Renuka Dhakal - Worksheet-1

#Getting Started with Image Processing with Python.

Introduction to Python Imaging Library(PIL)

2.1 Exercise - 1:

1. Read and display the image.

• Read the image using the Pillow library and display it.

```
from PIL import Image
# display image in colab
image_colored = Image.open("Lenna_(test_image).png")
display(image_colored)
```

• You can also use matplotlib to display the image.

```
from PIL import Image
import matplotlib.pyplot as plt
import numpy as np

# Open the image with PIL
image_colored = Image.open("Lenna_(test_image).png")
# Convert PIL image to numpy array
image_array = np.array(image_colored)

# Display the image using matplotlib
plt.imshow(image_array)
```

```
plt.axis('off') # Turn off axis numbers and ticks
plt.show()
```

1. Display only the top left corner of 100x100 pixels.

• Extract the top-left corner of the image (100x100 pixels) and display it using NumPy and Array Indexing.

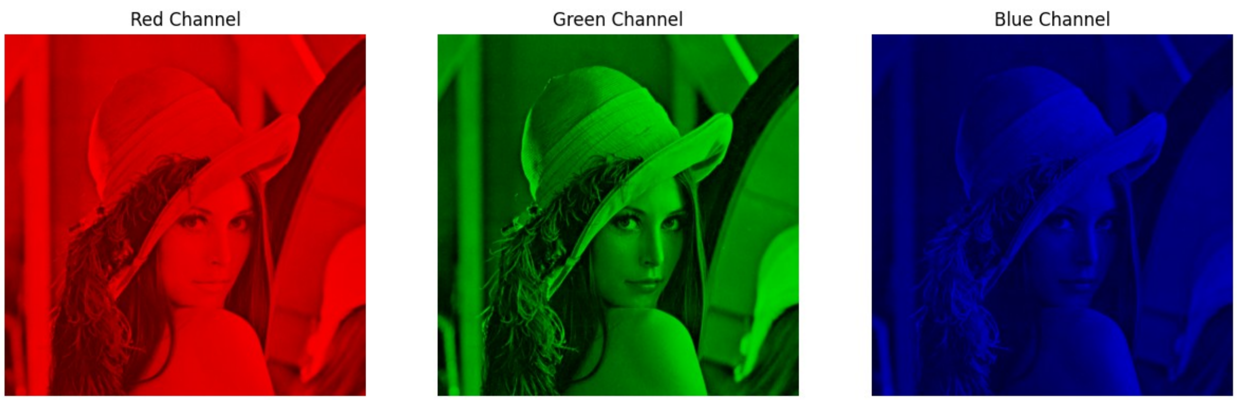
```
from PIL import Image
import matplotlib.pyplot as plt
import numpy as np # Import numpy
# Open the image with PIL
image_colored = Image.open("Lenna_(test_image).png")
# Convert PIL image to numpy array
image_array = np.array(image_colored)
# Extract the top-left 100x100 pixels
top_left_corner = image_array[:100, :100] # Selecting first 100 rows
and columns
# Display the extracted portion
plt.imshow(top_left_corner)
plt.axis('off') # Hide axis
plt.show()
```

1. Show the three color channels (R, G, B).

• Separate the image into its three color channels (Red, Green, and Blue) and display them individually, labeling each channel as R, G, and B.{Using NumPy.}

```
from PIL import Image
import matplotlib.pyplot as plt
import numpy as np # Import numpy
# Open the image with PIL
image_colored = Image.open("Lenna_(test_image).png")
# Convert PIL image to numpy array
image_array = np.array(image_colored)
# Extract Red, Green, and Blue channels
red_channel = image_array.copy()
green_channel = image_array.copy()
blue_channel = image_array.copy()
# Keep only the respective channel by setting other channels to 0
red_channel[:, :, 1:] = 0 # Set Green and Blue to 0, keeping only Red
green_channel[:, :, [0, 2]] = 0 # Set Red and Blue to 0, keeping only
Green
blue_channel[:, :, :2] = 0 # Set Red and Green to 0, keeping only Blue
# Display the three channels
fig, axes = plt.subplots(1, 3, figsize=(15, 5))
axes[0].imshow(red_channel)
axes[0].set_title("Red Channel")
axes[0].axis('off')
axes[1].imshow(green_channel)
```

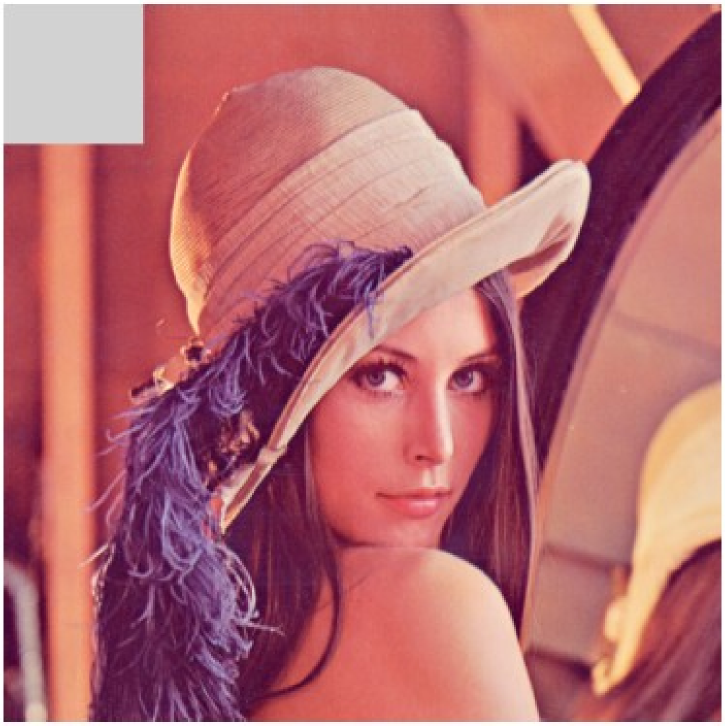
```
axes[1].set_title("Green Channel")
axes[1].axis('off')
axes[2].imshow(blue_channel)
axes[2].set_title("Blue Channel")
axes[2].axis('off')
plt.show()
```



