All questions refer to the robot arm sketched in the figure below. Answer the questions below with text, mathematical statements, and supporting sketches where appropriate. Include a commented copy of your Matlab code.

1. Build the robot kinematic simulation and submit a screenshot of the robot in the following two configurations. You can use the given "drawLine3D.m" and "draw_Coordinat3D.m" functions to plot and refer to "matlab_test3D.m" for instruction.

```
addpath('matlab_utils')
%Euler angles alpha, beta, and gamma
syms a b g

R = [cos(a)*cos(b), cos(a)*sin(b)*sin(g)-sin(a)*cos(g), ...
    cos(a)*sin(b)*cos(g)+sin(a)*sin(g); sin(a)*cos(b),...
    sin(a)*sin(b)*sin(g) + cos(a)*cos(g),...
    sin(a)*sin(b)*cos(g)-cos(a)*sin(g);...
    -sin(b), cos(b)*sin(g), cos(b)*cos(g)
    ];
syms p1 p2 p3
P = [p1;  p2; p3]
```

P =
$$\begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix}$$

```
T =[R, P;
    0 0 0 1]
```

T =
$$\begin{pmatrix} \cos(a)\cos(b) & \cos(a)\sin(b)\sin(g) - \cos(g)\sin(a) & \sin(a)\sin(g) + \cos(a)\cos(g)\sin(b) & p_1 \\ \cos(b)\sin(a) & \cos(a)\cos(g) + \sin(a)\sin(b)\sin(g) & \cos(g)\sin(a)\sin(b) - \cos(a)\sin(g) & p_2 \\ -\sin(b) & \cos(b)\sin(g) & \cos(b)\cos(g) & p_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(a) q = (0, 90 , 0, 30, 90, 0 )

```
[o1,R1, o2,R2, o3,R3, o4,R4, o5,R5, o6, R6, oee, Ree] = arm(0,pi/2,0,pi/6,pi/2,0)
```

o1 =
$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

R1 =
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

o2 =
$$\begin{pmatrix} 0 \\ 0 \\ \frac{3}{20} \end{pmatrix}$$

R2 =
$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

o3 =
$$\begin{pmatrix} 0 \\ 0 \\ -\frac{3}{20} \end{pmatrix}$$

R3 =
$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

o4 =
$$\begin{pmatrix} 0 \\ 0 \\ -\frac{3}{10} \end{pmatrix}$$

R4 =

$$\begin{pmatrix} 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ -1 & 0 & 0 \end{pmatrix}$$

o5 =

$$\begin{pmatrix} 0 \\ 0 \\ -\frac{2}{5} \end{pmatrix}$$

R5 =

$$\begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

o6 =

$$\begin{pmatrix} -\frac{7\sqrt{3}}{200} \\ \frac{7}{200} \\ -\frac{2}{5} \end{pmatrix}$$

R6 =

$$\begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

oee =

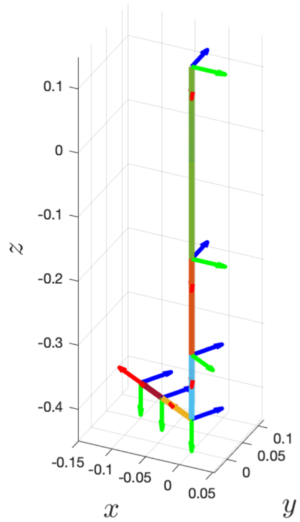
$$\begin{pmatrix} -\frac{3\sqrt{3}}{50} \\ \frac{3}{50} \\ -\frac{2}{5} \end{pmatrix}$$

