2 Assignment 2: Syntax of Propositional Logic (100 points)

2.1 Which of the following are well formed propositional formulas? (20 points)

(You may cross-check your answers using the program you wrote for quiz 3.)

1. $\vee pq$
2. $(\neg(p \to (q \wedge p)))$
3. $(\neg(p \to (q = p)))$
4. $((p \wedge (\neg q)) \vee (q \to r))$
5. $p\neg r$
6. $()$
7. (p)
8. q
9. $(\neg(\neg p))$

2.2 Parse Tree (20 points)

Build the simplified parse tree of the following formulas (brackets in the following formulas may be omitted by convention).

- $\neg p \wedge q \to r$
- $p \vee q \to \neg p \wedge r$
- $(p \to q) \wedge \neg(r \vee p \to q)$
- $(\neg((\neg(p \wedge q)) \vee (\neg r)))$

2.3 Propositional Formalization (20 points)

Let's consider a propositional language where

- A = ”Angelo comes to the party”
- B = ”Bruno comes to the party”
- C = ”Carlo comes to the party”
- D = ”David comes to the party”

Formalize the following sentences (you may ignore brackets by convention):

1. "If David comes to the party then Bruno and Carlo come too"
2. "Carlo comes to the party only if Angelo and Bruno do not come"
3. "David comes to the party if and only if Carlo comes and Angelo doesn't come"
4. "If David comes to the party, then, if Carlo doesn't come then Angelo comes"
5. "Carlo comes to the party provided that David doesn't come, but, if David comes, then Bruno doesn't come"
6. "A necessary condition for Angelo coming to the party, is that, if Bruno and Carlo aren't coming, David comes"
7. "Angelo, Bruno and Carlo come to the party if and only if David doesn't come, but, if neither Angelo nor Bruno come, then David comes only if Carlo comes"

2.4 Properties of formulas I (20 points)

Let $A \in Form(\mathscr{L}^P)$. Assume that m is the number of atoms in A, n is the total number of $\wedge, \vee, \rightarrow, \leftrightarrow$ in A. Prove that $m = n + 1$.

2.5 Properties of formulas II (20 points)

Let U, V, W be non-empty expressions in $\mathscr{L}^P$. Prove that UV and VW cannot both be formulas in $\mathscr{L}^P$.