Technical Summary For:
“A Formalization of Membrane Systems with Dynamically Evolving Structures”

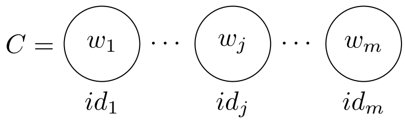
Ren Tristan A. de la Cruz

June 26, 2020

1 Formal Framework

1.1 Configurations

1. $C = \{(id_1, w_1), ..., (id_j, w_j)..., (id_m, w_m)\} \subseteq (\mathbb{N} \times O^\circ)^*$ - *basic configuration*

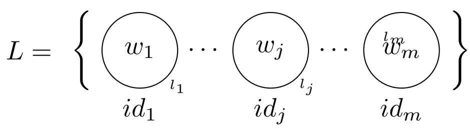


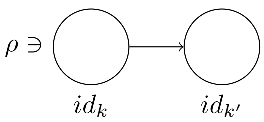
- $id_j \in \mathbb{N}$ - *cell id*
- $w_j \in O^\circ$ - *cell content*
- (id_j, w_j) - *cell*

2. $\mathbb{C} = \{C \mid size(C) > 0\}$

- $size(C)$ - size of basic configuration C
- $\mathbb{C}$ - set of all basic configurations with size greater than 0.

3. $\mathcal{C} = (L, \rho) = (L = \{(id_1, l_1, w_1), ..., (id_j, l_j, w_j), ...(id_m, l_m, w_m)\}, \rho)$





- $id_j \in \mathbb{N}$ - *id*
- $w_j \in O^\circ$ - *multiset* over O
- $l_j \in Lab$ - *label*
- (id_j, l_j, w_j) - *labelled cell*
- $L \in (\mathbb{N} \times Lab \times O^\circ)^*$ - list of labelled cells
- $\rho \subseteq \mathbb{N} \times \mathbb{N}$ - *relations* between cells (ids)
- $\mathcal{C} = (L, \rho)$ - *configuration*
- $\mathcal{C}_L = L$ and $\mathcal{C}_\rho = \rho$.

- $\overline{\mathcal{C}}_L \in \mathbb{C}$ - *projection* of $\mathcal{C}_L$ as basic configuration.

$$\overline{\mathcal{C}}_L = \left\{ \underset{id_1}{w_1} \cdots \underset{id_j}{w_j} \cdots \underset{id_m}{w_m} \right\}$$

4. $\mathfrak{C} = \{\mathcal{C} \mid \mathcal{C} = (L, \rho)\}$

   - $\mathfrak{C}$ is the set of all possible configurations.

1.2 Components of a Rule

0. $r = (Labels, \rho, Perm, For, Rewrite, Label\text{-}Rename, Delete, Delete\text{-}and\text{-}Move, Generate, Generate\text{-}and\text{-}Copy, Change\text{-}Relation)$

   - r is a rule.

1. $Labels(r) = (l_1, ..., l_j...., l_k) \in Lab^k$

$$Label(r) = \underset{1}{\bigcirc}_{l_1} \cdots \underset{j}{\bigcirc}_{l_j} \cdots \underset{k}{\bigcirc}_{l_k}$$

2. $\rho(r) \subseteq \mathbb{N}_k \times \mathbb{N}_k$

   - $\mathbb{N}_k = \{1, ..., j, ..., k\}$

3. $Perm(r) = \{P_1, ..., P_{j'}, ..., P_{\overline{p}}\} \subseteq \mathbb{C}_k$

$$Perm(r) = \Bigg\{ \; P_1 = \left\{ \underset{1}{p_{(1,1)}} \cdots \underset{j}{p_{(1,j)}} \cdots \underset{k}{p_{(1,k)}} \right\} \;, \; ...$$

$$P_{j'} = \left\{ \underset{1}{p_{(j',1)}} \cdots \underset{j}{p_{(j',j)}} \cdots \underset{k}{p_{(j',k)}} \right\} \;, \; ...$$

$$P_{\overline{p}} = \left\{ \underset{1}{p_{(\overline{p},1)}} \cdots \underset{j}{p_{(\overline{p},j)}} \cdots \underset{k}{p_{(\overline{p},k)}} \right\} \Bigg\}$$

