VP160 Recitation Class III

Zeyi Ren

UM-SJTU Joint Institute

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Force and Inertial Frame of Reference

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  e.g. force of friction
- Non-contact (field) forces: objects are not in contact with each other and exert forces on each other.
  e.g. gravitational force

Inertial frame of reference

In an inertial FoR, a physical object with zero net force acting on it moves with a constant velocity (which might be zero), or, equivalently, it is a frame of reference in which Newton's first law of motion holds.

Only holds in inertial FoR.

non-inertial FoR: in an inertial FoR, moves with a relative motion that is not a uniform linear motion (including $v=0$).

e.g. accelerating rocket / merry-go-round.

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In an inertial frame of reference, acceleration of a particle is directly proportional to the net force acting upon it, and inversely proportional to its mass.

$$\vec{F} = m\vec{a}$$

Using Newton's second law, normally we can derive the equation of motion:

$$\frac{d^2 r}{dt^2} = \frac{F(\dot{r}, r, t)}{m}$$

with $v(t_0) = v_0, r(t_0) = r_0$ known, it's an initial value problem to be solved.

$\ddot{r} + p\dot{r} + qr = f(t)$ General case in VV255/285

damped/driven oscillation, resonance.

Usually, can be simplified,

Seperation of variables:

$$m\frac{dv}{dt} = mg - kv^2$$

$$\int \frac{dv}{g - \frac{k}{m}v^2} = \int dt$$

$f(v) + C = t$ ⇐ plug in $V(t_0) = V_0$, solve C.

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How to solve initial value problem? 换元 . . .

Inhomogeneous Linear Systems

1.10.10. Theorem. The solution of the initial value problem

$$\frac{dx}{dt} = A(t)x + b(t), \qquad x(t_0) = x_0, \qquad (1.10.7)$$

is given by

$$x_{\mathrm{inhom}}(t) = x_{\mathrm{hom}}(t) + x_{\mathrm{part}}(t)$$

where $x_{\mathrm{hom}}(t)$ is the solution of the associated homogeneous initial value problem

$$\frac{dx_{\mathrm{hom}}}{dt} = A(t)x_{\mathrm{hom}}, \qquad x_{\mathrm{hom}}(t_0) = x_0,$$

and

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How to solve initial value problem?

Newton's Third Law

The mutual forces of action and reaction between two bodies are equal in magnitude and opposite in direction.

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$$\vec{F}_1 = -\vec{F}_2$$

Free-body diagram

A free body diagram consists of a diagrammatic representation of a single body or a subsystem of bodies isolated from its surroundings showing all the forces acting on it.

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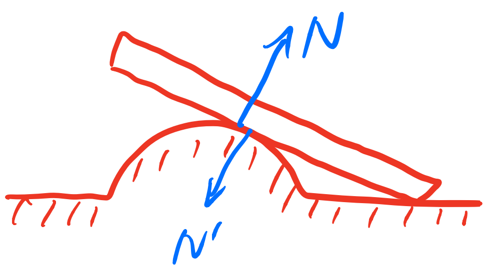

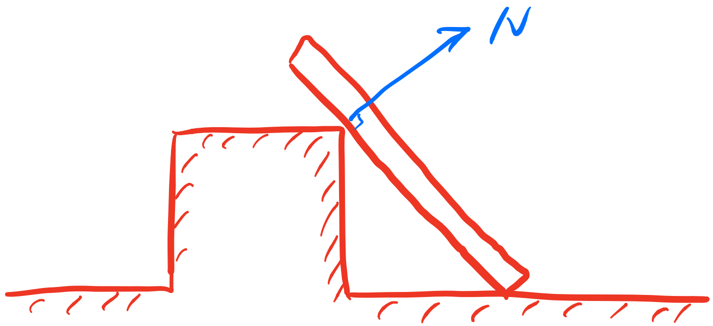

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   - cause: normal force

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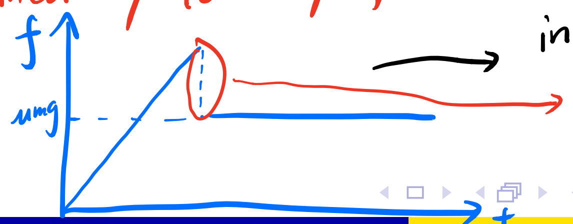
   - direction: perpendicular to the surface.
2. Friction force
   - cause: normal force
   - direction: in the opposite of $\vec{v}$. ← sliding friction.

static friction: determined by tendency of motion.

