VP160 Recitation Class IV

Non-inertial FoR

Zeyi Ren

UM-SJTU Joint Institute

June 7, 2021

Recall

The non-inertial FoR is a frame of reference that is not in a linear motion with uniform velocity relative to an inertial frame of reference.

Recall

The non-inertial FoR is a frame of reference that is not in a linear motion with uniform velocity relative to an inertial frame of reference.

Basic Formula

$$\vec{a'} = \vec{a} - \vec{a'_O} - \frac{d\vec{\omega}}{dt} \times \vec{r'} - 2(\vec{\omega} \times \vec{v'}) - \vec{\omega} \times (\vec{\omega} \times \vec{r'})$$

Recall

The non-inertial FoR is a frame of reference that is not in a linear motion with uniform velocity relative to an inertial frame of reference.

Basic Formula

$$\vec{a'} = \vec{a} - \vec{a'_O} - \frac{d\vec{\omega}}{dt} \times \vec{r'} - 2(\vec{\omega} \times \vec{v'}) - \vec{\omega} \times (\vec{\omega} \times \vec{r'})$$

$$m\vec{a'} = \vec{F} - m\vec{a'_O} - m\frac{d\vec{\omega}}{dt} \times \vec{r'} - 2m(\vec{\omega} \times \vec{v'}) - m\vec{\omega} \times (\vec{\omega} \times \vec{r'})$$

Recall

The non-inertial FoR is a frame of reference that is not in a linear motion with uniform velocity relative to an inertial frame of reference.

Basic Formula

$$\vec{a'} = \vec{a} - \vec{a'_O} - \frac{d\vec{\omega}}{dt} \times \vec{r'} - 2(\vec{\omega} \times \vec{v'}) - \vec{\omega} \times (\vec{\omega} \times \vec{r'})$$

$$m\vec{a'} = \vec{F} - m\vec{a'_O} - m\frac{d\vec{\omega}}{dt} \times \vec{r'} - 2m(\vec{\omega} \times \vec{v'}) - m\vec{\omega} \times (\vec{\omega} \times \vec{r'})$$

How to derive the formula?

Recall

Newton's Second Law doesn't hold in non-inertial FoR. To describe the motion in non-inertial FoR, we need to add the forces of inertia (pseudo-forces) into "Newton's Second Law" in non-inertial FoR.

Recall

Newton's Second Law doesn't hold in non-inertial FoR. To describe the motion in non-inertial FoR, we need to add the forces of inertia (pseudo-forces) into "Newton's Second Law" in non-inertial FoR.

Add $\mathbf{F}_{\mathbf{Unreal}}$:

$$\mathbf{F}' = \mathbf{F}_{\mathbf{Real}} + \mathbf{F}_{\mathbf{Unreal}}$$

Recall

Newton's Second Law doesn't hold in non-inertial FoR. To describe the motion in non-inertial FoR, we need to add the forces of inertia (pseudo-forces) into "Newton's Second Law" in non-inertial FoR.

Add $\mathbf{F}_{\mathbf{Unreal}}$:

$$\mathbf{F}' = \mathbf{F}_{\mathbf{Real}} + \mathbf{F}_{\mathbf{Unreal}}$$

to maintain the "Newton's Second Law" in non-inertial FoR:

$$\mathbf{F}' = m\mathbf{a}'$$

Recall

Newton's Second Law doesn't hold in non-inertial FoR. To describe the motion in non-inertial FoR, we need to add the forces of inertia (pseudo-forces) into "Newton's Second Law" in non-inertial FoR.