   - $\mathbb{C}_k \subseteq \mathbb{C}$ - *basic configurations* with cell ids in $\mathbb{N}_k$.
   - $p_{(j',j)} \in O^\circ$

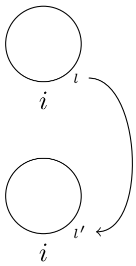
4. $For(r) = \{F_1, ..., F_{j'}, ..., F_{\overline{f}}\} \subseteq \mathbb{C}_k$

$$For(r) = \left\{ \begin{array}{l} F_1 = \left\{ \underset{1}{f_{(1,1)}} \cdots \underset{j}{f_{(1,j)}} \cdots \underset{k}{f_{(1,k)}} \right\} , \ \ldots \\ F_{j'} = \left\{ \underset{1}{f_{(j',1)}} \cdots \underset{j}{f_{(j',j)}} \cdots \underset{k}{f_{(j',k)}} \right\} , \ \ldots \\ F_{\overline{f}} = \left\{ \underset{1}{f_{(\overline{f},1)}} \cdots \underset{j}{f_{(\overline{f},j)}} \cdots \underset{k}{f_{(\overline{f},k)}} \right\} \end{array} \right\}$$

- $f_{(j',j)} \in O^{\circ}$

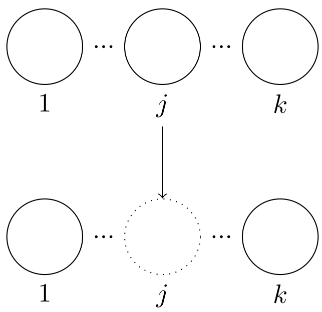
5. $Rewrite(r) = U \to V$
   - $U, V \in \mathbb{C}_k$
   - $Rewrite(r)$ is a general rewriting rule, rewriting a finite basic configuration U to another finite basic configuration V.

6. $Label\text{-}Rename(r) = \{..., (i, l'), ...\} \in (\mathbb{N}_k \times Lab)^*$



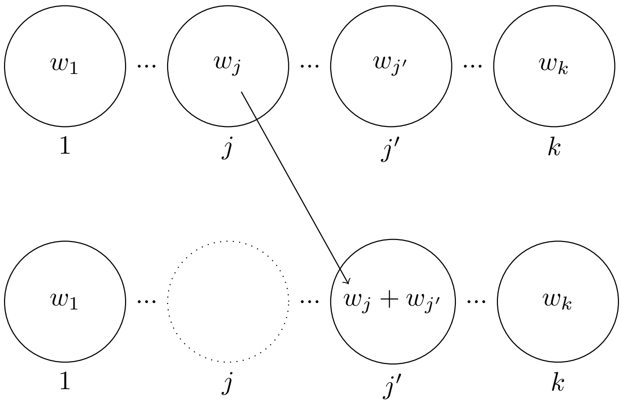
   - $i \in \mathbb{N}_k$ - id
   - $l' \in Lab$ - new label

7. $Delete(r) = \{..., j, ...\} \in \mathbb{N}_k^*$



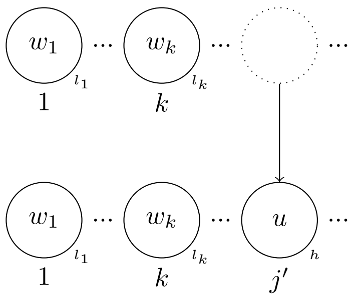
   - j - id of cell to be deleted

8. $Delete\text{-}Move(r) \in (\mathbb{N}_k \times \mathbb{N}_k)^*$



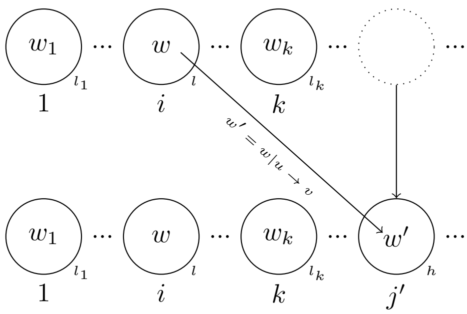
- (j, j') - *pair of ids*
- j - *id* of cell to be deleted
- j' - *id* of cell to receive the multiset
- *Delete-Move*(r)- *list of pairs* of ids

