VP160 Recitation Class VIII

Statics

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Equilibrium

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$$\tau_{ext} = 0$$

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  $\Rightarrow$ If the object is initially at rest, then it will remain at rest.

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If in a uniform gravitaional field (mostly):

$$\vec{\tau_{tot}} = \sum m_i \vec{r}_i \times \vec{g} = M\frac{\sum m_i \vec{r}_i}{\sum m_i} \times \vec{g} = M\vec{r}_c \times \vec{g} = \vec{r}_c \times \vec{G}$$

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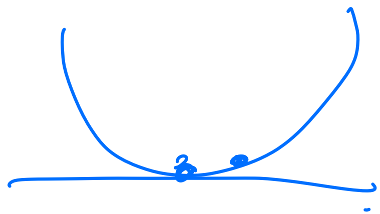
   - $\boxed{\frac{\partial U}{\partial q_i} = 0,}$ the system potential energy reaches an local minimum.
   - useful for low degree of freedom system.

0.



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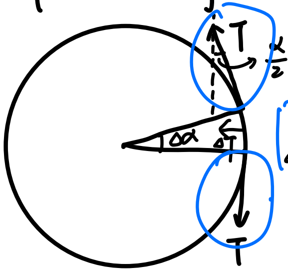
Equilibrium equations & Infinitesimal methods

Exercise 1

Find the tension force inside the string, as shown in the figure below. m and α are known.

For an infinitesimal of string



$\Delta T = (T \cdot \sin\frac{\Delta\alpha}{2})\,2 = \boxed{T\Delta\alpha}$

$\boxed{\sum \Delta N \sin\frac{\alpha}{2} = mg}$

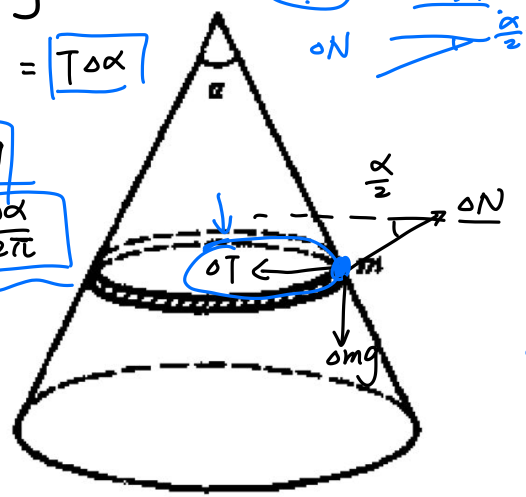
$\Delta N = \frac{mg}{\sin\frac{\alpha}{2}} \cdot \frac{\Delta\alpha}{2\pi}$

$X:\ \Delta N \cdot \cos\frac{\alpha}{2} = \Delta T$

$Y:\ \frac{mg}{\sin\frac{\alpha}{2}} \frac{\Delta\alpha}{2\pi} \cdot \cos\frac{\alpha}{2} = T\Delta\alpha$

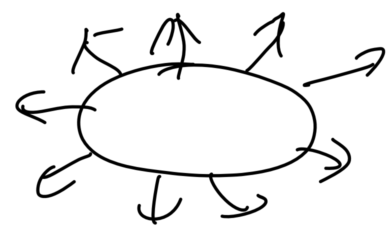
$\Rightarrow T = \frac{mg}{2\pi \tan\frac{\alpha}{2}} = \boxed{mg \cdot \frac{\cot\frac{\alpha}{2}}{2\pi}}$

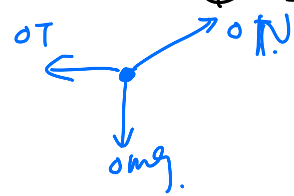
$\sum \Delta N = mg$



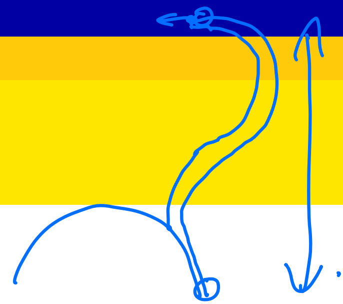
mg, $\boxed{N}$, T

ΔN.





