On Nested Application Condition and First Order Logic

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Abstract. Write me!!!

1 Outline

1. Nested conditions as tree of morphisms - Satisfaction
2. Nested condition as tree of cospans - Satisfaction
3. *Question: what's the relation with instantiation of Barbara's definition to Cospan(Graph) and the corresponding notion of satisfaction?*
4. Translating cospan-based conditions to standard ones [easy: take all pushouts and link them]
5. Translating standard conditions to cospan-based ones [inductive bottom-up contstruction; at each step needs to compute a unique interface taking the union]
6. Translation preserves Satisfaction
7. Morphisms of cospan-based conditions
8. Morphisms reflect Satisfaction
9. Algebra of nested conditions: we sketched it for the standard ones, how does it work for cospan-based ones?
10. The category of ranked cospan conditions is a FOL(?)-bicategory - morphisms among conditions with the same rank should be consistent with entailment.

2 The algebra of Nested Graph Conditions

Needed definitions:

1. Monoidal signature $\Sigma = \bigcup\{{}_n\Sigma_m\}_{n,m\in\mathbb{N}}$, arity $ar(\sigma) = n$ and coarity $coar(\sigma) = m$ iff $\sigma \in {}_n\Sigma_m$.
2. (Finite?) Edge-labeled hypergraphs: $G = \langle V, E, l : E \to \Sigma, s, t : E \to V^*\rangle$ with: for all $e \in E$ it holds $ar(l(e)) = |s(e)|$ and $coar(l(e)) = |t(e)|$.
3. Category of hypergraphs and their morphisms $\mathbf{HGraph_\Sigma}$.
4. Need a notation for the discrete graph (set) with n nodes. For example: $\underline{n} = \{1, \ldots, n\}$
5. Discrete cospan (aka *ranked hypergraph*, n, m-hypergraphs, n, m-ranked hypergraph ?): $\langle l, G, r\rangle$, with $l : \underline{n} \to V_G$, $r : \underline{m} \to V_G$.

6. *Positions*: finite strings of positive natural numbers ($\mathbb{N}^*_{>0}$) (including the empty ε, if possible concatenation is juxtaposition, w, iw, wj with $w \in \mathbb{N}^*_{>0}, i, j \in \mathbb{N}_{>0}$); *closed set of positions*: $W \subseteq \mathbb{N}^*_{>0}$ such that (1) it is finite, (2) it is prefix closed: $wi \in W \Rightarrow w \in W$, (3) it is downward closed (?) $wi \in W, i > 1 \Rightarrow w(i-1) \in W$.

Note 1. AC: We can define conditions first, and then ranked / typed ones

Definition 1 ((nested) conditions).

Note 2. AC: Several definitions possible.

1. *A* (nested) condition *C is a finite, non-empty, ordered*[3] *tree in the category of hypergraphs. Formally, $NC = \langle W, c : W \to |\mathbf{HGraph_\Sigma}|, a : W \setminus \{\varepsilon\} \to Mor(\mathbf{HGraph_\Sigma})\rangle$, where W is a closed set of positions, and for each $wi \in W$, $a(wi) : c(w) \to c(wi)$ is a morphism in $\mathbf{HGraph_\Sigma}$.*
   Typically, we denote hypergraph $c(w)$ as C_w. Therefore the root of C is C_ε.
2. *Recursive definiton using 'conditions' and 'predicates' as in the 2005 paper?*
3. *Coinductive definitions as in the paper with Barbara?*
4. *An n, m-ranked condition $\langle l, C, r\rangle$, is a condition C equipped with morphisms l, r making $\langle l, C_\varepsilon, r\rangle$ an n, m-ranked hypergraph .*

In order to equip ranked graph conditions with Peircean Bicategorical structure, we define the needed algebraic operations on them, and prove that the relevant axioms hold, up to *** (semantical equivalence?).

Definition 2 (operations on conditions).

- *The* discharger $! : 1 \to 0$ *is the ranked condition*

$$\langle id_{\underline{1}}, \langle \{\varepsilon\}, \varepsilon \mapsto \underline{1}, \emptyset\rangle, ?_{\underline{1}} : \underline{0} \to \underline{1}\rangle$$

- *The* co-discharger $*** : 0 \to 1$ *is the ranked condition*

$$\langle ?_{\underline{1}} : \underline{0} \to \underline{1}, \langle \{\varepsilon\}, \varepsilon \mapsto \underline{1}, \emptyset\rangle, id_{\underline{1}}\rangle$$

References

[3] AC: we order the children of each node just to give them a name using a position, but intuitively they should form a set.