1. Modify the top 100 × 100 pixels to a value of 210 and display the resulting image:

• Modify the pixel values of the top-left 100 × 100 region to have a value of 210 (which is a light gray color), and then display the modified image.

```
from PIL import Image
import matplotlib.pyplot as plt
import numpy as np # Import numpy
# Open the image with PIL
image_colored = Image.open("Lenna_(test_image).png")
# Convert PIL image to numpy array
image_array = np.array(image_colored)
# Modify the top-left 100x100 region to 210 (light gray)
image_array[:100, :100] = 210
# Convert back to image format and display
plt.imshow(image_array)
plt.axis('off') # Hide axis
plt.show()
```



2.2 Exercise - 2:

1. Load and display a grayscale image.

• Load a grayscale image using the Pillow library.

```
from PIL import Image
import matplotlib.pyplot as plt

# Load the image in grayscale mode
image_gray = Image.open("/content/cameraman.png").convert("L")
```

• Display the grayscale image using matplotlib.

```
# Display using matplotlib
plt.imshow(image_gray, cmap="gray") # Ensure grayscale colormap
plt.axis("off") # Hide axes
plt.title("Grayscale Image - Cameraman")
plt.show()
```

1. Extract and display the middle section of the image (150 pixels). • Extract a 150 pixel section from the center of the image using NumPy array slicing. • Display this cropped image using matplotlib.

• Extract a 150 pixel section from the center of the image using NumPy array slicing.

• Display this cropped image using matplotlib.

```
from PIL import Image
import matplotlib.pyplot as plt
import numpy as np # Import numpy
# Open the image in grayscale mode
image_gray = Image.open("cameraman.png").convert("L")
# Convert PIL image to numpy array
image_array = np.array(image_gray)
# Get image dimensions
height, width = image_array.shape
# Calculate the middle region (150 pixels in height)
start_y = (height - 150) // 2 # Starting Y-coordinate
end_y = start_y + 150 # Ending Y-coordinate
# Extract the middle section (150 pixels)
middle_section = image_array[start_y:end_y, :]
# Display the extracted section using matplotlib
plt.imshow(middle_section, cmap="gray")
plt.axis("off") # Hide axis
```

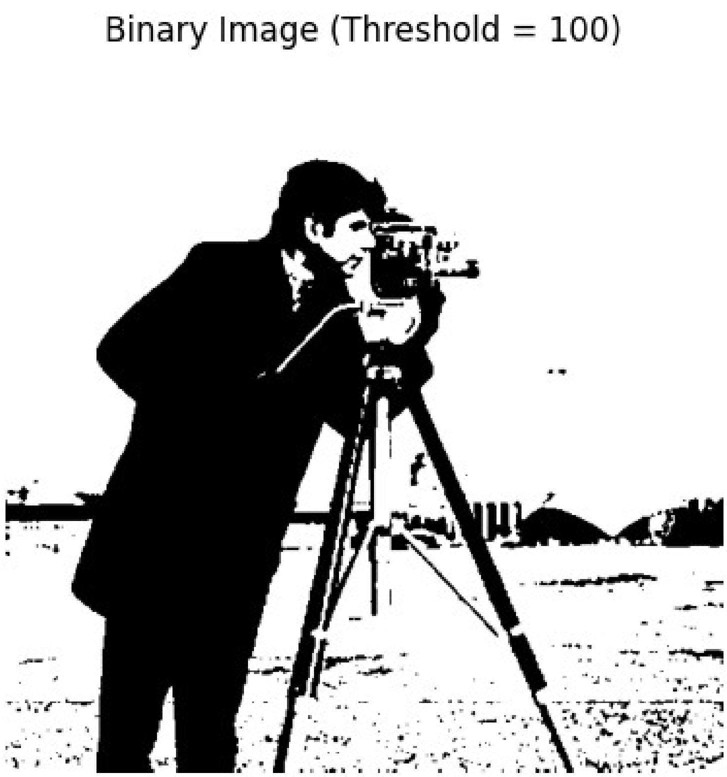
```
plt.title("Middle Section (150 pixels)")
plt.show()
```

1. Apply a simple threshold to the image (e.g., set all pixel values below 100 to 0).

• Apply a threshold to the grayscale image: set all pixel values below 100 to 0, and all values above 100 to 255 (creating a binary image).

