VP160 Recitation Class III

Zeyi Ren

UM-SJTU Joint Institute

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Force and Inertial Frame of Reference

Force

Force represents **interaction** between two objects or an object and its environment.

Two types of forces:

- Contact forces: objects are in contact with each other and exert forces on each other.
  e.g. force of friction
- Non-contact (field) forces: objects are not in contact with each other and exert forces on each other.
  e.g. gravitational force

Inertial frame of reference

In an inertial FoR, a physical object with zero net force acting on it moves with a constant velocity (which might be zero), or, equivalently, it is a frame of reference in which Newton's first law of motion holds.

Newton's Law

Newton's First Law

An object at rest will stay at rest, and an object in motion will stay in motion unless acted on by a net external force.

$$\sum \vec{F} = 0 \Leftrightarrow \vec{a} = 0$$

Newton's Second Law

In an inertial frame of reference, acceleration of a particle is directly proportional to the net force acting upon it, and inversely proportional to its mass.

$$\vec{F} = m\vec{a}$$

Using Newton's second law, normally we can derive the equation of motion:

$$\frac{d^2 r}{dt^2} = \frac{F(\dot{r}, r, t)}{m}$$

with $v(t_0) = v_0, r(t_0) = r_0$ known, it's an initial value problem to be solved.

How to solve initial value problem?

Newton's Third Law

The mutual forces of action and reaction between two bodies are equal in magnitude and opposite in direction.

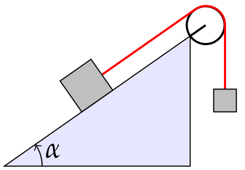
$$\vec{F}_1 = -\vec{F}_2$$

Free-body diagram

A free body diagram consists of a diagrammatic representation of a single body or a subsystem of bodies isolated from its surroundings showing all the forces acting on it.

Normally, you need to consider:

1. Normal force
   - direction: perpendicular to the surface.
2. Friction force
   - cause: normal force
   - direction: in the opposite of $\vec{v}$.
3. Tension force
   - direction: along the rope/rod/spring.
4. Weight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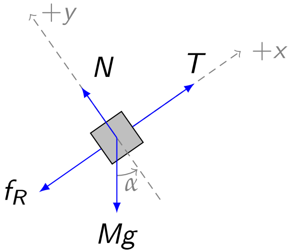
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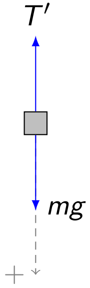
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Background

Statics and particles in equilibrium:

$$\sum \vec{F} = 0$$

Exercise 1

Application of free-body diagram and "Isolation Method"

Two identical smooth balls A and B are suspended from a fixed point O by two ropes of the same length. The two balls also support a smooth ball C of the same weight with A and B, as shown in the Fig. The system are now at an equilibrium. **Find** the relationship between α and β.

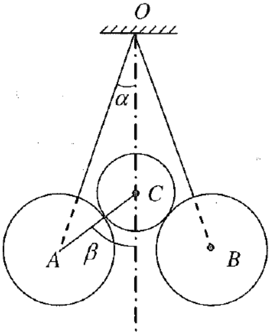


Figure 1. Exercise 1.

Exercise 2

Force of friction and "Whole Method"

Two blocks with mass m_1 and m_2 are stacked on the horizontal desk. Another block with mass m connected to m_1 and m_2 with an inextensible rope is put onto a pulley system. The system is showed in Fig.2. Suppose the friction coefficient between m_1 and m_2 is μ, and the desk is smooth enough to neglect friction. **Find:**

What conditions does the system need to satisfy if there is no relative sliding between m_1 and m_2.

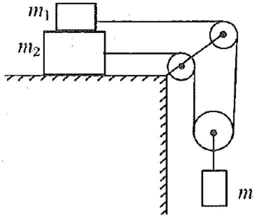


Figure 2. Exercise 2.

Exercise 3

Mass m hangs on a massless rope in a car moving with
(a) constant velocity $\mathbf{v}$,
(b) constant acceleration $\mathbf{a}$
on a horizontal surface. What is the angle the rope forms with the vertical direction.
Discuss the problems (a) (b) if the car slides (without friction) down a plane inclined at an angle α.

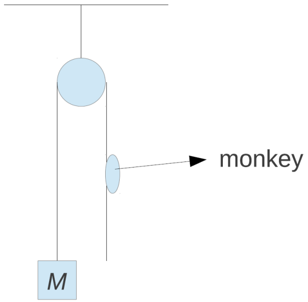
Exercise 4

A monkey with mass m holds a rope hanging over a frictionless pulley attached to mass M (see figure). Discuss motion of the system if the monkey

(a) does not move with respect to the rope,

(b) climbs up the rope with constant velocity $\mathbf{v_0}$ with respect to the rope,

(c) climbs up the rope with constant acceleration $\mathbf{a_0}$ with respect to the rope.

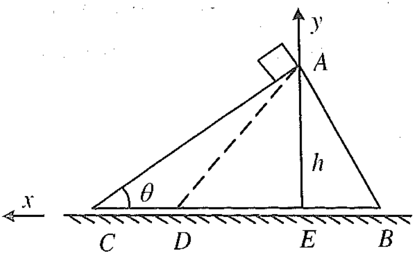


Exercise 5

Relative Motion and Newton's Second law

See in Fig.1, a split ABC with mass M, height h is placed on the horizontal plane. The inclination angle of AC is θ. A small object with mass m begin to slide down from A with initial velocity 0. Omitting the friction of each contact surface. **find**:

(a) The displacement of M when m reaches the ground,
(b) in the ground FoR, the acceleration $\mathbf{a_1}$ of M.
(c) in the m FoR(small object), the acceleration $\mathbf{a_2'}$ of M,
(d) in the ground FoR, the acceleration $\mathbf{a_2}$ of m,
(e) the normal force N between m and M,
(f) the normal force R between M and the ground.



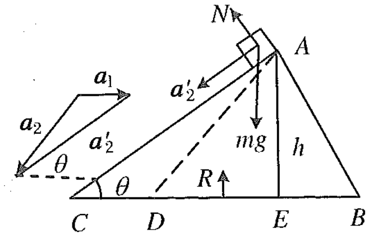


Figure 4. Exercise 5.

Motion with Air/Fluid Drag

Consider a particle with linear drag $\mathbf{F} = -k\mathbf{v}$ and initial velocity $\mathbf{v_0} = v_0 cos(\alpha)\hat{n}_x + v_0 sin(\alpha)\hat{n}_y$, what's its trajectory?

Two recommended ways:

1. decompose the drag force
2. decompose the velocity

What if quadratic drag force?

Reference

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