Technical Summary For:
"A Formal Framework for Static (Tissue) P Systems"

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1 Multisets

1. $V = \{a_1, ..., a_k\}$ - Alphabet
2. $m : V \to \mathbb{N}$ - Finite multiset over V
3. $m(a)$ - Number of instances of object a
4. $|m| = \sum_{a \in V} m(a)$ - Size of multiset m
5. $a_1^{m(a_1)}...a_k^{m(a_k)}$ - Representation of multiset m
6. $supp(m) = \{a \in V \mid m(a) \geq 1\}$ - Support of multiset m
7. $\langle V, \mathbb{N} \rangle$ - Set of all finite multiset over V
8. $m : V \to \mathbb{N} \cup \{\infty\}$ - Multiset over V (includes infinite multiset)
9. $\langle V, \mathbb{N}_\infty \rangle$ - Set of all multiset over V
10. $[x \leq y] := [x, y \in \langle V, \mathbb{N} \rangle] \wedge [\forall a \in V][x(a) \leq y(a)]$
11. $[z = x + y] := [x, y, z \in \langle V, \mathbb{N} \rangle] \wedge [\forall a \in V][z(a) = x(a) + y(a)]$
12. $z = x - y := [x, y, z \in \langle V, \mathbb{N} \rangle] \wedge [y \leq x] \wedge [\forall a \in V][z(a) = x(a) - y(a)]$
13. $[P \leq Q] := [P, Q \in \langle V, \mathbb{N} \rangle^n] \wedge [P = (p_1, ..., p_n)] \wedge [Q = (q_1, ..., q_n)] \wedge [\forall i, 1 \leq i \leq n][p_i \leq q_i]$
14. $[R = P + Q] := [P, Q, R \in \langle V, \mathbb{N} \rangle^n] \wedge [P = (p_1, ..., p_n)] \wedge [Q = (q_1, ..., q_n)] \wedge [R = (r_1, ..., r_n)] \wedge$ $[\forall i, 1 \leq i \leq n][r_i = p_i + q_i]$
15. $[R = P - Q] := [P, Q, R \in \langle V, \mathbb{N} \rangle^n] \wedge [P = (p_1, ..., p_n)] \wedge [Q = (q_1, ..., q_n)] \wedge [R = (r_1, ..., r_n)] \wedge$ $[Q \leq P] \wedge [\forall i, 1 \leq i \leq n][r_i = p_i - q_i]$

2 Network of Cells

1. $\Pi = (n, V, w, Inf, R)$
2. n - Number of cells
3. V - Alphabet of objects
4. $w = (w_1, ..., w_n)$ - vector of finite multisets, $w_i \in \langle V, \mathbb{N} \rangle$ - finite multiset associated with cell i
5. $Inf = (Inf_1, ..., Inf_n)$ - vector of sets, $Inf_i \subseteq V$ - set of symbols occuring infinitely often in cell i
6. $R = \{r_i\}$ - Set of interfactive rules

2.1 Interactive Rule

1. $r_i : (X \to Y; P, Q)$ - Interactive rule
2. $X = (x_1, ..., x_n)$, $Y = (y_1, ..., y_n)$, $[X, Y \in \langle V, \mathbb{N}\rangle^n]$,
   - X contains the 'consumed' multisets. Y contains the 'produced' multisets.
3. $P = (p_1, ..., p_n)$, $Q = (q_1, ..., q_n)$, $[\forall i, 1 \leq i \leq n][p_i, q_i \subseteq \langle V, \mathbb{N}\rangle]$
   - p_i, q_i are finite sets of multisets (over V), $\forall i,\ 1 \leq i \leq n$.
   - p_i contains all the required multisets for cell i, q_i contains all the forbidden multisets for cell i.