• Display the resulting binary image.

```
from PIL import Image
import matplotlib.pyplot as plt
import numpy as np # Import numpy
# Open the image in grayscale mode
image_gray = Image.open("cameraman.png").convert("L")
# Convert PIL image to numpy array
image_array = np.array(image_gray)
# Apply thresholding: Set values < 100 to 0, and >= 100 to 255
threshold_value = 100
binary_image = np.where(image_array < threshold_value, 0,
255).astype(np.uint8)
# Display the binary image using matplotlib
plt.imshow(binary_image, cmap="gray")
plt.axis("off") # Hide axis
plt.title("Binary Image (Threshold = 100)")
plt.show()
```

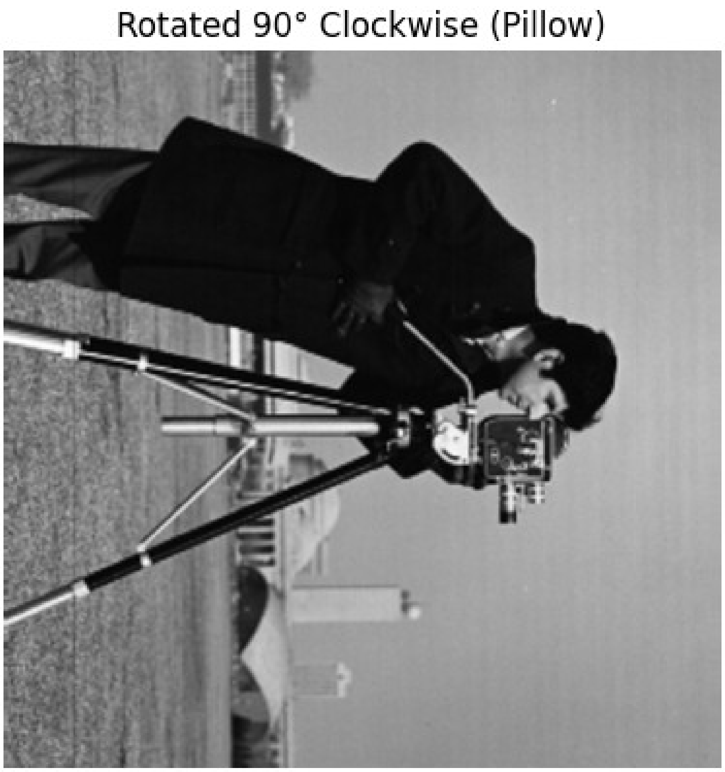


1. Rotate the image 90 degrees clockwise and display the result.

• Rotate the image by 90 degrees clockwise using the Pillow rotate method or by manipulating the image array.

• Display the rotated image using matplotlib.

```
from PIL import Image
import matplotlib.pyplot as plt
# Open the image in grayscale mode
image_gray = Image.open("cameraman.png").convert("L")
# Rotate 90 degrees clockwise using Pillow (-90 degrees
counterclockwise)
rotated_image = image_gray.rotate(-90, expand=True)
# Display the rotated image
plt.imshow(rotated_image, cmap="gray")
plt.axis("off") # Hide axis
plt.title("Rotated 90° Clockwise (Pillow)")
plt.show()
```



1. Convert the grayscale image to an RGB image.

• Convert the grayscale image into an RGB image where the grayscale values are replicated across all three channels (R, G, and B).

• Display the converted RGB image using matplotlib.

```
from PIL import Image
import matplotlib.pyplot as plt
# Open the image in grayscale mode
image_gray = Image.open("cameraman.png").convert("L")
# Convert grayscale to RGB using Pillow
image_rgb = image_gray.convert("RGB")
# Display the converted RGB image
plt.imshow(image_rgb)
plt.axis("off") # Hide axis
plt.title("Converted RGB Image (Pillow)")
plt.show()
```

3 Image Compression and Decompression using PCA.

1. Load and Prepare Data:

• Fetch an image of you choice.{If colour convert to grayscale} • Center the dataset - Standaridze the Data. • Calculate the covaraince matrix of the Standaridze data.

```
from PIL import Image
import numpy as np
import matplotlib.pyplot as plt
# Step 1: Load the image and convert it to grayscale
image_gray = Image.open("PeakyBlinders.jpg").convert("L") # Convert to
grayscale
# Convert grayscale image to NumPy array
image_array = np.array(image_gray)
# Step 2: Standardize the data (Center the dataset)
mean_pixel = np.mean(image_array, axis=0) # Compute mean along each
column (pixel)
std_pixel = np.std(image_array, axis=0) # Compute std along each
column (pixel)
standardized_image = (image_array - mean_pixel) / std_pixel #
Standardization
# Step 3: Reshape the standardized image for PCA (flatten the 2D image
into 1D vectors for each pixel row)
reshaped_image = standardized_image.reshape(-1, image_array.shape[1])
# Flatten rows into columns
```

```
# Step 4: Compute the covariance matrix (rows are samples, columns are
features)
cov_matrix = np.cov(reshaped_image, rowvar=False)
# Display grayscale image
plt.imshow(image_gray, cmap="gray")
plt.axis("off")
plt.title("Grayscale Image - Floyd")
plt.show()
# Print covariance matrix and top eigenvalues
print("Covariance Matrix:\n", cov_matrix)
```


```
Covariance Matrix:
 [[ 1.00139276  0.80646639  0.56012674 ... -0.11743611 -0.1327011
  -0.12981554]
 [ 0.80646639  1.00139276  0.88558743 ... -0.02562895 -0.03518278
  -0.00564036]
 [ 0.56012674  0.88558743  1.00139276 ...  0.13362055  0.11880263
   0.14795092]
 ...
 [-0.11743611 -0.02562895  0.13362055 ...  1.00139276  0.87716478
   0.78268033]
 [-0.1327011  -0.03518278  0.11880263 ...  0.87716478  1.00139276
   0.89237222]
 [-0.12981554 -0.00564036  0.14795092 ...  0.78268033  0.89237222
   1.00139276]]
```

