3 Assignment 3: Semantics of Propositional Logic (100 points)

For 3.1, 3.2, and 3.3, write down the answer and also briefly explain why.

3.1 For each formula, whether it is tautology, contradiction, or neither? (20 points)

- $(p \to q) \lor (p \to \neg q)$
- $(p \land \neg q) \land (\neg p \lor q)$
- $(p \to q) \leftrightarrow (q \to p)$
- $(p \to r) \land (q \to r) \leftrightarrow (p \lor q \to r)$
- $(p \lor q \to r) \lor p \lor q$

3.2 Whether the following logical equivalences are correct? (20 points)

- $p \to (q \land \neg q) \equiv \neg p$
- $(p \lor q) \land (\neg p \to \neg q) \equiv q$
- $((p \to q) \to q) \to q \equiv p \to q$
- $(p \land q) \lor r \equiv (p \to \neg q) \to r$

3.3 Whether the following logical consequences are correct? (20 points)

- $\neg p \models p \land \neg q \to p \land q$
- $(p \to q) \models \neg p \to \neg q$
- $(p \to q) \land \neg q \models \neg p$
- $p \to q \land r \models (p \to q) \to r$

3.4 Prove (15 points)

- $(A \to B) \lor (A \to C) \not\models A \to (B \land C)$
- $A \to (B \lor C) \not\models (A \to B) \land (A \to C)$
- $(A \land B) \to C \not\models (A \to C) \land (B \to C)$

3.5 Formalizing problems (15 points)

Aladdin finds two trunks A and B in a cave. He knows that each of them either contains a treasure or a fatal trap.

> On trunk A is written: "At least one of these two trunks contains a treasure."
>
> On trunk B is written: "In A there's a fatal trap."

Aladdin knows that either both the inscriptions are true, or they are both false.

1. Formalize the puzzle in propositional logic.
2. Can Aladdin choose a trunk being sure that he will find a treasure? If this is the case, which trunk should he open? Please find the solution using a truth table.

3.6 Adequate Sets (10 points)

Prove that $\{\rightarrow, \neg\}$ is an adequate set of connectives.