



K-means Clustering of Student Behavioral Patterns and Advanced Visualization Methods of Learning Technology Data

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Reilly Norum

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Report Submitted to:

Professor Lane Harrison
Computer Science

Professor Erin Ottmar
Learning Sciences and Technologies

Worcester Polytechnic Institute

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Summary of the project

This MQP presents two studies that examine middle school students' mathematical and behavioral patterns as they solve problems in an online algebraic learning game, *From Here to There! (FH2T)*. This game was designed and developed by members of our research team, and our previous studies showed that the game was effective in improving students' mathematical understanding (Hulse et al. 2019; Ottmar et al, 2015; Chan et al., 2021; Decker-Woodrow et al., under review). Over the past several years, a number of randomized controlled trials have been conducted on this game to test its impact and efficacy in middle school math classrooms. While the larger studies have focused on efficacy, other studies have leveraged the large amounts of secondary data collected from the technology as students solve problems. For example, a recent study using the log data in the game (Lee et al., under review) showed that four subgroups of students emerged based on in-game behavioral patterns, and a subgroup of the students who reattempted the same problem multiple times more often showed the largest learning gains.

The first study (Study 1) included in this MQP project provides a replication of this prior study (Lee et al., under review) using a larger and more recent data set ($N = 760$) that was conducted during the 2020-21 school year. The second study (Study 2) also replicated Lee et al.'s methodological approaches but a different set of variables were used. In both studies, I applied k-means clustering analysis to clickstream data collected in the game and then examined how students' behavioral patterns varied across the clusters using data visualizations.

Specifically, Study 1 aimed to answer the following research questions:

1. Do the same amount of clusters emerge based on students' behavioral patterns in the game?
2. Do the different clusters show similar changes in their understanding of mathematical equivalence?
3. Do the students' problem-solving processes and solution strategies show similar variance across clusters?

The results of the k-means cluster analysis showed that four distinct subgroups of the students emerged based on four variables of students' behaviors in the game; the number of problems completed, the proportion of reattempts, resets, and average pause time. Clusters were labeled based on their progress in the game and individually examined to see what type of student made up each cluster. Pretest and posttest scores were included in correlation analysis, showing that both pretest scores and the number of problems completed were positively correlated with posttest scores. Despite not including pretest scores in the cluster analysis, there were distinct differences in pretest scores across the clusters. Study 2 aimed to address the following research questions:

1. How many clusters emerge based on students' mathematical strategies used in the game?
2. Can the given clusters based on strategies used in the game significantly predict changes in students' mathematical understanding?
3. How do students' mathematical understanding vary across the given clusters?

The results of the k-means cluster analysis showed that four distinct student profiles emerged based on errors, hint usage, first step efficiency, and first step validity. It was identified that a clear difference between clusters was their help-seeking tendencies (usage of hints). Similar to Study 1, pretest and posttest scores were positively correlated, and there was also a strong positive correlation between pretest scores and the validity of the students' first step. A linear regression analysis was then conducted to see if posttest scores were significantly explained by cluster results, both before and after controlling for prior knowledge. The results indicated, as expected, the "high knowledge" group performed significantly better on the posttest both before and after controlling for prior knowledge, and the "low knowledge non-help seeking" group performed significantly better on the posttest compared to the control group.

1.0 Introduction

Mathematical learning has been a struggle for students of all ages and according to the National Assessment of Educational Progress, only 41% of fourth-graders and 34% of eighth-graders are proficient in mathematics (Hussar et al., 2020). Technology-based educational games have been introduced to support students' mathematical learning and address this issue. Previous studies have shown that well-designed educational games have a positive effect on student learning (Ke, 2013; Novak & Tassel, 2015).

1.1 Use of Hints

In traditional classrooms, help seeking tendencies can be seen as asking a question during class, reaching out to the instructor outside of class, or asking peers for assistance. However, with the emergence of online educational technologies, help seeking tendencies can look slightly different. While for some students, in the traditional classroom setting, asking for help can be anxiety inducing; however, online educational technologies offer a way for students to receive immediate assistance without the social pressure.

The help provided from online educational games differs from what is typically seen in the classroom. In online educational games, students are able to receive immediate help at the touch of a button. Types of assistance in online educational games range from context-specific hints (Anderson et al., 1995; Stamper et al., 2011) and worked problems, to support materials such as videos or glossaries (Aleven & Koedinger, 2000; Whitehill & Seltzer, 2016).

There are certain drawbacks to offering help on demand. It has been known that students often ignore the help or use them in ways that are not likely to promote learning (Aleven et al., 2003). However, two studies conducted on ASSISTments (Razzaq & Heffernan, 2010; Kehrner et al., 2013) found that when on-demand hints were available, students who requested more hints tended to perform better on algebra assignments, and receiving immediate feedback allowed for

more learning to occur. Although these results suggest that on-demand assistance may enable students to learn more, the extent of the benefits is still unclear.

1.2 Errors

There are several different ways to detect these misconceptions, or *mathematical errors* that students make during algebraic problem-solving can be a signal of these misconceptions (Russell et al., 2009). Further, students' mathematical errors do not often occur at random, instead, they may be systematically derived from their previous experiences or learned strategies. Thus, it is critical to better understand where and when mathematical errors occur in algebraic problem-solving processes to support students in correcting their misunderstandings. There are three types of errors that can occur when solving a problem in FH2T. *Shaking errors* occur when students tap on an operator to perform a mathematically invalid operation, such as adding before multiplying in $3+4*5$. *Snapping errors* occur when students drag a term to commute it within the expression (e.g., dragging and dropping 5 in front of 3 in $3+4*5$). *Keypad errors* occur when students use the keypad to substitute a term (e.g., 8) with a nonequivalent expression (e.g., $4+5$). Exploring students' errors in FH2T may provide insights into their misconceptions in algebra.

1.3 Pauses

In order to solve mathematical problems, students must be able to recognize the structure of a problem and apply the correct procedures. However, it can be seen that students often try to race through problems without taking time to implement efficient strategies (Schoenfeld, 1992). A recent study consisting of fifth- and sixth- graders showed that students' behavior varied (i.e. whether or not they took time to understand the problem) which was, in turn, associated with more accurate judgment of problem-solving performance (García, Rodríguez, Gonzalez-Castro, et al., 2016).

Many studies refer to students' metacognition, or their knowledge of one's own cognitive processes and the ability to regulate and monitor the processes (Flavell, 1976). It can be seen that when students regulate their behaviors strategically, for example focusing on the task and planning their actions, they show metacognition skills which ultimately contribute to success in mathematical tasks (Garofalo & Lester, 1985). This can be seen in a study conducted in 2019 with 524 upper elementary school students. It was observed that fifth- and sixth- graders who spent more time on a problem were more likely to correctly solve the problem (García et al., 2019) and showed more planning strategies than their peers that solved the problem incorrectly. A separate study showed that prompting students to think about what, when, and why certain strategies should be used improved ninth-graders' mathematical learning ability in an online environment (Kramarski & Gutman, 2006). The same relationship can be seen in a study of fifth-graders where those who received intervention, involving understanding the problem and planning strategies, showed improvement in their problem-solving abilities (Vula et al., 2017).