2.2 Configurations

1. $C = (u'_1, ..., u'_n)$ - Configuration, $[\forall i, 1 \leq i \leq n][u'_i \in \langle V, \mathbb{N}_\infty\rangle]$
2. $Inf^\infty = (Inf_1^\infty, ..., Inf_n^\infty)$ - Vector of infinite multisets
   - $Inf_i^\infty = b_1^\infty \cdots b_k^\infty$, $b_i \in Inf_i$, $\forall i,\ 1 \leq i \leq k = |Inf_i|$
3. $C^f = (u_1, ..., u_n)$ - Finite part of configuration C
   - $u'_i = u_i + Inf_i^\infty$, $u_i \cap Inf_i = \emptyset$, $\forall i,\ 1 \leq i \leq n$
   - $C_0^f = w = (w_1, ..., w_n)$ - finite parts of the initial configuration C.
   - $C_0 = w + Inf^\infty$ - full initial configuration of Π.

2.3 Eligibility of an Rule and Applicability of a Multiset of Rules

1. $eligibe(r, C) := [\forall i,\ 1 \leq i \leq n][[x_i \subseteq u_i] \wedge [\forall p \in p_i][p \subseteq u_i] \wedge [\forall q \in q_i][q \not\subseteq u_i]]$
   - Eligibility of rule r with respect to configuration C.
   - $C^f = (u_1, ..., u_n)$ - vector of finite multisets (u_i). Multisets in the cells.
   - $X = (x_1, ..., x_n)$ - vector of multisets (x_i). Multisets to be 'consumed'.
   - $P = (p_1, ..., p_n)$ - vector of multisets (p_i). Sets of required multisets for all cells.
   - $Q = (q_1, ..., q_n)$ - vector of multisets (q_i). Sets of required multisets for all cells.
2. $[\exists j][x_j \cap (V - Inf_j) \neq \emptyset]$. There is at least one multiset x_j in X such that least one symbol appearing in x_j that is no in Inf_j.
3. $Eligible(\Pi, C) = \{r \in R \mid eligible(r, C)\}$. Set of all eligible rules with respect to configuration C.

2.4 Marking Algorithm

- $r_i : (X_i \to Y_i; P_i, Q_i)$ - interaction rule.
- $X_i = (x_{i,1}, ..., x_{i,n})$ - vector of multisets.
- $X'_i = (x'_{i,1}, ..., x'_{i,n})$ - vector of multisets.
- $x_{i,j} = x'_{i,j} + Inf_j^\infty$. $x'_{i,j} \cap Inf_j = \emptyset$
- $C^f = (v_1, ..., v_n)$ - vector of multisets. Finite part of the configuration.
- $R = \{r_1, ..., r_h\}$ - set of eligible rules r_i.
- $R' \in \langle R, \mathbb{N}\rangle$ - finite multiset of eligible rules.

1. $Mark_0(\Pi, C, R') = (\lambda, ..., \lambda)$. $i = 1$.
2. If $X'_i \leq (C^f - Mark_{i-1}(\Pi, C, R'))$
   - `TRUE`: $Mark_i(\Pi, C, R') = (C^f - Mark_{i-1}(\Pi, C, R')) - X'_i$
   - `FALSE`: return `false`;
3. If $i = k$
   - `TRUE`: return `true` and $Mark(\Pi, C, R') = Mark_k(\Pi, C, R')$
   - `FALSE`: $i = i + 1$. Go back to step 2.

2.5 Applicability of a Multiset of Rules

1. $appl(R', C) \Leftrightarrow$ marking algorithm returns `true` and $Mark(\Pi, C, R')$ - Applicability of multiset of rules R' with respect to configuration C.

2. $Appl(\Pi, C) = \{R' \in \langle R, \mathbb{N}\rangle \mid R = Eligible(\Pi, C), appl(R', C)\}$ - Set of all applicable multisets of rules with respect to configuration C.

3. $Apply(\Pi, C, R') = C - Mark(\Pi, C, R') + \sum_{i=1}^{k} Y_i'$ - New configuration when rules in R' are applied.

2.6 Derivation Modes

1. $Appl(\Pi, C, \delta) \subseteq Appl(\Pi, C)$ - Set of applicable multisets of rules in δ-mode.

2. $Appl(\Pi, C, asyn) = Appl(\Pi, C)$ - *Asynchronous* Mode ($\delta = asyn$).

3. $Appl(\Pi, C, sequ) = \{R' \in Appl(\Pi, C) \mid |R'| = 1\}$ - *Sequential* Mode ($\delta = sequ$) - One rule per step.

