VP160 Recitation Class IV

Non-inertial FoR

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Recall

The non-inertial FoR is a frame of reference that is not in a linear motion with uniform velocity relative to an inertial frame of reference.

Basic Formula

$$\vec{a'} = \vec{a} - \vec{a'_O} - \frac{d\vec{\omega}}{dt} \times \vec{r'} - 2(\vec{\omega} \times \vec{v'}) - \vec{\omega} \times (\vec{\omega} \times \vec{r'})$$

$$m\vec{a'} = \vec{F} - m\vec{a'_O} - m\frac{d\vec{\omega}}{dt} \times \vec{r'} - 2m(\vec{\omega} \times \vec{v'}) - m\vec{\omega} \times (\vec{\omega} \times \vec{r'})$$

How to derive the formula?

Recall

Newton's Second Law doesn't hold in non-inertial FoR. To describe the motion in non-inertial FoR, we need to add the forces of inertia (pseudo-forces) into "Newton's Second Law" in non-inertial FoR.

Add $\mathbf{F}_{\textbf{Unreal}}$:

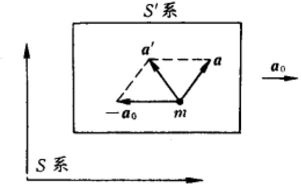
$$\mathbf{F}' = \mathbf{F}_{\textbf{Real}} + \mathbf{F}_{\textbf{Unreal}}$$

to maintain the "Newton's Second Law" in non-inertial FoR:

$$\mathbf{F}' = m\mathbf{a}'$$

$$\mathbf{F}_{\textbf{Unreal}} = -m\vec{a}'_O - m\frac{d\vec{\omega}}{dt} \times \vec{r}' - 2m(\vec{\omega} \times \vec{v}') - m\vec{\omega} \times (\vec{\omega} \times \vec{r}')$$

Drift "Force" $-m\vec{a_{O'}}$



Newton's Second Law in inertia FoR S:

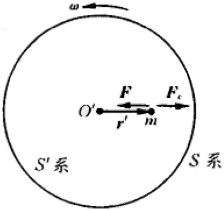
$$\mathbf{F} = m\mathbf{a}$$

Acceleration in non-inertial FoR S':

$$\mathbf{a}' = \mathbf{a} + (-\mathbf{a_0})$$

$$\Rightarrow \mathbf{F}' = m\mathbf{a}' = m\mathbf{a} + m(-\mathbf{a_0}) = \mathbf{F} + m(-\mathbf{a_0})$$

Centrifugal "Force" $-m\vec{\omega} \times (\vec{\omega} \times \vec{r'})$



Uniform circular motion in S :

$$\mathbf{a} = -\omega^2 \mathbf{r}', \quad \mathbf{F} = m\mathbf{a} = -m\omega^2 \mathbf{r}'$$

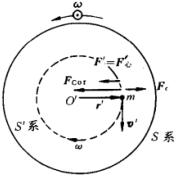
In S':

$$\mathbf{a}' = 0, \quad \mathbf{F}' = m\mathbf{a}' = 0$$

Thus, we need

$$\mathbf{F_c} = m\omega^2 \mathbf{r}' \quad \text{s.t.} \quad \mathbf{F}' = \mathbf{F} + \mathbf{F_c} = 0$$

Coriolis "Force" $-2m(\vec{\omega} \times \vec{v}')$



m stays still in S:

$$\mathbf{F} = 0$$

m moves in a uniform circular motion in S':

$$\mathbf{a}' = -\omega^2 \mathbf{r}', \quad \mathbf{F}' = m\mathbf{a}' = -m\omega^2 \mathbf{r}'$$

Thus, we need (Don't forget the centrifugal force we added)

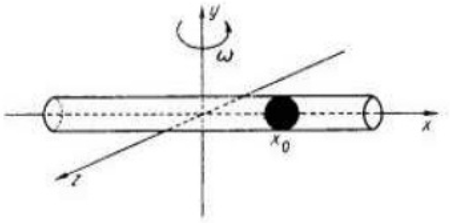
$$\mathbf{F}_{\mathbf{Cor}} = -2m\omega^2 \mathbf{r}' \quad \text{s.t.} \quad \mathbf{F}' = \mathbf{F} + \mathbf{F}_{\mathbf{c}} + \mathbf{F}_{\mathbf{Cor}} = 0 + m\omega^2 \mathbf{r}' + (-2m\omega^2 \mathbf{r}') \\ = m\mathbf{a}'$$

Euler "force" $-m\frac{d\vec{\omega}}{dt} \times \vec{r'}$

- Need to be considered when ω is time-variant.
- Also called Tangential inertial forces.
- Conventionally, we use $\vec{\beta}$ to denote the angular acceleration $\frac{d\vec{\omega}}{dt}$.

Exercise 1

A particle with mass m is inside a pipe that rotates with constant angular velocity ω about an axis perpendicular to the pipe.The kinetic coefficient of friction is equal to μ_k. Write down (do not solve!) the equation of motion for this particle in the non-inertial frame of reference of the rotating pipe. There is no gravitational force in this problem.



Exercise 2

If we let go an object $100m$ above the equator. Ignore the air drag, calculate the deviation caused by Coriolis Force.

Reference

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