Add $\mathbf{F}_{\mathbf{Unreal}}$:

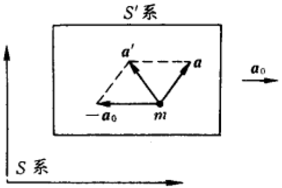
$$\mathbf{F}' = \mathbf{F}_{\mathbf{Real}} + \mathbf{F}_{\mathbf{Unreal}}$$

to maintain the "Newton's Second Law" in non-inertial FoR:

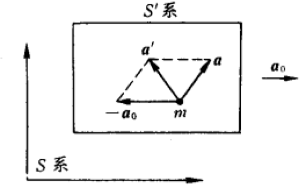
$$\mathbf{F}' = m\mathbf{a}'$$

$$\mathbf{F}_{\mathbf{Unreal}} = -m\vec{a}'_O - m\frac{d\vec{\omega}}{dt} \times \vec{r}' - 2m(\vec{\omega} \times \vec{v}') - m\vec{\omega} \times (\vec{\omega} \times \vec{r}')$$

Drift "Force" $-m\vec{a}_{O'}$



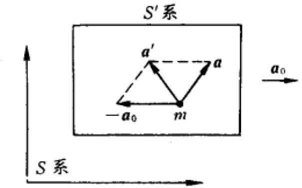
Drift "Force" $-m\vec{a}_{O'}$



Newton's Second Law in inertia FoR S:

$$\mathbf{F} = m\mathbf{a}$$

Drift "Force" $-m\vec{a_{O'}}$



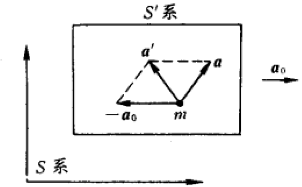
Newton's Second Law in inertia FoR S:

$$\mathbf{F} = m\mathbf{a}$$

Acceleration in non-inertial FoR S':

$$\mathbf{a}' = \mathbf{a} + (-\mathbf{a_0})$$

Drift "Force" $-m\vec{a_{O'}}$



Newton's Second Law in inertia FoR S:

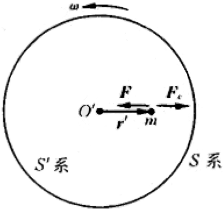
$$\mathbf{F} = m\mathbf{a}$$

Acceleration in non-inertial FoR S':

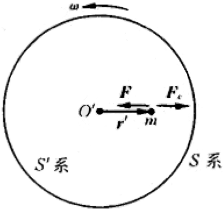
$$\mathbf{a}' = \mathbf{a} + (-\mathbf{a_0})$$

$$\Rightarrow \mathbf{F}' = m\mathbf{a}' = m\mathbf{a} + m(-\mathbf{a_0}) = \mathbf{F} + m(-\mathbf{a_0})$$

Centrifugal "Force" $-m\vec{\omega} \times (\vec{\omega} \times \vec{r'})$



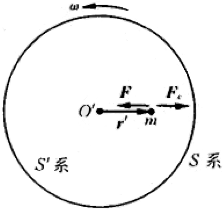
Centrifugal "Force" $-m\vec{\omega} \times (\vec{\omega} \times \vec{r'})$



Uniform circular motion in S :

$$\mathbf{a} = -\omega^2 \mathbf{r}', \quad \mathbf{F} = m\mathbf{a} = -m\omega^2 \mathbf{r}'$$

Centrifugal "Force" $-m\vec{\omega} \times (\vec{\omega} \times \vec{r'})$



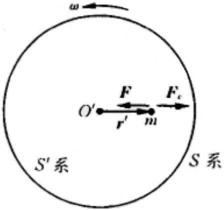
Uniform circular motion in S :

$$\mathbf{a} = -\omega^2 \mathbf{r}', \quad \mathbf{F} = m\mathbf{a} = -m\omega^2 \mathbf{r}'$$

In S':

$$\mathbf{a}' = 0, \quad \mathbf{F}' = m\mathbf{a}' = 0$$

Centrifugal "Force" $-m\vec{\omega} \times (\vec{\omega} \times \vec{r'})$



Uniform circular motion in S :

$$\mathbf{a} = -\omega^2 \mathbf{r}', \quad \mathbf{F} = m\mathbf{a} = -m\omega^2 \mathbf{r}'$$