Ree =

$$\begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

```
% draw figure
figure
hold on
grid on
drawLine3D(o1,o2);
drawCoordinate3DScale(R2,o2, 0.05);
drawLine3D(o2,o3);
drawCoordinate3DScale(R3,o3, 0.05);
drawLine3D(o3,o4);
drawCoordinate3DScale(R4,o4, 0.05);
drawLine3D(o4,o5);
drawCoordinate3DScale(R5,o5, 0.05);
drawLine3D(o5,o6);
drawCoordinate3DScale(R6,o6, 0.05);
drawLine3D(o6,oee);
drawCoordinate3DScale(Ree,oee, 0.05);
% R = eye(3);
xlabel('$x$','interpreter','latex','fontsize',20)
ylabel('$y$','interpreter','latex','fontsize',20)
zlabel('$z$','interpreter','latex','fontsize',20)
axis equal
fig=gcf;
fig.Position(3:4)=[550,400];


view(25,30)
```




(b) q = (0 , 120 , 0 , 60, 90, 0 )

```
[o1,R1, o2,R2, o3,R3, o4,R4, o5,R5, o6, R6, oee, Ree] = arm(0,degtorad(120),0,degtorad(60),degtorad(90),0)
```

o1 =

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

R1 =

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

o2 =

$$\begin{pmatrix} 0 \\ 0 \\ \frac{3}{20} \end{pmatrix}$$

R2 =

$$\begin{pmatrix} -\frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{3}}{2} & 0 & -\frac{1}{2} \end{pmatrix}$$

o3 =

$$\begin{pmatrix} -\frac{3}{20} \\ 0 \\ \frac{3}{20} - \frac{3\sqrt{3}}{20} \end{pmatrix}$$

R3 =

$$\begin{pmatrix} -\frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{3}}{2} & 0 & -\frac{1}{2} \end{pmatrix}$$

o4 =

$$\begin{pmatrix} -\frac{9}{40} \\ 0 \\ \frac{3}{20} - \frac{9\sqrt{3}}{40} \end{pmatrix}$$

R4 =

$$\begin{pmatrix} -\frac{1}{2} & \frac{3}{4} & \frac{\sqrt{3}}{4} \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{4} & -\frac{1}{4} \end{pmatrix}$$

o5 =

$$\begin{pmatrix} -\frac{11}{40} \\ 0 \\ \frac{3}{20} - \frac{11\sqrt{3}}{40} \end{pmatrix}$$

R5 =

$$\begin{pmatrix} -\frac{\sqrt{3}}{4} & \frac{3}{4} & -\frac{1}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ \frac{1}{4} & -\frac{\sqrt{3}}{4} & -\frac{\sqrt{3}}{2} \end{pmatrix}$$

o6 =

$$\begin{pmatrix} -\frac{7\sqrt{3}}{400} - \frac{11}{40} \\ \frac{7\sqrt{3}}{200} \\ \frac{67}{400} - \frac{11\sqrt{3}}{40} \end{pmatrix}$$

R6 =

$$\begin{pmatrix} -\frac{\sqrt{3}}{4} & \frac{3}{4} & -\frac{1}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ \frac{1}{4} & -\frac{\sqrt{3}}{4} & -\frac{\sqrt{3}}{2} \end{pmatrix}$$

oee =

$$\begin{pmatrix} -\frac{3\sqrt{3}}{100} - \frac{11}{40} \\ \frac{3\sqrt{3}}{50} \\ \frac{9}{50} - \frac{11\sqrt{3}}{40} \end{pmatrix}$$