With all of these studies in mind, it can be seen that taking time to understand the problem and develop a plan before solving could promote more efficient problem-solving skills.

Prior research suggests that pause time may be an indicator of thinking and planning how to solve the problem. This idea can be seen in studies of all ages. A 1995 study showed that adults who paused longer before their first move in the Tower of Hanoi task (rearranging disks according to specified rules) completed the task with fewer moves and reported using the time for planning purposes (Welsh et al., 1995). In the same way, a study of university students from 2010 found that students who spent more time on developing a plan before attempting the question performed better on critical thinking tasks that involved hypothesis testing and argument analysis (Ku & Ho, 2010). In an online science intelligent tutoring system, researchers used the number and duration of pauses as indicators of students' cognitive engagement (Gobert et al., 2015).

Overall, the studies above suggest that a longer pause time could indicate that the student is taking time to think through the problem and plan a strategy to solve it, ultimately, showing greater mathematical learning.

1.4 Dosage and Student Progress

Algebra is an important concept for students to learn in order to set them up for future success. By middle school, many students begin to struggle in mathematics and learning the new concepts, specifically algebraic ideas. In elementary school, students are taught math concretely with tangible objects to assist in creating a connection between math and the real world (Bruner et al. 1966). Once algebra is introduced, the abstract nature of variables allows for less possible tangible connections (Booth et al. 2014). The transition from concrete to abstract concepts in middle school can be identified as a reason for students falling behind. When the students begin to struggle, they often become disengaged and continue to struggle with basic algebraic concepts (Stein et al. 2011).

To prepare students for algebraic learning, some researchers suggest introducing algebraic concepts in early elementary school (National Council of Teachers of Mathematics (NCTM) 2000; Stephens et al. 2015). Research suggests that children begin to develop the ability to reason algebraically before they begin formal schooling (Doig and Ompok 2010). Developmentally, students are able to be taught algebraic concepts as long as they are scaled down to meet their skill level (Bay-Williams 2001; Carpenter et al. 2005; Carraher et al. 2006). By introducing students to algebraic ideas early, as students progress in their mathematical thinking, they may be better prepared for more difficult concepts (Bransford and Schwartz 1999; Koedinger et al. 2008; NCTM 2000). This idea has caused national organizations to encourage early intervention programs in elementary school to improve student algebra readiness (National Council of Teachers of Mathematics (NCTM) 2014).

1.5 From Here To There! : The Mathematical Game Context

From Here to There! (FH2T, <https://graspablemath.com/projects/fh2t>), is a gamified version of Graspable Math that was developed based on perceptual learning theories to help students' conceptual and procedural learning in algebra. The game allows students to dynamically manipulate and transform numbers and mathematical expressions using various gestures on screen.

The goal of the game is to transform a given expression into the mathematically equivalent goal using algebraic actions. As seen in Figure 1, each problem consists of two equivalent mathematical expressions, a start state (e.g., $121 \cdot 144$) and a goal state (e.g., $11 \cdot 132 \cdot 12$). Students must transform the starting expression into the goal state using gesture-like actions, such as moving, tapping, splitting, or decomposing numbers and expressions. This system provides a fluid visualization to show students how their actions change the problem at hand. If students complete the problem in the most efficient way, they receive three clovers (i.e., with the minimum required number of steps to reach the goal state). However, if the student uses more than the fewest steps possible, the number of clovers is decreased.

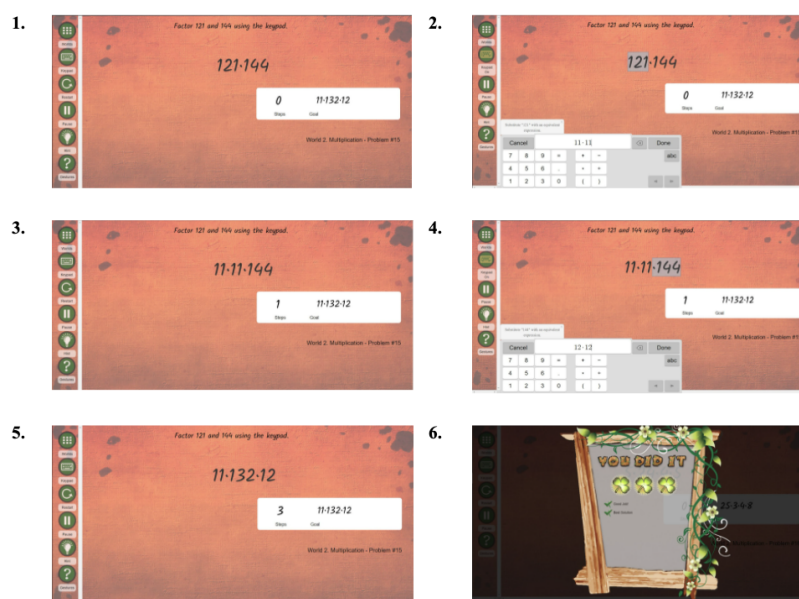


Figure 1. A sample problem and students' action in the game

Each level includes the opportunity to reset or reattempt the problem as many times as the student wants. If the restart button on the left side of the screen is pressed, the mathematical expression and number of steps are reset to the initial state for the student to start from the beginning. Students can also revisit a problem anytime to try to solve the problem in a more efficient way. FH2T consists of 14 worlds (a total of 252 problems) that cover various mathematical concepts. Students are able to advance to the next world once they have completed 14 consecutive problems in the world before.

Several studies over the past decade aimed to examine the efficacy of FH2T on middle school students. A preliminary study with 130 middle school students showed that, after four

30-minute intervention sessions, students in the fluid visualizations condition (similar to FH2T) improved their scores and the students in the manual calculations group did not (Ottmar et al., 2015). Additionally, a randomized control trial (RCT) with 475 middle school students showed that, after four 30 minute intervention sessions, students who used FH2T scored higher on the posttest compared to their peers (Chan et al., 2021). Lastly, a study consisting of 185 second grade students using an elementary version of FH2T showed that completing more unique problems in the game was associated with higher posttest scores. This effect was significant above and beyond the prior knowledge students possessed prior to the intervention sessions, as measured by the pretest (Hulse et al., 2019).

1.5.1 The Data Logging in FH2T

As students solved problems in the game, the system automatically logged files of all students' touch- or mouse-based actions with timestamps. While many indicators were available, these studies focus on eight behaviors in the game. Definitions and descriptions are provided in Table 1 below.

