VP160 Recitation Class I

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UM-SJTU Joint Institute

May 17, 2021

Before We Start

What's in RC

1. Basically two parts, conceptual part and exercises.
2. A brief review of concepts.
3. Exercises problems with hand-written notes, including some useful models that you may use in your assignments and exams.

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The fewer fundamental principles you use to solve problems, the better your understanding of physics.

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   - $a \times 10^n$ $(1 \leqslant |a| < 10)$
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   - Some commonly-used unit prefixes:

p	n	μ	m	c	k	M	G
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e.g.

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e.g.

Input: 1m; Output: $\frac{1}{1000}$

3. Basic Units & Derived Units
    - SI system of units:

Quantity	L	m	t	I	T	n	lv
Unit	m	kg	s	A	K	mol	cd

Check how these basic units are defined.

kg: the weight under which the plank constant

$$h = 6.62607015 \times 10^{-34}\ J \cdot s$$

$$J \cdot s = (kg)(m \cdot s^{-2})(m)\, s = kg \cdot m^2 \cdot s^{-1}$$

$$\frac{1}{299792458}\, s \times c$$

$$9192631770 \times T_{Cs133}$$

so kg is defined.

Dimensional Analysis: System of Units

1. We can first select some physical quantities as the "**basic quantities**" and specify a "**unit for measurement**" for each basic quantity, the other physical quantities' units can be derived from the relation between them and the fundamental quantities. These physical quantities are called **derived quantities** and their units It's called **derived unit**.

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   $N \leftarrow kg \cdot m \cdot s^{-2}$

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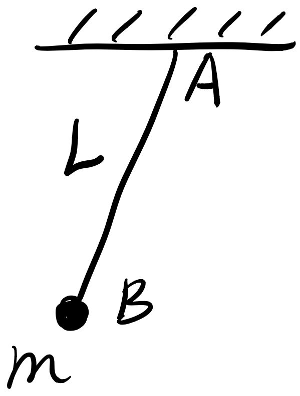
   e.g. The dimensional quantity of a particle of mass m is written as: $M = [m]$.

Exercise 1

A simple pendulum consists of a light inextensible string AB with length l, with the end A fixed, and a point mass m attached to B. The pendulum oscillates with a small amplitude, and the period of oscillation is T. It is suggested that T is proportional to the product of powers of m, l and g, where g is the acceleration due to gravity. Use dimensional analysis to find this relationship.



$$\begin{cases} [m] = M \ (= kg) \\ [l] = L \ (= m) \\ [g] = L \cdot T^{-2} \ (= m \cdot s^{-2}) \end{cases} \qquad [T] = T \ (= s) \quad (*)$$

$g = 9.8 \, m \cdot s^{-2}$

We assume that $T = k \cdot m^x l^y g^z$, k is a const. (no unit)

$$\overset{(*)}{\Longrightarrow} \quad T = M^x L^y (L \cdot T^{-2})^z$$

$$= M^x L^{y+z} \cdot T^{-2z}$$

$$\Rightarrow \begin{cases} x = 0 \\ y + z = 0 \\ -2z = 1 \end{cases} \Rightarrow \begin{cases} x = 0 \\ y = \frac{1}{2} \\ z = -\frac{1}{2} \end{cases}$$

Thus, $T \propto l^{\frac{1}{2}} g^{-\frac{1}{2}}$

$$T \propto \sqrt{\frac{l}{g}} \qquad \left(T = 2\pi \sqrt{\frac{l}{g}} \right)$$

Uncertainty

1. Because of limitations of measurement devices, imperfect measurement procedures and randomness of environmental conditions, as well as human factors related to the experimenter himself, no measurement can ever be perfect. Its result may therefore only be treated as an estimate of what we call the "exact value" of a physical quantity. The experiment may both overestimate and underestimate the value of the physical quantity, and it is crucial to provide a measure of the error, or better uncertainty, that a result of the experiment carries (cited from "Introduction to Measurement Data Analysis" in VP141).

