Assignment No-1

Number System

Divisibility

1) Which one of the following numbers is divisible by 99?
a) 3572404 b) 135792 c) 913464 d) 114345

Ans:- To check which of the given numbers is divisible by 99, we need to follow the divisibility rule for 99.
The rule for divisibility by 99 is that a number is divisible by 99 if and only if it is divisible by both 9 and 11.
1) Divisibility by 9:
A number is divisible by 9 if the sum of its digits is divisible by 9.
2) Divisibility by 11:
A number is divisible by 11 if the difference between the sum of its alternate digits (starting from the rightmost digit) is divisible by 11.
Let's apply these rules to each option:

a) 3572404
Divisibility by 9: 3 + 5 + 7 + 2 + 4 + 0 + 4 = 25 (Not divisible by 9)
Divisibility by 11: 4 - 0 + 2 - 7 + 5 - 3 = 1 (Not divisible by 11)
Not divisible by 99.

b) 135792
Divisibility by 9: 1 + 3 + 5 + 7 + 9 + 2 = 27 (Divisible by 9)
Divisibility by 11: 2 - 9 + 7 - 5 + 3 - 1 = -3 (Not divisible by 11)
Not divisible by 99.

c) 913464
Divisibility by 9: 9 + 1 + 3 + 4 + 6 + 4 = 27 (Divisible by 9)
Divisibility by 11: 4 - 6 + 4 - 3 + 1 - 9 = -9 (Divisible by 11)
Divisible by 99.

d) 114345
Divisibility by 9: 1 + 1 + 4 + 3 + 4 + 5 = 18 (Divisible by 9)
Divisibility by 11: 5 - 4 + 3 - 4 + 1 - 1 = 0 (Divisible by 11)
Divisible by 99.

The numbers that are divisible by 99 are:
c) 913464
d) 114345

2) If n is an integer, what is the remainder when (2n + 2)2 is divided by 4?

Ans:-Let's simplify the expression (2n + 2)^2 and find the remainder when it is divided by 4.

(2n + 2)^2 = (2n + 2) * (2n + 2) = 4n^2 + 4n + 4.

Now, let's find the remainder when 4n^2 + 4n + 4 is divided by 4.

When any even number is divided by 4, the remainder is 0 because every even number is divisible by 2 twice.

Therefore, the remainder when (2n + 2)^2 is divided by 4 is 0.

3) Find two nearest numbers to 19506 which are divisible by 9?

Ans:-To find the two nearest numbers to 19506 that are divisible by 9, we can proceed as follows:

Step 1: Find the nearest multiple of 9 below 19506:
The nearest multiple of 9 below 19506 is 19506 - (19506 % 9) = 19506 - 6 = 19500.

Step 2: Find the nearest multiple of 9 above 19506:
The nearest multiple of 9 above 19506 is 19506 + (9 - (19506 % 9)) = 19506 + 3 = 19509.

Therefore, the two nearest numbers to 19506 that are divisible by 9 are 19500 and 19509.

4) What is the value of M and N respectively if M39048458N is divisible by 8 and 11, where M and N are single digit integers?

Ans:- To determine the values of M and N, we need to consider the divisibility rules for both 8 and 11.

Divisibility by 8:
A number is divisible by 8 if the number formed by its last three digits is divisible by 8.

Divisibility by 11:
A number is divisible by 11 if the difference between the sum of its alternate digits (starting from the leftmost digit) is either 0 or divisible by 11.

Now, let's analyze the given number: M39048458N

Divisibility by 8:
The last three digits are "58". To be divisible by 8, "58" must be divisible by 8. The only single-digit number that makes this true is M = 2, as 58 is divisible by 8 (58 ÷ 8 = 7 remainder 2).

Divisibility by 11:
The sum of the alternate digits is 3 + 0 + 8 + 4 + N = 15 + N. For a single-digit N, the only value that makes 15 + N divisible by 11 is N = 8 (15 + 8 = 23, which is divisible by 11).

So, M = 2 and N = 8 to make the number M39048458N divisible by both 8 and 11.

5) How many pairs of X and Y are possible in the number 763X4Y2, if the number is divisible by 9?

Ans:-To determine the pairs of X and Y that make the number 763X4Y2 divisible by 9, we need to apply the divisibility rule for 9.

A number is divisible by 9 if the sum of its digits is divisible by 9.

Let's calculate the sum of the digits in the number 763X4Y2:

$7 + 6 + 3 + X + 4 + Y + 2 = 22 + X + Y$

For 763X4Y2 to be divisible by 9, the sum of the digits must be divisible by 9. So, we have:

$22 + X + Y \equiv 0 \pmod 9$

To find the possible pairs of X and Y, we need to check which values of X and Y satisfy the congruence above.

Let's consider the possible values for X and Y:

1. X = 1, Y = 1: 22 + 1 + 1 = 24 (Not divisible by 9)
2. X = 1, Y = 2: 22 + 1 + 2 = 25 (Not divisible by 9)
3. X = 1, Y = 3: 22 + 1 + 3 = 26 (Not divisible by 9)
4. X = 1, Y = 4: 22 + 1 + 4 = 27 (Divisible by 9)
5. X = 1, Y = 5: 22 + 1 + 5 = 28 (Not divisible by 9)
6. X = 1, Y = 6: 22 + 1 + 6 = 29 (Not divisible by 9)
7. X = 1, Y = 7: 22 + 1 + 7 = 30 (Divisible by 9)
8. X = 1, Y = 8: 22 + 1 + 8 = 31 (Not divisible by 9)
9. X = 1, Y = 9: 22 + 1 + 9 = 32 (Not divisible by 9)

From the above analysis, there are two pairs of X and Y that make the number 763X4Y2 divisible by 9:

1. X = 1, Y = 4
2. X = 1, Y = 7

6) When the integer n is divided by 8, the remainder is 3. What is the remainder if 6n is divided by 8?

Ans:-Let's consider the integer n when it is divided by 8 with a remainder of 3:

$n \equiv 3 \pmod 8$

Now, we need to find the remainder when 6n is divided by 8. We can express 6n as:

$6n = 6 * n$

To find the remainder when 6n is divided by 8, we can substitute the value of n from the congruence relation above:

$6n \equiv 6 * (3) \equiv 18 \equiv 2 \pmod 8$

So, the remainder when 6n is divided by 8 is 2.

7) If the product 4864 x 9P2 is divisible by 12, then what is the value of P?

Ans:-For a number to be divisible by 12, it must be divisible by both 3 and 4.

Let's first check the divisibility by 4:

To check if 4864 is divisible by 4, we only need to check if the last two digits (64) are divisible by 4. Since 64 is divisible by 4, 4864 is also divisible by 4.

Next, we need to check the divisibility by 3:

To check if 9P2 is divisible by 3, we need to find the sum of its digits and see if it is divisible by 3.

$9 + P + 2 = 11 + P$

For 9P2 to be divisible by 3, 11 + P must be divisible by 3.

For 11 + P to be divisible by 3, P must be equal to 1 or 4.

So, the possible values of P are 1 and 4 for the product 4864 x 9P2 to be divisible by 12.

8) If the number 7X86038 is exactly divisible by 11, then the smallest whole number in place of X?

Ans:-To check if the number 7X86038 is divisible by 11, we can use the divisibility rule for 11.

The divisibility rule for 11 states that a number is divisible by 11 if the difference between the sum of its alternate digits (starting from the leftmost digit) is either 0 or divisible by 11.

Let's calculate the difference between the sum of the alternate digits:

7 + 8 + 0 + 3 - (X + 6 + 8) = 18 - (X + 14) = 4 - X

For 7X86038 to be divisible by 11, the difference 4 - X must be divisible by 11. The possible values of X that make this true are X = 4 and X = 15.

However, we are looking for the smallest whole number in place of X. So, the smallest value for X is X = 4, which makes the number 7X86038 exactly divisible by 11.

9) If an integer n is divisible by 3, 5 and 12, what is the next larger integer divisible by all these numbers?
a) n 2 b) n + 180 c) 2n d) n + 60

Ans:-To find the next larger integer that is divisible by 3, 5, and 12, we need to find the least common multiple (LCM) of these numbers.

The LCM of three numbers can be calculated by finding the product of the highest powers of each prime factor that appears in the factorization of the numbers.

Let's find the prime factorizations of 3, 5, and 12:

3 = 3^1
5 = 5^1
12 = 2^2 * 3^1

Now, to find the LCM, we take the highest powers of each prime factor:

LCM = 2^2 * 3^1 * 5^1 = 60

So, the next larger integer that is divisible by 3, 5, and 12 is 60 more than n, which means the correct option is:

d) n + 60

10) What is the product of the largest and the smallest possible values of M for which a number 5M83M4M1 is divisible by 9?

Ans:-To determine the possible values of M for which the number 5M83M4M1 is divisible by 9, we need to apply the divisibility rule for 9.

A number is divisible by 9 if the sum of its digits is divisible by 9.

Let's calculate the sum of the digits in the number 5M83M4M1:

5 + M + 8 + 3 + M + 4 + M + 1 = 21 + 3M

For 5M83M4M1 to be divisible by 9, the sum of the digits must be divisible by 9. So, we have:

21 + 3M ≡ 0 (mod 9)

To find the possible values of M, we need to check which values of M satisfy the congruence above.

21 + 3M can be divisible by 9 when M = 1, 4, or 7.

Now, to find the product of the largest and smallest possible values of M:

Product = Largest M * Smallest M
Product = 7 * 1
Product = 7

So, the product of the largest and smallest possible values of M for which the number 5M83M4M1 is divisible by 9 is 7.

Factors

1) What is the number of prime factors in 6^4 x 8^6 x 10^8 x 14^10 x 22^12

Ans:-To find the number of prime factors in the expression 6^4 x 8^6 x 10^8 x 14^10 x 22^12, we first need to factorize each number into its prime factors and then count the occurrences of each prime factor.

Let's factorize each number:

6^4 = 2^4 * 3^4
8^6 = 2^18

10^8 = 2^8 * 5^8
14^10 = 2^10 * 7^10
22^12 = 2^12 * 11^12

Now, we can write the expression in terms of its prime factors:

6^4 x 8^6 x 10^8 x 14^10 x 22^12 = (2^4 * 3^4) x (2^18) x (2^8 * 5^8) x (2^10 * 7^10) x (2^12 * 11^12)

To find the total number of prime factors, we just need to count the occurrences of each prime factor:

For the prime factor 2, it appears 4 + 18 + 8 + 10 + 12 = 52 times.
For the prime factor 3, it appears 4 times.
For the prime factor 5, it appears 8 times.
For the prime factor 7, it appears 10 times.
For the prime factor 11, it appears 12 times.

Adding them up: 52 + 4 + 8 + 10 + 12 = 86

So, the expression 6^4 x 8^6 x 10^8 x 14^10 x 22^12 has a total of 86 prime factors.

2) N= a^4xb^3xc^7. Find the number of perfect square factors of N where a,b,c are three distinct prime numbers.

Ans:-To find the number of perfect square factors of N, we need to consider the exponents of its prime factors and determine which combinations result in perfect squares.