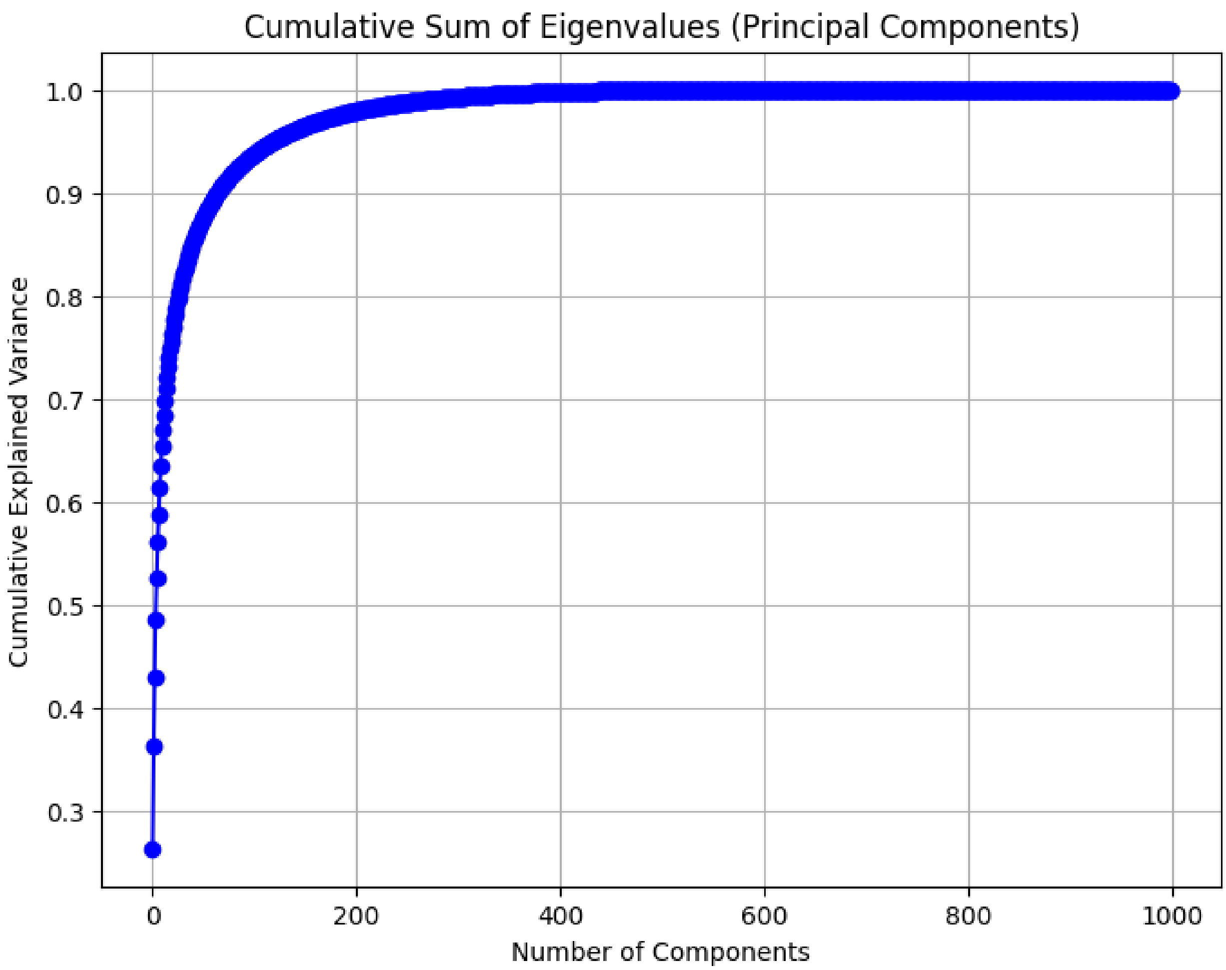
1. Eigen Decomposition and Identifying Principal Components:

