Recitation class topics
(week of 5 and 12 July)

Short review

(a) Angular momentum (single particle): angular momentum and torque; 2nd law of dynamics for rotational (orbital motion); aerial velocity; central forces; moment of inertia for a particle about a point

(b) Angular momentum (system of particles): conservation of angular momentum (internal forces do not change total angular momentum)

(c) Rigid body: definition; angular momentum of a rigid body (in general not parallel to angular velocity)

(d) Tensor of inertia; principal axes and principal moments of inertia; diagonalization: eigenvalues and eigenvectors and their interpretation

(e) Kinetic energy in rotational motion: general case and rotation about a principal axis

(f) Calculation of moments of inertia about a fixed axis; Steiner's theorem

(g) 2nd law of dynamics for rotational motion (rotation about a principal axis); combined motion (translation and rotation): kinetic energy and 2nd law of dynamics (please emphasize that the axis of rotation has to be a principal axis and cannot change direction)

(h) Work and power in rotational motion

(i) Conservation of angular momentum

(j) Gyroscopes and precession (please just discuss an example)

(k) Statics of a rigid body: conditions, center of gravity

Topics for discussion

(a) Please emphasize again that if $\mathbf{L}$ =const for a particle, then its motion is planar.

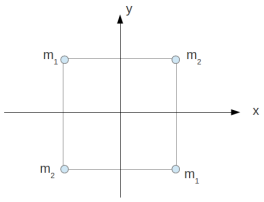
(b) Questions: 9.1, 9.4, 9.6+10, 9.8 (!), 9.16, 9.22

(c) Questions: 10.5, 10.7, 10.15, 10.18, 10.22, 10.27, 10.29; 11.1, 11.2, 11.5, 11.12, 11.16

Problems and exercises

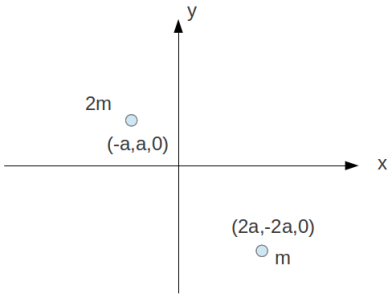
1. Exercises/Problems: 10.1, 10.3
2. A square with side length $2a$, lies in plane $z = 0$ and has masses m_1 and m_2 in its vertices (figure).
   (a) Find the components of the tensor of inertia with respect to axes x, y, z.
   (b) Diagonalize this tensor (find principal moments of inertia and directions of the principal axes).

You can easily get the result here from symmetry, but go through the diagonalization.



3. Do the same for the system of masses in the figure below. Solve the problem using symmetry first, and then check by doing calculations.

   [You will have a degenerate eigenvalue here, i.e. two vectors belonging to one eigenvalue. Physically, it means two principal axes have the same moment of inertia]



4. Using symmetry, find the principal axes and corresponding principal moments of inertia for: (a) thin disk (two axes contained in the disk $MR^2/4$; perpendicular axis $MR^2/2$), (b) thin-walled hollow sphere (all three $2MR^2/3$), (c) torus with mean radius R and the radius of cross-section r (two axes crossing the torus $M(4R^2+5r^2)/8$, perpendicular axis $M(R^2+3r^2/4)$).

   Which of these objects are symmetrical tops? spherical tops?

5. Exercises/Problems: 9.18, 9.21, 9.44, 9.55, 9.60, 9.83, 9.90

6. Exercises/Problems: 10.7, 10.13, 10.26 (!), 10.32, 10.43, 10.53 (!), 10.54 (!), 10.60, 10.70, 10.73 (!), 10.91 (!), 10.99, 11.2

7. A wedge with mass M and angle α rests on a frictionless horizontal surface. A cylinder with mass m rolls down the wedge without slipping. Find the acceleration of the wedge.

   [It's more convenient to solve this problem in the frame of reference associated with the wedge. Answer: $a = \frac{g \sin 2\alpha}{[3(M-m)/m]-2\cos^2\alpha}$]

8. A ball with mass m, moving with in the horizontal direction with speed v, hits the upper edge of a rectangular box with dimensions $l \times l \times 2l$. Assuming that the box can rotate about a fixed axis containing the edge AA', and the collision of the ball with the box is elastic (and the ball moves back in the horizontal direction after the collision), find

   (a) angular velocity the box starts moving with at the moment of collision [answer: $\omega_0 = \frac{2v}{I_{AA'}/ml+l}$],

   (b) equation of motion of the box after the collision [$I_{AA'}\ddot{\alpha} + Mgl\frac{\sqrt{5}}{2}\cos\alpha = 0$],

   (c) the minimum speed of the ball needed to put the box in the upright position [$v = (\frac{I_{AA'}}{ml}+l)\frac{1}{2}\sqrt{\frac{mg}{l}(\sqrt{5}-1)}$].