N = a^4 * b^3 * c^7

For a factor to be a perfect square, the exponents of its prime factors should be even. Let's consider the possible combinations of exponents that result in even exponents:

1. Exponents of a, b, and c are even:
   a^4 * b^2 * c^6 (Perfect square factor)

2. Exponents of a and b are even, and c is odd:
   a^4 * b^2 * c^7 (Not a perfect square factor)

3. Exponents of a and c are even, and b is odd:
   a^4 * b^3 * c^6 (Not a perfect square factor)

4. Exponents of b and c are even, and a is odd:
   a^1 * b^2 * c^6 (Not a perfect square factor)

5. Exponents of a is even, and b and c are odd:

a^4 * b^3 * c^7 (Not a perfect square factor)

6. Exponents of b is even, and a and c are odd:
   a^1 * b^2 * c^7 (Not a perfect square factor)

7. Exponents of c is even, and a and b are odd:
   a^1 * b^3 * c^6 (Not a perfect square factor)

8. Exponents of a, b, and c are odd:
   a^1 * b^3 * c^7 (Not a perfect square factor)

So, there is only one combination that results in a perfect square factor: a^4 * b^2 * c^6.

The number of perfect square factors of N = a^4 * b^2 * c^6 is 1.

3) How many factors of 12^3x30^4x35^2 are even numbers?

Ans:-To find the number of factors of the given expression that are even numbers, we need to consider the exponents of the prime factors and how they contribute to the factors being even.

Let's express the given expression in terms of its prime factors:

12^3 = 2^3 * 3^3
30^4 = 2^4 * 3^4 * 5^4
35^2 = 5^2 * 7^2

Now, to find the total number of factors of the expression, we can multiply the (exponent + 1) of each prime factor and then take the product:

Total factors = (3 + 1) * (4 + 1) * (2 + 1) = 4 * 5 * 3 = 60

Now, let's find the number of factors that are even numbers. A factor is even if it contains at least one power of 2, which means the exponent of 2 should be at least 1.

Factors with exponent of 2 for 2: 1 * (1 + 1) * (0 + 1) = 1 * 2 * 1 = 2
Factors with exponent of 3 for 2: 1 * (1 + 1) * (1 + 1) = 1 * 2 * 2 = 4

Now, we need to find the factors of the other prime factors (3, 5, and 7) that contain at least one power of 2 (even numbers):

Factors with exponent of 1 for 3, 5, and 7: (1 + 1) * (4 + 1) * (2 + 1) = 2 * 5 * 3 = 30
Factors with exponent of 2 for 3, 5, and 7: (2 + 1) * (4 + 1) * (2 + 1) = 3 * 5 * 3 = 45

Now, we add up all these cases to find the total number of factors that are even:

2 + 4 + 30 + 45 = 81

So, there are 81 factors of 12^3 * 30^4 * 35^2 that are even numbers.

4) If N=2^^, M= 2^4×3^2×5, then find the number of factors of N that are common with the factors of M.

Ans:-To find the common factors of N and M, we need to first factorize N and then find the factors that are common with the factors of M.

N = 2^2^2 (2 raised to the power of 2^2 = 2^4)

M = 2^4 * 3^2 * 5

Now, let's find the common factors of N and M. These are the factors that appear in both N and M:

Common factors = GCD(N, M)

To find the greatest common divisor (GCD) of N and M, we need to consider the highest power of each prime factor that appears in both N and M:

For 2: The highest power is 2^4 (common in N and M)
For 3: The highest power is 3^2 (common in N and M)
For 5: The highest power is 5^1 (common in M)

So, the common factors of N and M are 2^4 * 3^2 * 5.

Now, let's find the number of factors for the common factors:

Number of factors of common factors = (4+1) * (2+1) * (1+1) = 5 * 3 * 2 = 30

Therefore, there are 30 common factors of N and M.

5) N is the smallest number that has 5 factors. How many factors does (N - 1) have?

Ans:-Let's find the prime factorization of N and then determine the number of factors it has. Since N has 5 factors, it must be of the form p^4 or p^2 * q, where p and q are distinct prime numbers.

Case 1: N = p^4

In this case, N has (4+1) = 5 factors (1, p, p^2, p^3, and p^4).

Case 2: N = p^2 * q

In this case, N has (2+1) * (1+1) = 6 factors (1, p, p^2, q, p*q, and p^2*q).

Now, let's find the smallest N with 5 factors.

The smallest N with 5 factors is 2^4 = 16 (Case 1: p=2).

Now, we need to find (N - 1) and determine how many factors it has.

N - 1 = 16 - 1 = 15

Let's find the prime factorization of (N - 1) = 15:

15 = 3^1 * 5^1

Now, we can determine the number of factors of (N - 1):

Number of factors of (N - 1) = (1+1) * (1+1) = 2 * 2 = 4

So, (N - 1) has 4 factors.

6) If both 112 and 34 are factors of the number Ax4^3x6^2x13^11,then what is the smallest possible value of A?

Ans:-To find the smallest possible value of A, we need to consider the prime factorization of the given number and find the factors that are divisible by both 112 and 34.

Let's express the given number in terms of its prime factors:

Ax4^3x6^2x13^11 = A * 2^3 * 3^2 * 13^11

Now, let's find the factors that are divisible by both 112 and 34.

Factor of 112 = 2^4 * 7^1
Factor of 34 = 2^1 * 17^1

For a factor to be divisible by both 112 and 34, it must have at least the same or higher powers of 2, 7, and 17 as in their respective factorizations.

So, the smallest value of A will be the product of the highest powers of 2, 7, and 17 present in the factorizations of 112 and 34.

Smallest value of A = 2^1 * 7^1 * 17^1 = 2 * 7 * 17 = 238

Therefore, the smallest possible value of A is 238.

7) Find the total number of factors of 10!

Ans:-To find the total number of factors of 10!, we need to factorize 10! and then count the number of factors.

$10! = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
$= 2^8 \times 3^4 \times 5^2 \times 7$

The number of factors of any number N can be calculated by taking the product of (exponent + 1) of each prime factor in the factorization of N.

Total factors of $10! = (8 + 1) \times (4 + 1) \times (2 + 1) \times (1 + 1)$
$= 9 \times 5 \times 3 \times 2$
$= 270$

Therefore, the total number of factors of 10! is 270.

8) How many factors of $2^7 \times 3^6 \times 5^4 \times 7^3$ are even perfect squares?

Ans:-To find the factors of the given expression that are even perfect squares, we need to consider the exponents of each prime factor and determine which combinations result in even exponents (for perfect squares).

The given expression is: $2^7 \times 3^6 \times 5^4 \times 7^3$

For a factor to be an even perfect square, the exponents of its prime factors should be even.

Let's consider the possible combinations of even exponents for each prime factor:

1. Exponents of 2, 3, 5, and 7 are even:
   $2^6 \times 3^6 \times 5^4 \times 7^2$ (Even perfect square factor)

2. Exponents of 2 and 5 are even, and 3 and 7 are odd:
   $2^6 \times 3^6 \times 5^4 \times 7^3$ (Not an even perfect square factor)

3. Exponents of 2 and 3 are even, and 5 and 7 are odd:
   $2^6 \times 3^6 \times 5^3 \times 7^2$ (Not an even perfect square factor)

4. Exponents of 2 and 7 are even, and 3 and 5 are odd:
   $2^6 \times 3^4 \times 5^4 \times 7^2$ (Not an even perfect square factor)

5. Exponents of 3, 5, and 7 are even, and 2 is odd:
   $2^7 \times 3^6 \times 5^4 \times 7^2$ (Not an even perfect square factor)

6. Exponents of 2 and 3 are even, and 5 and 7 are odd:
   $2^6 \times 3^6 \times 5^4 \times 7^2$ (Not an even perfect square factor)

7. Exponents of 2 and 5 are even, and 3 and 7 are odd:
   $2^6 \times 3^6 \times 5^4 \times 7^3$ (Not an even perfect square factor)

8. Exponents of 2 and 3 are even, and 5 and 7 are odd:
   $2^6 \times 3^6 \times 5^3 \times 7^2$ (Not an even perfect square factor)

9. Exponents of 2 and 7 are even, and 3 and 5 are odd:
   $2^6 \times 3^4 \times 5^4 \times 7^2$ (Not an even perfect square factor)

So, there is only one combination that results in an even perfect square factor: $2^6 \times 3^6 \times 5^4 \times 7^2$.

The number of factors of the expression $2^7 \times 3^6 \times 5^4 \times 7^3$ that are even perfect squares is 1.

9) In how many ways can 480 be written as a product of two natural numbers?

Ans:-To find the number of ways 480 can be written as a product of two natural numbers, we need to consider all the possible factor pairs of 480.

Let's find the factorization of 480:

$480 = 2^5 * 3 * 5$

Now, we can form factor pairs by multiplying two factors together. The two factors should be divisors of 480.

The divisors of 480 are:
1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 16, 20, 24, 30, 32, 40, 48, 60, 80, 96, 120, 160, 240, 480

Now, let's find the number of factor pairs by pairing up these divisors:

1 * 480 = 480
2 * 240 = 480
3 * 160 = 480
4 * 120 = 480
5 * 96 = 480

6 * 80 = 480
8 * 60 = 480
10 * 48 = 480
12 * 40 = 480
15 * 32 = 480
16 * 30 = 480
20 * 24 = 480

There are 12 different ways to write 480 as a product of two natural numbers.

10) How many factors of 2^5x3^4 x 5^3 are not the factors of 2^3x5^4 x 7^5

Ans:-To find the factors that are not common between 2^5 * 3^4 * 5^3 and 2^3 * 5^4 * 7^5, we can first find the prime factorization of each number and then determine which factors are unique to each number.

Prime factorization of 2^5 * 3^4 * 5^3:
2^5 * 3^4 * 5^3 = 32 * 81 * 125

Prime factorization of 2^3 * 5^4 * 7^5:
2^3 * 5^4 * 7^5 = 8 * 625 * 16807

There are 41 factors that are not common between the two numbers.

Unit digits (Cyclicity)

1) What is the unit digit in the product (3^65 x 6^59 x 7^71)?

Ans:-To find the unit digit in the product (3^65 x 6^59 x 7^71), we only need to consider the unit digits of each term in the product. The unit digit of a number is the digit in the one's place.

Let's calculate the unit digits for each term:

1. 3^65:
The unit digit of 3^1 is 3.
The unit digit of 3^2 is 9.
The unit digit of 3^3 is 7.
The unit digit of 3^4 is 1.

The pattern repeats every 4 powers, and 65 is one more than a multiple of 4 (65 = 4 * 16 + 1). Therefore, the unit digit of 3^65 is the same as the unit digit of 3^1, which is 3.

2. 6^59:
The unit digit of 6^1 is 6.
The unit digit of 6^2 is 6.
The pattern repeats every 2 powers, and 59 is an odd number. Therefore, the unit digit of 6^59 is the same as the unit digit of 6^1, which is 6.

3. 7^71:
The unit digit of 7^1 is 7.
The unit digit of 7^2 is 9.
The unit digit of 7^3 is 3.
The unit digit of 7^4 is 1.
The pattern repeats every 4 powers, and 71 is one more than a multiple of 4 (71 = 4 * 17 + 3). Therefore, the unit digit of 7^71 is the same as the unit digit of 7^3, which is 3.

Now, let's multiply the unit digits:

Unit digit = (3 * 6 * 3) = 54

The unit digit of the product is 4.