• Compute Eigen Values and Eigen Vectors. • Sort the eigenvalues in descending order and choose the top k eigenvectors corresponding to the highest eigenvalues. • Identify the Principal Components with the help of cumulative Sum plot.

```
from PIL import Image
import numpy as np
import matplotlib.pyplot as plt
# Step 1: Load the image and convert it to grayscale
image_gray = Image.open("PeakyBlinders.jpg").convert("L") # Convert to
grayscale
# Convert grayscale image to NumPy array
image_array = np.array(image_gray)
# Step 2: Standardize the data (Center the dataset)
mean_pixel = np.mean(image_array, axis=0) # Compute mean along each
column (pixel)
std_pixel = np.std(image_array, axis=0) # Compute std along each
column (pixel)
standardized_image = (image_array - mean_pixel) / std_pixel #
Standardization
# Step 3: Reshape the standardized image for PCA (flatten the 2D image
into 1D vectors for each pixel row)
reshaped_image = standardized_image.reshape(-1, image_array.shape[1])
# Flatten rows into columns
# Step 4: Compute the covariance matrix (rows are samples, columns are
features)
cov_matrix = np.cov(reshaped_image, rowvar=False)
# Step 5: Compute eigenvalues and eigenvectors for PCA
eigenvalues, eigenvectors = np.linalg.eig(cov_matrix)
# Step 6: Sort eigenvalues and eigenvectors in descending order of
eigenvalue size
sorted_indices = np.argsort(eigenvalues)[::-1]
sorted_eigenvalues = eigenvalues[sorted_indices]
sorted_eigenvectors = eigenvectors[:, sorted_indices]
# Step 7: Calculate the cumulative sum of eigenvalues
cumulative_sum = np.cumsum(sorted_eigenvalues) /
np.sum(sorted_eigenvalues)
# Step 8: Plot the cumulative sum of eigenvalues
plt.figure(figsize=(8, 6))
plt.plot(cumulative_sum, marker='o', linestyle='-', color='b')
plt.title("Cumulative Sum of Eigenvalues (Principal Components)")
plt.xlabel("Number of Components")
plt.ylabel("Cumulative Explained Variance")
plt.grid(True)
plt.show()
# Step 9: Print sorted eigenvalues and top components
print("Sorted Eigenvalues:\n", sorted_eigenvalues)
print("\nTop 5 Eigenvectors:\n", sorted_eigenvectors[:, :5])
# Optionally, choose the top k components where the cumulative
```

```
variance is close to 1
k = np.argmax(cumulative_sum >= 0.95) # For example, choose components
explaining 95% of variance
print(f"Number of Components for 95% Variance: {k+1}")
```

```
/usr/local/lib/python3.11/dist-packages/matplotlib/cbook.py:1709:
ComplexWarning: Casting complex values to real discards the imaginary
part
  return math.isfinite(val)
/usr/local/lib/python3.11/dist-packages/matplotlib/cbook.py:1345:
ComplexWarning: Casting complex values to real discards the imaginary
part
  return np.asarray(x, float)
```




```
Sorted Eigenvalues:
 [ 2.63524943e+02+0.00000000e+00j  9.94013248e+01+0.00000000e+00j
  6.76555169e+01+0.00000000e+00j  5.54707699e+01+0.00000000e+00j
  4.13607478e+01+0.00000000e+00j  3.35154201e+01+0.00000000e+00j
  2.76468790e+01+0.00000000e+00j  2.61964594e+01+0.00000000e+00j
```

```
 2.05694798e+01+0.00000000e+00j  1.88562777e+01+0.00000000e+00j
 1.70451573e+01+0.00000000e+00j  1.41289294e+01+0.00000000e+00j
 1.34716997e+01+0.00000000e+00j  1.23330230e+01+0.00000000e+00j
 1.11120637e+01+0.00000000e+00j  9.85852968e+00+0.00000000e+00j
 8.67623826e+00+0.00000000e+00j  8.30665250e+00+0.00000000e+00j
 7.59733132e+00+0.00000000e+00j  7.21033746e+00+0.00000000e+00j
 7.04644060e+00+0.00000000e+00j  6.30878046e+00+0.00000000e+00j
 6.09011825e+00+0.00000000e+00j  5.73036720e+00+0.00000000e+00j
 5.59190639e+00+0.00000000e+00j  4.84309247e+00+0.00000000e+00j
 4.67373717e+00+0.00000000e+00j  4.57966094e+00+0.00000000e+00j
 4.35424320e+00+0.00000000e+00j  4.19536792e+00+0.00000000e+00j
 4.08121976e+00+0.00000000e+00j  3.88023814e+00+0.00000000e+00j
 3.78198493e+00+0.00000000e+00j  3.64182681e+00+0.00000000e+00j