Table 1. Table of variables used in cluster analysis

Variables	Operational definitions	Measures
<i>Study 1</i>		
Behavioral patterns (Independent variables for clustering)		
Number of problems completed	The distinct number of problems completed	Sum of the distinct number of problems completed during the intervention
Average number of resets	Hitting the reset button in the game to set the equation back to an initial state and restart the problem	$\frac{\text{the total number of resets}}{\text{the total number of problems completed}}$
Proportions of reattempts	Revisiting to solve the same problem again after completing it	$\frac{\text{the total number of reattempts across problems}}{\text{the distinct number of problems completed}}$
Average pause time	The amount of time students spent before taking their first action in their first attempt to reach the goal state	$\frac{\text{the time spent before taking the first action}}{\text{the total time spent to complete the problem}}$
<i>Study 2</i>		
Behavioral patterns (Independent variables for clustering)		
Percent error	The percent of problems where the student made at least 1 error	$\frac{\text{number of problems with at least 1 error}}{\text{the total number of problems seen}}$
Percent hint usage	The percent of seen problems where a hint was used	$\frac{\text{number of problems where a hint was used}}{\text{the total number of problems completed}}$

Average first efficiency	The average number of problems that the student completed on the first attempt	$\frac{\text{number of problems completed on first attempt}}{\text{the distinct number of problems completed}}$
Percent validity of first step	The percent of problems where the first interaction is a valid step	$\frac{\text{number of problems with valid first step}}{\text{the total number of problems attempted}}$
Outcome variables of interest		
Understanding of mathematical equivalence (for pretest and posttest)	Students' knowledge of mathematical equivalence	The sum of correctness on 6 items (correct=1, incorrect=0), Ranges between 0 and 6

1.6 The Larger Classroom Based Study

The data being used for these studies was collected as part of a larger randomized controlled study being conducted by the MAPLE Lab at WPI and funded by the US Department of Education Institute for Education Sciences (IES). This study tests the impact of three educational technology interventions on algebraic understanding in middle schoolers across four conditions: (a) the game used in this paper, From Here to There (FH2T), (b) DragonBox 12+, (c) Immediate Feedback, and (d) Active Control. Both the FH2T and DragonBox conditions represent use of game-based applications. The Immediate Feedback condition is an online homework system called ASSISTments. Active Control, in this study, is meant to represent traditional homework assignments while still using technology. Students were assigned a condition, and after nine 30-minute intervention sessions over the school year, it was seen that students in the FH2T and DragonBox conditions had significantly higher posttest scores compared to their peers (Decker-Woodrow et. al., submitted). This effect remained when controlling for prior knowledge. It was seen that the Immediate Feedback condition had significantly higher posttest scores compared to the Active Control condition but this did not hold true when controlling for prior knowledge. It was concluded that FH2T and DragonBox are effective digital games to support middle school algebraic learning. For the purpose of this paper, we are looking at only FH2T.

2.0 Study 1

The following paper was submitted and accepted to the International Conference of the Learning Sciences (ICLS) 2022 conference.

Norum, R., Lee, J. E., & Ottmar, E. (2022). Student profiling on behavioral patterns in an online mathematics game: Clustering using K-means. *Proceedings of the 16th International Conference of the Learning Sciences*. Hiroshima, Japan: International Society of the Learning Sciences.

Student Profiling on Behavioral Patterns in an Online Mathematics Game: Clustering Using K-Means

Reilly Norum, renorum@wpi.edu
Ji-Eun Lee, jlee13@wpi.edu
Erin Ottmar, erottmar@wpi.edu
Worcester Polytechnic Institute

Abstract: This preliminary study examined whether distinct student profiles ($N = 760$) emerged based on their behavioral patterns in an online algebraic learning game. We applied k-means clustering analysis to clickstream data collected in the game and then examined how students' behavioral patterns varied across the clusters using data visualization. The results identified four groups of students based on their in-game behaviors, showing that there was a large variation in their behavioral patterns for engaging with the game.

Introduction

With the rapid development of game technology, there has been an increasing interest in the use of online educational games in mathematics learning. Previous studies have reported that well-designed online games are effective in improving students' mathematical knowledge, skills, and engagement (Chang et al., 2016). However, due to the greater flexibility and interactivity of online educational games compared to other types of educational technology tools, their effects largely depend on many factors, in particular, student behavioral patterns in games (Martin et al., 2015). Thus, it is important to understand how individual students behave differently in the game and how their different behavioral types relate to learning outcomes in order to provide more personalized learning environments (Vandewaetere et al., 2011).

Our team has developed an online mathematics learning game to help middle-school students' algebraic learning. Our previous work has shown that the game is effective in enhancing students' algebraic understanding, but the efficacy varies depending on students' behavioral patterns in the game (Authors, in review). In this study, we aim to replicate the findings of our previous work with a larger dataset that includes a more diverse sample in terms of instructional level and race/ethnicity. Our study addresses the following research questions: 1. How many different clusters emerge based on students' behavioral patterns in the game? 2. How do the students' behavioral patterns vary across the clusters?

Method

The game was developed based on perceptual learning theories to help students' algebraic learning. The goal of the game is to transform the expression into the mathematically equivalent but perceptually different target goal using various gesture actions. The system allows students to dynamically manipulate and transform numbers and mathematical symbols on the screen so that students can identify algebraic structures easily and think flexibly. Our sample consisted of 760 (55% male, 45% female) 7th-grade students from ten middle schools located in the Southern U.S. The students played the game individually for nine 30-minutes intervention sessions. For cluster analysis, we included four variables of in-game behaviors that correlated to learning outcomes identified in our earlier work (Authors, in review). These variables were the total number of problems completed, the average number of resets, the proportion of reattempts (i.e.,

revisiting the completed problem again), and the average pause time (i.e., the amount of time spent before making the first action).

Results

The elbow method was used to determine the optimal number of clusters, suggesting four clusters. The k-means cluster algorithm divided the entire sample into four clusters of students based on their behavioral patterns: Cluster 1 ($n = 188$, 24.7%), Cluster 2 ($n = 186$, 24.5%), Cluster 3 ($n = 76$, 10%), and Cluster 4 ($n = 310$, 40.8%). Figure 2 shows the box plots of four variables of student behavioral patterns by cluster.

The clusters were labeled based on the most dominant characteristics that appeared for each cluster: Slow Progressors (Cluster 1) included students who completed the least number of problems, with the lowest values for the proportion of reattempts and the average number of resets. Fast Progressors (Cluster 2) included students who completed the largest number of problems but the low values of the average number of resets, the proportion of reattempts, and average pause time. These students sped through the game, completing as many problems as possible, rather than taking time to think about their first step. Slow-Steady Progressors (Cluster 3) included students who had the highest proportion of reattempts and the average number of resets. This cluster seemed to carefully progress through the problems, resetting and retrying problems along the way. Intermediate Progressors (Cluster 4) included students who showed middle values for all variables.