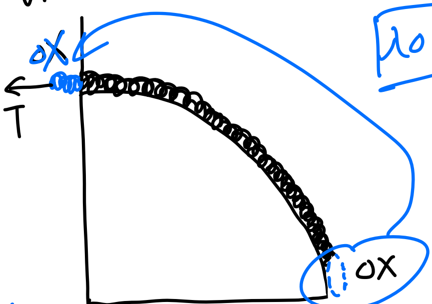
Virtual Work



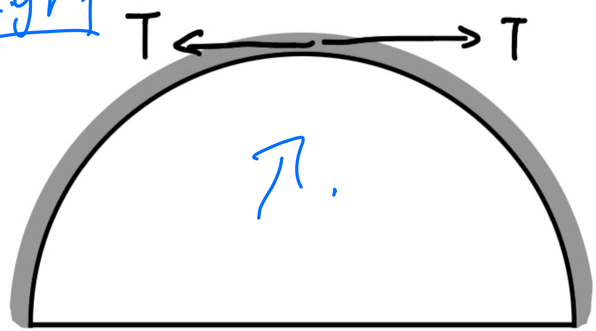
Exercise 2

A half cylinder is placed on the horizontal plane, and is covered by a uniform chain with length πr and linear density λ. Find the tensile force of the chain at the top of the cylinder.

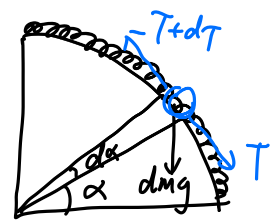
① virtual work



$\boxed{\lambda \delta x g r}$



②



$$\boxed{T\delta x + \delta W_g = 0}$$

$$\delta W_g = \boxed{-\delta U_g} = \lambda g \delta x \cdot r$$

$$\Rightarrow T = \lambda g r$$

$$dT = dmg\cos\alpha$$

$$\boxed{dT = \lambda r d\alpha g \cos\alpha}$$

$$T = \int_0^{\frac{\pi}{2}} \lambda g r \cos\alpha \, d\alpha$$

$$= \lambda g r \sin\alpha \Big|_0^{\frac{\pi}{2}} = \boxed{\lambda g r}$$

Extended materials on virtual work (for your interest)

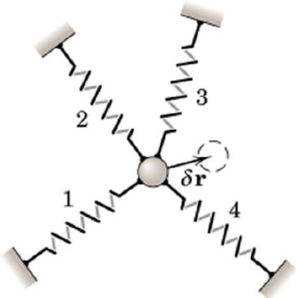
$\delta S = 0. \Rightarrow$ ← path.

$\boxed{\delta S} = 0$

$\boxed{\frac{\partial S}{\partial q_i} = 0}$

Virtual Displacement

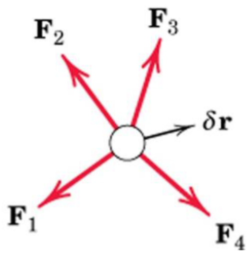
⇒ **Virtual Displacement** is not experienced but only **assumed to exist** so that **various possible equilibrium positions** may be **compared** to determine the **correct one**



- *Imagine* the small *virtual displacement* of particle (**δr**) which is acted upon by several forces.
- The corresponding *virtual work*,

$$\delta U = \vec{F}_1 \cdot \delta\vec{r} + \vec{F}_2 \cdot \delta\vec{r} + \vec{F}_3 \cdot \delta\vec{r} = \left(\vec{F}_1 + \vec{F}_2 + \vec{F}_3\right) \cdot \delta\vec{r}$$
$$= \vec{R} \cdot \delta\vec{r}$$

$= 0.$



Virtual Displacement

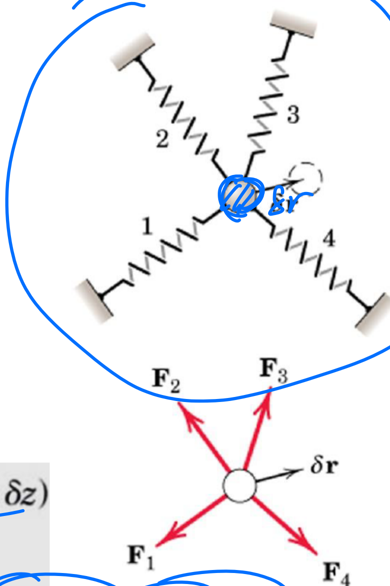
Equilibrium of a Particle

Total virtual work done on the particle due to virtual displacement $\delta\mathbf{r}$:

$$\delta U = \mathbf{F}_1 \cdot \delta\mathbf{r} + \mathbf{F}_2 \cdot \delta\mathbf{r} + \mathbf{F}_3 \cdot \delta\mathbf{r} + \cdots = \Sigma\mathbf{F} \cdot \delta\mathbf{r}$$