 3.40222937e+00+0.00000000e+00j  3.26468111e+00+0.00000000e+00j
 3.22516173e+00+0.00000000e+00j  3.10197320e+00+0.00000000e+00j
 2.96632850e+00+0.00000000e+00j  2.85265995e+00+0.00000000e+00j
 2.79369705e+00+0.00000000e+00j  2.68575326e+00+0.00000000e+00j
 2.55596373e+00+0.00000000e+00j  2.50350454e+00+0.00000000e+00j
 2.39888081e+00+0.00000000e+00j  2.34992852e+00+0.00000000e+00j
 2.30494457e+00+0.00000000e+00j  2.28643102e+00+0.00000000e+00j
 2.26844572e+00+0.00000000e+00j  2.12026362e+00+0.00000000e+00j
 2.03086688e+00+0.00000000e+00j  2.00741173e+00+0.00000000e+00j
 1.96053092e+00+0.00000000e+00j  1.93469917e+00+0.00000000e+00j
 1.86353326e+00+0.00000000e+00j  1.79719932e+00+0.00000000e+00j
 1.78041709e+00+0.00000000e+00j  1.70514319e+00+0.00000000e+00j
 1.67817190e+00+0.00000000e+00j  1.61762498e+00+0.00000000e+00j
 1.58296817e+00+0.00000000e+00j  1.56795512e+00+0.00000000e+00j
 1.53539859e+00+0.00000000e+00j  1.50046883e+00+0.00000000e+00j
 1.49280279e+00+0.00000000e+00j  1.43771905e+00+0.00000000e+00j
 1.40257448e+00+0.00000000e+00j  1.35041511e+00+0.00000000e+00j
 1.33589755e+00+0.00000000e+00j  1.31565067e+00+0.00000000e+00j
 1.29068069e+00+0.00000000e+00j  1.24120055e+00+0.00000000e+00j
 1.22946219e+00+0.00000000e+00j  1.21135511e+00+0.00000000e+00j
 1.19487314e+00+0.00000000e+00j  1.14201072e+00+0.00000000e+00j
 1.13193856e+00+0.00000000e+00j  1.12401063e+00+0.00000000e+00j
 1.07175557e+00+0.00000000e+00j  1.05336689e+00+0.00000000e+00j
 1.05029650e+00+0.00000000e+00j  1.04524943e+00+0.00000000e+00j
 1.01020804e+00+0.00000000e+00j  1.00430229e+00+0.00000000e+00j
 9.87150366e-01+0.00000000e+00j  9.72525796e-01+0.00000000e+00j
 9.66515422e-01+0.00000000e+00j  9.46544796e-01+0.00000000e+00j
 9.30216233e-01+0.00000000e+00j  8.89176014e-01+0.00000000e+00j
 8.78472919e-01+0.00000000e+00j  8.70698829e-01+0.00000000e+00j
 8.48539012e-01+0.00000000e+00j  8.46190214e-01+0.00000000e+00j
 8.31504655e-01+0.00000000e+00j  8.19295733e-01+0.00000000e+00j
 7.94138622e-01+0.00000000e+00j  7.89880773e-01+0.00000000e+00j
 7.82228338e-01+0.00000000e+00j  7.66333332e-01+0.00000000e+00j
 7.58144006e-01+0.00000000e+00j  7.49851998e-01+0.00000000e+00j
 7.46326774e-01+0.00000000e+00j  7.34366772e-01+0.00000000e+00j
 7.16054882e-01+0.00000000e+00j  6.98418160e-01+0.00000000e+00j
```

```
 6.95712506e-01+0.00000000e+00j  6.90578548e-01+0.00000000e+00j
 6.75978434e-01+0.00000000e+00j  6.65955895e-01+0.00000000e+00j
 6.51045030e-01+0.00000000e+00j  6.48235825e-01+0.00000000e+00j
 6.38623663e-01+0.00000000e+00j  6.35542602e-01+0.00000000e+00j
 6.23897153e-01+0.00000000e+00j  6.16175958e-01+0.00000000e+00j
 6.06695703e-01+0.00000000e+00j  5.90674912e-01+0.00000000e+00j
 5.83203844e-01+0.00000000e+00j  5.81939284e-01+0.00000000e+00j
 5.68566160e-01+0.00000000e+00j  5.59347796e-01+0.00000000e+00j
 5.53972623e-01+0.00000000e+00j  5.47576623e-01+0.00000000e+00j
 5.38552786e-01+0.00000000e+00j  5.28808110e-01+0.00000000e+00j
 5.24763332e-01+0.00000000e+00j  5.22489208e-01+0.00000000e+00j
 5.15458925e-01+0.00000000e+00j  5.02955915e-01+0.00000000e+00j
 4.93683789e-01+0.00000000e+00j  4.88870501e-01+0.00000000e+00j
 4.84541755e-01+0.00000000e+00j  4.79288438e-01+0.00000000e+00j
 4.70723249e-01+0.00000000e+00j  4.60410730e-01+0.00000000e+00j
 4.55857428e-01+0.00000000e+00j  4.50350988e-01+0.00000000e+00j
 4.43260010e-01+0.00000000e+00j  4.37454621e-01+0.00000000e+00j
 4.33181697e-01+0.00000000e+00j  4.27841990e-01+0.00000000e+00j
 4.25262366e-01+0.00000000e+00j  4.20334388e-01+0.00000000e+00j
 4.17262666e-01+0.00000000e+00j  4.13500848e-01+0.00000000e+00j
 4.07314437e-01+0.00000000e+00j  4.01134008e-01+0.00000000e+00j
 3.97049043e-01+0.00000000e+00j  3.95696036e-01+0.00000000e+00j
 3.89707280e-01+0.00000000e+00j  3.85599676e-01+0.00000000e+00j
 3.75802891e-01+0.00000000e+00j  3.69584995e-01+0.00000000e+00j