9. *Generate*$(r) = \{..., (j', h, u), ...\} \in (\mathbb{N}' \times Lab \times O^\circ)^*$



- j' - *primed id* - new id
- h - label
- u - multiset

10. *Generate-Copy*$(r) = \{..., (j', h, i, (u, v)), ...\} \in (\mathbb{N}' \times Lab \times \mathbb{N} \times (O^\circ \times O^\circ))^*$



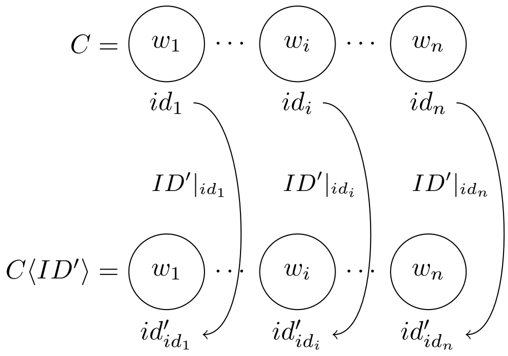
- $j' \in \mathbb{N}'$ - primed id
- $h \in Lab$ - label
- $i \in \mathbb{N}'_k$ - cell id
- $w_1, ..., w, ..., w_k \in O^\circ$ - multisets
- $u, v \in O^\circ$ - multisets
- (u, v) - written as $u \to v$

11. *Change-Relation*

- *Change-Relation* is a *graph transducer* that updates the relation ρ.
- This transducer should be recursive and it can only add and remove edges.

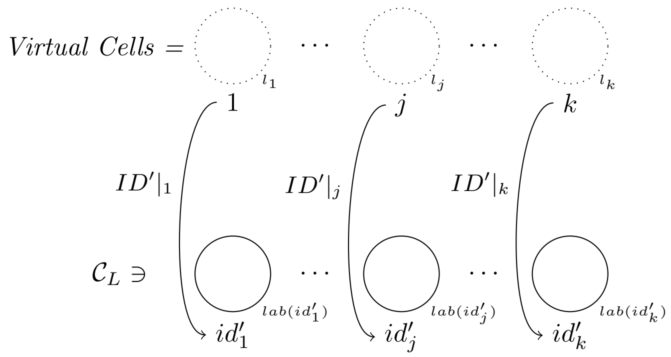
1.3 Eligibility of an Id Vector

1. $ID' = (id'_1, ..., id'_i, ..., id'_{n'}) \in \mathbb{N}^{n'}$



   - $ID'|_x = id'_x$
   - $ID'|_{id_i} = id'_{id_i}$
   - $C = \{(id_1, w_n), ..., (id_i, w_i), ...(id_n, w_n)\}$
   - $C\langle ID' \rangle = \{(ID'|_{id_1}, w_n), ..., (ID'|_{id_i}, w_i), ...(ID'|_{id_n}, w_n)\}$
   - $C\langle ID' \rangle = \{(id'_{id_1}, w_n), ..., (id'_{id_i}, w_i), ...(id'_{id_n}, w_n)\}$
   - $|ID'| = n' = max\{id_i\}$
   - $|C| = n$
   - $|C| = n \leq n' = |ID'|$

2. $Eligible(ID', r, \mathcal{C})$



   - $ID' = (id'_1, ..., id'_j, ..., id'_k) \in \mathbb{N}^k$
   - r is a *k-degree interaction rule*
   - $r\langle ID' \rangle$ - *instantiated rule* - replace virtual ids with ids in ID'
   - $Labels(r) = (l_1, ..., l_j, ..., l_k)$
   - $\mathcal{C} = (L, \rho)$
   - $lab : \mathbb{N} \to Lab$
   - $Eligible(ID', r, \mathcal{C}) \Leftrightarrow :$
     - $\forall x, y \in \{1, ..., k\} : (x \neq y) \to (id'_x \neq id'_y)$ [id uniqueness]
     - $\forall x \in \{1, ..., k\} : l_x = lab(id'_x)$ [label consistency]
     - $\forall x, y, \in \{1, ..., k\} : (x, y) \in \rho(r) \to (id'_x, id'_y) \in \mathcal{C}_\rho$ [relations consistency]
   - $\mathcal{I}_\mathcal{C}(r) = \{ID' \mid Eligible(ID', r, \mathcal{C})\}$