$X = x_0 \pm \varepsilon$

with a probability of m% in $[x-\varepsilon, x+\varepsilon]$

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2. The detailed calculation will be encountered in VP141. The principles of uncertainty analysis will be explained in VE401.

Significant Figures

1. Experimental uncertainty should almost always be rounded to one significant figure. The only exception is when the uncertainty has a leading digit of 1, then we can keep two significant figures (you may encounter this in VP141).

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Example

1. 1.7392 ($SF = 5$)
2. 0.0970 ($SF = 3$)
3. 3.7×10^5 ($SF = 2$)
4. 5.00×10^3 ($SF = 3$)

Back-of-the-envelop Problems

Definition

A quick estimation of some physical quantities. Also called “Fermi problems”. Named after the American physicist Enrico Fermi.

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Tips

Distance between the earth and the sun; the age of the universe, etc.

1. Try to remember the order of magnitude of some important constant.
2. This type of questions may occur in exams.

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Exercise 2.1

True or False: The power of China Railway High-speed (CRH) is about 500kW.

(False) Check it by yourself $P = Fv$

Exercise 2.2 (just for fun)

Fermi's estimation of nuclear explosion: Fermi held a scrap of paper at the height of his head and released while at the same time, an atomic bomb exploded in New Mexico, the scrap of paper fell on the floor with a horizontal displacement of $2.5m$. How many tons of TNT this explosion is equivalent to?

Just for fun!

$E = mc^2$ $\quad [E] = kg\cdot(m/s)^2 = kg\cdot m^2\cdot s^{-2}$

The explosion cloud: 

From New Mexico to where Fermi were.

$t \approx 40s.$

What we have: ρ, r, t

Using dimesional analysis: $E \propto \rho\cdot r^5 t^{-2}$ $\quad \rho_{air} \approx 1.3\, kg\cdot m^{-3}$

$\Rightarrow r \propto E^{\frac{1}{5}} t^{\frac{2}{5}} \rho^{-\frac{1}{5}}$

$V = \frac{dr}{dt} \propto E^{\frac{1}{5}} \cdot \frac{2}{5} t^{-\frac{3}{5}} \rho^{-\frac{1}{5}}$

$E^{\frac{1}{5}} \propto V \cdot t^{\frac{3}{5}} \rho^{\frac{1}{5}}$

Then E can be estimated.

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Scalar	**Vector**
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Vectors in $\mathbb{R}^n$

$\vec{u} = (u_1, u_2, ..., u_n)^T$

Basic Vector operations

- Addition & Subtraction

$$\vec{u} \pm \vec{v} = (u_1 \pm v_1, u_2 \pm v2, u_3 \pm v_3)$$

- Scalar Multiplication

$$\lambda\vec{u} = (\lambda u_1, \lambda u_2, \lambda u_3)$$

- Dot Product

$$\vec{u} \cdot \vec{v} = |u||v|cos\theta = u_1 v_1 + u_2 v_2 + u_3 v_3$$

- Orthogonal Projection Vector of the vector $\vec{u}$ onto the vector $\vec{v}$

$$\frac{\vec{u} \cdot \vec{v}}{|\vec{v}|} \cdot \frac{\vec{v}}{|\vec{v}|}$$

Basic Vector operations

- Cross Product
  - Magnitude: $|\vec{u} \times \vec{v}| = |\vec{u}||\vec{v}|sin\theta$
  - Direction: determined by **Right Hand Rule**
  - Matrix expression(Using determinant):

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$
$$= (u_2 v_3 - u_3 v_2)\hat{i} + (u_3 v_1 - u_1 v_3)\hat{j} + (u_1 v_2 - u_2 v_1)\hat{k}$$

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- Triple Product
  - Scalar Triple Product:

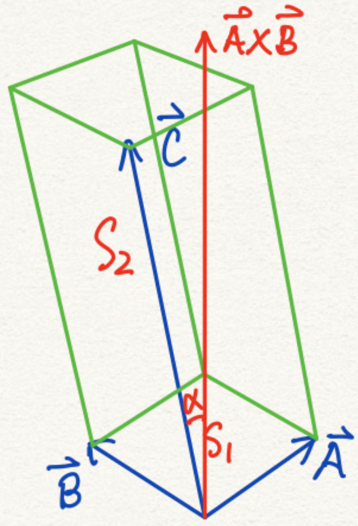
$$\vec{u} \cdot (\vec{v} \times \vec{w}) = \vec{v} \cdot (\vec{w} \times \vec{u}) = \vec{w} \cdot (\vec{u} \times \vec{v})$$

  - Vector Triple Product:

$$\vec{u} \times (\vec{v} \times \vec{w}) = \vec{v}(\vec{u} \cdot \vec{w}) - \vec{w}(\vec{u} \cdot \vec{v})$$

An interesting rule : $(\vec{A}\times\vec{B})\cdot\vec{C} = (\vec{C}\times\vec{A})\cdot\vec{B} = (\vec{B}\times\vec{C})\cdot\vec{A}$

explaining :



$|(\vec{A}\times\vec{B})| = S_1$

$(\vec{A}\times\vec{B})\cdot\vec{C} = S_1\cdot|\vec{C}|\cdot\cos\alpha$

$= S_1\cdot h_1 = V$

$(\vec{B}\times\vec{C})\cdot\vec{A} = \vec{S_2}\cdot\vec{A}\cdot\cos\alpha' = V$

Exercise 3.1

Consider two vectors $\vec{u} = 3\hat{n}_x + 4\hat{n}_y$, and $\vec{w} = 6\hat{n}_x + 16\hat{n}_y$. Find:
(a) the components of the vector $\vec{w}$ that are parallel and perpendicular to the vector $\vec{u}$,
(b) the angle between $\vec{w}$ and $\vec{u}$.

(a) $|\vec{w}_{\parallel}| = \dfrac{\vec{w}\cdot\vec{u}}{|\vec{u}|} = \dfrac{82}{5}$

$\vec{w}_{\parallel} = |\vec{w}_{\parallel}| \cdot \hat{u} = \dfrac{82}{5} \cdot \left(\dfrac{3}{5}, \dfrac{4}{5}\right)$

$\vec{w}_{\perp} = \vec{w} - \vec{w}_{\parallel} = \left(-\dfrac{96}{25}, \dfrac{72}{25}\right)$

(b) $16.3°$.

Exercise 3.2

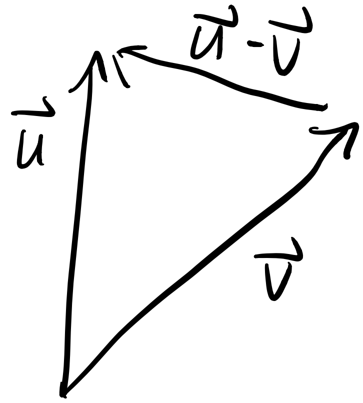
Check that in Cartesian coordinates, the two expression equations of dot product of two vectors $\vec{u}$ and $\vec{v}$ are equivalent:

$$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$$

$$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}|cos\theta$$

Using consine law, we have

$$2|\vec{u}|\cdot|\vec{v}|\cos\theta = |\vec{u}|^2 + |\vec{v}|^2 - |\vec{u} - \vec{v}|^2$$



$$2|\vec{u}|\cdot|\vec{v}|\cos\theta = u_1^2 + u_2^2 + u_3^2 + v_1^2 + v_2^2 + v_3^2$$
$$- (u_1 - v_1)^2 - (u_2 - v_2)^2 + (u_3 - v_3)^2$$

$$= 2u_1v_1 + 2u_2v_2 + 2u_3v_3$$

$\square$

Exercise 3.3

Prove:

$$\vec{u} \times (\vec{v} \times \vec{w}) + \vec{v} \times (\vec{w} \times \vec{u}) + \vec{w} \times (\vec{u} \times \vec{v}) = 0$$

Then check **under what circumstance** the associativity of vector triple product stands, that is

$$\vec{u} \times (\vec{v} \times \vec{w}) = (\vec{u} \times \vec{v}) \times \vec{w}$$

(In general, this equation does not hold)

(a) Just using the equation of vector triple product

(b) Notice $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$, using the derived formula.