2) Find unit digit of product (173)45*(152)77*(777)999

Ans:-To find the unit digit of the product (173)^45 * (152)^77 * (777)^999, we only need to consider the unit digits of each term in the product. The unit digit of a number is the digit in the one's place.

Let's calculate the unit digits for each term:

1. (173)^45:
The unit digit of 173 is 3.
Since 3^1 is 3, the unit digit of (173)^45 is also 3.

2. (152)^77:
The unit digit of 152 is 2.
Since 2^1 is 2, the unit digit of (152)^77 is also 2.

3. (777)^999:
The unit digit of 777 is 7.
Since 7^1 is 7, the unit digit of (777)^999 is also 7.

Now, let's multiply the unit digits:

Unit digit = (3 * 2 * 7) = 42

The unit digit of the product is 2.

3) What is the unit's digit of the number 6^256–4^256?

Ans:-To find the unit digit of the number 6^256 - 4^256, we need to consider the unit digits of each term separately and then subtract them.

Let's calculate the unit digits for each term:

1. 6^256:
The unit digit of 6^1 is 6.
The unit digit of 6^2 is 6.
The unit digit of 6^3 is 6.
The pattern repeats every 4 powers, and 256 is a multiple of 4 (256 = 4 * 64). Therefore, the unit digit of 6^256 is the same as the unit digit of 6^4, which is 6.

2. 4^256:
The unit digit of 4^1 is 4.
The unit digit of 4^2 is 6.
The unit digit of 4^3 is 4.
The unit digit of 4^4 is 6.
The pattern repeats every 4 powers, and 256 is a multiple of 4 (256 = 4 * 64). Therefore, the unit digit of 4^256 is the same as the unit digit of 4^4, which is 6.

Now, let's subtract the unit digits:

Unit digit = (6 - 6) = 0

The unit digit of the number 6^256 - 4^256 is 0.

4) Find the unit's digit in 264^102+264^103

Ans:-To find the unit digit in 264^102 + 264^103, we need to consider the unit digits of each term separately and then add them.

Let's calculate the unit digits for each term:

1. 264^102:
The unit digit of 264 is 4.
Since 4^1 is 4, the unit digit of 264^102 is also 4.

2. 264^103:
The unit digit of 264 is 4.
Since 4^1 is 4, the unit digit of 264^103 is also 4.

Now, let's add the unit digits:

Unit digit = (4 + 4) = 8

The unit digit in 264^102 + 264^103 is 8.

5) What is the unit digit of (316)^3n + 1?

Ans:-To find the unit digit of (316)^3n + 1, we need to consider the unit digit of the expression (316)^3n and then add 1 to it.

Let's calculate the unit digit of (316)^3n:

The unit digit of 316 is 6.
Now, let's find the unit digit of 6 raised to various powers:

6^1 = 6
6^2 = 36 (unit digit is 6)
6^3 = 216 (unit digit is 6)
6^4 = 1296 (unit digit is 6)

The pattern repeats every 4 powers, and 3n is a multiple of 4 for any positive integer n. Therefore, the unit digit of (316)^3n will be 6 for any positive integer n.

Now, let's add 1 to the unit digit:

Unit digit = 6 + 1 = 7

The unit digit of (316)^3n + 1 is 7.

6) What is the unit digit in (7^95 - 3^58)?

Ans:-To find the unit digit in (7^95 - 3^58), we need to consider the unit digits of each term separately and then subtract them.

1. 7^95:
The unit digit of 7 is 7.
Now, let's find the unit digit of 7 raised to various powers:

7^1 = 7
7^2 = 49 (unit digit is 9)
7^3 = 343 (unit digit is 3)
7^4 = 2401 (unit digit is 1)

The pattern repeats every 4 powers, and 95 is one less than a multiple of 4 (95 = 4 * 23 + 3). Therefore, the unit digit of 7^95 is the same as the unit digit of 7^3, which is 3.

2. 3^58:
The unit digit of 3 is 3.
Now, let's find the unit digit of 3 raised to various powers:

3^1 = 3
3^2 = 9
3^3 = 27 (unit digit is 7)
3^4 = 81 (unit digit is 1)

The pattern repeats every 4 powers, and 58 is two less than a multiple of 4 (58 = 4 * 14 + 2). Therefore, the unit digit of 3^58 is the same as the unit digit of 3^2, which is 9.

Now, let's subtract the unit digits:

Unit digit = (3 - 9) = -6

The unit digit in (7^95 - 3^58) is -6. However, we need to find the positive unit digit, so we take the last digit (ignore the negative sign) and add 10 to it:

Positive unit digit = 10 - 6 = 4

The positive unit digit in (7^95 - 3^58) is 4.

7) What is the rightmost non-zero digit of the number 30^2720

Ans:- To find the rightmost non-zero digit of the number 30^2720, we need to find the rightmost non-zero digit of 30^2720, which is the same as finding the rightmost non-zero digit of (3^2720 * 10^2720).

Now, let's focus on the term 3^2720:

The rightmost non-zero digit of 3^1 is 3.
The rightmost non-zero digit of 3^2 is 9.
The rightmost non-zero digit of 3^3 is 7.
The rightmost non-zero digit of 3^4 is 1.

The pattern repeats every 4 powers, and 2720 is a multiple of 4 (2720 = 4 * 680). Therefore, the rightmost non-zero digit of 3^2720 is the same as the rightmost non-zero digit of 3^4, which is 1.

Now, let's consider the term 10^2720:

10^1 = 10
10^2 = 100
10^3 = 1000
10^4 = 10000

... and so on.

The rightmost non-zero digit of any power of 10 is always 1.

Now, we need to multiply the rightmost non-zero digits of 3^2720 and 10^2720:

Rightmost non-zero digit = 1 * 1 = 1

Therefore, the rightmost non-zero digit of the number 30^2720 is 1.

8) What will be the last digit of the number obtained by multiplying the numbers 81*82*83*84*86*87*88*89?

Ans:-To find the last digit of the number obtained by multiplying the numbers 81 * 82 * 83 * 84 * 86 * 87 * 88 * 89, we can calculate the last digit of each number individually and then multiply them.

Let's calculate the last digit of each number:

1. Last digit of 81: 1
2. Last digit of 82: 2
3. Last digit of 83: 3
4. Last digit of 84: 4
5. Last digit of 86: 6
6. Last digit of 87: 7
7. Last digit of 88: 8
8. Last digit of 89: 9

Now, let's multiply the last digits:

Last digit = 1 * 2 * 3 * 4 * 6 * 7 * 8 * 9

To find the last digit of the product, we can ignore the digit position greater than the unit's place, as they won't affect the last digit. So, we only need to consider the last digit of the product of the last digits:

Last digit = 1 * 2 * 3 * 4 * 6 * 7 * 8 * 9 = 24192

Now, we only need the unit's place digit, which is the last digit:

Last digit = 2

Therefore, the last digit of the number obtained by multiplying 81 * 82 * 83 * 84 * 86 * 87 * 88 * 89 is 2.

9) Find the last three-digits of the product: 12345x54321

Ans:-To find the last three digits of the product 12345 x 54321, we can multiply the numbers and then take the last three digits of the result.

12345 x 54321 = 670592745

Now, we only need the last three digits, which are 745.

Therefore, the last three digits of the product 12345 x 54321 are 745.

10) Find the last digit of 1^5+2^5+3^5……9^5

Ans:-To find the last digit of the sum 1^5 + 2^5 + 3^5 + ... + 9^5, we need to calculate each term separately and then sum them up.

Let's find the last digit of each term:

1^5 = 1
2^5 = 32 (last digit is 2)
3^5 = 243 (last digit is 3)
4^5 = 1024 (last digit is 4)
5^5 = 3125 (last digit is 5)
6^5 = 7776 (last digit is 6)
7^5 = 16807 (last digit is 7)
8^5 = 32768 (last digit is 8)
9^5 = 59049 (last digit is 9)

Now, let's sum up the last digits:

Last digit = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45

The last digit of the sum 1^5 + 2^5 + 3^5 + ... + 9^5 is 5.

Remainders

1) What is the remainder when 7^25 is divided by 6?

Ans:-To find the remainder when 7^25 is divided by 6, we can use the concept of modular arithmetic.

Note that for any positive integer n, (a^n) % m is equivalent to the remainder when a^n is divided by m.

Let's calculate the remainders for smaller powers of 7:

7^1 = 7 (Remainder when divided by 6 is 1)

7^2 = 49 (Remainder when divided by 6 is 1)
7^3 = 343 (Remainder when divided by 6 is 1)
7^4 = 2401 (Remainder when divided by 6 is 1)
7^5 = 16807 (Remainder when divided by 6 is 1)

The pattern repeats with a remainder of 1 for every power of 7. So, for 7^25:

7^25 = (7^5)^5 = (16807)^5

Now, let's find the remainder when (16807)^5 is divided by 6:

(16807)^5 = 4437053121

Remainder when divided by 6 = 4437053121 % 6 = 1

Therefore, the remainder when 7^25 is divided by 6 is 1.

2) What is the remainder when 3^45 is divided by 8?

Ans:-To find the remainder when 3^45 is divided by 8, we can use the concept of modular arithmetic.

Note that for any positive integer n, (a^n) % m is equivalent to the remainder when a^n is divided by m.

Let's calculate the remainders for smaller powers of 3:

3^1 = 3 (Remainder when divided by 8 is 3)
3^2 = 9 (Remainder when divided by 8 is 1)
3^3 = 27 (Remainder when divided by 8 is 3)
3^4 = 81 (Remainder when divided by 8 is 1)

The pattern repeats with a remainder of 3 and 1 alternately for every power of 3. Since 45 is an odd number, the remainder when 3^45 is divided by 8 will be the same as the remainder when 3^1 is divided by 8, which is 3.

Therefore, the remainder when 3^45 is divided by 8 is 3.

3) Find the remainder when 49^6 is divided by 6.

Ans:-To find the remainder when 49^6 is divided by 6, we can use the concept of modular arithmetic.

Note that for any positive integer n, (a^n) % m is equivalent to the remainder when a^n is divided by m.

Let's calculate the remainder for 49^6:

49^1 = 49 (Remainder when divided by 6 is 1)
49^2 = 2401 (Remainder when divided by 6 is 1)
49^3 = 117649 (Remainder when divided by 6 is 1)
49^4 = 5764801 (Remainder when divided by 6 is 1)
49^5 = 282475249 (Remainder when divided by 6 is 1)
49^6 = 13841287201

Now, let's find the remainder when 13841287201 is divided by 6:

13841287201 % 6 = 1

Therefore, the remainder when 49^6 is divided by 6 is 1.

4) What is the remainder when 14^15^16 is divided by 5?

Ans:-To find the remainder when 14^15^16 is divided by 5, we can use the concept of modular arithmetic.

Note that for any positive integers a, b, and m, (a^b)^c % m is equivalent to the remainder when a^(b*c) is divided by m.

Let's calculate the remainder for 14^(15^16):

First, calculate 15^16:
15^1 = 15 (Remainder when divided by 5 is 0)
15^2 = 225 (Remainder when divided by 5 is 0)
15^3 = 3375 (Remainder when divided by 5 is 0)
15^4 = 50625 (Remainder when divided by 5 is 0)
15^5 = 759375 (Remainder when divided by 5 is 0)
... and so on.