1.4 Applicability of a Multiset of Rules

- $R = \{r_1, ..., r_i, ..., r_n\}$ - *multiset* of rules
- $\mathcal{I}_{\mathcal{C}}(r_i) = \{ID'_{i,1}, ..., ID'_{i,j_i}, ..., ID'_{i,k_i}\}$
- $RI = \{r_1\langle ID'_{1,j_1}\rangle, ..., r_i\langle ID'_{i,j_i}\rangle, ..., r_n\langle ID'_{n,j_n}\rangle\}$ - *multiset of instantiated rules*

$applicable(RI, \mathcal{C}) \Leftrightarrow \forall r_i \in R : \mathcal{I}_{\mathcal{C}}(r_i) \neq \emptyset : \forall r_i\langle ID'_{i,j_i}\rangle \in RI :$

1. $\forall P \in Perm(r_i\langle ID'_{i,j_i}\rangle) \cup \{DP\} : P \subseteq \overline{\mathcal{C}}_L$
   - $DP = \{(id_i, u) \mid (j', h, id_i, u \to v) \in Generate\text{-}and\text{-}Copy(r_i\langle ID'_{i,j_i}\rangle)\}$

$$Perm(r_i\langle ID'_{i,j_i}\rangle) = \Big\{ \; P_1 = \Big\{ \underset{ID'_{i,j_i}|_1}{p_{(1,1)}} \cdots \underset{ID'_{i,j_i}|_j}{p_{(1,j)}} \cdots \underset{ID'_{i,j_i}|_k}{p_{(1,k)}} \Big\}, \; ...$$

$$P_{j'} = \Big\{ \underset{ID'_{i,j_i}|_1}{p_{(j',1)}} \cdots \underset{ID'_{i,j_i}|_j}{p_{(j',j)}} \cdots \underset{ID'_{i,j_i}|_k}{p_{(j',k)}} \Big\}, \; ...$$

$$P_{\overline{p}} = \Big\{ \underset{ID'_{i,j_i}|_1}{p_{(\overline{p},1)}} \cdots \underset{ID'_{i,j_i}|_j}{p_{(\overline{p},j)}} \cdots \underset{ID'_{i,j_i}|_k}{p_{(\overline{p},k)}} \Big\} \; \Big\}$$

$$DP = \Big\{ \; \cdots \underset{id_i}{u} \cdots \; \Big\}$$

$$\overline{\mathcal{C}}_L = \Big\{ \underset{id_1}{w_1} \cdots \underset{id_j}{w_j} \cdots \underset{id_n}{w_n} \Big\}$$

2. $\forall F \in For(r_i\langle ID'_{i,j_i}\rangle) : F \subseteq \overline{\mathcal{C}}_L$

$$For(r_i\langle ID'_{i,j_i}\rangle) = \Big\{ \; F_1 = \Big\{ \underset{ID'_{i,j_i}|_1}{f_{(1,1)}} \cdots \underset{ID'_{i,j_i}|_j}{f_{(1,j)}} \cdots \underset{ID'_{i,j_i}|_k}{f_{(1,k)}} \Big\}, \; ...$$

$$F_{j'} = \Big\{ \underset{ID'_{i,j_i}|_1}{f_{(j',1)}} \cdots \underset{ID'_{i,j_i}|_j}{f_{(j',j)}} \cdots \underset{ID'_{i,j_i}|_k}{f_{(j',k)}} \Big\}, \; ...$$

$$F_{\overline{f}} = \Big\{ \underset{ID'_{i,j_i}|_1}{f_{(\overline{f},1)}} \cdots \underset{ID'_{i,j_i}|_j}{f_{(\overline{f},j)}} \cdots \underset{ID'_{i,j_i}|_k}{f_{(\overline{f},k)}} \Big\} \; \Big\}$$