Under pulling $F = kt$,



in many problems, neglect the gap, simply $f \le \mu N$

static friction

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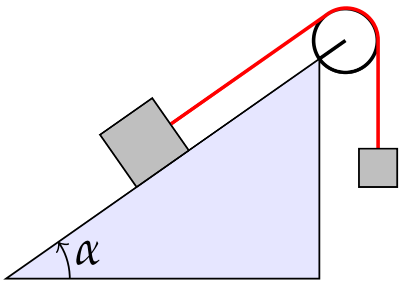
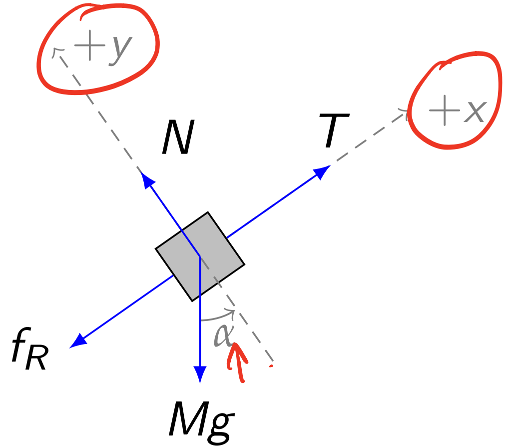
   - direction: along the rope/rod/spring.
4. Weight

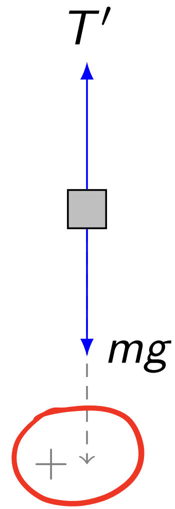
Mark all forces!



α



$+y$
$+x$
N
T
f_R
α
Mg



T'
mg
$+$

Background

Statics and particles in equilibrium:

$$\sum \vec{F} = 0$$

$\Longleftrightarrow$

equilibrium in x, y, z directions:
$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum F_z = 0 \end{cases}$$

Background

Statics and particles in equilibrium:

$$\sum \vec{F} = 0$$

Exercise 1

Application of free-body diagram and "Isolation Method"

Two identical smooth balls A and B are suspended from a fixed point O by two ropes of the same length. The two balls also support a smooth ball C of the same weight with A and B, as shown in the Fig. The system are now at an equilibrium. **Find** the relationship between α and β.

A:

x: $T\sin\alpha = N_1 \sin\beta$ ①

y: $T\cos\alpha = mg + N_1\cos\beta$ ②

C:

y: $2N_1\cos\beta = mg$ ③

①② eliminate T:

$$\tan\alpha = \frac{N_1\sin\beta}{mg+N_1\cos\beta} \quad ④$$

③ plug N_1 into ④

$$\tan\alpha = \frac{\sin\beta}{\frac{mg}{\frac{mg}{2\cos\beta}}+\cos\beta} = \frac{1}{3}\tan\beta$$