The pattern repeats with a remainder of 0 for every power of 15. So, 15^16 will also have a remainder of 0 when divided by 5.

Now, calculate 14^(15^16):

14^(15^16) = 14^0 (Since 15^16 is divisible by 5)

Any number raised to the power of 0 is 1. So, 14^(15^16) is 1.

Now, find the remainder when 1 is divided by 5:

1 % 5 = 1

Therefore, the remainder when 14^15^16 is divided by 5 is 1.

5) Find the remainder when 67^99 is divided by 7.

Ans:-To find the remainder when 67^99 is divided by 7, we can use the concept of modular arithmetic.

Note that for any positive integer a, a^b % m is equivalent to the remainder when a^b is divided by m.

Let's calculate the remainder for 67^99:

First, let's find the remainder when 67 is divided by 7:

67 % 7 = 4

Now, let's find the remainder when 67^99 is divided by 7:

Since 67 % 7 = 4, we can rewrite 67 as (7 * 9 + 4):

67 = 7 * 9 + 4

Now, let's use the property of modular arithmetic that states: (a * b) % m = ((a % m) * (b % m)) % m

(67^99) % 7 = ((7 * 9 + 4)^99) % 7

Now, let's consider the term (7 * 9 + 4)^99:

Using the binomial theorem, we can expand (7 * 9 + 4)^99 as follows:

(7 * 9 + 4)^99 = (7^99 * 9^0 * C(99, 0)) + (7^98 * 9^1 * C(99, 1) * 4) + (7^97 * 9^2 * C(99, 2) * 4^2) + ... + (7^0 * 9^99 * C(99, 99) * 4^99)

Since 7^99 is divisible by 7 (7 * some_integer), all the terms (7^99 * 9^k * C(99, k) * 4^k) for k from 1 to 99 are divisible by 7.

Therefore, the remainder when (7 * 9 + 4)^99 is divided by 7 is simply the last term (7^0 * 9^99 * C(99, 99) * 4^99):

(7 * 9 + 4)^99 % 7 = 4^99

Now, let's find the remainder when 4^99 is divided by 7:

4^1 = 4
4^2 = 16 (Remainder when divided by 7 is 2)

4^3 = 64 (Remainder when divided by 7 is 1)
4^4 = 256 (Remainder when divided by 7 is 4)
4^5 = 1024 (Remainder when divided by 7 is 1)

The pattern repeats with a remainder of 4, 2, 1, and 4 for every power of 4. Since 99 is three more than a multiple of 4 (99 = 4 * 24 + 3), the remainder when 4^99 is divided by 7 is the same as the remainder when 4^3 is divided by 7, which is 1.

Therefore, the remainder when 67^99 is divided by 7 is 1.

6) What is the remainder when 73x75x78x57x197x37 is divided by 34.

Ans:-To find the remainder when 73 x 75 x 78 x 57 x 197 x 37 is divided by 34, we can use the concept of modular arithmetic and the properties of remainders.

Let's find the remainders for each individual factor when divided by 34:

1. 73 % 34 = 5
2. 75 % 34 = 7
3. 78 % 34 = 10
4. 57 % 34 = 23
5. 197 % 34 = 23
6. 37 % 34 = 3

Now, let's find the remainder when the product of these remainders is divided by 34:

Remainder = (5 x 7 x 10 x 23 x 23 x 3) % 34

Now, we can simplify the product:

(5 x 7 x 10 x 23 x 23 x 3) = 138045

Now, let's find the remainder when 138045 is divided by 34:

Remainder = 138045 % 34 = 9

Therefore, the remainder when 73 x 75 x 78 x 57 x 197 x 37 is divided by 34 is 9.

7) Let N = 1421*1423*1425. What is the remainder when N is divided by 12?

Ans:-To find the remainder when N = 1421 * 1423 * 1425 is divided by 12, we can first find the remainders of each individual factor when divided by 12, and then calculate the remainder of the product.

1. 1421 % 12 = 5
2. 1423 % 12 = 7

3. 1425 % 12 = 9

Now, let's find the remainder when the product of these remainders is divided by 12:

Remainder = (5 x 7 x 9) % 12

Now, we can simplify the product:

(5 x 7 x 9) = 315

Now, let's find the remainder when 315 is divided by 12:

Remainder = 315 % 12 = 3

Therefore, the remainder when N = 1421 * 1423 * 1425 is divided by 12 is 3.

8) Find the remainder when 2^256 is divided by 17.

Ans:-To find the remainder when 2^256 is divided by 17, we can use the concept of modular arithmetic and Euler's theorem.

Euler's theorem states that if a and m are coprime positive integers (i.e., gcd(a, m) = 1), then a^(φ(m)) ≡ 1 (mod m), where φ(m) is Euler's totient function and represents the count of positive integers less than m that are coprime with m.

Since 17 is a prime number, φ(17) = 17 - 1 = 16.

Now, we need to find the remainder of 2^256 divided by 17. To do this, we'll reduce the exponent 256 modulo φ(17):

256 % φ(17) = 256 % 16 = 0

Now, we can use Euler's theorem:

2^256 ≡ 2^0 ≡ 1 (mod 17)

Therefore, the remainder when 2^256 is divided by 17 is 1.

9) The remainder of 39^97! /40 is:
a) 39 b) 0 c) 1 d) None of these

Ans:-To find the remainder of 39^97! divided by 40, we need to use the concept of modular arithmetic.

Since 39 and 40 are not coprime (they share a common factor of 1), we cannot use Euler's theorem directly. However, we can use Euler's totient function to simplify the expression.

Let's consider the Euler's totient function of 40 (denoted as φ(40)):

$φ(40) = 40 * (1 - 1/2) * (1 - 1/5) = 40 * (1/2) * (4/5) = 16$

Now, we can reduce the exponent 97! modulo φ(40):

97! % φ(40) = 97! % 16 = 0

Since 97! is divisible by 16, we can rewrite the expression as follows:

39^97! / 40 = (39^16)^k * (39^r) / 40

where k is the quotient when 97! is divided by 16, and r is the remainder.

Now, let's find the value of 39^16 modulo 40:

$39^{16} \equiv 1 \pmod{40}$

Therefore, the expression simplifies to:

(39^16)^k * (39^r) / 40 ≡ 1^k * (39^r) / 40 ≡ 39^r / 40

Now, we just need to find the remainder when 39^r is divided by 40, where r is the remainder when 97! is divided by 16.

Since r = 0 (as shown earlier), we have:

$39^r \equiv 39^0 \equiv 1 \pmod{40}$

Therefore, the remainder of 39^97! / 40 is 1 (option c).

10) Find the remainder on dividing 1!+2!+3!.....+100! by 7?

Ans:-To find the remainder when 1! + 2! + 3! + ... + 100! is divided by 7, we need to calculate the sum first and then find its remainder when divided by 7.

1! = 1
2! = 2
3! = 6
4! = 24
5! = 120
6! = 720

7! = 5040
8! = 40320
9! = 362880
10! = 3628800
... and so on

Let's find the remainders of each factorial when divided by 7:

1! % 7 = 1
2! % 7 = 2
3! % 7 = 6
4! % 7 = 3
5! % 7 = 5
6! % 7 = 6
7! % 7 = 0
8! % 7 = 0
9! % 7 = 0
10! % 7 = 0
... and so on

From the 7th factorial onward, all factorials will have a remainder of 0 when divided by 7.

Now, let's calculate the sum of the remainders up to 100!:

Remainder = (1 + 2 + 6 + 3 + 5 + 6) + (0 + 0 + 0 + 0 + ... + 0)

The first bracket contains the remainders up to 6!, and the second bracket represents the remainders of all the factorials from 7! to 100!.

The sum of the remainders up to 6! is 1 + 2 + 6 + 3 + 5 + 6 = 23.

Now, let's find the remainder of the sum of the remainders when divided by 7:

Remainder = 23 % 7 = 2

Therefore, the remainder when 1! + 2! + 3! + ... + 100! is divided by 7 is 2.

Factorials

1) What is the highest power of 21 that divides 20!?

Ans:-To find the highest power of 21 that divides 20!, we need to determine how many times 21 can be factored out from 20!.

To find the highest power of 21 that divides 20!, we can find the highest power of 21's prime factors, which are 3 and 7.

First, let's determine the number of factors of 3 in 20!:

20! = 20 × 19 × 18 × ... × 3 × 2 × 1

The number of factors of 3 in 20! is equal to the number of multiples of 3 in the set {1, 2, 3, ..., 20}. There are six multiples of 3 in this set: 3, 6, 9, 12, 15, and 18.

Next, let's determine the number of factors of 7 in 20!:

20! = 20 × 19 × 18 × ... × 7 × 6 × ... × 1

The number of factors of 7 in 20! is equal to the number of multiples of 7 in the set {1, 2, 3, ..., 20}. There is one multiple of 7 in this set: 7.

Now, we need to find the highest power of 21 that divides 20!. Since 21 = 3 × 7, we can combine the results for the factors of 3 and 7.

Since there are six multiples of 3 and one multiple of 7 in 20!, the highest power of 21 that divides 20! is 21^(min(6, 1)) = 21^1 = 21.

Therefore, the highest power of 21 that divides 20! is 21.

2) What is the highest power of 32 that divides 31!?

Ans:-To find the highest power of 32 that divides 31!, we need to determine how many times 32 can be factored out from 31!.

The prime factorization of 32 is 2^5 (2 raised to the power of 5).

To find the highest power of 32 that divides 31!, we need to count the number of multiples of 2^5 (32) in the set {1, 2, 3, ..., 31}, as each multiple of 32 contributes to the highest power of 32 in the factorial.

The multiples of 32 in the set {1, 2, 3, ..., 31} are: 32.

Therefore, the highest power of 32 that divides 31! is 32^1 = 32.

3) Find the largest number less than 28 which divides 28!?

Ans:-To find the largest number less than 28 which divides 28!, we need to determine the greatest common divisor (GCD) of 28!.

The factorial 28! is the product of all positive integers from 1 to 28:

28! = 28 × 27 × 26 × ... × 3 × 2 × 1

To find the largest number less than 28 that divides 28!, we need to find the largest number that is a factor of all the individual numbers from 1 to 28.

To do this, we can find the prime factorization of each number from 1 to 28 and determine the highest power of each prime that appears in all the factorizations.

Prime factorization of each number from 1 to 28:
1 = 1
2 = 2
3 = 3
4 = 2^2
5 = 5
6 = 2 × 3
7 = 7
8 = 2^3
9 = 3^2
10 = 2 × 5
11 = 11
12 = 2^2 × 3
13 = 13
14 = 2 × 7
15 = 3 × 5
16 = 2^4
17 = 17
18 = 2 × 3^2
19 = 19
20 = 2^2 × 5
21 = 3 × 7
22 = 2 × 11
23 = 23
24 = 2^3 × 3
25 = 5^2
26 = 2 × 13
27 = 3^3
28 = 2^2 × 7

Now, let's determine the highest power of each prime that appears in the factorization of 28!:

- The highest power of 2 is 2^4.
- The highest power of 3 is 3^3.
- The highest power of 5 is 5^2.
- The highest power of 7 is 7^1.
- For all other primes (11, 13, 17, 19, 23), the highest power is 1.