3. $\left(\bigcup_{i=1}^{n} U_i\right) \subseteq \overline{\mathcal{C}}_L$
    - $Rewrite(r\langle ID'_{i,j_i}\rangle) = U_i \rightarrow V_i$
    - $U_i, V_i \in \mathbb{C}$

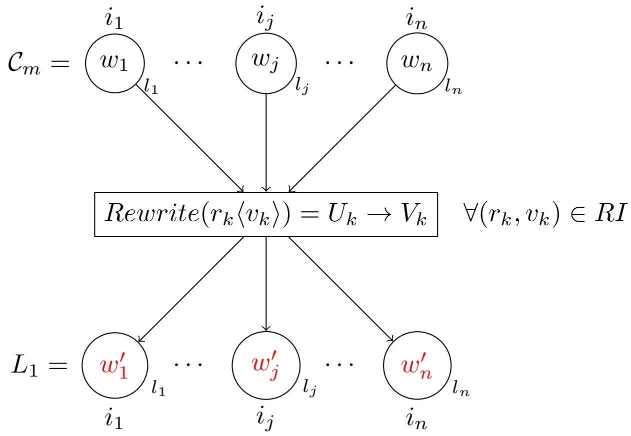
$$U_i = \left\{ \cdots, \underset{id_j}{u}, \cdots \right\}$$

4. $\forall id_s, i, k: \ [(id_s, l) \in \mathit{Label\text{-}Rename}(r_i\langle ID'_{i,j_i}\rangle) \ \wedge \ (id_s, l') \in \mathit{Label\text{-}Rename}(r_k\langle ID'_{k,j_k}\rangle)] \rightarrow l = l'$
5. $\forall i, k: \mathit{Change\text{-}Relation}(r_k)(\mathit{Change\text{-}Relation}(r_i)(\mathcal{C}_\rho)) = \mathit{Change\text{-}Relation}(r_i)(\mathit{Change\text{-}Relation}(r_k)(\mathcal{C}_\rho))$

- $Applicable(R, \mathcal{C}) = \{RI \mid applicable(RI, \mathcal{C})\}$
- $Applicable(\Pi, \mathcal{C}) = \bigcup_{Applicable(R,\mathcal{C}) \neq \emptyset} Applicable(R, \mathcal{C})$

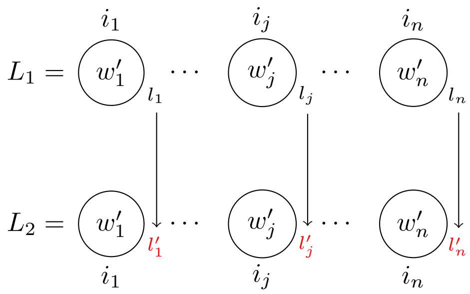
1.5 Applying a Multiset of Rules

1. $L_1 = \{(i_1, l_1, w'_1)...(i_n, l_n, w'_n)\}$



- $$w'_j = w_j - \left(\bigcup_{(r_k,v_k)\in RI} U_k|_j\right) + \left(\bigcup_{(r_k,v_k)\in RI} V_k|_j\right)$$
- $RI = \{(r_1, v_1)...(r_n, v_n)\}$
- $\mathcal{C} = (\{(i_1, l_1, w_1)...(i_n, l_n, w_n)\}, \rho)$
- $Rewrite(r_k\langle v_k\rangle) = U_k \rightarrow V_k$
- $U_k, V_k \in \mathbb{C}_k$

2. $L_2 = \{(i_1, l'_1, w'_1)...(i_n, l'_n, w'_n)\}$



- $$l'_j = \begin{cases} e_s, & \text{if } \exists (r_k, v_k) \in RI \text{ such that } (j, e_s) \in \mathit{Label\text{-}Rename}(r_k\langle v_k\rangle) \\ l_j, & \text{otherwise} \end{cases}$$

3. $L_c = L_c(r_1) \cdot \cdots \cdot L_c(r_n)$

$$L_c(r_k) = \underset{m_1}{u_1}{}_{h_1} \quad \cdots \quad \underset{m_t}{u_t}{}_{h_t}$$

- $L_c(r_k) = \{(m_1, h_1, u_1) \cdots (m_t, h_t, u_t)\}$
- $\mathit{Generate}(r_k) = \{(1', h_1, u_1) \cdots (t', h_t, u_t)\}$