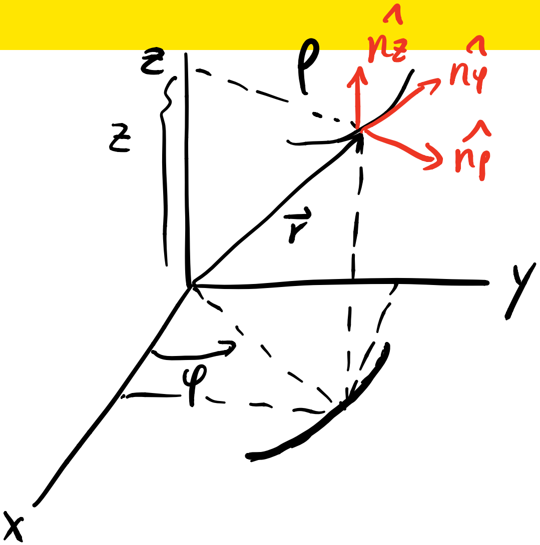
$\Rightarrow \vec{v} \times (\vec{w} \times \vec{u}) = 0.$

Or you can simply calculate using vector triple product again.

Cartesian Coordinate

1. Coordinates: x, y, z
2. Unit Vectors: $\hat{n}_x, \hat{n}_y, \hat{n}_z$
3. Position Vector: $\vec{r} = x\hat{n}_x + y\hat{n}_y + z\hat{n}_z$

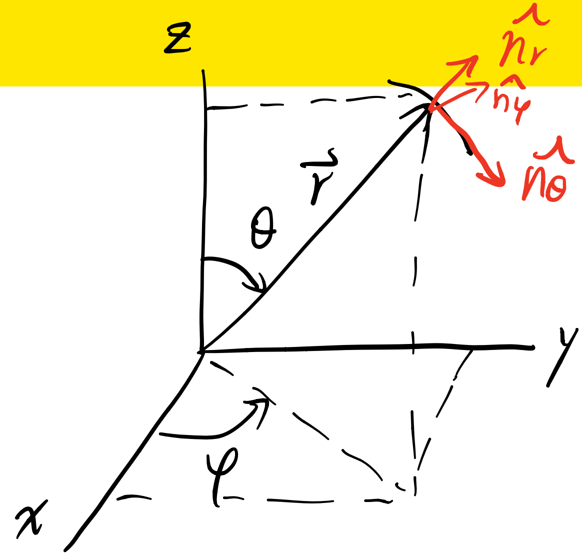
Cylindrical Coordinate



1. Coordinates: ρ, ϕ, z
2. Unit Vectors: $\hat{n}_\rho, \hat{n}_\phi, \hat{n}_z$
3. Position Vectors: $\vec{r} = \rho\hat{n}_\rho + z\hat{n}_z$
4. Relationship with Cartesian Coordinates:

$$\begin{cases} \rho = \sqrt{x^2 + y^2} \\ \phi = \arctan(y/x) \\ z = z \end{cases} \qquad \begin{cases} x = \rho\cos(\theta) \\ y = \rho\sin(\phi) \\ z = z \end{cases}$$

Spherical Coordinate



1. Coordinates: r, ϕ, θ
2. Unit Vectors: $\hat{n}_r, \hat{n}_\phi, \hat{n}_\theta$
3. Position Vector: $\vec{r} = r\hat{n}_r$
4. Relationship with Cartesian Coordinate:

$$\begin{cases} r = \sqrt{x^2 + y^2 + z^2} \\ \phi = \arctan(y/x) \\ \theta = \arctan(\sqrt{x^2 + y^2}/z) \end{cases} \qquad \begin{cases} x = \rho \sin(\theta)\cos(\phi) \\ y = r\sin(\theta)\sin(\phi) \\ z = r\cos(\theta) \end{cases}$$

