上 海 交 通 大 学 试 卷

（ 2019 - 2020 Academic Year/ Summer Semester ）

# Final Exam

# VP160 Honors Physics I

# 4 August 2020

# 16:00-17:40

**You are to abide by the University of Michigan-Shanghai Jiao Tong University Joint Institute (UM-SJTU JI) Honor Code. Please attach the Honor Code pledge to your solutions.**

- **Please carefully read instructions in the exam paper.**
- **Part I of the exam is to be completed on Canvas within a given time limit and submitted before the specified time.**
- **Solutions to exam questions in Parts II and III must be submitted as a pdf file to Canvas assignment 'Midterm Exam' no later than 17:50 (4 August 2020).**
- **In case you have a question, please raise your hand or send a zoom message to the instructor/TA.**

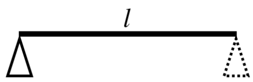
## PART I: Quiz *(10 questions, total 20 points)*

*Please visit Canvas/Quizzes to complete this part. There is a time limit of 20 minutes for this part and it must be completed before 4.30 p.m.*

## PART II: Computational/Conceptual Questions *(5 questions, total 26 points)*

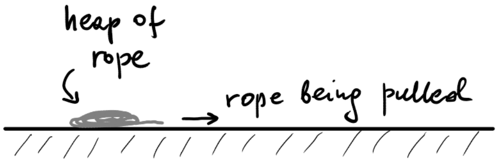
*Please remember to explain your answers!*

**Question 1.** (*4 points*) A uniform rod of length $l$ and mass $m$ rests on supports at its ends. The right support is quickly removed (see the figure).

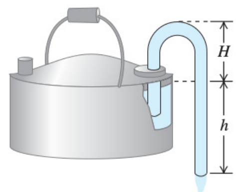



*What is the force on the left support immediately thereafter?* The acceleration due to gravity $g$ is given.

**Question 2.** (*5 points*) A rope of constant linear mass density $\lambda$ lies coiled on the frictionless floor. You grab an end and pull it horizontally with constant force of magnitude $F$. *What is the position of the end of the rope, as a function of time (while it is unraveling)?*



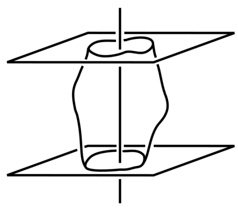

**Question 3.** (*6 points*) A siphon, as shown in the figure below, is used for removing liquids from containers. To establish the flow, the tube must be initially filled with fluid. Let the fluid have density $\varrho$, and let the atmospheric pressure be $p_{\text{atm}}$. Assume that the cross-sectional area of the tube is constant at all points along it. The lower end of the siphon is at a distance $h$ below the surface of the liquid in the container.




A characteristic feature of a siphon is that the fluid initially flows "uphill." *What is the greatest height $H$ that the high point of the tube can have, if flow is still to occur?*

The acceleration due to gravity is $g$. Also, assume that the container has a very large diameter, and ignore any effects of viscosity.

**Question 4.** (*4 points*) A piece of moldable material with constant bulk density, can be formed into any shape. We want to form it into a body to be situated between two parallel planes. The goal is to have the moment of inertia of the body about a fixed axis of rotation, normal to the planes, be as small as possible.



*What shape should the body take?* Justify your answer.

**Question 5.** (*7 points*) *Can you suggest a realistic arrangement of mass, confined in a sphere of a finite radius, such that in a certain region within that sphere a uniform (i.e., constant both in magnitude and direction) gravitational field is produced?*

Explain your answer.

*Continued on the next page!*

## PART III: Problems *(4 problems, total 54 points)*

*Solve the following problems. Show all your work.*

**Problem 1.** (*11 points*) Consider the following force field

$$\mathbf{F}(\mathbf{r}) = 2(xz^2 - y)\hat{n}_x - 2(x + 3yz)\hat{n}_y + (2x^2z - 3y^2)\hat{n}_z,$$

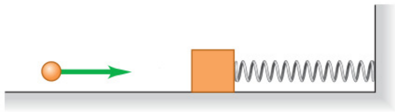
defined in the whole space. The numerical factors have appropriate units, so that the force is expressed in Newtons.

(a) Check whether the field $\mathbf{F}$ is conservative. If so, find the corresponding potential energy.

(b) Use the simplest method to calculate work done by the field $\mathbf{F}$ on a particle that moves from the origin to the point $(1, 1, 0)$ along a straight line.

**Problem 2.** (*13 points*) One end of a massless spring is attached to a wall and the other to a block with mass $2m$ resting on a smooth horizontal surface, as shown in the figure below. The spring does not obey Hooke's law. Namely, the magnitude of the force needed keep the spring deformed (stretched or compressed) by length $l$ is $F = kl^3$, where $k > 0$ is a constant.

Initially, the spring is neither stretched nor compressed. A ball with mass $m$, moving with speed $v_0$, hits the block in an elastic collision and the block starts oscillating.



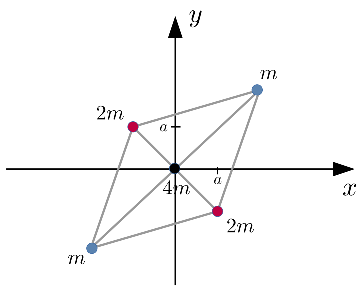
What is the amplitude of these oscillations? What is their period?

**Problem 3.** (*14 points*) Four point masses are arranged into a parallelogram in the plane $z = 0$, so that the masses $m$ and $2m$ are placed in its vertices $(\pm 2a, \pm 2a, 0)$ and $(\mp a, \pm a, 0)$, respectively (see the figure below). Additionally there is a point mass $4m$ placed at the origin. The masses are permanently connected to each other with massless rigid rods.

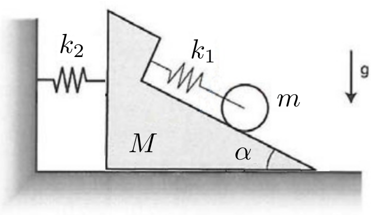
(a) Find the components of the tensor of inertia with respect to the axes $x, y, z$ (the $z$ axis points out of page).

(b) Use symmetry to determine the directions of principal axes.

(c) Diagonalize the tensor from part (a), i.e. find principal moments of inertia and the directions of principal axes to verify the results of part (b).




**Problem 4.** (*16 points*) A wedge with mass $M$ is placed on a frictionless horizontal surface. The wedge is inclined at an angle $\alpha$ to the horizontal. A uniform cylinder of mass $m$, and radius $R$ can roll on the wedge without slipping. An axle, that coincides with the symmetry axis of the cylinder, is connected to the top of wedge by a massless spring, obeying Hooke's law with the spring constant $k_1$. The wedge is connected to a non-moving wall by another massless spring, satisfying Hooke's law with the spring constant $k_2$. The acceleration due to gravity is $g$.




(a) How many degrees of freedom does the system have? Define the corresponding generalized coordinates.

(b) Find expressions for the kinetic energy and the potential energy for the system.

(c) Use the Lagrange formalism to find the equations of motion of the system. (Do not attempt to solve them!)

[End of the exam paper]