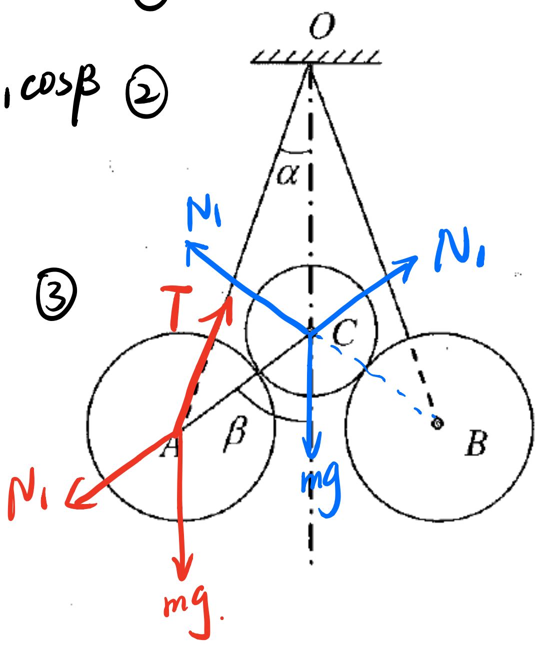


Figure 1. Exercise 1.

$$\boxed{\tan\beta = 3\tan\alpha}$$

Exercise 2

Force of friction and "Whole Method"

Two blocks with mass m_1 and m_2 are stacked on the horizontal desk. Another block with mass m connected to m_1 and m_2 with an inextensible rope is put onto a pulley system. The system is showed in Fig.2. Suppose the friction coefficient between m_1 and m_2 is μ, and the desk is smooth enough to neglect friction. **Find:**

What conditions does the system need to satisfy if there is no relative sliding between m_1 and m_2.

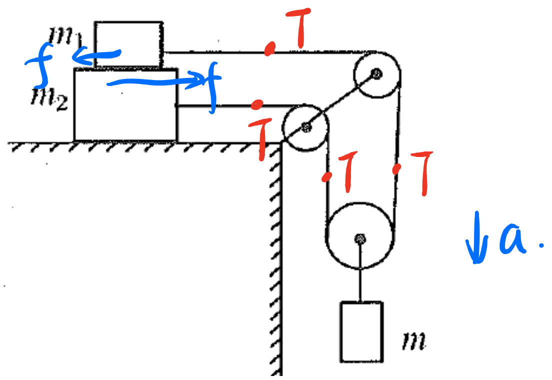


Figure 2. Exercise 2.

Suppose there's no friction between m_1 and m_2.

m_1, m_2, m treat as "whole" 整体.

$$mg = (m + m_1 + m_2)\, a$$

$$a = \frac{mg}{m + m_1 + m_2}$$

Then "Isolation"

for m:

$$mg - 2T = ma$$

$$\Rightarrow T = \frac{m_1 + m_2}{2} a = \frac{(m_1+m_2)mg}{2(m+m_1+m_2)}$$

for m_1:

$$T - f = m_1 a$$

$$\Rightarrow f = \frac{m_2 - m_1}{2} a = \frac{m(m_2-m_1)g}{2(m+m_1+m_2)}$$

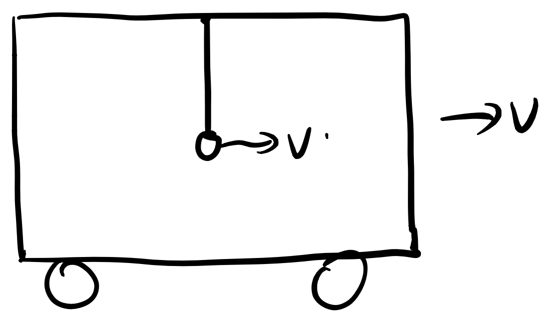
No friction: $f \leq \mu N = \mu m_1 g$

$$\Rightarrow \mu \geq \frac{m(m_2-m_1)}{2m_1(m+m_1+m_2)}$$

Exercise 3

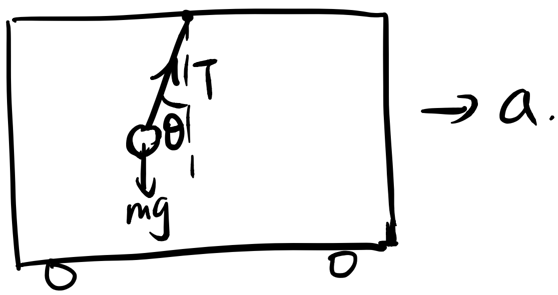
Mass m hangs on a massless rope in a car moving with
(a) constant velocity $\mathbf{v}$,
(b) constant acceleration $\mathbf{a}$
on a horizontal surface. What is the angle the rope forms with the vertical direction.
Discuss the problems (a) (b) if the car slides (without friction) down a plane inclined at an angle α.