4. $L'_c = L'_c(r_1) \cdot \cdots \cdot L'_c(r_n)$

$$\begin{tikzcd}
L_2 \ni n_1 \; w'_{n_1} \arrow[d] & \cdots & w'_{n_s} \; n_s \arrow[d] \\
m_{t+1} \; (w'_{n_1})_{e_1} \arrow[d, "w''_{n_1} = w'_{n_1} - u_1 + v_1"'] & \cdots & (w'_{n_s})_{e_s} \; m_{t+s} \arrow[d, "w''_{n_s} = w'_{n_s} - u_s + v_s"] \\
L'(r_k) = \; m_{t+1} \; (w''_{n_1})_{e_1} & \cdots & (w''_{n_s})_{e_s} \; m_{t+s}
\end{tikzcd}$$

- $L'_c(r_k) = \{(m_{t+1}, e_1, w'_{n_1} - u_1 + v_1) ... (m_{t+s}, e_s, w'_{n_s} - u_s + v_s)\}$
- $\mathit{Generate\text{-}and\text{-}Copy}(r_k) = \{(1', e_1, n_1, u_1 \to v_1) ... (s', e_s, n_s, u_s \to v_s)\}$
- $(i_j, l'_j, w'_j) \in L_2$

5. $L_3 = L_2 \cdot L_c \cdot L'_c$
6. $L_4 = \{(i_1, l'_1, w''_1) \cdots (i_n, l'_n, w''_n)\}$
   - $$w''_j = w'_j + \left(\bigcup_{\text{last}(i_k) = i_j} w'_k \right)$$
   - $(i_j, l'_j, w'_j) \in L_3$
   - $\mathbf{p} = (p_1, ..., p_j, ..., p_n)$ - vector of "destination" cell ids for multisets from cells that may be deleted.
   - $p_j = p(i_j)$ - destination cell for multiset from cell i_j.
   - $$p(i_j) = \begin{cases} *, & \text{if } \exists (r_k, v_k) \in RI \text{ such that } i_j \in \mathit{Delete}(r_k\langle v_k\rangle) \\ e, & \text{if } \exists (r_k, v_k) \in RI \text{ such that } (i_j, e) \in \mathit{Delete\text{-}and\text{-}Move}(r_k\langle v_k\rangle) \\ i_j, & \text{otherwise} \end{cases}$$
   - $\mathit{Delete}(r) \in \mathbb{N}_k^*$ is the list of cell ids to be deleted.
   - $\mathit{Delete\text{-}and\text{-}Move}(r) \in (\mathbb{N}_k \times \mathbb{N}_k)^*$. $(i, j) \in \mathit{Delete\text{-}and\text{-}Move}(r)$ means to delete cell i and move its multiset to cell j.
   - If $p_k = p(i_k) = i_k$, then cell i_k will not be deleted.
   - If $p_k = p(i_k) \neq i_k$ (either $p(i_k) = *$ or $p(i_k) = e$), then cell i_k will be deleted.
   - If $p_k = i_k$, then there is a sequence $x_1, ..., x_{j-1}, ..., x_m$ of cell ids where:

- $x_1 = p_k$
- $x_j = p(x_{j-1})$ for $2 \leq j \leq m$
- $x_m = z$

There is a chain of "delete-and-move" and z is the last cell id to not be deleted or to be deleted and not moved.

- $last(i_j) = z$

7. $L_5 = \{(i_1, l'_1, w''_1)...(i_{n_1}, l'_{n_1}, w''_{n_1})\}$ where $(i_j, l'_j, w''_j) \in L_4$ and $p_j = i_j$.

   - L_5 contains L_4 cells with the deleted cells removed.

8. $Apply(RI, \mathcal{C}) = (L_2, \mathcal{C}'_\rho)$

   - $\mathcal{C}'_\rho$ is the updated 'parent' relation after $CREATE\text{-}NODES$, $DELETE\text{-}NODES$, and $Change\text{-}Relation(r_k\langle v_k \rangle)$ have been computed for all $(r_k, v_k) \in RI$ on $\mathcal{C}_\rho$.