Introduction

Part 1: Pen and paper exercise

Suppose we have three colored boxes r (red), b (blue), and g (green). Box r contains 3 apples, 4 oranges, and 3 limes, box b contains 1 apple, 1 orange and no limes, and box g contains 3 apples, 3 oranges, and 4 limes. A box is chosen at random with probabilities $p(r) = 0.2$, $p(b) = 0.2$, $p(g) = 0.6$, and a piece of fruit is removed from the box with equal probability of selecting any of the items in the box.

1) What is the probability of selecting an apple?

Given:

- box probabilities: $p(r) = 0.2$, $p(b) = 0.2$, $p(g) = 0.6$
- items in box are selected with equal probability:
  - Red box: $p(apple|r) = \frac{3}{3+4+3} = 0.3$, $p(orange|r) = 0.4$, $p(lime|r) = 0.3$
  - Blue box: $p(apple|b) = 0.5$, $p(orange|b) = 0.5$, $p(lime|b) = 0$
  - Green box: $p(apple|g) = 0.3$, $p(orange|g) = 0.3$, $p(lime|g) = 0.4$

Wanted: $p(apple)$

Solution:

$$\begin{aligned} p(apple) &= p(r, apple) + p(b, apple) + p(g, apple) \\ &= p(r) \cdot p(apple|r) + p(b) \cdot p(apple|b) + p(g) \cdot p(apple|g) \\ &= 0.2 \cdot 0.3 + 0.2 \cdot 0.5 + 0.6 \cdot 0.3 \\ &= 0.34 \end{aligned}$$

2) If we observe that the selected fruit is in fact an orange, what is the probability that it came from the green box?

Wanted: $p(g|orange)$

Solution:

$$p(g|orange) = \frac{p(g, orange)}{p(orange)} = \frac{p(orange|g) \cdot p(g)}{p(orange)} = \frac{0.3 \cdot 0.6}{0.36} = 0.5$$

with

$$\begin{aligned} p(orange) &= p(r) \cdot p(orange|r) + p(b) \cdot p(orange|b) + p(g) \cdot p(orange|g) \\ &= 0.2 \cdot 0.4 + 0.2 \cdot 0.5 + 0.6 \cdot 0.3 \\ &= 0.36 \end{aligned}$$