The original expression is:

$$P_h[X = i] \approx \frac{\sqrt{2}}{\sqrt{\pi h}\left(1 - \frac{i^2}{h^2}\right)^{\frac{h+1}{2}}\left(\frac{h+i}{h-i}\right)^{\frac{i}{2}}}$$

We will approximate this expression for large h, aiming to reduce it to a Gaussian form.

Step 1: Approximation of $\left(1 - \frac{i^2}{h^2}\right)^{\frac{h+1}{2}}$

We begin by simplifying the term:

$$\left(1 - \frac{i^2}{h^2}\right)^{\frac{h+1}{2}}$$

For large h, we assume that $\frac{i^2}{h^2}$ is small. We can then use the binomial expansion (or a Taylor series expansion) for small x, which states:

$$(1 - x)^n \approx e^{-nx} \text{ for small } x$$

Here, $x = \frac{i^2}{h^2}$ and $n = \frac{h+1}{2}$, so we approximate:

$$\left(1 - \frac{i^2}{h^2}\right)^{\frac{h+1}{2}} \approx e^{-\frac{h+1}{2} \cdot \frac{i^2}{h^2}}$$

For large h, the factor $\frac{h+1}{2}$ is approximately $\frac{h}{2}$, so this simplifies further to:

$$e^{-\frac{h}{2} \cdot \frac{i^2}{h^2}} = e^{-\frac{i^2}{2h}}$$

Thus, the term $\left(1 - \frac{i^2}{h^2}\right)^{\frac{h+1}{2}}$ is approximated by:

$$e^{-\frac{i^2}{2h}}$$

Step 2: Approximation of $\left(\frac{h+i}{h-i}\right)^{\frac{i}{2}}$

Next, we simplify the term:

$$\left(\frac{h+i}{h-i}\right)^{\frac{i}{2}}$$

For large h, we can use the logarithmic approximation. First, express the ratio as:

$$\frac{h+i}{h-i} = 1 + \frac{2i}{h-i}$$

For large h, $h - i \approx h$, so we approximate the ratio as:

$$\frac{h+i}{h-i} \approx 1 + \frac{2i}{h}$$

Now, we apply the logarithmic expansion for small x, which states:

$$\log(1+x) \approx x \text{ for small } x$$

Thus:

$$\log\left(\frac{h+i}{h-i}\right) \approx \frac{2i}{h}$$

Exponentiating both sides, we obtain:

$$\left(\frac{h+i}{h-i}\right)^{\frac{i}{2}} \approx e^{\frac{i}{2}\cdot\frac{2i}{h}} = e^{\frac{i^2}{h}}$$

Thus, the term $\left(\frac{h+i}{h-i}\right)^{\frac{i}{2}}$ is approximated by:

$$e^{\frac{i^2}{h}}$$

Step 3: Combining the Results

Now we combine the results from Step 1 and Step 2. The original expression becomes:

$$P_h[X = i] \approx \frac{\sqrt{2}}{\sqrt{\pi h}} e^{-\frac{i^2}{2h}} e^{\frac{i^2}{h}}$$

Simplifying the exponents:

$$P_h[X = i] \approx \frac{\sqrt{2}}{\sqrt{\pi h}} e^{-\frac{i^2}{2h}}$$

This is a Gaussian distribution with mean 0 and variance proportional to h.