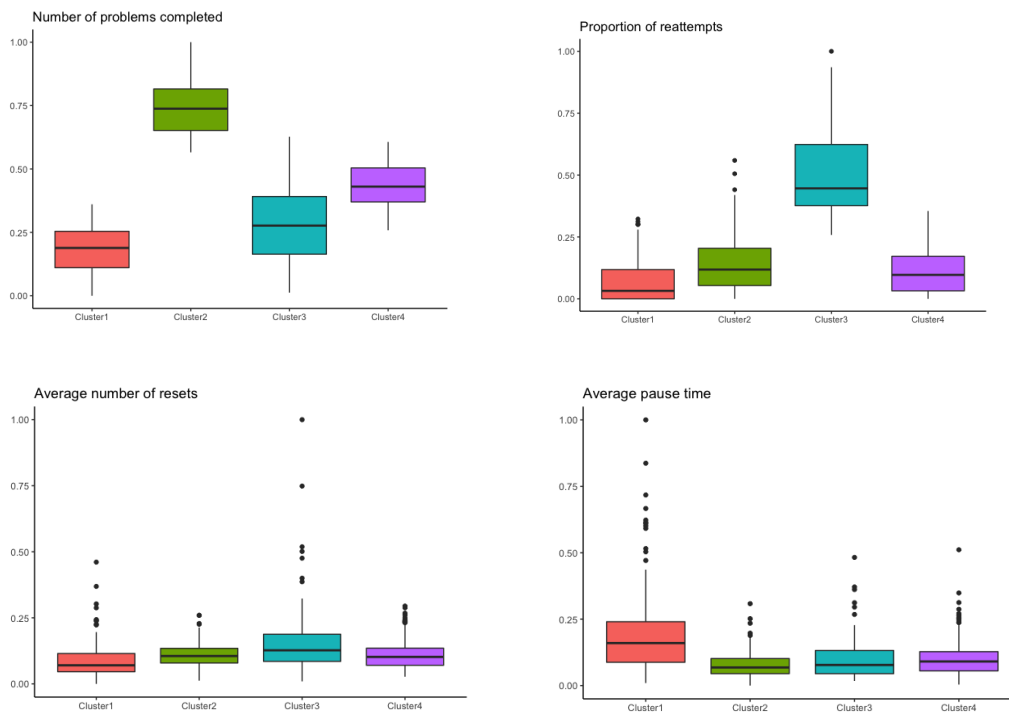


Figure 1

*Cluster results depicting the distribution of variables of behavioral patterns for four clusters
(Note: Red: Cluster 1- Slow Progressors. Green: Cluster 2- Fast Progressors. Turquoise: Cluster 3- Slow-Steady Progressors. Purple: Cluster 4- Intermediate Progressors)*

Discussion

This preliminary study examined whether distinct student profiles emerged based on their behavioral patterns in an online algebraic learning game. The results of the k-means clustering analysis identified four distinct groups of students based on their in-game behaviors, showing that there was a large variation in their behavioral patterns for engaging with the game. These student profiles identified from the cluster analysis can serve as a basis for building an adaptive learning environment (Vandewaetere et al., 2011). Further research should be done to investigate how these different behavioral patterns correlate to mathematics performance.

2.1 Further Information about Methodology to Conduct Study 1

While the above paper was accepted as a short paper at ICLS, there are a number of core components that were not included in the paper. Below, I provide more information about the participants, the methods, and the results of the project. The game in the paper above is the same game used in Study 2, described on page 6.

Means, standard deviations, minimum and maximum values, and correlations coefficients for all variables used in the study are presented in Table 2. A Pearson's product-moment correlation analysis was conducted to examine the relationships between variables. Two variables showed statistically significant and strong positive correlations with posttest score: pretest scores ($r(759) = .67, p < .001$) and number of distinct problems completed in the game ($r(759) = .59, p < .001$). Two variables had a small negative correlation with posttest score, average number of resets ($r(759) = -.09, p < .05$) and average pause time before first interaction with the problem ($r(759) = -.15, p < .001$). The proportion of reattempts did not have a statistically significant association with the posttest scores. From the initial analysis, students' posttest scores may be associated with in-game behaviors.

One variable had a moderate negative correlation with the number of distinct problems completed in the game, the average time for the student to first interact with the problem ($r(759) = -.38, p < .001$).

Table 2. Descriptive statistics and correlations of sample ($N=760$)

$(N = 760)$						
Variable	1	2	3	4	5	6
1. Posttest scores	-					
2. Pretest scores	.67***	-				
3. o_distinct_completed	.59***	.49***	-			
4. o_percent_gobacks	-.02	.02	-.04	-		
5. o_avg_resets	-.09*	-.15***	-.04	.34***	-	

6.	o_avg_time_interaction_first	-.15***	-.07	-.38***	-.10**	-.14***	-
	<i>M</i>	4.48	4.66	111.00	.14	.86	16.14
	<i>SD</i>	2.93	2.67	55.03	.15	.55	9.33
	Min.	0.00	0.00	5.00	0.00	.04	5.55
	Max.	10.00	10.00	249	0.93	7.44	93.88

*** $p < .001$, ** $p < .01$, * $p < .05$

The elbow method was used to determine the optimal number of clusters. As shown in Figure 2, the “fviz_nbclust” R function suggested four cluster solutions (dotted line), and the elbow point also indicated that $k = 4$ would produce the optimal cluster results.

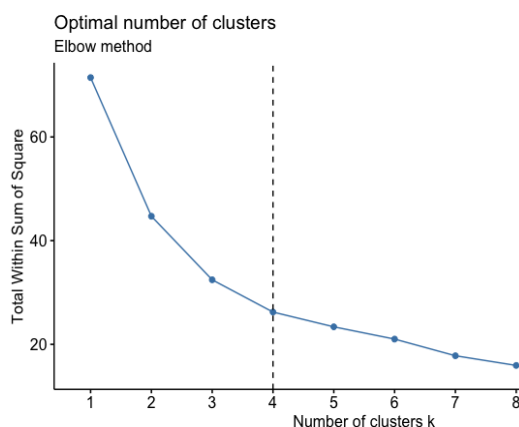


Figure 2. The elbow method for determining the optimal number of clusters.

Table 3. Descriptive statistics of each variable by cluster

Variable	<i>Cluster 1:</i> <i>Slow</i> <i>Progressors</i>		<i>Cluster 2:</i> <i>Fast</i> <i>Progressors</i>		<i>Cluster 3:</i> <i>Slow-Steady</i> <i>Progressors</i>		<i>Cluster 4:</i> <i>Intermediate</i> <i>Progressors</i>	
Number of students in the cluster (%)	188 (24.7%)		186 (24.5%)		76 (10%)		310 (40.8%)	
Number of problems completed, <i>M</i> (<i>SD</i>)	49.91 (21.75)	L	186.55 (25.90)	H	73.20 (35.25)	L	112.67 (19.95)	M
Average number of resets, <i>M</i> (<i>SD</i>)	.70 (0.48)	L	.85 (0.31)	M	1.30 (1.17)	H	.93 (0.37)	M
Proportion of reattempts, <i>M</i> (<i>SD</i>)	.06 (0.07)	L	.13 (0.09)	L	.48 (0.16)	H	.11 (0.09)	L
Average pause time (seconds), <i>M</i> (<i>SD</i>)	229.57 (139.45)	H	124.4 (41.43)	L	150.02 (79.30)	L	138.44 (44.69)	L

Note. H, M, and L indicate that the mean for the cluster was high, medium, or low within the data.