 3.67881860e-01+0.00000000e+00j  3.66016131e-01+0.00000000e+00j
 3.59918098e-01+0.00000000e+00j  3.58138778e-01+0.00000000e+00j
 3.51494200e-01+0.00000000e+00j  3.44496380e-01+0.00000000e+00j
 3.41012015e-01+0.00000000e+00j  3.38384620e-01+0.00000000e+00j
 3.35542804e-01+0.00000000e+00j  3.30241022e-01+0.00000000e+00j
 3.28071056e-01+0.00000000e+00j  3.20238964e-01+0.00000000e+00j
 3.18680372e-01+0.00000000e+00j  3.11070480e-01+0.00000000e+00j
 3.07694800e-01+0.00000000e+00j  3.05050302e-01+0.00000000e+00j
 3.03260466e-01+0.00000000e+00j  3.02008220e-01+0.00000000e+00j
 2.96369608e-01+0.00000000e+00j  2.93806580e-01+0.00000000e+00j
 2.90116582e-01+0.00000000e+00j  2.83865097e-01+0.00000000e+00j
 2.82060199e-01+0.00000000e+00j  2.78600586e-01+0.00000000e+00j
 2.77002437e-01+0.00000000e+00j  2.71577082e-01+0.00000000e+00j
 2.70487901e-01+0.00000000e+00j  2.68697471e-01+0.00000000e+00j
 2.65806708e-01+0.00000000e+00j  2.62851421e-01+0.00000000e+00j
 2.61247738e-01+0.00000000e+00j  2.57749320e-01+0.00000000e+00j
 2.54761410e-01+0.00000000e+00j  2.51358532e-01+0.00000000e+00j
 2.49256375e-01+0.00000000e+00j  2.47657356e-01+0.00000000e+00j
 2.46003072e-01+0.00000000e+00j  2.39435143e-01+0.00000000e+00j
 2.37651776e-01+0.00000000e+00j  2.36334706e-01+0.00000000e+00j
 2.35202296e-01+0.00000000e+00j  2.28638049e-01+0.00000000e+00j
 2.28235279e-01+0.00000000e+00j  2.26874651e-01+0.00000000e+00j
 2.22220737e-01+0.00000000e+00j  2.19537089e-01+0.00000000e+00j
 2.16040114e-01+0.00000000e+00j  2.14455784e-01+0.00000000e+00j
 2.11986400e-01+0.00000000e+00j  2.09364953e-01+0.00000000e+00j
```

```
2.07621801e-01+0.00000000e+00j  2.07354898e-01+0.00000000e+00j
2.02493085e-01+0.00000000e+00j  2.00125942e-01+0.00000000e+00j
1.99560699e-01+0.00000000e+00j  1.95801783e-01+0.00000000e+00j
1.93921367e-01+0.00000000e+00j  1.91522991e-01+0.00000000e+00j
1.90700841e-01+0.00000000e+00j  1.89105068e-01+0.00000000e+00j
1.88208073e-01+0.00000000e+00j  1.85477862e-01+0.00000000e+00j
1.83510328e-01+0.00000000e+00j  1.81649431e-01+0.00000000e+00j
1.80182253e-01+0.00000000e+00j  1.78649518e-01+0.00000000e+00j
1.75412721e-01+0.00000000e+00j  1.74436509e-01+0.00000000e+00j
1.73412776e-01+0.00000000e+00j  1.71101971e-01+0.00000000e+00j
1.69368924e-01+0.00000000e+00j  1.66424855e-01+0.00000000e+00j
1.64646069e-01+0.00000000e+00j  1.62150633e-01+0.00000000e+00j
1.59974186e-01+0.00000000e+00j  1.58776175e-01+0.00000000e+00j
1.57729772e-01+0.00000000e+00j  1.57024090e-01+0.00000000e+00j
1.55338243e-01+0.00000000e+00j  1.53696390e-01+0.00000000e+00j
1.52463585e-01+0.00000000e+00j  1.49538843e-01+0.00000000e+00j
1.48575335e-01+0.00000000e+00j  1.47477918e-01+0.00000000e+00j
1.47197012e-01+0.00000000e+00j  1.45134909e-01+0.00000000e+00j
1.43497787e-01+0.00000000e+00j  1.41927679e-01+0.00000000e+00j
1.40663625e-01+0.00000000e+00j  1.37999226e-01+0.00000000e+00j
1.37495862e-01+0.00000000e+00j  1.36124066e-01+0.00000000e+00j
1.35125237e-01+0.00000000e+00j  1.34058092e-01+0.00000000e+00j
1.32506638e-01+0.00000000e+00j  1.31178857e-01+0.00000000e+00j
1.29602176e-01+0.00000000e+00j  1.28315366e-01+0.00000000e+00j
1.27864359e-01+0.00000000e+00j  1.26049250e-01+0.00000000e+00j
1.23110062e-01+0.00000000e+00j  1.22787792e-01+0.00000000e+00j
1.21912938e-01+0.00000000e+00j  1.20148289e-01+0.00000000e+00j
1.18985213e-01+0.00000000e+00j  1.18424257e-01+0.00000000e+00j
1.17370331e-01+0.00000000e+00j  1.15656166e-01+0.00000000e+00j
1.15444383e-01+0.00000000e+00j  1.13921666e-01+0.00000000e+00j
1.12943484e-01+0.00000000e+00j  1.11821562e-01+0.00000000e+00j
1.11261574e-01+0.00000000e+00j  1.10265398e-01+0.00000000e+00j
1.10169612e-01+0.00000000e+00j  1.06938831e-01+0.00000000e+00j
1.06520760e-01+0.00000000e+00j  1.04309862e-01+0.00000000e+00j
1.04044198e-01+0.00000000e+00j  1.02431480e-01+0.00000000e+00j
1.00740127e-01+0.00000000e+00j  9.98685857e-02+0.00000000e+00j