4. $Appl(\Pi, C, max) = \{R' \in Appl(\Pi, C) \mid \not\exists R'' \in Appl(\Pi, C),\ R'i \not\subseteq R''\}$ - *Maximally Parallel* Mode ($\delta = max$) - Adding any rule to a maximally parallel R' will result in an inapplicable multiset of rules.

5. $Appl(\Pi, C, min) = \{R' \in Appl(\Pi, C) \mid \not\exists R'' \in Appl(\Pi, C),\ R' \subseteq R'', \exists j, (R'' - R') \cap R_j \neq \emptyset, R' \cap R_j = \emptyset\}$ - *Minimally Parallel* Mode ($\delta = min$) - There is no partition $R = R_1 \cup R_2 \cup \cdots \cup R_h$. Rule set R is be partitioned. $R' \subseteq R''$. R'' 'extends' R'.

6. $\delta \in \{asyn, sequ, max, min\}$ - Basic derivation modes

7. $Appl(\Pi, C, max_{rule}\delta) = \{R' \in Appl(\Pi, C, \delta) \mid \not\exists R'' \in Appl(\Pi, C, \delta)\ |R''| > |R'|\}$ - *Maximum Rules* δ-Mode.

8. $Appl(\Pi, C, max_{set}\delta) = \{R' \in Appl(\Pi, C, \delta) \mid \not\exists R'' \in Appl(\Pi, C, \delta)\ ||R''|| > ||R'||\}$ - *Maximum Sets (Partitions)* δ-Mode.

9. $Appl(\Pi, C, all_{set}\delta) = \{R' \in Appl(\Pi, C, \delta) \mid \forall j, 1 \leq j \leq h, (R_j \cap \bigcup_{X \in Appl(\Pi, C)} X \neq \emptyset) \rightarrow (R_j \cap R' \neq \emptyset)\}$ - *All Set* δ-Mode.

2.7 Transition

1. $[C \Rightarrow_{(\Pi,\Delta)} C'] \Leftrightarrow [[\exists R' \in Appl(\Pi, C, \Delta)][C' = Apply(\Pi, C, R')]]$ - *Transition* in Δ-mode.

2. $[C \Rightarrow^{*}_{(\Pi,\Delta)} C']$ - Transitive closure and reflexive nature of the transition relation $\Rightarrow_{(\Pi,\Delta)}$

3. $[accessible(C, \Pi, \Delta)] \Leftrightarrow [C_0 \Rightarrow_{(\Pi,\Delta)} C]$ - Accessibility of configuration C in Δ-mode.

4. $Acc(\Pi, \Delta) = \{C \mid accessible(C, \Pi, \Delta)\}$ - Set of all accessible configurations in Δ-mode derivation.

5. $[Deterministic(\Pi, \Delta)] \Leftrightarrow [\forall C \in Acc(\Pi, \Delta)][Appl(\Pi, C, \Delta)| \leq 1]$ - Determinism of system Π under Δ-mode derivation.

2.8 Halting Conditions

1. $H(\Pi, \Delta) = \{C' \in Acc(\Pi, \Delta) \mid Appl(\Pi, C', \Delta) = \emptyset\}$ - Set of *total halting* configurations. Under Δ derivation mode, accessible configurations where there are no applicable multisets of rules.

2. $A(\Pi, \Delta) = \{C' \in Acc(\Pi, \Delta) \mid Appl(\Pi, C', \Delta) \neq \emptyset, \forall R' \in Appl(\Pi, C', \Delta), Apply(\Pi, C', R') = C'\}$ - Set of *adult halting* configurations. Under Δ derivation mode, accessible configurations where for all applicable multiset of rules R' applying R' to configuration C' result to C'.

3. $h(\Pi, \Delta) = \{C' \in Acc(\Pi, \Delta) \mid \not\exists R' \in Appl(\Pi, C', \Delta), \forall i, 1 \leq i \leq h, R' \cap R_j \neq \emptyset\}$ - Set of *partial halting* configurations. Under Δ derivation mode, accessible configurations where there are no applicable multiset of rules R' such that R' contains rules from all partitions R_j.