VP160 Midterm Review Class

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   - k (unit prefix) m (unit)
   - Some commonly-used unit prefixes:

p	n	μ	m	c	k	M	G
10^{-12}	10^{-9}	10^{-6}	10^{-3}	10^{-2}	10^3	10^6	10^9

CASIO fx-991CN X $\triangleright$ $\boxed{OPTN}$ $\triangleright$ 3 : 工程符号

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   Input: 1m; Output: $\frac{1}{1000}$
3. Basic Units & Derived Units
   - SI system of units:

Quantity	L	m	t	I	T	n	lv
Unit	m	kg	s	A	K	mol	cd

Dimensional Analysis: System of Units

1. We can first select some physical quantities as the "**basic quantities**" and specify a "**unit for measurement**" for each basic quantity, the other physical quantities' units can be derived from the relation between them and the fundamental quantities. These physical quantities are called **derived quantities** and their units It's called **derived unit**.

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2. A set of units derived in this way, is called a **system of units**.
3. We often use capital letter to represent a "dimensional quantity", and use $[x]$ to represent the "dimensional quantity" of specific physical quantity x.
   e.g. The dimensional quantity of a particle of mass m is written as: $M = [m]$.

Dimensional Analysis: Method of Undetermined Coefficients

Exercise 1

A simple pendulum consists of a light inextensible string AB with length l, with the end A fixed, and a point mass m attached to B. The pendulum oscillates with a small amplitude, and the period of oscillation is T. It is suggested that T is proportional to the product of powers of m, l and g, where g is the acceleration due to gravity. Use dimensional analysis to find this relationship.

Back-of-the-envelop Problems

Definition

A quick estimation of some physical quantities.

Tips

1. Try to remember the order of magnitude of some important constant.
2. This type of questions may occur in exams.

Basic Vector operations

- Addition & Subtraction

$$\vec{u} \pm \vec{v} = (u_1 \pm v_1, u_2 \pm v2, u_3 \pm v_3)$$

- Scalar Multiplication

$$\lambda\vec{u} = (\lambda u_1, \lambda u_2, \lambda u_3)$$

- Dot Product

$$\vec{u} \cdot \vec{v} = |u||v|cos\theta = u_1 v_1 + u_2 v_2 + u_3 v_3$$

- Orthogonal Projection Vector of the vector $\vec{u}$ onto the vector $\vec{v}$

$$\frac{\vec{u} \cdot \vec{v}}{|\vec{v}|} \cdot \frac{\vec{v}}{|\vec{v}|}$$

Basic Vector operations

- Cross Product
  - Magnitude: $|\vec{u} \times \vec{v}| = |\vec{u}||\vec{v}|sin\theta$
  - Direction: determined by **Right Hand Rule**
  - Matrix expression(Using determinant):

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$
$$= (u_2 v_3 - u_3 v_2)\hat{i} + (u_3 v_1 - u_1 v_3)\hat{j} + (u_1 v_2 - u_2 v_1)\hat{k}$$

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- Scalar Triple Product:

$$\vec{u} \cdot (\vec{v} \times \vec{w}) = \vec{v} \cdot (\vec{w} \times \vec{u}) = \vec{w} \cdot (\vec{u} \times \vec{v})$$

1D Kinematics

- Average vs. Instantaneous Quantities

$$v_{x,A} = \frac{x(t+\Delta t) - x(t)}{\Delta t} \quad a_{x,A} = \frac{v(t+\Delta t) - v(t)}{\Delta t}$$

- Relationships

$$x = x(t)$$
$$v = v(t) = \frac{d}{dt}x(t)$$
$$a = a(t) = \frac{d}{dt}v(t) = \frac{d^2}{dt^2}x(t)$$

- Relative motion

$$x = x_r + x'; \quad v = v_r + v'; \quad a = a_r + a'$$

Kinematics in 3D: Cylindrical Coordinates

Basic Formulas in Cylindrical Coordinates

$$\vec{r} = \rho\hat{n}_\rho + z\hat{n}_z$$
$$\vec{v} = \dot{\rho}\hat{n}_\rho + \rho\dot{\phi}\hat{n}_\phi + \dot{z}\hat{n}_z$$
$$\vec{a} = (\ddot{\rho} - \rho\dot{\phi}^2)\hat{n}_\rho + (\rho\ddot{\phi} + 2\dot{\rho}\dot{\phi})\hat{n}_\phi + \ddot{z}\hat{n}_z$$

Tips:

if $z = 0$, they become kinematics formulas in polar coordinates(see next slide).