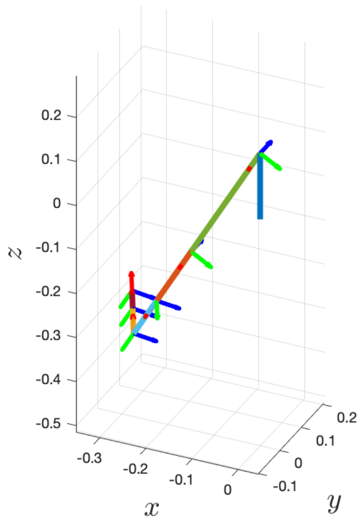
Ree =

$$\begin{pmatrix} -\frac{\sqrt{3}}{4} & \frac{3}{4} & -\frac{1}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ \frac{1}{4} & -\frac{\sqrt{3}}{4} & -\frac{\sqrt{3}}{2} \end{pmatrix}$$

```
% draw figure
figure
hold on
grid on
drawLine3D(o1,o2);
drawCoordinate3DScale(R2,o2, 0.05);
drawLine3D(o2,o3);
drawCoordinate3DScale(R3,o3, 0.05);
drawLine3D(o3,o4);
drawCoordinate3DScale(R4,o4, 0.05);
drawLine3D(o4,o5);
drawCoordinate3DScale(R5,o5, 0.05);
drawLine3D(o5,o6);
drawCoordinate3DScale(R6,o6, 0.05);
drawLine3D(o6,oee);
drawCoordinate3DScale(Ree,oee, 0.05);
% R = eye(3);
xlabel('$x$','interpreter','latex','fontsize',20)
ylabel('$y$','interpreter','latex','fontsize',20)
zlabel('$z$','interpreter','latex','fontsize',20)
axis equal
fig=gcf;
```

```
fig.Position(3:4)=[550,400];

view(25,30)
```




2. Find the corresponding end-effector (i.e EE) SE(3) for the following two configurations

(a) q = (0, 90, 90, 30, 90, 0)

```
[o1,R1, o2,R2, o3,R3, o4,R4, o5,R5, o6, R6, oee, Ree] = arm(0,degtorad(90),degtorad(90),degtorad(30),degtorad(90),0)
```

o1 =

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

R1 =

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

o2 =

$$\begin{pmatrix} 0 \\ 0 \\ \frac{3}{20} \end{pmatrix}$$

R2 =

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

o3 =

$$\begin{pmatrix} 0 \\ 0 \\ -\frac{3}{20} \end{pmatrix}$$

R3 =

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

o4 =

$$\begin{pmatrix} -\frac{3}{20} \\ 0 \\ -\frac{3}{20} \end{pmatrix}$$

R4 =

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$$

o5 =

$$\begin{pmatrix} -\frac{1}{4} \\ 0 \\ -\frac{3}{20} \end{pmatrix}$$

R5 =

$$\begin{pmatrix} 0 & 0 & -1 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \end{pmatrix}$$

o6 =

$$\begin{pmatrix} -\frac{1}{4} \\ \frac{7}{200} \\ \frac{7\sqrt{3}}{200} - \frac{3}{20} \end{pmatrix}$$

R6 =

$$\begin{pmatrix} 0 & 0 & -1 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \end{pmatrix}$$

oee =

$$\begin{pmatrix} -\frac{1}{4} \\ \frac{3}{50} \\ \frac{3\sqrt{3}}{50} - \frac{3}{20} \end{pmatrix}$$

Ree =

$$\begin{pmatrix} 0 & 0 & -1 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \end{pmatrix}$$

```
digits(8)

Tee = [Ree, oee;
    0 0 0 1];
disp(vpa(Tee))
```

$$\begin{pmatrix} 0 & 0 & -1.0 & -0.25 \\ 0.5 & 0.8660254 & 0 & 0.06 \\ 0.8660254 & -0.5 & 0 & -0.046076952 \\ 0 & 0 & 0 & 1.0 \end{pmatrix}$$

(b) q = (0, 60, 45, 60, 90, 0)

```
[o1,R1, o2,R2, o3,R3, o4,R4, o5,R5, o6, R6, oee, Ree] = arm(0,degtorad(60),degtorad(45),degtorad(60),degtorad(90),0)
```

o1 =

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

R1 =

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

o2 =

$$\begin{pmatrix} 0 \\ 0 \\ \frac{3}{20} \end{pmatrix}$$

R2 =

$$\begin{pmatrix} \frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{3}}{2} & 0 & \frac{1}{2} \end{pmatrix}$$

o3 =

$$\begin{pmatrix} \frac{3}{20} \\ 0 \\ \frac{3}{20} - \frac{3\sqrt{3}}{20} \end{pmatrix}$$