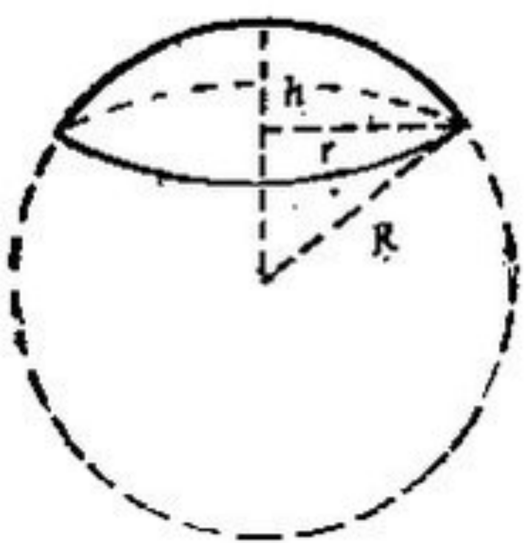
Exercise 4.1

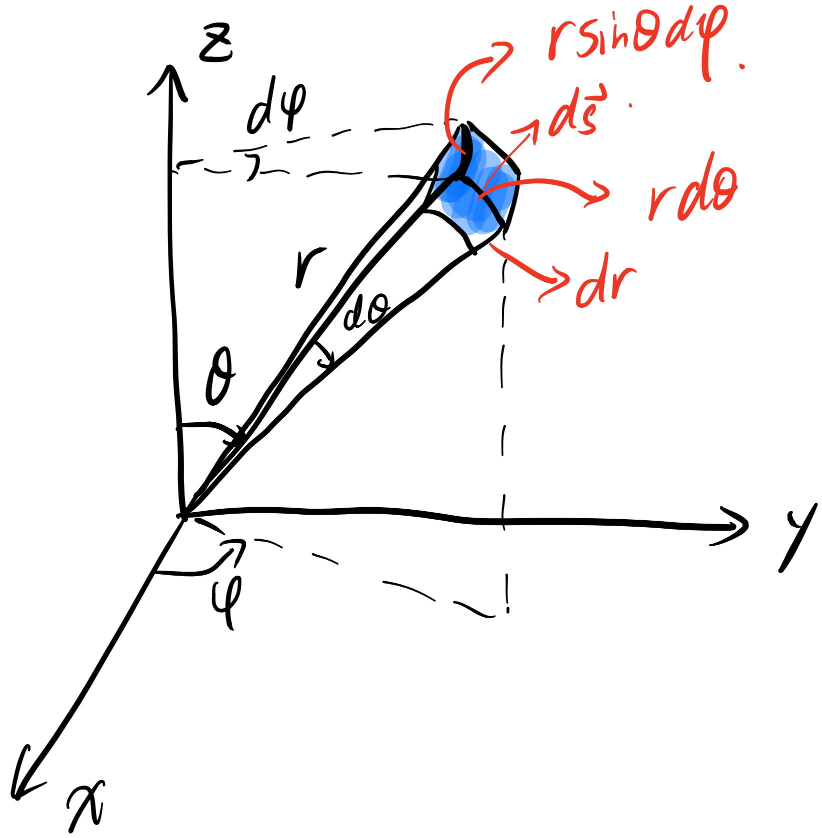
Derive the above relation equations.