Expressing $\sum\mathbf{F}$ in terms of scalar sums and $\delta\mathbf{r}$ in terms of its component virtual displacements in the coordinate directions:

$$\begin{aligned}\delta U = \Sigma\mathbf{F} \cdot \delta\mathbf{r} &= (\mathbf{i}\,\Sigma F_x + \mathbf{j}\,\Sigma F_y + \mathbf{k}\,\Sigma F_z) \cdot (\mathbf{i}\,\delta x + \mathbf{j}\,\delta y + \mathbf{k}\,\delta z) \\ &= \Sigma F_x\,\delta x + \Sigma F_y\,\delta y + \Sigma F_z\,\delta z = 0\end{aligned}$$



The sum is zero since $\sum\mathbf{F} = 0$, which gives $\sum F_x = 0$, $\sum F_y = 0$, $\sum F_z = 0$

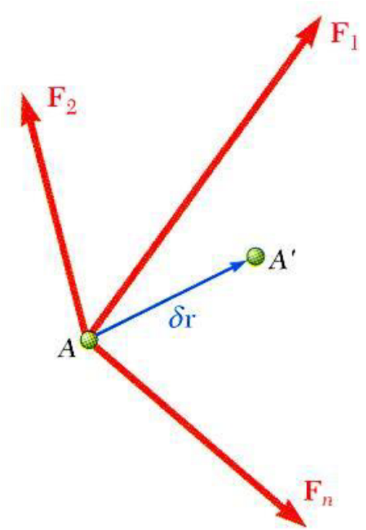
Alternative Statement of the equilibrium: $\boxed{\delta U = 0}$

This condition of **zero virtual work** for **equilibrium** is **both necessary and sufficient** since we can apply it to the **three mutually perpendicular directions** → **3 conditions of equilibrium**

Virtual Work

***Principle of Virtual Work*:**

- If a particle is in equilibrium, the total virtual work of forces acting on the particle is zero for any virtual displacement.



- If a rigid body is in equilibrium
  - total virtual work of external forces acting on the body is zero for any virtual displacement of the body
- If a system of connected rigid bodies remains connected during the virtual displacement
  - the work of the external forces need be considered
    - since work done by internal forces (equal, opposite, and collinear) cancels each other.

$\vec{r} \times \vec{F}_1$
$+ \vec{r} \times \vec{F}_2 = 0$

$= 0$

Equilibrium equations

τ

Exercise 3

Three cylinders have same mass and radius. Friction coefficient between two cylinders is μ_1, between cylinder and ground is μ_2. Find the minimum of μ_1 and μ_2 ~~respectively~~, so that the system is in static.

τ:

$\tau_2 = f_1 R - f_2 R = 0$

$\Rightarrow f_1 = f_2$

$\tau_1 = N_1 \cdot l + mg \cdot l - N_2 \cdot l = 0$

$\Rightarrow \boxed{N_1 + mg = N_2}$

Whole method: $\boxed{N_2 = \frac{3}{2} mg}$ ①

$\Rightarrow \boxed{N_1 = \frac{1}{2} mg}$

F:

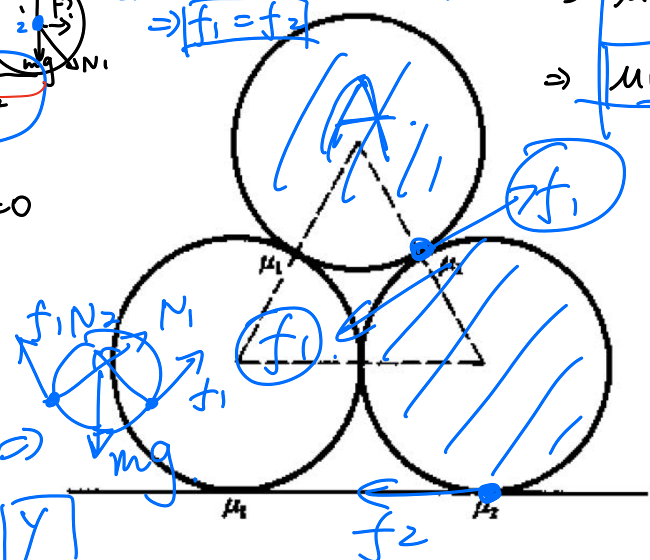
$\boxed{2N_1 \cos 30^\circ + 2f_1 \sin 30^\circ = mg}$ ②

