VP160 Recitation Class VIII

Statics

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Equilibrium

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$\Rightarrow$ If the object is initially at rest, then it will remain at rest.

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If in a uniform gravitaional field (mostly):

$$\vec{\tau_{tot}} = \sum m_i \vec{r_i} \times \vec{g} = M\frac{\sum m_i \vec{r_i}}{\sum m_i} \times \vec{g} = M\vec{r_c} \times \vec{g} = \vec{r_c} \times \vec{G}$$

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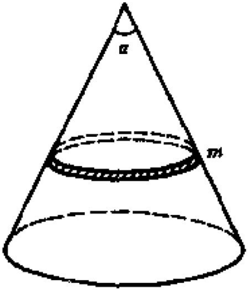
    - $\frac{\partial U}{\partial q_i} = 0$, the system potential energy reaches an local minimum.
    - useful for low degree of freedom system.

Equilibrium equations & Infinitesimal methods

Exercise 1

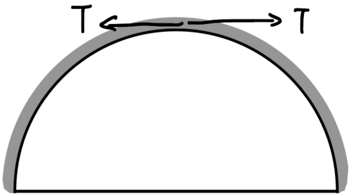
Find the tension force inside the strain, as shown in the figure below. m and α are known.



Virtual Work

Exercise 2

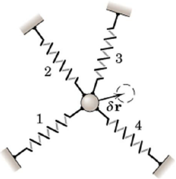
A half cylinder is placed on the horizontal plane, and is covered by a uniform chain with length πr and linear density λ. Find the tensile force of the chain at the top of the cylinder.

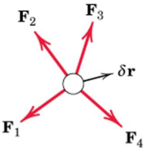


Extended materials on virtual work (for your interest)

Virtual Displacement

Virtual Displacement is not experienced but only **assumed to exist** so that **various possible equilibrium positions** may be **compared** to determine the **correct one**





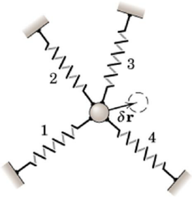
- *Imagine* the small *virtual displacement* of particle (**δr**) which is acted upon by several forces.
- The corresponding *virtual work*,

$$\delta U = \vec{F}_1 \cdot \delta\vec{r} + \vec{F}_2 \cdot \delta\vec{r} + \vec{F}_3 \cdot \delta\vec{r} = \left(\vec{F}_1 + \vec{F}_2 + \vec{F}_3\right) \cdot \delta\vec{r}$$
$$= \vec{R} \cdot \delta\vec{r}$$

Virtual Displacement

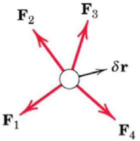
Equilibrium of a Particle

Total virtual work done on the particle due to virtual displacement $\delta \mathbf{r}$:



$$\delta U = \mathbf{F}_1 \cdot \delta\mathbf{r} + \mathbf{F}_2 \cdot \delta\mathbf{r} + \mathbf{F}_3 \cdot \delta\mathbf{r} + \cdots = \Sigma\mathbf{F} \cdot \delta\mathbf{r}$$

Expressing $\sum \mathbf{F}$ in terms of scalar sums and $\delta \mathbf{r}$ in terms of its component virtual displacements in the coordinate directions:



$$\begin{aligned}\delta U &= \Sigma\mathbf{F} \cdot \delta\mathbf{r} = (\mathbf{i}\,\Sigma F_x + \mathbf{j}\,\Sigma F_y + \mathbf{k}\,\Sigma F_z) \cdot (\mathbf{i}\,\delta x + \mathbf{j}\,\delta y + \mathbf{k}\,\delta z) \\ &= \Sigma F_x\,\delta x + \Sigma F_y\,\delta y + \Sigma F_z\,\delta z = 0\end{aligned}$$

The sum is zero since $\sum \mathbf{F} = 0$, which gives $\sum F_x = 0$, $\sum F_y = 0$, $\sum F_z = 0$

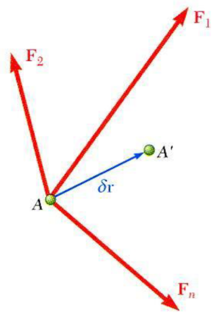
Alternative Statement of the equilibrium: $\delta U = 0$

This condition of **zero virtual work** for **equilibrium** is **both necessary and sufficient** since we can apply it to the **three mutually perpendicular directions** → **3 conditions of equilibrium**

Virtual Work

Principle of Virtual Work:

- If a particle is in equilibrium, the total virtual work of forces acting on the particle is zero for any virtual displacement.