(a)



inertial FoR.

(b)



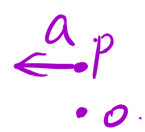
x: $T\sin\theta = ma$.

y: $T\cos\theta = mg$

$\Rightarrow \theta = \arctan\left(\frac{a}{g}\right)$

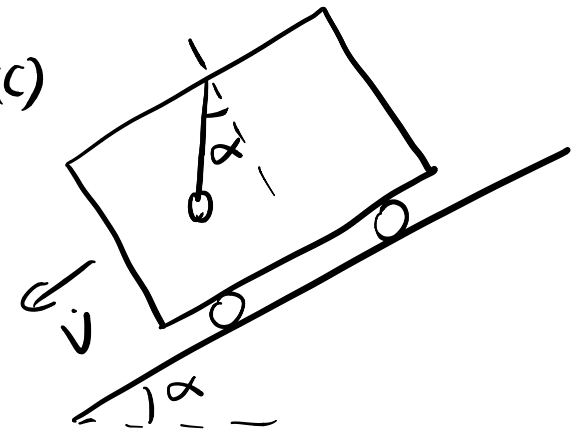
Inertia Force :

ground FoR : $F = ma$.



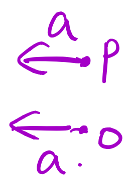
$\boxed{F - ma = 0}$

(c)



accelerating FoR :

$\underset{F-ma}{F'} = m\underset{0.}{a'}$

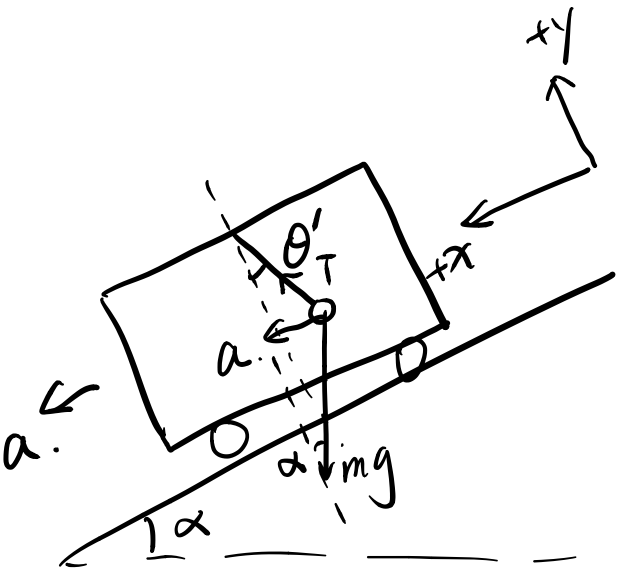


in o's eye $a_p = 0$

$-m\vec{a}$ is called inertia force.

虚假力,并不真实存在,但考虑能够直接在 accelerating FoR 中应用牛顿定律(本来不在 non-inertial 中成立)

(d)