Now, we can find the largest number less than 28 that divides 28! by taking the product of the highest powers of each prime:

Largest number = 2^4 × 3^3 × 5^2 × 7^1 = 16 × 27 × 25 × 7 = 302400.

Therefore, the largest number less than 28 which divides 28! is 302400.

4) Find the number of zeroes at the end of 97!

Ans:-To find the number of zeros at the end of 97!, we need to determine the number of factors of 10 in its prime factorization. Since a trailing zero in a number is the result of a factor of 10, we need to count the occurrences of factors of 10 in 97!.

The prime factorization of 10 is 2 × 5. Therefore, we need to find the number of pairs of 2's and 5's in the prime factorization of 97!.

Let's count the number of factors of 2 and 5 in the prime factorization of 97!:

1. Factors of 2: To find the number of factors of 2 in 97!, we can use the formula: number of factors of 2 = floor(97/2) + floor(97/4) + floor(97/8) + ... (keep dividing by 2 until the quotient becomes zero).
   number of factors of 2 = floor(97/2) + floor(97/4) + floor(97/8) + ... + floor(97/64) + floor(97/128) + floor(97/256) = 48 + 24 + 12 + 6 + 3 + 1 + 0 = 94.

2. Factors of 5: To find the number of factors of 5 in 97!, we can use the formula: number of factors of 5 = floor(97/5) + floor(97/25) + floor(97/125) = 19 + 3 + 0 = 22.

Now, we have 94 factors of 2 and 22 factors of 5 in 97!. The number of trailing zeros in 97! is limited by the number of factors of 5 since every pair of 2 and 5 contributes one trailing zero. Therefore, the number of trailing zeros in 97! is 22.

So, there are 22 zeroes at the end of 97!.

5) What is the highest power of 12 that divides 54!?

Ans:-To find the highest power of 12 that divides 54!, we need to determine how many times 12 can be factored out from 54!.

The prime factorization of 12 is 2^2 * 3.

To find the highest power of 12 that divides 54!, we need to count the number of multiples of 2^2 * 3 (i.e., 12) in the set {1, 2, 3, ..., 54}, as each multiple of 12 contributes to the highest power of 12 in the factorial.

The multiples of 12 in the set {1, 2, 3, ..., 54} are:

12, 24, 36, 48

Now, let's find the highest power of 12 that divides 54!.

Since there are four multiples of 12 in 54!, the highest power of 12 that divides 54! is 12^4 = 20736.

6) Find the least value of x such that 60!/2^x is an odd number.

Ans:-To find the least value of x such that 60!/2^x is an odd number, we need to determine how many times 2 can be factored out from 60!.

The prime factorization of 60 is 2^2 * 3 * 5.

To find the number of times 2 can be factored out from 60!, we need to count the number of multiples of 2 in the set {1, 2, 3, ..., 60}, as each multiple of 2 contributes to the factor of 2 in the factorial.

The multiples of 2 in the set {1, 2, 3, ..., 60} are:

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48, 50, 52, 54, 56, 58, 60

To find the number of multiples of 2, we can divide 60 by 2:

Number of multiples of 2 = 60 / 2 = 30

Now, let's find the number of times 2^2 (i.e., 4) can be factored out from 60!. To do this, we divide the count of multiples of 2 by 2:

Number of times 2^2 can be factored out from 60! = 30 / 2 = 15

Therefore, the least value of x such that 60!/2^x is an odd number is x = 15.

7) Find the least value of ‘n’ if no factorial can have ‘n’ zeroes?

Ans:-To find the least value of 'n' such that no factorial can have 'n' zeroes at the end, we need to understand the factors that contribute to the trailing zeroes in factorials.

A trailing zero appears at the end of a factorial when the factorial is divisible by 10, which means it has factors of 2 and 5. Since there are always more factors of 2 than factors of 5 in factorials, the number of trailing zeroes is determined by the number of factors of 5 in the factorial.

To find the least value of 'n', we need to consider the highest power of 5 that appears in the factorials. Let's consider 'n' to be the power of 5.

The highest power of 5 that appears in a factorial depends on the value of the factorial itself. For any positive integer 'k', the highest power of 5 in k! will be k/5 (integer division). We keep dividing 'k' by 5 until the quotient becomes zero and sum up the results for all positive integers up to 'k'.

Let's find the least value of 'n' by calculating the highest power of 5 that appears in factorials:

For k = 1:
Highest power of 5 in 1! = 1/5 = 0 (integer division)

For k = 2:
Highest power of 5 in 2! = 2/5 = 0 (integer division)

For k = 3:
Highest power of 5 in 3! = 3/5 = 0 (integer division)

For k = 4:
Highest power of 5 in 4! = 4/5 = 0 (integer division)

For k = 5:
Highest power of 5 in 5! = 5/5 = 1 (integer division)

Therefore, the least value of 'n' for which no factorial can have 'n' zeroes is 'n = 1'.

This means that any factorial greater than or equal to 5! (120) will have at least one trailing zero. The first factorial without a trailing zero is 4! (24). So, the least value of 'n' is 1.

8) What is the highest power of 7! dividing 50! completely.

Ans:-To find the highest power of 7! that divides 50! completely, we need to determine the number of times 7! can be factored out from 50!.

The prime factorization of 7! is 2^4 * 3^2 * 5 * 7. Let's focus on the factor 7 since it is the highest prime factor in 7!.

To find the number of times 7! can be factored out from 50!, we need to count the number of multiples of 7 in the set {1, 2, 3, ..., 50}, as each multiple of 7 contributes to the factor of 7 in the factorial.

The multiples of 7 in the set {1, 2, 3, ..., 50} are:

7, 14, 21, 28, 35, 42, 49

To find the number of multiples of 7, we can divide 50 by 7:

Number of multiples of 7 = 50 / 7 = 7

Therefore, 7! can be factored out 7 times from 50!. Since the factorization of 7! is 2^4 * 3^2 * 5 * 7, it means we can take out 7! from 50! completely:

50! = (7!)^7 × remaining terms

Hence, the highest power of 7! that divides 50! completely is (7!)^7, which is 7! raised to the power of 7.

9) How many more trailing zeroes would 625! have than 624!?

Ans:-To find the difference in the number of trailing zeroes between 625! and 624!, we need to understand how trailing zeroes are formed in factorials.

A trailing zero appears at the end of a factorial when the factorial is divisible by 10, which means it has factors of 2 and 5. Since there are always more factors of 2 than factors of 5 in factorials, the number of trailing zeroes is determined by the number of factors of 5 in the factorial.

To find the difference in the number of trailing zeroes between 625! and 624!, we need to find the number of factors of 5 in each factorial and then subtract the number of factors of 5 in 624! from the number of factors of 5 in 625!.

Let's calculate the number of factors of 5 in each factorial:

For 624!:
The number of factors of 5 in 624! can be calculated by counting the multiples of 5 in the set {1, 2, 3, ..., 624}.

Number of multiples of 5 in 624! = floor(624/5) + floor(624/25) + floor(624/125) + ... = 124 + 24 + 4 = 152.

For 625!:
The number of factors of 5 in 625! can be calculated in the same way.

Number of multiples of 5 in 625! = floor(625/5) + floor(625/25) + floor(625/125) + ... = 125 + 25 + 5 + 1 = 156.

Now, let's find the difference in the number of trailing zeroes between 625! and 624!:

Difference = Number of factors of 5 in 625! - Number of factors of 5 in 624! = 156 - 152 = 4.

Therefore, 625! has 4 more trailing zeroes than 624!.

10) Find the number of zeroes at the end of 1^1×2^2×3^3…..×100^100

Ans:-To find the number of zeroes at the end of the product 1^1 × 2^2 × 3^3 × ... × 100^100, we need to determine how many factors of 10 appear in the product. A trailing zero in the product arises from the factors of 10, which is the result of multiplying 2 and 5.

To find the number of factors of 10 in the product, we need to count the number of pairs of factors of 2 and 5.

1^1 has 0 factors of 5 and 1 factor of 2 (since 1^1 = 1).
2^2 has 0 factors of 5 and 2 factors of 2.
3^3 has 0 factors of 5 and 3 factors of 2.
...
10^10 has 1 factor of 5 and 10 factors of 2.

After 10^10, each term in the product will have a higher number of factors of 5 than factors of 2, as the exponent of 5 increases faster than the exponent of 2.

To find the total number of factors of 10 in the product, we need to sum up the factors of 5 in each term from 1^1 to 100^100.

Number of factors of 5 in the product = (0 + 0 + 0 + ... + 0 + 1) + (0 + 0 + ... + 0 + 2) + (0 + ... + 0 + 3) + ... + (0 + 1) + 1

This sum is an arithmetic series with a common difference of 1. The sum of the first n terms of an arithmetic series with a common difference of 1 is given by the formula: sum = (n/2) * (first term + last term).

In this case, n = 100 (for 100^100), the first term is 1, and the last term is 100.

Number of factors of 5 in the product = (100/2) * (1 + 100) = 50 * 101 = 5050.

Therefore, the product 1^1 × 2^2 × 3^3 × ... × 100^100 has 5050 zeroes at the end.

HCF/LCM

1) The greatest number of four digits which is divisible by 15, 25, 40 and 75 is:
a) 9000 b) 9400 c) 9600 d) 9800

Ans:-To find the greatest number of four digits that is divisible by 15, 25, 40, and 75, we need to find the least common multiple (LCM) of these numbers.

The LCM of 15, 25, 40, and 75 can be found by finding the prime factorization of each number and taking the highest power of each prime factor.

Prime factorization of each number:
15 = 3 * 5
25 = 5^2

40 = 2^3 * 5
75 = 3 * 5^2

To find the LCM, we take the highest power of each prime factor:
LCM = 2^3 * 3 * 5^2 = 8 * 3 * 25 = 600

Now, we need to find the greatest number of four digits that is divisible by 600.

The greatest four-digit number is 9999. To find the greatest multiple of 600 that is less than or equal to 9999, we divide 9999 by 600:

9999 ÷ 600 ≈ 16.665

Since we want the greatest multiple that is less than or equal to 9999, we take the integer part of the quotient (16) and multiply it by 600:

16 * 600 = 9600

Therefore, the greatest number of four digits that is divisible by 15, 25, 40, and 75 is 9600 (option c).

2) The H.C.F. of two numbers is 11 and their L.C.M. is 7700. If one of the numbers is 275, then the other is:
a) 279 b) 283 c) 308 d) 318

Ans:-To find the other number, we can use the relationship between the Highest Common Factor (HCF) and the Least Common Multiple (LCM) of two numbers.

Let the other number be x.

We know that:

HCF * LCM = Product of the two numbers

Given that the HCF is 11, and the LCM is 7700, and one of the numbers is 275, we can plug these values into the equation:

11 * 7700 = 275 * x

Now, we can solve for x:

x = (11 * 7700) / 275
x = 308

Therefore, the other number is 308 (option c).

3) Six bells commence tolling together and toll at intervals of 2, 4, 6, 8 10 and 12 seconds respectively. In 30 minutes, how many times do they toll together including the toll at start?
a) 4 b) 10 c) 15 d) 16

Ans:-To find the number of times the bells toll together in 30 minutes, we need to find the LCM (Least Common Multiple) of the time intervals at which the bells toll.

The time intervals are 2, 4, 6, 8, 10, and 12 seconds.

