CS433 Assignment

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1 Task 0

We just used the standard encoding without any optimizations as given in the paper. For $k = 2$, it gave UNSAT for various values of n. We can also use a lemma in the paper which states:
If the lower bound is $\geq k$ and $D_{r,c,k}$ is not satisfying for some r, c. Then the lower bound is $\geq k + 1$.
Here $D_{r,c,k}$ is true iff there's a k color packing of the graph made by nodes i such that $|i| \leq r$ and 0 gets color c.
For $k = 3$, the coloring we found was $\ldots, 3, 1, 2, 1, 3, 1, 2, 1, \ldots$.

2 Task 1

2.1 The Variables

Each vertex $v = (x, y)$ in the grid and each color $c \in \{1, \ldots, k\}$ is associated with a Boolean variable $x_{(v,c)}$, which is true iff v has color c.

2.2 Grid Construction

The grid consists of all integer coordinate points (x, y) satisfying $|x| + |y| \leq r$, forming a diamond-shaped structure centered at the origin. For example, with $r = 2$, the grid includes points such as $(0, 0)$, $(1, 0)$, $(0, 1)$, and $(-1, 0)$.

2.3 Core Constraints

2.3.1 Unique Color Assignment

Each vertex must be assigned exactly one color. This is enforced by the clause:

$$\bigvee_{c=1}^{k} x_{v,c}$$

Additionally, a vertex cannot have multiple colors simultaneously.

2.3.2 Packing Constraints

For any color c, two vertices assigned this color must be at least $c + 1$ units apart in Manhattan distance. Specifically, for any v_1 and v_2 where $d_{\text{manhattan}}(v_1, v_2) \leq c$, the following constraint applies:

$$\neg x_{v_1,c} \vee \neg x_{v_2,c}$$

2.4 Advanced Optimizations for $k > 3$

2.4.1 Regional Variables

For colors $c \geq 4$, additional "plus"-shaped regions S_v are defined, centered at each vertex v. A regional variable $r_{S,c}$ is introduced, ensuring:

$$r_{S,c} \Rightarrow \bigvee_{v \in S} x_{v,c}$$

$$x_{v,c} \Rightarrow r_{S,c}, \quad \forall v \in S$$

2.4.2 Region Packing Constraints

To maintain proper spacing, active regions for the same color must be at least $c + 1$ units apart:

$$\neg r_{S_1,c} \vee \neg r_{S_2,c}, \quad \text{if } d(S_1, S_2) < c + 1$$

2.4.3 At-Least-One-Distance (ALOD) Condition

Each vertex must have at least one neighbor (including itself) colored with 1:

$$\bigvee_{u \in N(v)} x_{u,1}$$

2.4.4 Symmetry Breaking

To reduce redundant search space, if a color appears uniquely in a symmetric region, its placement is restricted to a single section, such as an upper quadrant.

2.5 Packing Chromatic Number Bounds

2.5.1 Lower Bound

We could show that till k = 12, a finite graph couldn't get colored in the mentioned way.

2.5.2 Upper Bound

For radius 12 and k = 15, we found a satisfiable coloring for a finite graph.