①, ②

$\Rightarrow \boxed{f_1 = mg\left(1 - \frac{\sqrt{3}}{2}\right)}$

$\boxed{f_1 R - f_2 R = 0}$

$\Rightarrow \boxed{f_1 = f_2}$



$\Rightarrow \boxed{\mu_1 N_1 \geq f_1 \quad \mu_2 N_2 \geq f_2}$

$\Rightarrow \boxed{\mu_1 \geq 2 - \sqrt{3}, \quad \mu_2 \geq \frac{2}{3}\left(1 - \frac{\sqrt{3}}{2}\right)}$

$N_2 \quad N_2$

Derivation of energy

Exercise 4

Find θ when the system is in static. Assume $l = 50cm$, $m = 50g$, $r = 8cm$, $M = 200g$.

$D.O.F. = 1 \Rightarrow q_1 = \theta$

$$U_l = -\left(mg\frac{l}{2}\cos\theta\right)2 - \left(mg\cdot\frac{3}{2}l\cos\theta\right)2 = -4mgl\cos\theta$$

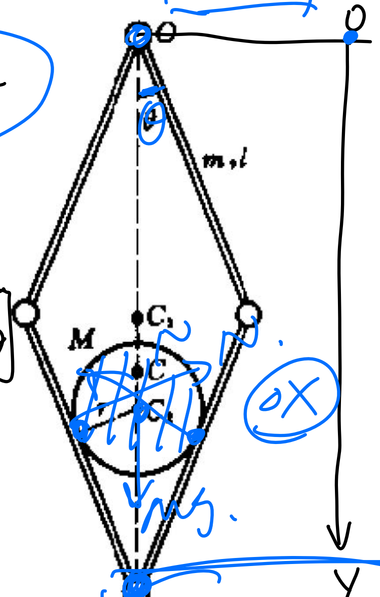
$$U_M = -Mg\left(2l\cos\theta - \frac{r}{\sin\theta}\right)$$

$$U(\theta) = -g\left[4ml\cos\theta + 2Ml\cos\theta - M\frac{r}{\sin\theta}\right] = -g\left[0.3\cos\theta - \frac{0.016}{\sin\theta}\right]$$

$$\frac{\partial U(\theta)}{\partial\theta} = 0$$

$$\Rightarrow -0.3\sin\theta - 0.016(-1)\sin^{-2}\theta\cos\theta = 0$$

$\boxed{U=0.}$



$$\boxed{\frac{\partial U(\theta)}{\partial\theta} = 0}$$

$0 < \theta < \frac{\pi}{2}$

oX

$\Rightarrow$

$$\theta = 0.376\ \text{rad.}$$

Stress

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Strain

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$$\text{elastic modulus} = \frac{\text{stress}}{\text{strain}}$$

P.

$[P]$.

Young's modulus: tensile stress divided by tensile strain

$$Y = \frac{\frac{F\perp}{A}}{\frac{\Delta l}{L}}$$

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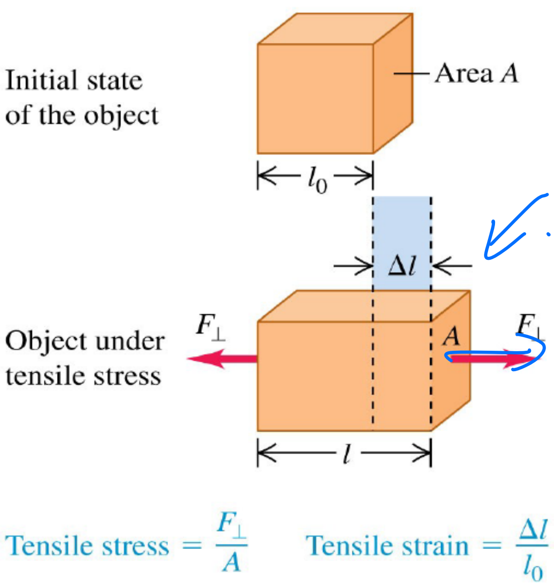

