GeeksQuiz

Computer science mock tests for geeks

Binary Tree | Set 2 (Properties)

We have discussed Introduction to Binary Tree in set 1. In this post, properties of binary are discussed.

1) The maximum number of nodes at level 'l' of a binary tree is 2^{l-1}.

Here level is number of nodes on path from root to the node (including root and node). Level of root is 1.
This can be proved by induction.
For root, l = 1, number of nodes = 2^{1-1} = 1
Assume that maximum number of nodes on level l is 2^{l-1}
Since in Binary tree every node has at most 2 children, next level would have twice nodes, i.e. 2 * 2^{l-1}

2) Maximum number of nodes in a binary tree of height 'h' is 2^h – 1.

Here height of a tree is maximum number of nodes on root to leaf path. Height of a leaf node is considered as 1.
This result can be derived from point 2 above. A tree has maximum nodes if all levels have maximum nodes. So maximum number of nodes in a binary tree of height h is 1 + 2 + 4 + .. + 2^{h-1}. This is a simple geometric series with h terms and sum of this series is 2^h – 1.
In some books, height of a leaf is considered as 0. In this convention, the above formula becomes 2^{h+1} – 1

3) In a Binary Tree with N nodes, minimum possible height or minimum number of levels is $\lceil Log_2(N+1) \rceil$

This can be directly derived from point 2 above. If we consider the convention where height of a leaf node is considered as 0, then above formula for minimum possible height becomes $\lceil Log_2(N+1) \rceil - 1$

4) A Binary Tree with L leaves has at least $\lceil Log_2L \rceil + 1$ levels

A Binary tree has maximum number of leaves when all levels are fully filled. Let all leaves be at level l, then below is true for number of leaves L.

```
    L   <=  2^(l-1)  [From Point 1]
    Log2L <=  l-1
    l >=   ⌈ Log2L ⌉ + 1
```

5) In Binary tree, number of leaf nodes is always one more than nodes with two children.

```
    L = T + 1
 Where L = Number of leaf nodes
       T = Number of internal nodes with two children
```

See Handshaking Lemma and Tree for proof.

In the next article on tree series, we will be discussing different types of Binary Trees and their properties.

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Category: Data Structures

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BeautifulCode • 2 days ago

for #1 : Here level is number of nodes on path from root to the node (including root and node)

this definition is wrong .

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Garima Dhanania • 3 days ago

Proof of Property 5:

It is trivially true for n=1 as L=1 and T=0 so 1=0+1
It is also true for n=2 as L=1 and T=0 so 1=0+1
For n=3, either L=1 and T=0 when all keys are in a line, else L=2 and T=1 when the root has 2 children.
Assuming property to be true for all k<=n-1.
To prove it is also true for n.

If n(root) has only 1 child, its subtree will have n-1 elements. Total L=no of leaves in n-1 subtree and total T=no of nodes with 2 children in n-1 subtree(as the nth key(root) has only 1 child). So by hypothesis it holds for n keys.

If n has 2 children, say k in left subtree and n-k-1 in right subtree. Then by induction hypothesis L1=T1+1 and L2=T2+1. Total L=L1+L2
=T1+1+T2+1
=(T1+T2+1)+1
=T +1
where T is total number of nodes with 2 children which equals sum of such nodes in left subtree,right subtree and root(has 2 children) itself.
Hence Proved.

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Anup Rai • 9 days ago

For the property that number of leaves is one greater then number of nodes with two children we can also give this argument.

For each node with single child it will end with a leaf of a node with two child so we can shrink them all to make a tree with only nodes with two children or leafs without any loss in the number of leaves(as single child nodes are just extending the path).
Now for every node with two children we can see that it adds two extra leaves and remove one of the leaf of its parent(by having changing itself from a leaf to a parent of two)
.
Now each node with does this it adds two leaves and remove one.
Its just the root which adds two and remove non as it has no parent.
So for each node other then root no. of leaves = 1
1*(n-1)=n-1
for root leaves =2
n-1 +2 =n+1
hence proved.

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Rajan Kalra • 10 days ago

Max number of nodes shown in property 2 holds for internal nodes only and not the total no of nodes in a binary tree. Total no of nodes in a binary tree is 2^h+1 - 1. Correct me if I am wrong!

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Saurabh • 19 days ago

Isn't the above mentioned proof for minimum number of levels for given number of leaves wrong I mean isn't the inequality sign must be inverted.

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GeeksforGeeks Mod → Saurabh • 19 days ago

Thanks for pointing this out. We have corrected the inequality signs.

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