4 Assignment 4: PL Proof Systems (100 points)

4.1 The Hilbert-style proof system (20 points)

Use the Hilbert-style proof system to prove the following :

1. $\vdash (\neg A \to A) \to A$
2. $\{\neg A\} \vdash (\neg B \to A) \to B$

4.2 ND proof system (50 points)

Use the Natural Deduction proof system to the prove the following:

1. $\neg(\neg p \vee q) \vdash p$
2. $p \wedge q \to r \vdash p \to (q \to r)$
3. $(p \vee q) \vee r \vdash p \vee (q \vee r)$
4. $p \wedge (q \vee r) \vdash (p \wedge q) \vee (p \wedge r)$
5. $\neg(p \vee q) \vdash \neg p \wedge \neg q$

4.3 Formalization (10 points)

If the train arrives late and there are no taxis at the station, then John is late for his meeting. John is not late for his meeting. The train did arrive late. Therefore, there were taxis at the station.

- Formalize this problem into a set of premises and a conclusion.
- Prove the validity of this argument using the ND proof system.

4.4 CNF Reduction (10 points)

Use logical equivalence recipe (instead of truth table) to reduce the following formula to CNF.

$$(\neg p \to q) \to (q \to \neg r)$$

4.5 Resolution (10 points)

Transform the following set of formulas into clausal form and refute using resolution.

$$\{p, p \to ((q \vee r) \wedge \neg(q \wedge r)), p \to ((s \vee t) \wedge \neg(s \wedge t)), s \to q, \neg r \to t, t \to s\}$$