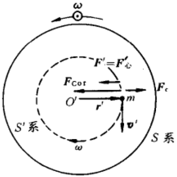
In S':

$$\mathbf{a}' = 0, \quad \mathbf{F}' = m\mathbf{a}' = 0$$

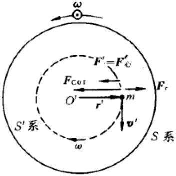
Thus, we need

$$\mathbf{F_c} = m\omega^2 \mathbf{r}' \quad \text{s.t.} \quad \mathbf{F}' = \mathbf{F} + \mathbf{F_c} = 0$$

Coriolis "Force" $-2m(\vec{\omega} \times \vec{v}')$



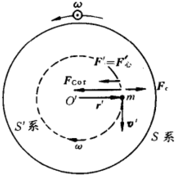
Coriolis "Force" $-2m(\vec{\omega} \times \vec{v}')$



m stays still in S:

$$\mathbf{F} = 0$$

Coriolis "Force" $-2m(\vec{\omega} \times \vec{v}')$



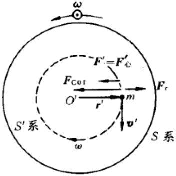
m stays still in S:

$$\mathbf{F} = 0$$

m moves in a uniform circular motion in S':

$$\mathbf{a}' = -\omega^2 \mathbf{r}', \quad \mathbf{F}' = m\mathbf{a}' = -m\omega^2 \mathbf{r}'$$

Coriolis "Force" $-2m(\vec{\omega} \times \vec{v}')$



m stays still in S:

$$\mathbf{F} = 0$$

m moves in a uniform circular motion in S':

$$\mathbf{a}' = -\omega^2 \mathbf{r}', \quad \mathbf{F}' = m\mathbf{a}' = -m\omega^2 \mathbf{r}'$$

Thus, we need (Don't forget the centrifugal force we added)

$$\mathbf{F_{Cor}} = -2m\omega^2 \mathbf{r}' \quad \text{s.t.} \quad \mathbf{F}' = \mathbf{F} + \mathbf{F_c} + \mathbf{F_{Cor}} = 0 + m\omega^2 \mathbf{r}' + (-2m\omega^2 \mathbf{r}') = m\mathbf{a}'$$

Euler "force" $-m\frac{d\vec{\omega}}{dt} \times \vec{r}'$

- Need to be considered when ω is time-variant.

Euler "force" $-m\frac{d\vec{\omega}}{dt} \times \vec{r'}$

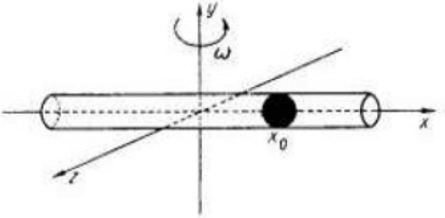
- Need to be considered when ω is time-variant.
- Also called Tangential inertial forces.

Euler "force" $-m\frac{d\vec{\omega}}{dt} \times \vec{r'}$

- Need to be considered when ω is time-variant.
- Also called Tangential inertial forces.
- Conventionally, we use $\vec{\beta}$ to denote the angular acceleration $\frac{d\vec{\omega}}{dt}$.

Exercise 1

A particle with mass m is inside a pipe that rotates with constant angular velocity ω about an axis perpendicular to the pipe.The kinetic coefficient of friction is equal to μ_k. Write down (do not solve!) the equation of motion for this particle in the non-inertial frame of reference of the rotating pipe. There is no gravitational force in this problem.



Exercise 2

If we let go an object $100m$ above the equator. Ignore the air drag, calculate the deviation caused by Coriolis Force.

Reference

Yigao Fang.
VP160 Recitation Slides.
2020

Haoyang Zhang.
VP160 Recitation Slides.
2020

Yousheng Shu (舒幼生).
Mechanics (力学)
Peking University Press, 2005