R3 =

$$\begin{pmatrix} \frac{\sqrt{2}}{4} - \frac{\sqrt{2}\sqrt{3}}{4} & 0 & \frac{\sqrt{2}\sqrt{3}}{4} + \frac{\sqrt{2}}{4} \\ 0 & 1 & 0 \\ -\frac{\sqrt{2}\sqrt{3}}{4} - \frac{\sqrt{2}}{4} & 0 & \frac{\sqrt{2}}{4} - \frac{\sqrt{2}\sqrt{3}}{4} \end{pmatrix}$$

o4 =

$$\begin{pmatrix} \frac{3\sqrt{2}}{80} - \frac{3\sqrt{2}\sqrt{3}}{80} + \frac{3}{20} \\ 0 \\ \frac{3}{20} - \frac{3\sqrt{2}}{80} - \frac{3\sqrt{3}}{20} - \frac{3\sqrt{2}\sqrt{3}}{80} \end{pmatrix}$$

R4 =

$$\begin{pmatrix} \frac{\sqrt{2}}{4} - \sigma_1 & \frac{\sqrt{3}\left(\sigma_1 + \frac{\sqrt{2}}{4}\right)}{2} & \frac{\sqrt{2}\sqrt{3}}{8} + \frac{\sqrt{2}}{8} \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\sigma_1 - \frac{\sqrt{2}}{4} & -\frac{\sqrt{3}\left(\sigma_1 - \frac{\sqrt{2}}{4}\right)}{2} & \frac{\sqrt{2}}{8} - \frac{\sqrt{2}\sqrt{3}}{8} \end{pmatrix}$$

where

$$\sigma_1 = \frac{\sqrt{2}\sqrt{3}}{4}$$

o5 =

$$\begin{pmatrix} \frac{\sqrt{2}}{16} - \frac{\sqrt{2}\sqrt{3}}{16} + \frac{3}{20} \\ 0 \\ \frac{3}{20} - \frac{\sqrt{2}}{16} - \frac{3\sqrt{3}}{20} - \frac{\sqrt{2}\sqrt{3}}{16} \end{pmatrix}$$

R5 =

$$\begin{pmatrix} -\frac{\sqrt{2}\sqrt{3}}{8} - \frac{\sqrt{2}}{8} & \frac{\sqrt{3}\left(\sigma_1 + \frac{\sqrt{2}}{4}\right)}{2} & \frac{\sqrt{2}}{4} - \sigma_1 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ \frac{\sqrt{2}\sqrt{3}}{8} - \frac{\sqrt{2}}{8} & -\frac{\sqrt{3}\left(\sigma_1 - \frac{\sqrt{2}}{4}\right)}{2} & -\sigma_1 - \frac{\sqrt{2}}{4} \end{pmatrix}$$

where

$$\sigma_1 = \frac{\sqrt{2}\sqrt{3}}{4}$$

o6 =

$$\begin{pmatrix} \frac{43\sqrt{2}}{800} - \frac{57\sqrt{2}\sqrt{3}}{800} + \frac{3}{20} \\ \frac{7\sqrt{3}}{200} \\ \frac{3}{20} - \frac{57\sqrt{2}}{800} - \frac{3\sqrt{3}}{20} - \frac{43\sqrt{2}\sqrt{3}}{800} \end{pmatrix}$$

R6 =

$$\begin{pmatrix} -\frac{\sqrt{2}\ \sqrt{3}}{8}-\frac{\sqrt{2}}{8} & \frac{\sqrt{3}\left(\sigma_1+\frac{\sqrt{2}}{4}\right)}{2} & \frac{\sqrt{2}}{4}-\sigma_1 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ \frac{\sqrt{2}\ \sqrt{3}}{8}-\frac{\sqrt{2}}{8} & -\frac{\sqrt{3}\left(\sigma_1-\frac{\sqrt{2}}{4}\right)}{2} & -\sigma_1-\frac{\sqrt{2}}{4} \end{pmatrix}$$

where

$$\sigma_1 = \frac{\sqrt{2}\ \sqrt{3}}{4}$$

oee =

$$\begin{pmatrix} \frac{19\sqrt{2}}{400}-\frac{31\sqrt{2}\ \sqrt{3}}{400}+\frac{3}{20} \\ \frac{3\sqrt{3}}{50} \\ \frac{3}{20}-\frac{31\sqrt{2}}{400}-\frac{3\sqrt{3}}{20}-\frac{19\sqrt{2}\ \sqrt{3}}{400} \end{pmatrix}$$