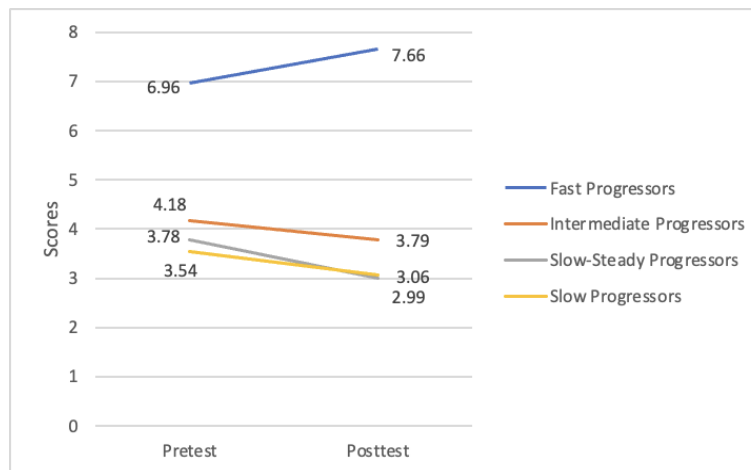
From the cluster analysis, the sample ($N = 760$) was divided into four groups of students based on their behavioral patterns: Cluster 1 ($n = 188$, 24.7%), Cluster 2 ($n = 186$, 24.5%), Cluster 3 ($n = 76$, 10%), and Cluster 4 ($n = 310$, 40.8%). Table 3 shows the descriptive statistics of each variable by clusters with relative descriptions of the values (low, medium, high) within the data. Figure 1 (above) shows the box plots of four variables of behavioral patterns by cluster.

We then explored whether there is a significant difference in students' understanding of mathematical equivalence at baseline according to their behavioral patterns identified in the cluster analysis. Table 4 shows the means, standard deviations, and medians of pretest and posttest scores. It can be seen that despite not including pretest scores in the cluster analysis, there were distinct differences in the pretest scores of the four clusters: fast progressors had the highest pretest scores ($M=6.96$, $SD=2.41$, $MED=8.00$) and slow progressors had the lowest pretest scores ($M=3.54$, $SD=2.18$, $MED=3.00$).

Table 4. Descriptive statistics for four clusters

Cluster	Pretest			Posttest		
	M	SD	Mdn	M	SD	Mdn
Fast Progressors ($n = 186$)	6.96	2.41	8.00	7.66	2.30	8.00
Intermediate Progressors ($n = 310$)	4.18	2.35	4.00	3.79	2.41	3.00
Slow-Steady Progressors ($n = 76$)	3.78	2.27	3.00	2.99	1.98	3.00
Slow Progressors ($n = 188$)	3.54	2.18	3.00	3.06	2.16	3.00

Figure 3. Slope graph of average pretest and posttest score by cluster



3.0 Study 2

3.1 Introduction

While Study 1 focused on several behavioral indicators, Study 2 focused more on mathematical indicators of knowledge. In this paper, I utilized the same methodological approaches and replaced the four variables with student errors, hint usage, efficiency of strategy, and the mathematical validity of students' first actions. The goal of this paper was to determine whether we could utilize these in game variables to indicate student profiles that could be indicative of their mathematical knowledge. K-means clustering was used to establish student profiles and a regression analysis with and without prior knowledge accounted for was run.

3.1.1 Participants

Our sample (N=760) was drawn from a randomized controlled study conducted in 2020-2021, which examined the efficacy of three different educational technologies with four different conditions. The initial sample consisted of 1,119 students from six middle schools located in the Southern United States who were assigned to the FH2T condition. The sample was further refined to 760 students as 359 students were missing information vital to the study, such as pretest and posttest score. Of the 760 students included for further analysis (55% male, 45% female), all of the students were in the seventh grade. The students were in varying levels of mathematics classes with 85% of students in advanced, 8% of students in on-level, and 7% of students in support.

3.1.2 Materials

As described on page 6.

3.1.3 Measures

While the original study focused on student behavioral patterns in game, this study focuses on mathematical strategy/understanding. Four variables were selected for this study:

1. `o_percent_error` - the percentage of problems attempted where the student made at least one error
2. `o_percentage_hint` - the percentage of attempted problems where a hint was used
3. `o_avg_first_efficiency` - the average efficiency of all the first step attempts of problems attempted
4. `o_percent_interaction_step_first` - the percentage of problems where the first interaction in the first attempt was a valid step

Each variable was normalized using min-max scaling in order to prevent one of the variables with a larger scale from over influencing the cluster solutions (Kassambara, 2017).

3.1.4 Data Analysis

In order to provide insight into the first research question, a k-means clustering analysis was performed using the “cluster” package in R. This method is one of the most commonly used unsupervised machine learning algorithms to categorize data into groups based on similar characteristics. The algorithm compares the distances between data points and a specified k number of centroids, or cluster centers, that are iteratively calculated using the observed data.

To identify clusters, Euclidean distance was used in conjunction with the Hartigan-Wong algorithm. This combination defines the total within-cluster variation as the sum of squared distances between items and the corresponding centroid. The optimal number of clusters was verified using the “fviz_nbclust” function in the “factoextra” R package. This function implemented the elbow method by identifying k as the point where the reduction in the total within-cluster sum of squares drops significantly and produces an angle, or elbow, in the graph. Ultimately, k is selected based on the number of clusters that best describes the data such that adding more clusters would not improve the explanation of variance among and between groups of data points.

To run a regression analysis, four dummy variables were created to represent which cluster each student belongs to (1 being they are in the cluster, 0 being they are not). The “lm.beta” package was used to create two models, one without prior knowledge and one with prior knowledge accounted for.

3.2 Results

3.2.1 Descriptive Statistics and Correlation Analysis

Means, standard deviations, minimum and maximum values, and correlations coefficients for all variables used are presented in Table 5 below. A Pearson’s product-moment correlation

analysis was conducted to examine the relationships between variables.