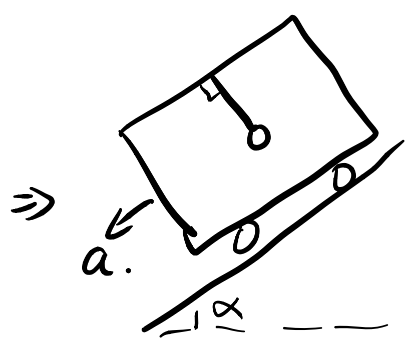
x: $T\sin\theta' + mg\sin\alpha = ma$

y: $T\cos\theta' = mg\cos\alpha$

$\Rightarrow \theta' = \arctan\left(\frac{a - g\sin\alpha}{g\cos\alpha}\right)$

here $a = g\sin\alpha$ $\Rightarrow \theta' = 0$

$\Rightarrow$



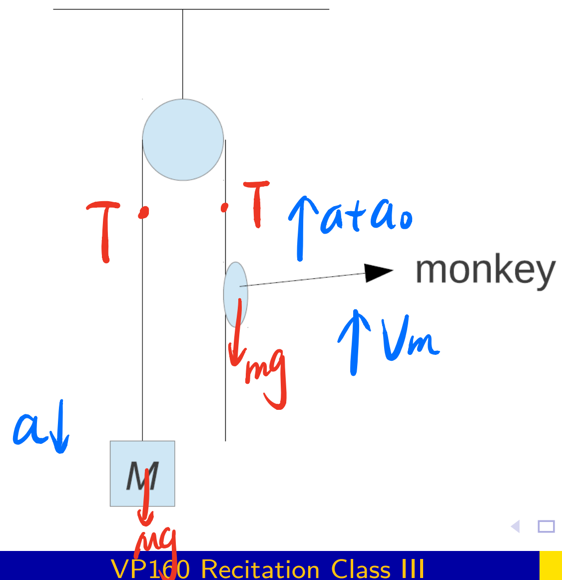
Exercise 4

A monkey with mass m holds a rope hanging over a frictionless pulley attached to mass M (see figure). Discuss motion of the system if the monkey

(a) does not move with respect to the rope,

(b) climbs up the rope with constant velocity $\mathbf{v_0}$ with respect to the rope,

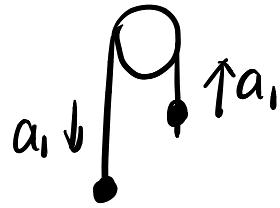
(c) climbs up the rope with constant acceleration $\mathbf{a_0}$ with respect to the rope.



(a) $Mg - T = Ma_1$

$T - mg = ma_1$

$\Rightarrow a_1 = \dfrac{(M-m)g}{M+m}$



(b) $a' = a_1$.

$V_m = V_0 + V_{rope} = V_0 + a't = V_0 + \dfrac{(M-m)gt}{M+m}$

(c) "Isolation"

for m:

$T - mg = m(a_0 + a)$

for M:

$Mg - T = Ma$

$\Rightarrow a = \dfrac{(M-m)g - ma_0}{M+m}$ → acceleration of rope/M.

$a + a_0 = \dfrac{(M-m)g + Ma_0}{M+m}$ → acceleration of m

$v_m = (a+a_0)t = \dfrac{(M-m)g + Ma_0}{M+m} t$ → speed of m.

Exercise 5

Relative Motion and Newton's Second law

See in Fig.1, a split ABC with mass M, height h is placed on the horizontal plane. The inclination angle of AC is θ. A small object with mass m begin to slide down from A with initial velocity 0. Omitting the friction of each contact surface. **find**:

~~(a) The displacement of M when m reaches the ground~~,

(b) in the ground FoR, the acceleration $\mathbf{a_1}$ of M. ✓

(c) in the M FoR(big object), the acceleration $\mathbf{a}_2'$ of m, ✓

(d) in the ground FoR, the acceleration $\mathbf{a_2}$ of m,

(e) the normal force N between m and M,

(f) the normal force R between M and the ground.

a_1: in ground FoR, a of ABC
a_2': in ABC FoR, a of m
a_2: in ground FoR, a of m.