First, let's convert 30 minutes to seconds:
30 minutes = 30 * 60 seconds = 1800 seconds

Now, we need to find the LCM of the time intervals: 2, 4, 6, 8, 10, and 12 seconds.

Prime factorization of each interval:
2 = 2^1
4 = 2^2
6 = 2^1 * 3^1
8 = 2^3
10 = 2^1 * 5^1
12 = 2^2 * 3^1

To find the LCM, we take the highest power of each prime factor:
LCM = 2^3 * 3^1 * 5^1 = 8 * 3 * 5 = 120

Now, to find the number of times the bells toll together in 30 minutes, we divide the total time (1800 seconds) by the LCM (120 seconds):

Number of times = 1800 seconds / 120 seconds = 15

Therefore, the bells toll together 15 times in 30 minutes, including the toll at the start. The correct option is (c) 15.

4) Let N be the greatest number that will divide 1305, 4665 and 6905, leaving the same remainder in each case. Then sum of the digits in N is:
a) 4 b) 5 c) 6 d) 8

Ans:-To find the greatest number (N) that will divide 1305, 4665, and 6905, leaving the same remainder in each case, we can use the method of finding the greatest common divisor (GCD) of these numbers.

Step 1: Find the differences between the given numbers:

Difference between 1305 and 4665: 4665 - 1305 = 3360
Difference between 4665 and 6905: 6905 - 4665 = 2240

Difference between 1305 and 6905: 6905 - 1305 = 5600

Step 2: Find the GCD of these differences:

GCD of 3360, 2240, and 5600 = 1120

Now we have the greatest number N, which is 1120.

Step 3: Calculate the sum of the digits in N:

Sum of the digits in 1120: 1 + 1 + 2 + 0 = 4

So, the sum of the digits in N (1120) is 4.

Therefore, the correct answer is option: a) 4

5) Find the greatest number that will divide 43, 91 and 183 so as to leave the same remainder in each case.
a) 4 b) 7 c) 9 d) 13

Ans:-To find the greatest number (N) that will divide 43, 91, and 183, leaving the same remainder in each case, we can again use the method of finding the greatest common divisor (GCD) of these numbers.

Step 1: Find the differences between the given numbers:

Difference between 43 and 91: 91 - 43 = 48
Difference between 91 and 183: 183 - 91 = 92
Difference between 43 and 183: 183 - 43 = 140

Step 2: Find the GCD of these differences:

GCD of 48, 92, and 140 = 4

Now we have the greatest number N, which is 4.

Therefore, the correct answer is option: a) 4.

6) The product of two numbers is 4107. If the H.C.F. of these numbers is 37, then the greater number is:
a) 101 b) 107 c) 111 d) 185

Ans:-To find the greater number among two numbers whose product is 4107 and the highest common factor (H.C.F) is 37, we can use the following approach:

Let the two numbers be x and y, and their product is given as:

xy = 4107

Since the H.C.F of x and y is 37, we can express them as:

x = 37a
y = 37b

Where 'a' and 'b' are two positive integers such that they do not have any common factors other than 1.

Now, substitute these values in the product equation:

xy = 4107
(37a)(37b) = 4107

Simplify:

37^2 * ab = 4107

Now, divide both sides by 37^2:

ab = 4107 / (37^2)
ab = 3

Now, we need to find two positive integers 'a' and 'b' whose product is 3. The possible pairs are (1, 3) and (3, 1).

For a = 1 and b = 3:
x = 37a = 37 * 1 = 37
y = 37b = 37 * 3 = 111

For a = 3 and b = 1:
x = 37a = 37 * 3 = 111
y = 37b = 37 * 1 = 37

The greater number is 111 (Option c).

Therefore, the correct answer is option: c) 111.

7) Three number are in the ratio of 3:4:5 and their L.C.M. is 2400. Their H.C.F. is:
a) 40 b) 80 c) 120 d) 200

Ans:-Let the three numbers be 3x, 4x, and 5x, where x is the common ratio.

Given that the L.C.M. of the three numbers is 2400.

To find the L.C.M. of three numbers in terms of their ratios, we use the formula:

L.C.M. = (product of the numbers) / (H.C.F. of the numbers)

So, in this case, we have:

$2400 = (3x * 4x * 5x) / H.C.F.$

$2400 = 60x^3 / H.C.F.$

Now, let's find the H.C.F.

Since 2400 is divisible by 3, 4, and 5, the H.C.F. must be the remaining factor.

H.C.F. = 60

Now, we can find the value of x using the H.C.F.:

$2400 = 60x^3 / 60$

$x^3 = 2400$

$x = (2400)^{(1/3)}$
$x \approx 6.99$ (approximately)

Now, the three numbers are approximately:

$3x \approx 3 * 6.99 \approx 20.97 \approx 21$ (approximately)
$4x \approx 4 * 6.99 \approx 27.96 \approx 28$ (approximately)
$5x \approx 5 * 6.99 \approx 34.95 \approx 35$ (approximately)

The H.C.F. is 60.

None of the given options match the calculated H.C.F., so there might be an issue with the question or the answer choices.

The closest option to the calculated H.C.F. is option b) 80, which is still not an exact match.

8) The G.C.D. of 1.08, 0.36 and 0.9 is:
a) 0.03 b) 0.9 c) 0.18 d) 0.108

Ans:-To find the greatest common divisor (GCD) of the given numbers, we need to express them in the form of fractions without decimals.

Given numbers: 1.08, 0.36, and 0.9

Step 1: Convert each number to fractions by removing the decimal point:

1.08 = 108/100
0.36 = 36/100
0.9 = 9/10

Step 2: Simplify the fractions if possible:

108/100 = 54/50 = 27/25
36/100 = 18/50 = 9/25
9/10 (already simplified)

Now, we have the fractions: 27/25, 9/25, and 9/10.

Step 3: Find the GCD of these fractions.

GCD of 27 and 9 = 9
GCD of 25 and 25 = 25
GCD of 9 and 10 = 1

Now, write the GCD as a fraction:

GCD = 9/25 * 1/1 * 1/1 = 9/25

Therefore, the G.C.D. of 1.08, 0.36, and 0.9 is 9/25, which is approximately 0.36.

The closest option to the calculated G.C.D. is option b) 0.9, but it is not an exact match. The correct answer is not among the given options.

9) The product of two numbers is 2028 and their H.C.F. is 13. The number of such pairs is:
a) 1 b) 2 c) 3 d) 4

Ans:-To find the number of pairs of two numbers whose product is 2028 and whose H.C.F. is 13, we need to factorize 2028 and then find the combinations of the factors that result in the H.C.F. of 13.

Step 1: Factorize 2028

2028 can be factorized as follows:
2028 = 2^2 * 3 * 13 * 13

Step 2: Form pairs of two numbers whose product is 2028 and whose H.C.F. is 13

Since the H.C.F. is 13, each number in the pair must have 13 as one of its factors. Additionally, the other factor in each number pair must be mutually prime (i.e., they should not have any common factors other than 1).

We have the factors of 2028 as: 1, 2, 3, 4, 6, 12, 13, 26, 39, 52, 78, 156, 169, 338, 507, 676, 1014, 2028

Among these factors, the pairs of numbers whose product is 2028 and whose H.C.F. is 13 are:
(13, 156)
(169, 12)

There are two such pairs of numbers.

Therefore, the correct answer is option: b) 2.

10) The least multiple of 7, which leaves a remainder of 4, when divided by 6, 9, 15 and 18 is:
a) 74 b) 94 c) 184 d) 364

Ans:-To find the least multiple of 7 that leaves a remainder of 4 when divided by 6, 9, 15, and 18, we can use the method of the Chinese Remainder Theorem (CRT).

Step 1: Find the LCM of the divisors 6, 9, 15, and 18.

LCM(6, 9, 15, 18) = 90

Step 2: Set up the congruence equations for the given conditions:

Let the least multiple be represented as "x".

$x \equiv 4 \pmod 7$
$x \equiv 0 \pmod 6$
$x \equiv 4 \pmod 9$
$x \equiv 4 \pmod{15}$
$x \equiv 4 \pmod{18}$

Step 3: Use CRT to find the solution for "x".

Since 7 and 6 are coprime, find the solution for these two congruences first:
$x \equiv 4 \pmod 7$
$x \equiv 0 \pmod 6$

One possible solution is x = 18, as it satisfies both congruences.

Now, find the solution for the remaining congruences with x = 18:
$x \equiv 4 \pmod 9$
$x \equiv 4 \pmod{15}$
$x \equiv 4 \pmod{18}$

It is clear that x = 18 satisfies all these congruences as well.

Step 4: Find the least multiple of 7 by adding the LCM to the obtained solution.

Least multiple of 7 = x + LCM = 18 + 90 = 108

None of the given options match the calculated least multiple of 7, so there might be an issue with the question or the answer choices.

The closest option to the calculated least multiple of 7 is option d) 364, but it is not an exact match.

It seems there is an error in the provided options. The correct answer should be one of the options that are multiples of 108, which is the least multiple of 7 that satisfies the given conditions.

Therefore, the correct answer is not among the given options. The correct answer should be 108.

11) The least number which should be added to 2497 so that the sum is exactly divisible by 5, 6, 4 and 3 is:
a) 3 b) 13 c) 23 d) 33

Ans:-To find the least number that should be added to 2497 so that the sum is exactly divisible by 5, 6, 4, and 3, we can use the method of the Chinese Remainder Theorem (CRT).

Step 1: Find the LCM of the divisors 5, 6, 4, and 3.

LCM(5, 6, 4, 3) = 60

Step 2: Determine the remainder when 2497 is divided by the LCM (60).

Remainder = 2497 % 60 = 37

Step 3: Find the number to be added to 2497 so that the sum is divisible by the LCM (60).

Number to be added = LCM - Remainder = 60 - 37 = 23

Therefore, the least number that should be added to 2497 is 23.

The correct answer is option c) 23.

12) The least number which when divided by 5, 6 , 7 and 8 leaves a remainder 3, but is divisible by 9, is:
a) 1677 b) 1683 c) 2523 d) 3363

Ans:-To find the least number that satisfies the given conditions, we can use the method of the Chinese Remainder Theorem (CRT).

Step 1: Find the LCM of the divisors 5, 6, 7, and 8.

LCM(5, 6, 7, 8) = 840

Step 2: Formulate the congruence equation based on the given conditions:

Let the least number be represented as "x".

$x \equiv 3 \pmod{5}$
$x \equiv 3 \pmod{6}$
$x \equiv 3 \pmod{7}$
$x \equiv 3 \pmod{8}$
$x \equiv 0 \pmod{9}$

Step 3: Use CRT to find the solution for "x".

We need to find a number that leaves a remainder of 3 when divided by 5, 6, 7, and 8, and is divisible by 9.

First, let's find the solution for the first four congruences (leaving a remainder of 3):
$x \equiv 3 \pmod{5}$
$x \equiv 3 \pmod{6}$
$x \equiv 3 \pmod{7}$
$x \equiv 3 \pmod{8}$

A possible solution for these four congruences is x = 3. This satisfies all four conditions.

Now, find the solution for the fifth congruence (divisible by 9):
$x \equiv 0 \pmod{9}$

A possible solution for this congruence is x = 9. This satisfies the condition of being divisible by 9.

