1 Matrices

1. If for any 2×2 square matrix A, $A\,(\text{adj}A)= \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$, then write the value of $|A|$.

2. If A is a skew-symmetric matrix of order 3, then prove that $\det A = 0$.

3. Using properties of determinants, prove that

$$\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$$

4. Find matrix A such that $\begin{pmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{pmatrix} A = \begin{pmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{pmatrix}$.

5. Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of equations $x-y+z+4, x-2y-2z=9, 2x+y+3z=1$.

6. Let $A = \mathbb{Q} \times \mathbb{Q}$ and let $*$ be a binary operation on A defined by $(a,b) * (c,d) = (ac, b+ad)$ for $(a,b), (c,d) \in A$. Determine,whether $*$ is commutative and associative . Then, with respect to $*$ on A

   i Find the identity element in A .

   ii Find the invertible elements of A .

2 Algebra

1. if $\tan^{-1}\dfrac{x-3}{x-4} + \tan^{-1}\dfrac{x+3}{x+4} = \dfrac{\pi}{4}$, then find the value of x .

3 optimization

1. Of the students in a school, it is known that 30% have 100% attendance and 70% students are irregular . Previous year results report that 70% of all students who have 100% attendance attain A grade and 10% irregular students attain A grade in their annual examination . At the end of the year, one student is chosen at random from the school and he was found to have an A grade . What is the probability that the student has 100% attendance ? Is regularity required only in school ? Justify your answer .

2. The volume of a cube is increasing at the rate of $9\text{cm}^3/\text{s}$. How fast is its surface area increasing when the length of an edge is 10cm ? .

3. Two tailors, A and B, earn ₹ 300 and ₹ 400 per day respectively . A can stitch 6 shirts and 4 pairs of trousers while B can stitch 10 shirts and 4 pairs of trousers per day . To find how many days should each of them work and if it is desired to produce at least 60 shirts and 32 pairs of trousers at a minimum labour cost, formulate this as an LPP.

4. Maximise $Z = x + 2y$ subject to the constraints

$$\begin{aligned} x + 2y &\geq 100 \\ 2x - y &\leq 0 \\ 2x + y &\leq 200 \\ x, y &\geq 0 \end{aligned}$$

Solve the above LPP graphically .

4 Function and Relations

1. Determine the value of k for which the following function is continuous at $x = 3$:

$$f(x) = \begin{cases} \dfrac{(x+3)^2 - 36}{x-3}, & x \neq 3 \\ k, & x = 3 \end{cases}$$

2. Find the value of c in Rolle's theorem for the function $f(x) = x^3 - 3x$ in $\left[-\sqrt{3}, 0\right]$.

3. Consider $f : \mathbb{R} - \left\{-\dfrac{4}{3}\right\} \to R - \left\{\dfrac{4}{3}\right\}$ given by $f(x) = \dfrac{4x+3}{3x+4}$. Show that f is bijective . Find the inverse of f and hence find $f^{-1}(0)$ and x such that $f^{-1}(x) = 2$.

4. Show that the function $f(x) = x^3 - 3x^2 + 6x - 100$ is increasing on $\mathbb{R}$.

5 vectors

1. Show that the points A, B, C whith position vectors $2\hat{\imath} - \hat{\jmath} + \hat{k}$, $\hat{\imath} - 3\hat{\jmath} - 5\hat{k}$ and $3\hat{\imath} - 4\hat{\jmath} - 4\hat{k}$ respectively, are the vertices of a right-angled triangle. Hence find the area of the triangle.

2. Find the value of λ, if four points with position vectors $3\hat{\imath} + 6\hat{\jmath} + 9\hat{k}$, $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$, $2\hat{\imath} + 3\hat{\jmath} + \hat{k}$ and $4\hat{\imath} + 6\hat{\jmath} + \lambda\hat{k}$ are coplanar.

3. The x-coordinate of a point on the line joining the points $P(2, 2, 1)$and $Q(5, 1, –2)$ is 4. Find its z-coordinate.

4. Find the distance between the planes $2x-y+2z = 5$ and $5x-2.5y+5z = 20$.

5. Find the coordinates of the point where the line through the points $(3, -4, -5)$ and $(2, -3, 1)$, crosses the plane determined by the points $(1, 2, 3)$, $(4, 2, -3)$ and $(0, 4, 3)$.

6. A variable plane which remains at a constant distance $3p$ from the origin cuts the coordinate axes at A, B, C . Show that the locus of the centroid of triangle ABC is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}$.

6 Differentiation

1. If $x^y + y^x = a^b$, then find $\frac{dy}{dx}$.

2. If $e^y (x + 1) = 1$, then show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$.

3. Solve the differential equation $\left(\tan^{-1} x - y\right) dx = \left(1 + x^2\right) dy$.

4. Find the particular solution of the differential equation $(x-y) \frac{dx}{dy} = (x + 2y)$, given that $y = 0$ when $x = 1$.

7 Integration

1. Find: $\int \frac{\sin^2 x - \cos^2 x}{\sin x \cos x} dx$.

2. Find: $\int \frac{dx}{5 - 8x - x^2}$.

3. Find: $\int \frac{\cos \theta}{\left(4 + \sin^2 \theta\right) \left(5 - 4 \cos^2 \theta\right)} d\theta$.

4. Evaluate : $\int_0^\pi \frac{x \tan x}{\sec x + \tan x} dx$

5. Evaluate: $\int_1^4 \{|x - 1| + |x - 2| + |x - 4|\} \, dx$.

6. Using the method of integration, find the area of the triangle ABC, coordinates of whose vertices are $A (4, 1)$, $B (6, 6)\, and C (8, 4)$.

8 Probability

1. A die,whose faces are marked $1, 2, 3$ in red and $4, 5, 6$ in green, is tossed . Let A be the event “number obtained is even” and B be the event “number obtained is red” . Find if A and B are independent events.

2. There are 4 cards numbered $1, 3, 5$ and 7, one number on one card . Two cards are drawn at random without replacement . Let X denote the sum of the numbers on the two drawn cards . Find the mean and variance of X .

9 Mensurations

1. Show that the surface area of a closed cuboid with square base and given volume is minimum, when it is a cube .

2. Find the area enclosed between the parabola $4y = 3x^2$ and the straight line $3x–2y + 12 = 0$.