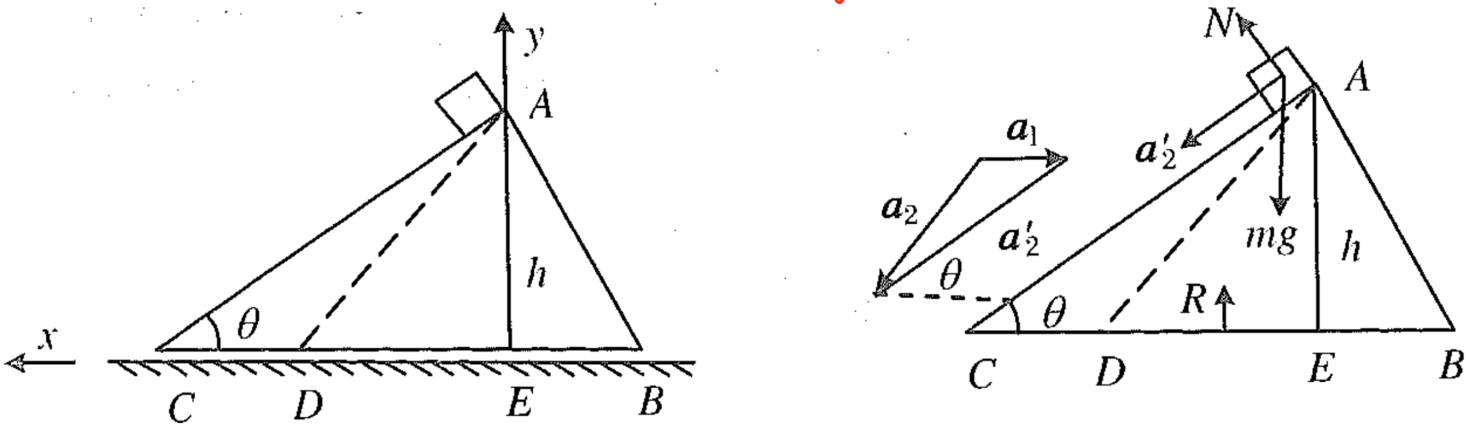
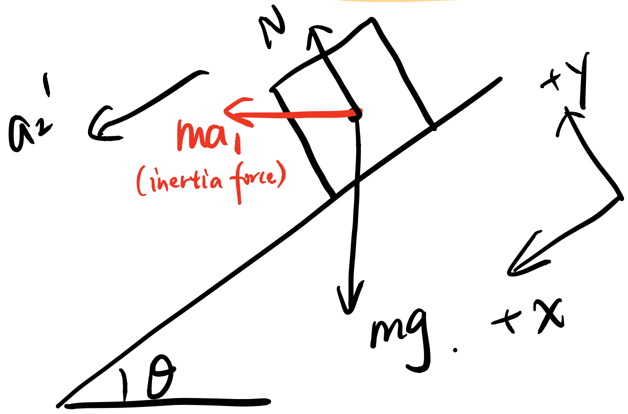


Figure 4. Exercise 5.

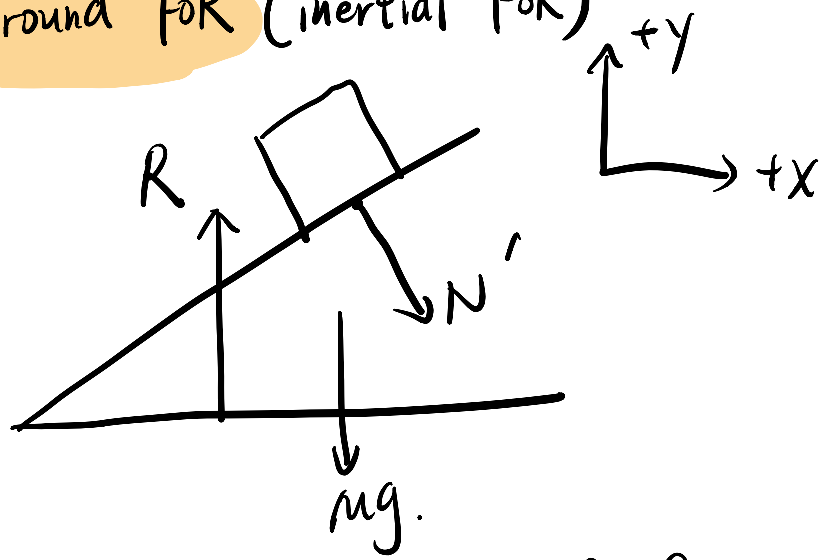
ABC FoR (accelerating in ground FoR)



$$x: \quad mg\sin\theta + ma_1\cos\theta = m a_2' \quad ①$$

$$y: \quad mg\cos\theta = ma_1\sin\theta + N. \quad ②$$

ground FoR (inertial FoR)