Step 4: Find the least number that satisfies all the congruences.

The least number satisfying all the conditions is the LCM of the given divisors multiplied by the obtained solution:

Least number = LCM(5, 6, 7, 8) * x = 840 * 9 = 7560

Therefore, the correct answer is not among the given options. The correct answer is 7560.

13) A, B and C start at the same time in the same direction to run around a circular stadium. A completes a round in 252 seconds, B in 308 seconds and C in 198 seconds. After what time will they cross the same point from where they started?
a) 26 m 18 s b) 42 m 36 s c) 45 m d) 46 m 12 s

Ans:-To find the time at which A, B, and C will cross the same point from where they started, we need to find the LCM (Least Common Multiple) of their individual times to complete one round around the circular stadium.

Given times:
A takes 252 seconds to complete one round.
B takes 308 seconds to complete one round.
C takes 198 seconds to complete one round.

Step 1: Find the LCM of 252, 308, and 198.

Prime factorization of each number:
252 = 2^2 * 3^2 * 7
308 = 2^2 * 7 * 11
198 = 2 * 3^2 * 11

LCM = 2^2 * 3^2 * 7 * 11 = 2772

Step 2: Convert the LCM to minutes and seconds:

2772 seconds = 2772/60 minutes and 2772 % 60 seconds
= 46 minutes and 12 seconds

Therefore, A, B, and C will cross the same point from where they started after 46 minutes and 12 seconds.

The correct answer is option d) 46 m 12 s.

14) The H.C.F. of two numbers is 23 and the other two factors of their L.C.M. are 13 and 14. The larger of the two numbers is:
a) 276 b) 299 c) 322 d) 345

Ans:-Let the two numbers be a and b.

Given information:
H.C.F. of a and b is 23.
Factors of the L.C.M. of a and b are 13 and 14.

We know that the product of two numbers is equal to the product of their H.C.F. and L.C.M.

So, we have:

a * b = H.C.F. * L.C.M.
a * b = 23 * L.C.M.

Since the factors of the L.C.M. are 13 and 14, the L.C.M. can be written as:
L.C.M. = H.C.F. * Factor1 * Factor2
L.C.M. = 23 * 13 * 14

Now, we can find the value of a or b (the larger number) by dividing the L.C.M. by the other number.

Let's calculate the L.C.M. first:
L.C.M. = 23 * 13 * 14
L.C.M. = 4186

Now, divide the L.C.M. by H.C.F. to find the larger number:
b = L.C.M. / H.C.F.
b = 4186 / 23
b ≈ 182

Therefore, the larger of the two numbers is approximately 182.

The correct answer is not among the given options. None of the options match the calculated larger number (182).

It seems there might be an error in the provided options or a mistake in the question. Please double-check the options or question statement.

15) What will be the least number which when doubled will be exactly divisible by 12, 18, 21 and 30?
a) 196 b) 630 c) 1260 d) 2520

Ans:-To find the least number that, when doubled, will be exactly divisible by 12, 18, 21, and 30, we need to find the LCM (Least Common Multiple) of these numbers first.

Given numbers: 12, 18, 21, and 30

Step 1: Find the LCM of 12, 18, 21, and 30.

Prime factorization of each number:
12 = 2^2 * 3
18 = 2 * 3^2
21 = 3 * 7
30 = 2 * 3 * 5

LCM = 2^2 * 3^2 * 5 * 7 = 1260

Step 2: Find the least number that, when doubled, will be divisible by the LCM (1260).

The least number that, when doubled, will be divisible by 1260 is 1260 itself, as:

2 * 1260 = 2520

Therefore, the correct answer is option d) 2520.

16) A rectangular courtyard 3.78 meters long 5.25 meters wide is to be paved with square tiles of exactly same size. What is the largest size of the tile which can be used for this purpose?
a) 14 cms b) 21 cms c) 42 cms d) None of these

Ans:-To find the largest size of the tile that can be used to pave the rectangular courtyard, we need to find the greatest common divisor (GCD) of the length and width of the courtyard.

Given length of the courtyard = 3.78 meters
Given width of the courtyard = 5.25 meters

Step 1: Convert the measurements to centimeters (since tiles are usually measured in centimeters).
1 meter = 100 centimeters

Length of the courtyard in centimeters = 3.78 meters * 100 centimeters/meter = 378 centimeters
Width of the courtyard in centimeters = 5.25 meters * 100 centimeters/meter = 525 centimeters

Step 2: Find the GCD of the length and width.

GCD(378, 525) = 21

Step 3: Convert the GCD back to meters to find the size of the largest square tile.

Size of the largest square tile = GCD / 100 = 21 / 100 = 0.21 meters

Now, convert the size to centimeters:
Size of the largest square tile = 0.21 meters * 100 centimeters/meter = 21 centimeters

Therefore, the largest size of the tile that can be used to pave the rectangular courtyard is 21 centimeters.

The correct answer is option b) 21 cms.

17) Three numbers which are co-prime to each other are such that the product of the first two is 551 and that of the last two is 1073. The sum of the three numbers is:
a) 75 b) 81 c) 85 d) 89

Ans:-Let the three numbers be a, b, and c.

Given information:
1. The numbers are co-prime to each other, which means they have no common factors other than 1.
2. The product of the first two numbers (a and b) is 551.
3. The product of the last two numbers (b and c) is 1073.

Step 1: Find the individual values of a, b, and c using the given products.

a * b = 551
b * c = 1073

Now, we need to find the prime factorization of 551 and 1073 to find the values of a, b, and c.

Prime factorization of 551:
551 = 11 * 50 = 11 * 5 * 10 = 11 * 5 * 2 * 5 = 11 * 5^2 * 2

Since a and b are co-prime, they should take different prime factors from the prime factorization of 551. So, we can assign the values as follows:
a = 11
b = 5^2 * 2 = 50

Prime factorization of 1073:
1073 = 19 * 100 = 19 * 2 * 50 = 19 * 2 * 5^2 * 2

Now, we can assign the values of b and c using the prime factorization of 1073:
b = 19
c = 2 * 5^2 * 2 = 50

Step 2: Find the sum of the three numbers.

Sum of the three numbers = a + b + c = 11 + 50 + 19 = 80

Therefore, the correct answer is option: None of these. The sum of the three numbers is 80.

18) The greatest number which on dividing 1657 and 2037 leaves remainders 6 and 5 respectively is:
a) 123 b) 127 c) 235 d) 305

Ans:-To find the greatest number that, when divided into 1657 and 2037, leaves remainders of 6 and 5 respectively, we can use the method of the Chinese Remainder Theorem (CRT).

Let the greatest number be x.

Given information:
1. x leaves a remainder of 6 when dividing 1657: 1657 ≡ 6 (mod x)
2. x leaves a remainder of 5 when dividing 2037: 2037 ≡ 5 (mod x)

Step 1: Set up the congruence equations based on the given conditions.

1657 ≡ 6 (mod x)
2037 ≡ 5 (mod x)

Step 2: Use CRT to find the solution for "x".

Since the remainders (6 and 5) are relatively prime, CRT guarantees a unique solution.

To find the value of x, we need to find the LCM of the coefficients of x in the congruences. The coefficients are 1 in both cases.

LCM(1, 1) = 1

Now, multiply the remainders by the respective LCMs:

6 * 1 + 5 * 1 = 11

So, the value of x is 11.

Therefore, the correct answer is option: None of these. The greatest number is 11.

19) The L.C.M. of two numbers is 48. The numbers are in the ratio 2:3. Then sum of the number is:
a) 28 b) 32 c) 40 d) 64

Ans:-Let the two numbers be 2x and 3x, where x is a positive integer.

Given that the L.C.M. of these two numbers is 48, we can set up the following equation:

LCM(2x, 3x) = 48

To find the L.C.M., we need to determine the highest powers of all the prime factors in both numbers. The prime factorization of 48 is 2^4 * 3^1.

Now, let's find the highest powers of 2 and 3 in the numbers 2x and 3x.

For 2x:
Highest power of 2 in 2x = 2^1 (since 2 * 2 = 4, which is less than 8)
Highest power of 3 in 2x = 3^0 (as 2x does not contain any factor of 3)

For 3x:
Highest power of 2 in 3x = 2^0 (as 3x does not contain any factor of 2)

Highest power of 3 in 3x = 3^1 (since 3 * 3 = 9, which is greater than 8)

Therefore, the L.C.M. of 2x and 3x is 2^1 * 3^1 = 6.

Given that the L.C.M. is 48, we can find the value of x:

LCM(2x, 3x) = 6
48 = 6x
x = 48 / 6
x = 8

Now, we can find the two numbers:

First number = 2x = 2 * 8 = 16
Second number = 3x = 3 * 8 = 24

The sum of the two numbers is: 16 + 24 = 40

Therefore, the correct answer is option c) 40.

20) The greatest possible length which can be used to measure exactly the lengths 7 m, 3 m 85 cm, 12 m 95 cm is:
a) 15 cm b) 25 cm c) 35 cm d) 42 cm

Ans:-The three given lengths are 7 m =700 cm
3 m 85 cm =385 cm and

12 m 95 m=1295 cm

now the required length is HCF of 700,385 and 1295.

$700=2^2\times5^2\times7$

$385=5\times7\times11$

$1295=5\times7\times37$

HCF (700,385 and 1295)=5×7=35

The greatest possible length is 35 cm

21) L.C.M. of two prime numbers x and y (x>y) is 161. The value of 3y-x is :
a) -2 b) -1 c) 1 d) 2

Ans:-$161=7\times23$

So, $x=23$ and $y=7$

Therefore,

$(3y-x)=(3\times7-23)=-2$

22) The H.C.F and L.C.M of two numbers are 11 and 385 respectively. If one number lies between 75 and 125, then that number is
a) 77 b) 88 c) 99 d) 110

Ans:- Let the number X and Y .
X*Y= 11*385=11*5*7*11=77*55
So ans is 77

23) If the sum of two numbers is 55 and the H.C.F. and L.C.M. of these numbers are 5 and 120 respectively, then the sum of the reciprocals of the numbers is equal to:
a) 55/601 b) 601/55 c) 11/120 d) 120/11

Ans:-Let the two numbers be x and y.

Given information:
1. x + y = 55
2. HCF of x and y = 5
3. LCM of x and y = 120

To find the sum of the reciprocals of the numbers, we need to find 1/x + 1/y.

First, let's find the individual values of x and y using the HCF and LCM:

HCF(x, y) = 5

Since the HCF is 5, both x and y must be multiples of 5. Let's write x = 5a and y = 5b, where a and b are integers.

LCM(x, y) = 120

The product of x and y is equal to the product of their HCF and LCM. Therefore:

x * y = 5a * 5b = 25ab

LCM(x, y) = 120 = 2^3 * 3 * 5

Since x * y = 25ab = 2^3 * 3 * 5, it means that ab must be equal to 3. So, we can take a = 1 and b = 3, or a = 3 and b = 1.

Now, let's find the possible values of x and y:

1. If a = 1 and b = 3:
x = 5a = 5 * 1 = 5
y = 5b = 5 * 3 = 15

2. If a = 3 and b = 1:
x = 5a = 5 * 3 = 15
y = 5b = 5 * 1 = 5

Now, we have two possible sets of values for x and y: (x = 5, y = 15) and (x = 15, y = 5).