Kinematics in 2D: Polar Coordinates

Basic Formulas in Polar Coordinates

$$\vec{r} = r\hat{n}_r \tag{1}$$

$$\vec{v} = \dot{r}\hat{n}_r + r\dot{\theta}\hat{n}_\theta \tag{2}$$

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{n}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{n}_\theta \tag{3}$$

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Relations with cartesian coordinates

$$x = rcos\theta, \ \ y = rsin\theta$$

$$d\vec{r} = dr\hat{n}_r + rd\theta\hat{n}_\theta, \ \ |d\vec{r}| = \sqrt{(dr)^2 + (rd\theta)^2}$$

Kinematics in 3D: Natural Coordinates

Basic Vectors

1. $\hat{n}_\tau$: along the direction of $\vec{v}$
2. $\hat{n}_n$ and $\hat{n}_b$: perpendicular to the direction of $\vec{v}$

$$\hat{n}_\tau = \frac{\vec{v}}{|\vec{v}|}, \ \hat{n}_n = \frac{\dot{\hat{n}}_\tau}{|\dot{\hat{n}}_\tau|}, \ \hat{n}_b = \hat{n}_\tau \times \hat{n}_n$$

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Basic Formulas

$$\vec{v} = v\hat{n}_\tau$$

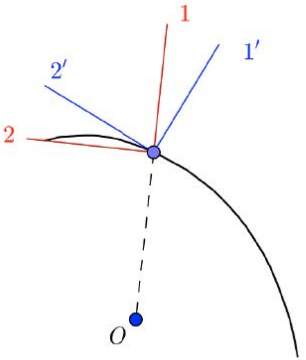
$$\vec{a} = \dot{v}\hat{n}_\tau + \frac{v^2}{R_c}\hat{n}_n$$

R_c means radius of curvature, what is radius of curvature?

Normal and Tangential, Radial and Transversal

CAUTION!

In general, radial $\neq$ normal, nor transverse $\neq$ tangential!



$1'$ normal direction

$2'$ tangential direction

1 radial direction

2 transverse direction

Useful Methods

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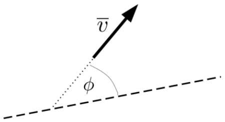
3. Integral by parts:

$$\int u'v = uv - \int v'u$$

Exercise 2

(12 points) A particle moves in a plane so that the angle between the particle's instantaneous velocity $\overline{v}$ and its instantaneous acceleration $\overline{a}$ is constant and equal to α. Let ϕ be the angle that the vector $\overline{v}$ forms with a fixed direction on that plane (see the figure). Initially, the speed of the particle $|\overline{v}(0)| = v_0$ and $\phi(0) = \phi_0$.



Find the instantaneous speed of the particle as a function of the angle ϕ.

Force and Inertial Frame of Reference

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Force represents **interaction** between two objects or an object and its environment.

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Inertial frame of reference

In an inertial FoR, a physical object with zero net force acting on it moves with a constant velocity (which might be zero), or, equivalently, it is a frame of reference in which Newton's first law of motion holds.

Using Newton's Law in Complex System

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1. Objects at relatively rest $\rightarrow$ see as an whole.
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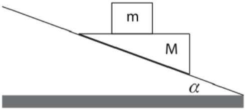
3. To maintain a relatively static condition: $|f| \leqslant \mu_s N$
4. Use inertia force in Non-inertia FoR (usually objects with constant acceleration $\vec{a}$), $\vec{F}' = m(-\vec{a})$.

Exercise 3

(12 points) A wedge with mass M is placed on a frictionless fixed plane inclined at an angle α. The upper surface of the wedge is horizontal. A block with mass m is placed on top of the wedge. The system is released from rest. Assuming that there is no friction between M and m, find the acceleration of the block m with respect to the wedge M just after the system is released from rest. The acceleration due to gravity g is known.



Motion with Air/Fluid Drag

Consider a particle with linear drag $\mathbf{F} = -k\mathbf{v}$ and initial velocity $\mathbf{v_0} = v_0 cos(\alpha)\hat{n_x} + v_0 sin(\alpha)\hat{n_y}$, what's its trajectory?

Two recommended ways:

1. decompose the drag force
2. decompose the velocity

What if quadratic drag force?