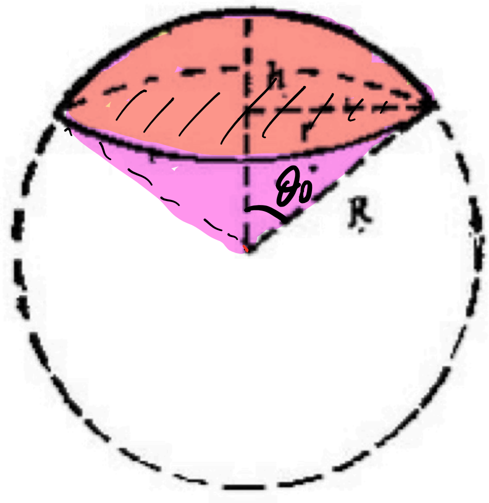
See lecture notes.

Exercise 4.2

Using Spherical Coordinate, calculate the volume of the solid-line part showed below







$$d\tau = r\sin\theta d\varphi \cdot rd\theta \cdot dr = r^2 \sin\theta\, dr\, d\theta\, d\varphi.$$

$$V^* = \int_0^R \int_0^{\theta_0} \int_0^{2\pi} r^2 \sin\theta\, dr\, d\theta\, d\varphi.$$

$$= \int_0^R r^2 dr \int_0^{\theta_0} \sin\theta d\theta \int_0^{2\pi} d\varphi.$$

$$= \frac{2\pi}{3} R^3 \cdot (1-\cos\theta_0) = \frac{2}{3}\pi R^3 \left(1 - \frac{\sqrt{R^2-r^2}}{R}\right)$$

$$V = V^* - V_{purple} = V^* - \frac{1}{3} \cdot \pi r^2 \cdot (R-h)$$

$$= \cdots$$

- Average vs. Instantaneous Quantities

$$v_{x,A} = \frac{x(t+\Delta t) - x(t)}{\Delta t} \quad a_{x,A} = \frac{v(t+\Delta t) - v(t)}{\Delta t}$$

- Relationships

$$x = x(t)$$
$$v = v(t) = \frac{d}{dt}x(t)$$
$$a = a(t) = \frac{d}{dt}v(t) = \frac{d^2}{dt^2}x(t)$$

- Relative motion

$$x = x_r + x'; \quad v = v_r + v'; \quad a = a_r + a'$$

Exercise 5.1

A car is moving in one direction along a straight line. Find the average velocity of the car if:
(a) it travels half of the journey time with velocity v_1 and the other half with velocity v_2,
(b) it covers half of the distance with velocity v_1 and the other one with velocity v_2.

(a) $\frac{V_1+V_2}{2}$

(b) $\frac{2V_1V_2}{V_1+V_2}$

Exercise 5.2

A particle is moving along a straight line with velocity $v_x(t) = -\beta A\omega e^{-\beta t} cos\omega t$, where A, ω, and β are positive constants. Assuming that $x(0) = 5m$.

(a) What are the units of these constants?

(b) Find acceleration $a_x(t)$ and position $x(t)$ of the particle.

(c) Sketch the graphs of $x(t)$, $v_x(t)$, and $a_x(t)$.

(d) What kind of motion may these results describe?

(a) $\omega t = \theta \Leftrightarrow [\omega] = \frac{1}{[t]}$ unit of ω: s^{-1}

βt = dimensionless $\Rightarrow [\beta] = \frac{1}{[t]}$ unit of β : s^{-1}

$[-\beta A \omega] = [v_x] \Rightarrow$ unit of A: $m \cdot s$

(b) $\Delta x = \int_0^t v_x(t)\,dt = \int_0^t -\beta A \omega e^{-\beta t} \cos\omega t$

$$= -\beta A \omega \underbrace{\int_0^t e^{-\beta t} \cos\omega t}_{=A}$$

Integration by parts:
$(uv)' = u'v + v'u$
$\int u'v = uv - \int v'u$

$$= -\beta A \omega \cdot \left[\frac{1}{\omega} \sin\omega t\, e^{-\beta t} \Big|_0^t - \int_0^t \frac{1}{\omega} \sin\omega t (-\beta) e^{-\beta t} \right]$$