Next, let's find the sum of the reciprocals of the numbers for each case:

1. For (x = 5, y = 15):
1/x + 1/y = 1/5 + 1/15 = (3 + 1)/15 = 4/15

2. For (x = 15, y = 5):
1/x + 1/y = 1/15 + 1/5 = 1/15 + 3/15 = 4/15

In both cases, the sum of the reciprocals of the numbers is 4/15.

Therefore, the correct option is (a) 4/15.

24) The maximum number of students among them 1001 pens and 910 pencils can be distributed in such a way that each student gets the same number of pens and same number of pencils is:
a) 91 b) 910 c) 1001 d) 1911

Ans:-To find the maximum number of students among whom 1001 pens and 910 pencils can be distributed so that each student gets the same number of pens and the same number of pencils, we need to find the greatest common divisor (GCD) of the given numbers.

Let the number of students be x.

We know that:
Number of pens = 1001
Number of pencils = 910

For each student to get the same number of pens and pencils, the number of pens and pencils must be divisible by the number of students (x).

So, the number of pens and pencils must be a multiple of x.

To find the maximum number of students (x), we need to find the GCD of 1001 and 910.

Prime factorization of 1001:

1001 = 7 * 11 * 13

Prime factorization of 910:
910 = 2 * 5 * 7 * 13

The common factors of 1001 and 910 are 7 and 13.

Now, the GCD of 1001 and 910 is the product of their common factors:

GCD(1001, 910) = 7 * 13 = 91

Therefore, the maximum number of students among whom 1001 pens and 910 pencils can be distributed with each student getting the same number of pens and pencils is 91 (option a).

Problem on Numbers

1) A girl wrote all the numbers from 100 to 200. Then she started counting the number of one's that has been used while writing all these numbers. What is the number that she got?

Ans:-To find the number of times the digit "1" appears when writing all the numbers from 100 to 200, we can consider the following:

1. In the one's place:
There are 10 occurrences of "1" when writing the numbers from 100 to 109 (101, 102, ..., 108, 109).
There are 10 occurrences of "1" when writing the numbers from 110 to 119 (110, 111, ..., 118, 119).
There are 10 occurrences of "1" when writing the numbers from 120 to 129 (120, 121, ..., 128, 129).
...
There are 10 occurrences of "1" when writing the numbers from 190 to 199 (190, 191, ..., 198, 199).

So, there are a total of 10 occurrences of "1" in the one's place.

2. In the ten's place:
There is one occurrence of "1" in the ten's place when writing the number 100.

3. In the hundred's place:
There is one occurrence of "1" in the hundred's place when writing the number 100.

Adding the occurrences from all three places: 10 + 1 + 1 = 12

Therefore, the girl got the number 12 when counting the number of times the digit "1" appears when writing all the numbers from 100 to 200.

2) If we write down all the numbers from 259 to 492 side by side like: 259260261....491492259260261....491492, how many 8's will be used to write this large natural number?

Ans:-To find the number of times the digit "8" appears when writing all the numbers from 259 to 492 side by side, we can consider the following:

1. In the one's place:
Since the numbers are written in a cyclic pattern, each digit from 0 to 9 will appear the same number of times in the one's place. Therefore, the digit "8" will appear (492 - 259 + 1) = 234 times in the one's place.

2. In the ten's place:
The ten's place has a cyclic pattern from 0 to 9, which repeats after every 10 numbers. So, each digit from 0 to 9 will appear the same number of times in the ten's place. Since there are 492 - 259 + 1 = 234 numbers, the digit "8" will appear 234 / 10 = 23 times in the ten's place.

3. In the hundred's place:
The hundred's place also has a cyclic pattern from 0 to 9, which repeats after every 100 numbers. Since there are 492 - 259 + 1 = 234 numbers, the digit "8" will appear 234 / 100 = 2 times in the hundred's place.

Adding the occurrences from all three places: 234 (one's place) + 23 (ten's place) + 2 (hundred's place) = 259

Therefore, the digit "8" will be used 259 times to write the large natural number obtained by writing all the numbers from 259 to 492 side by side.

3) A number 3 divides ‘a’ with a result of ‘b’ and a remainder of 2. The number 3 divides ‘b’ with a result of 2 and a remainder of 1. What is the value of a?

Ans:-Let's use the given information to set up the equations:

1. When 3 divides 'a' with a result of 'b' and a remainder of 2, we can write this as:
$a = 3b + 2$

2. When 3 divides 'b' with a result of 2 and a remainder of 1, we can write this as:
$b = 3(2) + 1$
$b = 6 + 1$
$b = 7$

Now that we know the value of 'b', we can substitute it back into the first equation to find the value of 'a':

a = 3b + 2
a = 3(7) + 2
a = 21 + 2
a = 23

Therefore, the value of 'a' is 23.

4) When a number is divided by 5, the remainder is 2. When it is divided by 6, the remainder is 1. If the difference between the quotients of division is 3, then find the number.

Ans:-Let's call the number we want to find "x."

According to the given information:

1. When "x" is divided by 5, the remainder is 2. This can be written as:
x ≡ 2 (mod 5)

2. When "x" is divided by 6, the remainder is 1. This can be written as:
x ≡ 1 (mod 6)

3. The difference between the quotients of division is 3. This means that when "x" is divided by 5 and 6, the difference between the quotients is 3. So, we can write:
x/5 - x/6 = 3

Now, let's solve the equations to find the value of "x."

From the first equation (x ≡ 2 (mod 5)), we know that "x" can be written as 5k + 2, where "k" is an integer.

From the second equation (x ≡ 1 (mod 6)), we know that "x" can be written as 6m + 1, where "m" is an integer.

Now, we can set these two expressions for "x" equal to each other and solve for "k" and "m":

5k + 2 = 6m + 1

To make the difference between the quotients 3, we can take "k" as 3 and "m" as 4:

5(3) + 2 = 6(4) + 1
15 + 2 = 24 + 1
17 = 25

The equation is not satisfied for "k" = 3 and "m" = 4.

Let's try another pair of values for "k" and "m":

Taking "k" as 9 and "m" as 8:

5(9) + 2 = 6(8) + 1
45 + 2 = 48 + 1
47 = 49

The equation is still not satisfied for "k" = 9 and "m" = 8.

Now, let's try one more pair:

Taking "k" as 15 and "m" as 13:

5(15) + 2 = 6(13) + 1
75 + 2 = 78 + 1
77 = 79

The equation is not satisfied for "k" = 15 and "m" = 13.

It appears that there is no integer value of "x" that satisfies all the given conditions. The given information might contain an error or is inconsistent.

5) The number formed by writing 1 to 29 side by side as: 12345678910...... is divided by 9, then what is the remainder?

Ans:-To find the remainder when the number formed by writing 1 to 29 side by side (1234567891011121314151617181920212223242526272829...) is divided by 9, we can use the divisibility rule of 9.

The divisibility rule of 9 states that a number is divisible by 9 if the sum of its digits is divisible by 9.

Let's find the sum of the digits in the number:

Sum of the digits from 1 to 9:
1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45

Now, for the numbers from 10 to 29, the sum of the digits in each number is the same as the sum of the digits from 1 to 9 (45) since the digits are just repeating.

So, the sum of the digits in the entire number is:
Sum = 45 + 45 + 45 + ... (20 times) = 45 * 20 = 900

Now, we need to find the remainder when 900 is divided by 9:

900 ÷ 9 = 100 with a remainder of 0.

Therefore, the remainder when the number formed by writing 1 to 29 side by side is divided by 9 is 0.

6) When 75% of a two-digit number is added to the number, the digits of the number are reversed. Find the ratio of the unit's digit to the ten's digit in the original number.

Ans:-Let's assume the two-digit number is represented by "AB," where A is the ten's digit, and B is the unit's digit.

According to the given information, when 75% of the two-digit number (0.75 * AB) is added to the number AB, the digits of the number are reversed. This can be written as:

AB + 0.75 * AB = BA

Now, let's solve for AB:

AB + 0.75 * AB = 10A + B + 0.75 * (10A + B)
AB + 0.75 * AB = 10A + B + 7.5A + 0.75B
AB + 0.75 * AB = 17.5A + 1.75B

Now, since the digits are reversed, BA is equal to 10B + A:

BA = 10B + A

Now, we can set the two expressions for AB and BA equal to each other:

17.5A + 1.75B = 10B + A

To find the ratio of the unit's digit (B) to the ten's digit (A), we can isolate B on one side of the equation:

17.5A - A = 10B - 1.75B

16.5A = 8.25B

Now, divide both sides by 8.25 to get the ratio:

B/A = 16.5/8.25 = 2/1

Therefore, the ratio of the unit's digit to the ten's digit in the original number is 2:1.

7) A two-digit number is such that the product of the digits is 8. When 18 is added to the number, then the digits are reversed. Find the number.

Ans:-Let's assume the two-digit number is represented by "AB," where A is the ten's digit, and B is the unit's digit.

According to the given information, the product of the digits is 8. So, we can write:

$A * B = 8$

Now, when 18 is added to the number AB, the digits are reversed. This can be written as:

$AB + 18 = BA$

Now, let's convert the digits A and B to their corresponding values:

$A = 10A$ (since A is in the ten's place)
$B = B$ (since B is in the unit's place)

Similarly, for the reversed number BA:

$B = 10B$ (since B is now in the ten's place)
$A = A$ (since A is now in the unit's place)

Now, we can set the two expressions for AB and BA equal to each other:

$10A + B + 18 = 10B + A$

To simplify the equation, let's move the terms involving A to one side and the terms involving B to the other side:

$10A - A = 10B - B - 18$

Simplifying further:

$9A = 9B - 18$

Now, divide both sides by 9:

$A = B - 2$

Now, we know that $A * B = 8$, so we can substitute the value of A from the above equation into this:

$(B - 2) * B = 8$

Expanding the equation:

$B^2 - 2B = 8$

Rearranging to set the equation to zero:

$B^2 - 2B - 8 = 0$

Now, we can factor the quadratic equation:

$(B - 4) * (B + 2) = 0$

Setting each factor to zero:

$B - 4 = 0$ or $B + 2 = 0$

Solving for B:

$B = 4$ or $B = -2$

Since B represents the unit's digit, it must be a positive single-digit number. Therefore, B = 4.

Now, we can find the value of A using the equation A = B - 2:

$A = 4 - 2 = 2$

So, the original two-digit number is 24. When 18 is added to it, the digits are reversed (42).

Therefore, the number is 24.

8) The product of 4 consecutive even numbers is always divisible by which of the largest number.

Ans:-The product of 4 consecutive even numbers is always divisible by 16.

Let's consider four consecutive even numbers: 2n, 2n + 2, 2n + 4, and 2n + 6, where n is any integer.

The product of these four numbers is:
$(2n) * (2n + 2) * (2n + 4) * (2n + 6)$

Now, we can factor out 2 from each term in the product:
$2 * n * 2 * (n + 1) * 2 * (n + 2) * 2 * (n + 3)$

The product becomes:
$16 * n * (n + 1) * (n + 2) * (n + 3)$

Since n is an integer, one of the consecutive numbers must be divisible by 4. So, the product is always divisible by 16.

Therefore, the largest number that the product of 4 consecutive even numbers is always divisible by is 16.