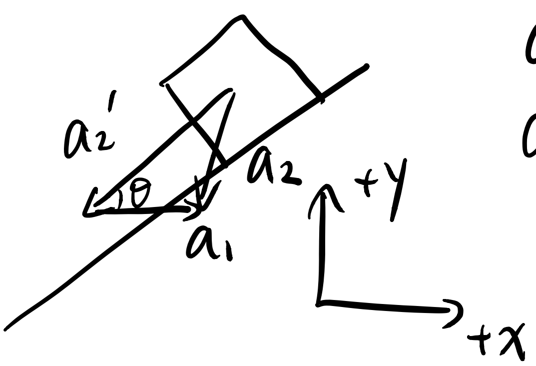
$$N' = N. \quad ③$$

$$x: \quad N'\sin\theta = Ma_1 \quad ④$$

$$y: \quad R = Mg + N'\cos\theta. \quad ⑤$$

②③④: $$a_1 = \frac{mg\sin\theta\cos\theta}{M + m\sin^2\theta}, \quad N = \frac{Mmg\cos\theta}{M + m\sin^2\theta}$$

plug into ①: $$a_2' = \frac{(M+m)g\sin\theta}{M + m\sin^2\theta}.$$



$$a_{2x} = a_1 - a_2'\cos\theta = \frac{-Mg\sin\theta\cos\theta}{M + m\sin^2\theta}$$

$$a_{2y} = -a_2'\sin\theta. = -\frac{(M+m)g\sin^2\theta}{M + m\sin^2\theta}.$$

$$\vec{a_2} = \frac{-Mg\sin\theta\cos\theta}{M + m\sin^2\theta}\hat{i} + \frac{-(M+m)g\sin^2\theta}{M + m\sin^2\theta}\hat{j}$$

⑤: $$R = \frac{M(m+M)g}{M + m\sin^2\theta}.$$

Motion with Air/Fluid Drag

Consider a particle with linear drag $\mathbf{F} = -k\mathbf{v}$ and initial velocity $\mathbf{v_0} = v_0 cos(\alpha)\hat{n_x} + v_0 sin(\alpha)\hat{n_y}$, what's its trajectory?

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Two recommended ways:

1. decompose the drag force

$$F_x = -kv_x, \quad F_y = -kv_y.$$

$$x: \quad -kv_x = m\ddot{x}$$

$$y: \quad -kv_y - mg = m\ddot{y}.$$

x:

$$m\frac{dv_x}{dt} = -kv_x.$$

$$\int_{v_0\cos\alpha}^{v_x} \frac{dv_x}{v_x} = \int_0^t \frac{-k}{m}\,dt.$$

$$\ln\frac{v_x}{v_0\cos\alpha} = -\frac{k}{m}t$$

$$\Rightarrow \quad v_x = v_0\cos\alpha\, e^{-\frac{k}{m}t}.$$

y:

$$m\frac{dv_y}{dt} = -kv_y - mg.$$

$$v_y' = v_y + \frac{mg}{k}.$$

$$\frac{dv_y}{dt} = -\frac{k}{m}v_y - g.$$

$$\frac{dv_y'}{dt} = -\frac{k}{m}v_y'$$

$$\int_{v_0\sin\alpha+\frac{mg}{k}}^{v_y'} \frac{dv_y'}{v_y'} = \int_0^t -\frac{k}{m}\,dt$$

$$\ln\frac{v_y'}{v_0\sin\alpha+\frac{mg}{k}} = -\frac{k}{m}t$$

$$\Rightarrow v_y = \left(v_0\sin\alpha + \frac{mg}{k}\right)e^{-\frac{k}{m}t} - \frac{mg}{k}$$

$$dx = v_x\,dt \Rightarrow \int_0^x dx = \int_0^t v_x\,dt$$

$$dy = v_y\,dt \Rightarrow \int_0^y dy = \int_0^t v_y\,dt$$

$$\Rightarrow \begin{cases} x = v_0\cos\alpha\,\frac{m}{k}\left(1-e^{-\frac{k}{m}t}\right) \\ y = \left(v_0\sin\alpha + \frac{mg}{k}\right)\frac{m}{k}\left(1-e^{-\frac{k}{m}t}\right) - \frac{mg}{k}t \end{cases}$$

