"

$\exists x \exists y \exists z (\neg(x=y) \wedge \neg(y=z) \wedge \neg(x=z))$

- "There are at most 2 elements"

  $\forall x \forall y \forall z ((x=y) \vee (y=z) \vee (x=z))$