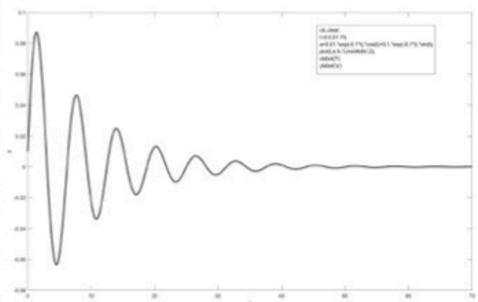
$$= -\beta A \omega \cdot \left[\frac{1}{\omega} \sin\omega t\, e^{-\beta t} + \frac{\beta}{\omega} \int_0^t \sin\omega t\, e^{-\beta t} \right]$$

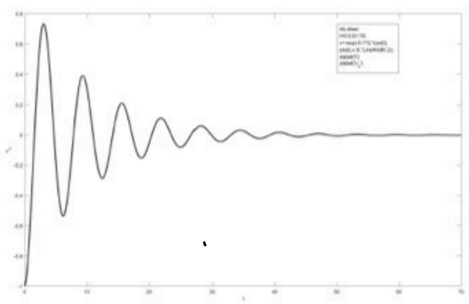
$$= -\beta A \omega \cdot \left[\frac{1}{\omega} \sin\omega t\, e^{-\beta t} + \frac{\beta}{\omega} \left[-\frac{1}{\omega} \cos\omega t\, e^{-\beta t} \Big|_0^t - \int_0^t -\frac{1}{\omega} \cos\omega t \cdot -\beta e^{-\beta t} \right] \right]$$

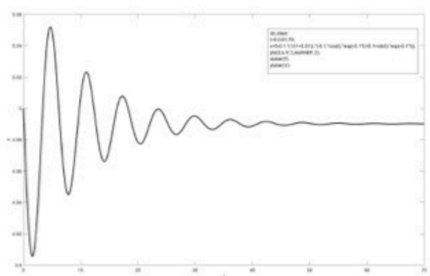
$A = [a_1 + a_2 - a_3 A]$

↳ $A = ?$

$$x(t) = 5 - \frac{\beta A \omega}{\beta^2 + \omega^2} \left[\beta (1 - e^{-\beta t} \cos\omega t) + \omega e^{-\beta t} \sin\omega t \right]$$







Underdamped Oscillation

Exercise 5.3

In a certain motion of a particle along a straight line, the acceleration turns out to be related to the position of the particle according to the formula $a_x = \sqrt{kx}$, where $k > 0$ is a constant and $x > 0$. How does the velocity depend on x, if we know that for $v_x(x_0) = v_0$?

Take the derivative using the chain rule:

$$a_x = \frac{dv}{dt} = \frac{dv}{dx}\cdot\frac{dx}{dt} = v\cdot\frac{dv}{dx} = \sqrt{kx}$$

$$v\,dv = \sqrt{kx}\,dx$$

Doing Integration on both sides, we have

$$\frac{1}{2}\left(v^2 - v_0^2\right) = \frac{2}{3}\sqrt{k}\left(x^{\frac{3}{2}} - x_0^{\frac{3}{2}}\right)$$

Exercise 5.4

A dripping water faucet steadily releases drops 1drop/s apart. As these drops fall, does the distance between them decrease, increase, or remain the same? (Try to find the answer without calculation)

The distance will increase

Exercise 5.5

A spare paddle drops from a fisherman ' s canoe. After one hour of paddling the fisherman realizes that the paddle is missing. He turn around and paddles his canoe back to find the paddle. Assume that the fisherman paddles always with the same speed $v = 10 km/h$ with respect to the river, the speed of the rivers current is $u = 6 km/h$. Find:
(a) the time that the fisherman takes to find the paddle;
(b) the distance between the places where the paddle drops and the fisherman find it.
Can you find the answers within a second?

Tips: Take the river as the frame of reference.

(a) One hour.

(b) $D = 2 \times 6 = 12 km.$

Reference

Yigao Fang.
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