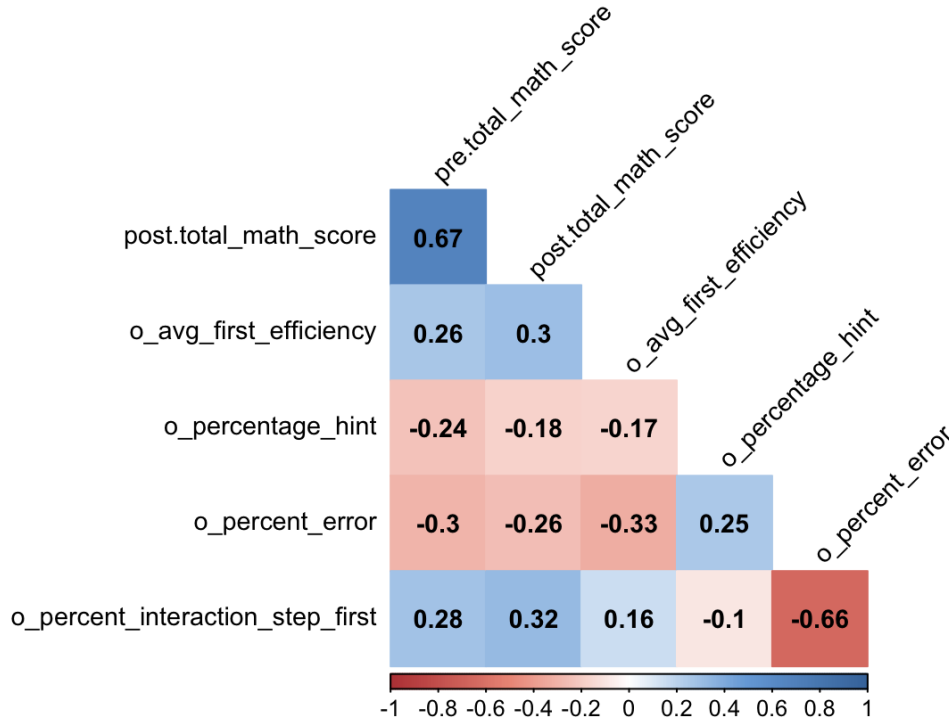


Figure 4. Correlation plot of variables used in cluster analysis

One variable showed statistically significant and strong positive correlation with posttest scores: pretest scores ($r(759) = .67, p < .001$). Two variables showed moderate, positive correlations with posttest score, average efficiency of the first step ($r(759) = .3, p < .001$) and percent interaction step first ($r(759) = 0.32, p < .001$). Two variables had a small negative correlation with posttest score, percent error ($r(759) = -.26, p < .001$) and percent hint ($r(759) = -.18, p < .001$). From the initial analysis, students' posttest scores may be associated with the mathematical understanding measures examined.

Table 5. Descriptive statistics and correlations of sample (N = 760)

Variable	1	2	3	4	5	6
1. Posttest scores	-					
2. Pretest scores	0.67***	-				
3. o_percent_error	-0.26***	-0.30***	-			
4. o_percentage_hint	-0.18***	-0.24***	0.25***	-		
5. o_avg_first_efficiency	0.30***	0.26***	-0.33***	-0.17***	-	
6. o_percent_interaction_step_first	0.32***	0.28***	-0.66***	-0.10**	0.16***	-
M	4.48	4.66	0.50	0.14	0.92	0.74

<i>SD</i>	2.93	2.67	0.10	0.11	0.04	0.07
Min.	0.00	0.00	0.2	0.00	0.61	0.44
Max.	10.00	10.00	0.94	0.67	1.01	1.00

*** $p < .001$, ** $p < .01$, * $p < .05$

3.2.2 Clustering

The elbow method was used to determine the optimal number of clusters before performing clustering. As shown in Figure 5, the “fviz_nbclust” R function suggested four cluster solutions (dotted line), and the elbow point also indicated that $k = 4$ would produce the optimal cluster results.

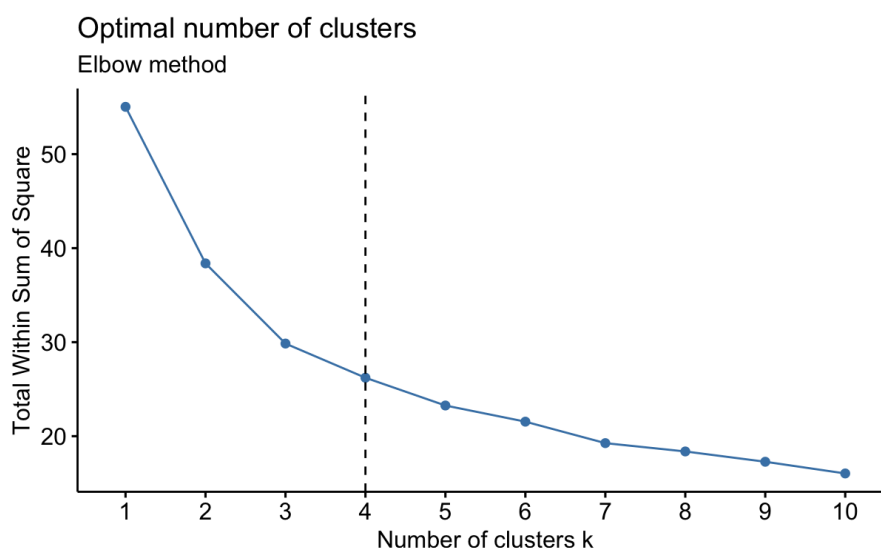


Figure 5. The elbow method showing the optimal number of clusters for our sample

Table 6. Descriptive statistics of each variable by cluster

Variable	<i>Cluster 1:</i> <i>Intermediate Knowledge</i> <i>Help Seekers</i>	<i>Cluster 2:</i> <i>High Knowledge</i>	<i>Cluster 3:</i> <i>Low Knowledge</i>	<i>Cluster 4:</i> <i>Intermediate</i> <i>Knowledge Not</i> <i>Seeking Help</i>
Number of students in the cluster (%)	133 (17.5%)	222 (29.2%)	105 (13.8%)	300 (39.5%)
Percent Error, M (SD)	.56 (.09)	.40 (.05)	L .63 (.08)	H .51 (.05)
Percent Hint, M (SD)	.33 (.10)	H .11 (.07)	L .11 (.07)	L .09 (.06)

Average First Efficiency, <i>M</i> (<i>SD</i>)	.91 (.04)	L	.94 (.04)	H	.89 (.05)	L	.93 (.03)	H
Percent Validity of First Step, <i>M</i> (<i>SD</i>)	.72 (.07)		.81 (.04)	H	.64 (.07)	L	.74 (.04)	

Note. H, M, and L indicate that the mean for the cluster was high, medium, or low within the data.

From the cluster analysis, the sample (N = 760) was divided into four groups of students based on their behavioral patterns: Cluster 1 (n = 114, 15%), Cluster 2 (n = 328, 43.16%), Cluster 3 (n = 102, 13.42%), and Cluster 4 (n = 216, 28.42%). Table 6 shows the descriptive statistics of each variable by cluster with relative descriptions of the values (low, medium, high) within the data. Figure 6 shows the box plots of four variables by cluster.