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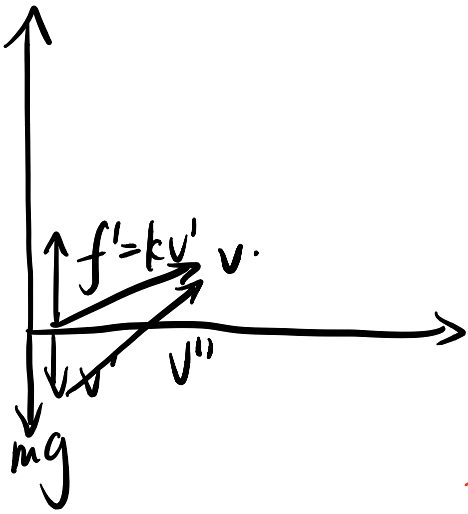
2. decompose the velocity



$\vec{v} = \vec{v}' + \vec{v}''$

$f' = kv' = mg$, $v' = \frac{mg}{k}$ equivalent with a.

$\frac{\vec{v}'}{|\vec{v}'|}$: So $\vec{v}'$ remain the same, in this direction, rectlinear motion.

$\frac{\vec{v}''}{|\vec{v}''|}$: $\vec{v}''$, only the drag force, with acceleration

$$a'' = \frac{kv''}{m}.$$

$$\frac{dv''}{dt} = -\frac{kv''}{m}. \quad \int_{v''(0)}^{v''} \frac{dv''}{v''} = \int_0^t -\frac{k}{m} dt$$

$$v''^2_{(0)} = (v\sin\theta + v')^2 + (v\cos\theta)^2$$

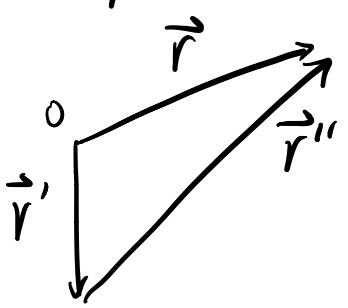
$$v''(0) = \sqrt{\left(v\sin\theta + \frac{mg}{k}\right)^2 + (v\cos\theta)^2}$$

$$v''(t) = \sqrt{\left(v\sin\theta + \frac{mg}{k}\right)^2 + (v\cos\theta)^2}\, e^{-\frac{k}{m}t}.$$

$$\Rightarrow \quad \vec{v} = \vec{v''(t)} + \vec{v}'$$

$$\vec{r} = \vec{r}'' + \vec{r}'.$$

$\vec{r}(t)$ can be solved.



this method can sometimes be more useful than the upper one when dealing linear force.

Motion with Air/Fluid Drag

Consider a particle with linear drag $\mathbf{F} = -k\mathbf{v}$ and initial velocity $\mathbf{v_0} = v_0 cos(\alpha)\hat{n_x} + v_0 sin(\alpha)\hat{n_y}$, what's its trajectory?

Two recommended ways:

1. decompose the drag force
2. decompose the velocity

What if quadratic drag force?

$$x:\quad m\ddot{x} = -k(\dot{x}^2+\dot{y}^2)\frac{\dot{x}}{\sqrt{\dot{x}^2+\dot{y}^2}}$$

$$y:\quad m\ddot{y} = -mg-k(\dot{x}^2+\dot{y}^2)\frac{\dot{y}}{\sqrt{\dot{x}^2+\dot{y}^2}}$$

can't do seperation of variables

You just need to know how to do it in 1D.

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