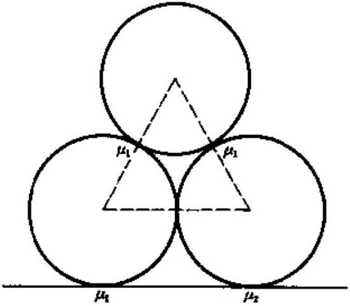


- If a rigid body is in equilibrium
  - total virtual work of external forces acting on the body is zero for any virtual displacement of the body
- If a system of connected rigid bodies remains connected during the virtual displacement
  - the work of the external forces need be considered
    - since work done by internal forces (equal, opposite, and collinear) cancels each other.

Equilibrium equations

Exercise 3

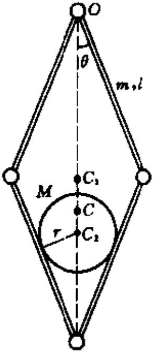
Three cylinders have same mass and radius. Friction coefficient between two cylinders is μ_1, between cylinder and ground is μ_2. Find the minimum of μ_1 and μ_2 respectively, so that the system is in static.



Derivation of energy

Exercise 4

Find θ when the system is in static. Assume $l = 50cm$, $m = 50g$, $r = 8cm$, $M = 200g$.



Stress

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$$\text{elastic modulus} = \frac{\text{stress}}{\text{strain}}$$

Young's modulus: tensile stress divided by tensile strain

$$Y = \frac{\frac{F\perp}{A}}{\frac{\Delta l}{L}}$$

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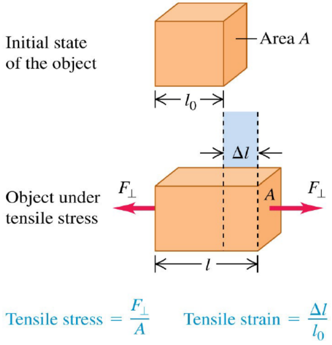

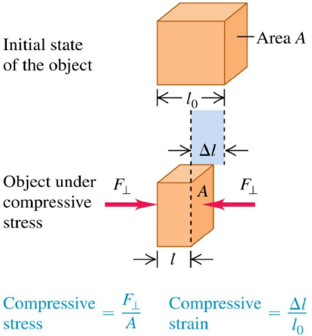

Bulk's modulus: bulk stress divided by bulk strain

$$B = -\frac{\Delta p}{\frac{\Delta v}{V}}$$

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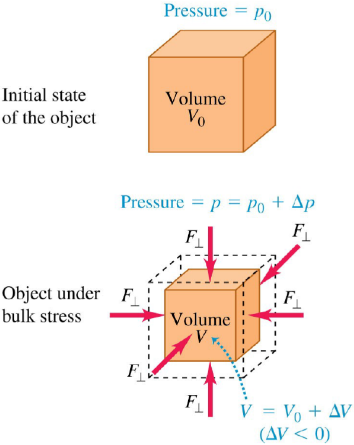

Shear modulus: shear stress divided by shear strain

$$S = \frac{\frac{F_{\parallel}}{A}}{\frac{X}{h}}$$

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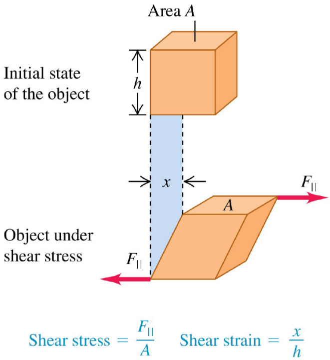

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Thanks!

Reference

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