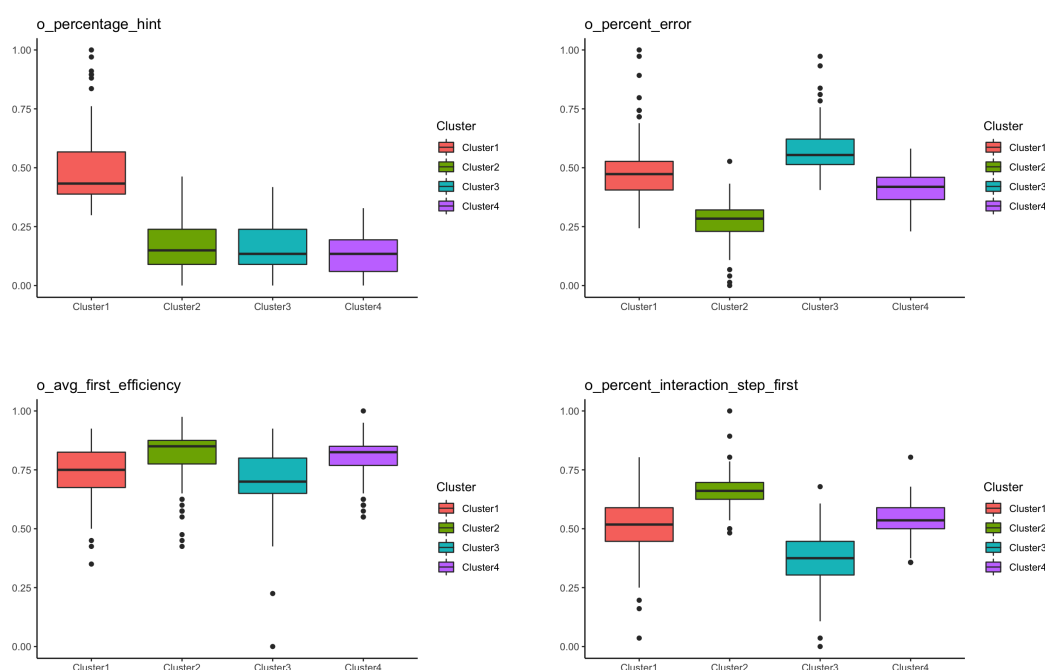


Figure 6: Cluster results depicting the distribution of variables of _patterns for four clusters (Note: Red: Cluster 1- *Intermediate Knowledge Help Seekers*. Green: Cluster 2- *High Knowledge*. Turquoise: Cluster 3- *Low Knowledge Not Seeking Help*. Purple: Cluster 4- *Intermediate Knowledge Not Seeking Help*.)

To interpret the clustering results, the clusters were labeled based on knowledge level and help seeking tendencies:

1. *Intermediate Knowledge Help Seekers* (Cluster 1, n = 133) included students who were on the lower end for first step efficiency, in the middle for percent validity of first step, in the middle for percent error, and highest for percentage of hint usage.
2. *High Knowledge* (Cluster 2, n = 222) included students who were highest in first step efficiency, highest percent validity for first step, lowest percent error and mid for percentage of hint usage. This cluster included the largest number of students (43.16%).

3. *Low Knowledge Not Seeking Help* (Cluster 3, n = 105) included students who had the lowest first step efficiency, lowest percent validity of first step, highest percent error, and low percentage of hint usage.
4. *Intermediate Knowledge Not Seeking Help* (Cluster 4, n = 300) included students who showed high first step efficiency, mid percent validity for first step, mid percent error, and lowest percentage of hint usage.

We then explored whether there is a significant difference in students' understanding of mathematical equivalence at baseline according to their behavioral patterns identified in the cluster analysis. Table 7 shows the means, standard deviations, and medians of pretest and posttest scores. It can be seen that despite not including pretest scores in the cluster analysis, there were distinct differences in the pretest scores of the four clusters: the high knowledge group had the highest pretest scores (M=5.62, SD=2.81, MED=6.00) and the low knowledge non-help seeking group had the lowest pretest scores (M=3.36, SD=2.03, MED=3.00).

Table 7. Descriptive statistics for four clusters

Cluster	Pretest			Posttest		
	M	SD	Mdn	M	SD	Mdn
<i>Intermediate Knowledge Help Seekers</i> (n = 133)	3.38	2.06	3.00	3.24	2.38	3.00
<i>High Knowledge</i> (n = 222)	5.62	2.81	6.00	5.50	3.00	5.00
<i>Low Knowledge Not Seeking Help</i> (n = 105)	3.36	2.03	3.00	2.93	2.08	3.00
<i>Intermediate Knowledge Not Seeking Help</i> (n = 300)	4.98	2.61	5.00	4.81	2.96	4.00

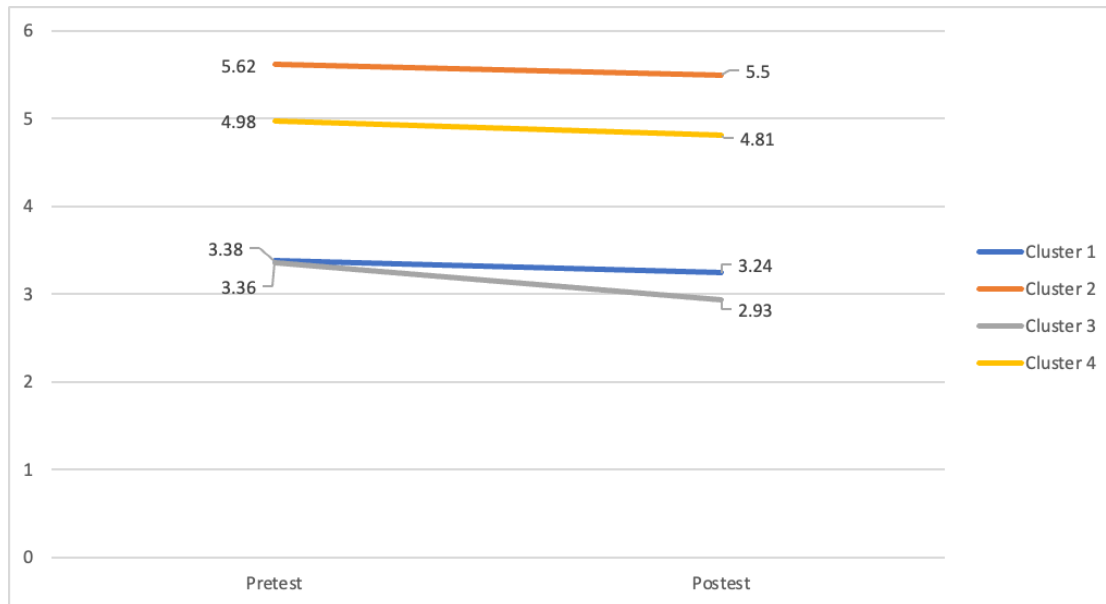


Figure 7. Slope graph of average pretest and posttest score by cluster



Figure 8. Parallel coordinate plot showing each student's change in score separated by cluster

Linear regression analyses were conducted to examine how much of the variance in students' posttest scores can be explained by cluster results both before and after controlling for prior knowledge (i.e. pretest scores). Four dummy variables were created to represent which group each student belongs to (1 = in group, 0 = not in group). Cluster 1 was selected as the reference group. Table 8 below shows the results of the linear regression analyses predicting posttest scores.