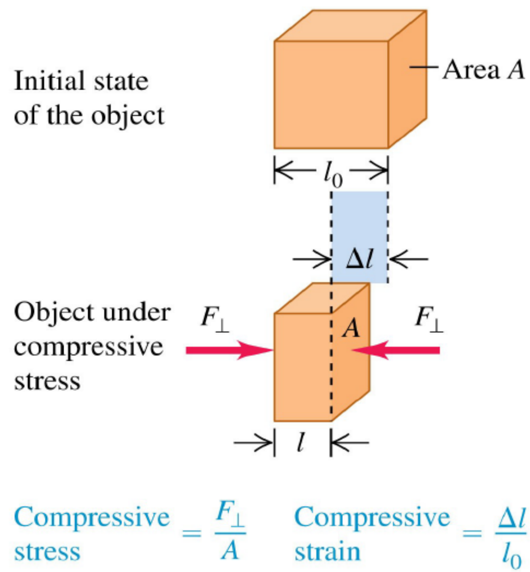

Bulk's modulus: bulk stress divided by bulk strain

$$B = -\frac{\Delta p}{\frac{\Delta v}{V}}$$

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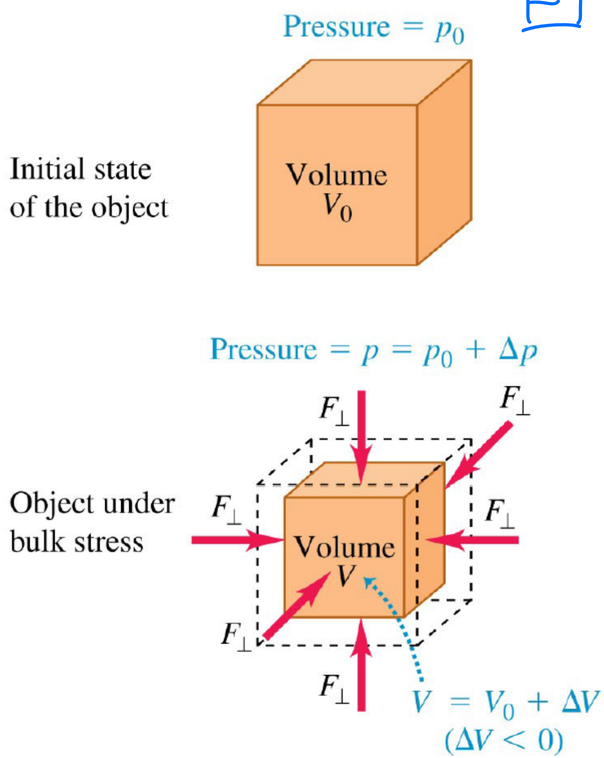
$$B = -\frac{\Delta p}{\frac{\Delta V}{V}}$$

$\delta V < 0$



Shear modulus: shear stress divided by shear strain

$$S = \frac{\frac{F_{\parallel}}{A}}{\frac{X}{h}}$$

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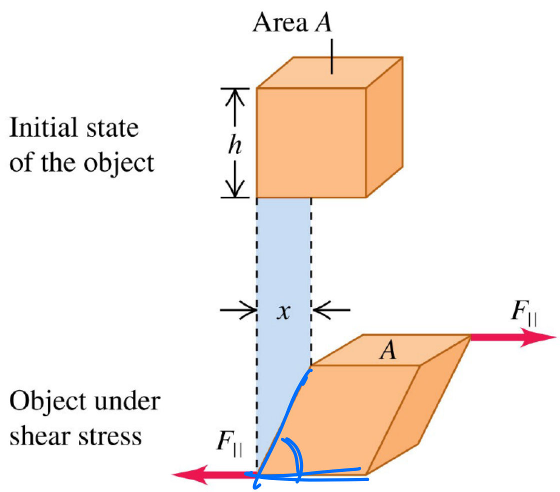

Shear stress $= \frac{F_{||}}{A}$ Shear strain $= \frac{x}{h}$

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Thanks!

Reference

Yigao Fang.
VP160 Recitation Slides.
2020

Haoyang Zhang.
VP160 Recitation Slides.
2020

Yousheng Shu (舒幼生).
Mechanics (力学)
Peking University Press, 2005