Ree =

$$\begin{pmatrix} -\frac{\sqrt{2}\ \sqrt{3}}{8}-\frac{\sqrt{2}}{8} & \frac{\sqrt{3}\left(\sigma_1+\frac{\sqrt{2}}{4}\right)}{2} & \frac{\sqrt{2}}{4}-\sigma_1 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ \frac{\sqrt{2}\ \sqrt{3}}{8}-\frac{\sqrt{2}}{8} & -\frac{\sqrt{3}\left(\sigma_1-\frac{\sqrt{2}}{4}\right)}{2} & -\sigma_1-\frac{\sqrt{2}}{4} \end{pmatrix}$$

where

$$\sigma_1 = \frac{\sqrt{2}\ \sqrt{3}}{4}$$

```
Tee = [Ree, oee;
    0 0 0 1];
disp(vpa(Tee))
```

$$\begin{pmatrix} -0.48296291 & 0.8365163 & -0.25881905 & 0.027339689 \\ 0.8660254 & 0.5 & 0 & 0.10392305 \\ 0.12940952 & -0.22414387 & -0.96592583 & -0.33575994 \\ 0 & 0 & 0 & 1.0 \end{pmatrix}$$

```
function [o1,R1, o2,R2, o3,R3, o4,R4, o5,R5, o6, R6, oee, Ree] = arm(th1,th2,th3,th4,th5,th6)
    syms theta
    Rx = [1, 0, 0;
        0, cos(theta), -sin(theta);
        0 sin(theta), cos(theta);
        ];
    Ry = [cos(theta), 0, sin(theta);
        0, 1, 0;
        -sin(theta), 0, cos(theta);
        ];
    Rz = [cos(theta), -sin(theta), 0;
        sin(theta), cos(theta), 0;
        0 0, 1;
        ];
    syms p1 p2 p3
    P = [p1;  p2; p3];
    Tx =[Rx, P;
        0 0 0 1];
    Ty =[Ry, P;
        0 0 0 1];
    Tz =[Rz, P;
        0 0 0 1];
    T01 = subs(Tz,{theta,p1, p2, p3},{th1,0,0,0});
    T12 = subs(Ty,{theta,p1, p2, p3},{th2,0,0,0.15});
    T23 = subs(Ty,{theta,p1, p2, p3},{th3,0.3,0,0});
    T34 = subs(Tx,{theta,p1, p2, p3},{th4,0.15,0,0});
```

```
    T45 = subs(Ty,{theta,p1, p2, p3},{th5,0.1,0,0});
    T56 = subs(Tx,{theta,p1, p2, p3},{th6,0.07,0,0});
    T6EE = subs(Tx,{theta,p1, p2, p3},{0,0.05,0,0});

    [R1, o1] = Textract(T01);

    T02 = Tmul(T01, T12);
    [R2, o2] = Textract(T02);

    T03 = Tmul(T02, T23);
    [R3, o3] = Textract(T03);

    T04 = Tmul(T03, T34);
    [R4, o4] = Textract(T04);

    T05 = Tmul(T04, T45);
    [R5, o5] = Textract(T05);

    T06 = Tmul(T05, T56);
    [R6, o6] = Textract(T06);

    T0EE = Tmul(T06, T6EE);
    oee = T0EE(1:3,4);
    Ree = T0EE(1:3,1:3);
end

function Tres = Tmul(T1, T2)
    Tres = T1 * T2;
end

function Tres = Tinv(T)
    R = T(1:3,1:3);
    p = T(1:3,4);

    Tres =[R.', -R.' * p;
    0 0 0 1];
end

function [R,p] = Textract(T)
    R = T(1:3,1:3);
    p = T(1:3,4);
end
```