Table 8. Results of the multiple regression analysis predicting posttest scores

Predictors	B	SE	β	t	p	R ²	Adjusted R ²
Model 1						.1105	.107
(Constant)	.324	.024		13.496***	<.001		
Cluster 4 (1=Yes, 0=No) <i>Intermediate Knowledge Non-Help Seekers</i>	.157	.029	.261	5.429***	<.001		
Cluster 2 (1=Yes, 0=No) <i>High Knowledge</i>	.226	.030	.352	7.456***	<.001		
Cluster 3 (1=Yes, 0=No) <i>Low Knowledge Not Seeking Help</i>	-.031	.036	-.036	-.850	.396		
Model 2						.4616	.4587
(Constant)	.090	.021		4.204***	<.001		
Prior Knowledge <i>Pretest Scores</i>	.693	.031	.630	22.186***	<.001		
Cluster 4 (1=Yes, 0=No) <i>Intermediate Knowledge Non-Help Seekers</i>	.046	.023	.076	1.988*	.047		
Cluster 2 (1=Yes, 0=No) <i>High Knowledge</i>	.071	.025	.110	2.886**	.004		
Cluster 3 (1=Yes, 0=No) <i>Low Knowledge Not Seeking Help</i>	-.030	.028	-.035	-1.057	.291		

*** $p < .001$, ** $p < .01$, * $p < .05$

As shown in Model 1 in Table 8, three variables of cluster results explained a statistically significant amount of variance in posttest scores ($F(3, 756)=31.32$, $p < .001$, $R^2 = 0.1105$, R^2 adjusted = 0.107). The results indicated that students in the “high knowledge” group performed significantly better on the posttest compared to students in other groups ($\beta = .352$, $t(759) = 7.456$, $p < .001$). The students in the “low knowledge not seeking help” group did not significantly differ from the “intermediate knowledge help-seeking” group ($\beta = -.036$, $t(759) = -.850$, $p = .396$).

In Model 2, students’ prior knowledge (pretest scores) was added to the model as a control variable. Four variables of cluster results explained a statistically significant amount of variance in posttest scores ($F(4, 755)=161.8$, $p < .001$, $R^2 = 0.4616$, R^2 adjusted = 0.4587). After controlling for students’ prior knowledge, the results again indicated that the “high knowledge” group performed significantly better on the posttest compared to the reference group (i.e., intermediate knowledge help seeking group) although the magnitude of the association decreased with the addition of prior knowledge ($\beta = .110$, $t(759)=2.886$, $p=.004$). The students in the “low knowledge not seeking help” group again did not differ significantly compared to the “intermediate knowledge help-seeking” group ($\beta = -.035$, $t(759)=-1.057$, $p=.291$). Lastly, the non-significant beta coefficient for the students in the “low knowledge non-help seeking” group suggests that they performed similarly well on the posttest as the students in the “intermediate knowledge help seeking” group.

4.0 Discussion

This MQP attempts to investigate the subgroups of students that have been created using certain behavioral patterns and in game behaviors.

For the first research question, I examined how students can be grouped into different clusters based on their behavioral patterns (percentage error, percentage hint, average first efficiency, and percent validity of first step) in game. The k-means cluster analysis identified four groups of students: Intermediate Knowledge Help Seekers (17.5%) who had the highest hint usage and the lowest first efficiency, High Knowledge (29.2%) who had the highest first efficiency and validity of first step and lowest percent error and hint usage, Low Knowledge (13.8%) who had the highest error but lowest hint usage, first step efficiency, and first step validity, and Intermediate Knowledge Non-Help Seekers (39.5%) who had the lowest hint usage and high first efficiency. Although a large number of students in the sample were in high level math classes (35.1%), we still saw some variability in their behavioral patterns. The behavioral patterns uniquely predicted student learning outcome (posttest score) beyond students' prior knowledge, suggesting that how students interact with the online educational game has an impact on their learning. This advocates for the consideration of adaptive learning environments that support different types of learners when developing and designing online educational games (Vandewaetere et al., 2011).

The second research question examined how students' different behavioral patterns influenced their understanding of mathematical equivalence. The results indicated that each group saw a decrease from their pretest to posttest scores, which was not expected based on past research. However, this data was collected during the start of the pandemic in 2020 when schools were in the midst of figuring out how to teach their students given these unique circumstances. We can assume that these decreases seen from pretest to posttest score are an anomaly and can be attributed to the changes and anxiety inducing times caused by the pandemic. This is consistent with new work that shows "COVID-19 slide" especially in relation to mathematics in middle schools as a result of the pandemic (Kuhfeld, 2020). Despite this, the "high knowledge" saw the least amount of change from pretest to posttest, a decrease of 0.12, and the "low knowledge" group saw the largest change, a decrease of 0.43.

The fast progressor group, who had the highest pretest score, the highest validity of first step and first step efficiency, and low error and hint usage also had the highest posttest scores. Although we did not see the expected increase in any of the groups, the average learning loss showed a similar relationship to the average learning gain that was expected. The "high knowledge" group had the lowest average learning loss, a decrease of 0.12, which could suggest that in addition to having the highest prior knowledge, this group retained the most information. The "low knowledge" group had the greatest average learning loss, and with the highest percent error, this could suggest that this group retained the least amount of information. The two "intermediate knowledge" groups had very similar average learning losses but the "intermediate knowledge non-help seekers" showed a slightly larger average learning loss. The "intermediate

knowledge” group who had help seeking tendencies saw a decrease of 0.14 while the group without help seeking tendencies saw a decrease of 0.17.

For the last research question, the makeup of each cluster was examined. The variables used focused on students’ mathematical ability and could be seen to be a proxy for how much mathematical knowledge students began with. In support of this, it was seen that the students with the lowest pretest scores had the highest average error and lowest validity of first step and efficiency. Alternatively, the students with the highest pretest scores had the lowest error, highest validity of first step, and highest first step efficiency. The other two clusters showed middle values for average error and first step efficiency. The difference being the first cluster had very high hint usage. As such, future studies could utilize the data from the logs as students play to provide indicators of their underlying mathematical knowledge. This could be especially useful if pretests of their knowledge, implemented previously, are not available for teachers or administrators.

Exploring how in-app behaviors and mathematical actions within games could provide insight into students problem solving in ways that traditionally are not available using more summative assessments. Moving forward, our team will explore additional ways that the log data can inform students' knowledge and serve as formative assessments of students' understanding.

5.0 Lessons Learned from the MQP Process

Through the work I have done on this project, I have learned valuable skills to take with me into the workforce. I was able to teach myself how to code in R, utilizing many libraries and packages in order to run models, collect statistics, and create visualizations. I was able to improve my writing skills for scientific research and even had part of this project accepted to a conference. I have had an incredible experience working with a large set of learning sciences data and being able to collaborate with my peers in the MAPLE Lab to really help this project come together. This project has helped me gain confidence in myself and realize the amount I have learned in the past four years at WPI.

6.0 References

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7.0 Additional Materials

The code for this project and the outputs from the code can be accessed here:

<https://github.com/renorum/MQP>

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