9.93904397e-02+0.00000000e+00j  9.87654116e-02+0.00000000e+00j
9.81385326e-02+0.00000000e+00j  9.61749635e-02+0.00000000e+00j
9.53830356e-02+0.00000000e+00j  9.48584267e-02+0.00000000e+00j
9.37534455e-02+0.00000000e+00j  9.35066604e-02+0.00000000e+00j
9.30856157e-02+0.00000000e+00j  9.04689163e-02+0.00000000e+00j
8.99770455e-02+0.00000000e+00j  8.93474978e-02+0.00000000e+00j
8.76133848e-02+0.00000000e+00j  8.69534625e-02+0.00000000e+00j
8.67747013e-02+0.00000000e+00j  8.63140367e-02+0.00000000e+00j
8.52817272e-02+0.00000000e+00j  8.42879380e-02+0.00000000e+00j
8.26620763e-02+0.00000000e+00j  8.17243693e-02+0.00000000e+00j
8.10774560e-02+0.00000000e+00j  8.05782815e-02+0.00000000e+00j
8.02733646e-02+0.00000000e+00j  7.89147840e-02+0.00000000e+00j
7.79153519e-02+0.00000000e+00j  7.68421941e-02+0.00000000e+00j
```

```
 7.64221990e-02+0.00000000e+00j  7.59482232e-02+0.00000000e+00j
 7.43322567e-02+0.00000000e+00j  7.37143803e-02+0.00000000e+00j
 7.26931391e-02+0.00000000e+00j  7.18028505e-02+0.00000000e+00j
 7.11403227e-02+0.00000000e+00j  7.08067775e-02+0.00000000e+00j
 6.98408277e-02+0.00000000e+00j  6.93802609e-02+0.00000000e+00j
 6.84719340e-02+0.00000000e+00j  6.81198867e-02+0.00000000e+00j
 6.77454737e-02+0.00000000e+00j  6.75137107e-02+0.00000000e+00j
 6.67321756e-02+0.00000000e+00j  6.60510052e-02+0.00000000e+00j
 6.52289608e-02+0.00000000e+00j  6.42162668e-02+0.00000000e+00j
 6.34822136e-02+0.00000000e+00j  6.31360024e-02+0.00000000e+00j
 6.26636723e-02+0.00000000e+00j  6.18945243e-02+0.00000000e+00j
 6.14678204e-02+0.00000000e+00j  6.13770976e-02+0.00000000e+00j
 6.11102290e-02+0.00000000e+00j  5.97760092e-02+0.00000000e+00j
 5.93148162e-02+0.00000000e+00j  5.83475939e-02+0.00000000e+00j
 5.80709407e-02+0.00000000e+00j  5.74845489e-02+0.00000000e+00j
 5.65739893e-02+0.00000000e+00j  5.61325308e-02+0.00000000e+00j
 5.54819041e-02+0.00000000e+00j  5.49939621e-02+0.00000000e+00j
 5.45120494e-02+0.00000000e+00j  5.40791454e-02+0.00000000e+00j
 5.29655582e-02+0.00000000e+00j  5.25161901e-02+0.00000000e+00j
 5.20280553e-02+0.00000000e+00j  5.12841774e-02+0.00000000e+00j
 5.07607223e-02+0.00000000e+00j  4.97823180e-02+0.00000000e+00j
 4.97263800e-02+0.00000000e+00j  4.95946375e-02+0.00000000e+00j
 4.93833161e-02+0.00000000e+00j  4.85263392e-02+0.00000000e+00j
 4.83484316e-02+0.00000000e+00j  4.77482733e-02+0.00000000e+00j
 4.74950997e-02+0.00000000e+00j  4.69107401e-02+0.00000000e+00j
 4.61877709e-02+0.00000000e+00j  4.50115931e-02+0.00000000e+00j
 4.47001184e-02+0.00000000e+00j  4.43477060e-02+0.00000000e+00j
 4.41314533e-02+0.00000000e+00j  4.38759638e-02+0.00000000e+00j
 4.36282780e-02+0.00000000e+00j  4.31427149e-02+0.00000000e+00j
 4.30684272e-02+0.00000000e+00j  4.25469023e-02+0.00000000e+00j
 4.18959447e-02+0.00000000e+00j  4.13032110e-02+0.00000000e+00j
 4.09448956e-02+0.00000000e+00j  4.06320478e-02+0.00000000e+00j
 4.03982972e-02+0.00000000e+00j  3.98359880e-02+0.00000000e+00j
 3.94244568e-02+0.00000000e+00j  3.90547840e-02+0.00000000e+00j
 3.87011070e-02+0.00000000e+00j  3.82817564e-02+0.00000000e+00j
 3.78095091e-02+0.00000000e+00j  3.73395834e-02+0.00000000e+00j
 3.69582895e-02+0.00000000e+00j  3.66977894e-02+0.00000000e+00j
 3.62999548e-02+0.00000000e+00j  3.56157209e-02+0.00000000e+00j
 3.53045361e-02+0.00000000e+00j  3.50468426e-02+0.00000000e+00j
 3.45726723e-02+0.00000000e+00j  3.43383702e-02+0.00000000e+00j
 3.40292632e-02+0.00000000e+00j  3.37858870e-02+0.00000000e+00j
 3.33269734e-02+0.00000000e+00j  3.32337547e-02+0.00000000e+00j
 3.28299060e-02+0.00000000e+00j  3.26584009e-02+0.00000000e+00j
 3.23300467e-02+0.00000000e+00j  3.20151567e-02+0.00000000e+00j
 3.17049042e-02+0.00000000e+00j  3.16705126e-02+0.00000000e+00j
 3.12340710e-02+0.00000000e+00j  3.04190160e-02+0.00000000e+00j
 3.00754025e-02+0.00000000e+00j  2.98068089e-02+0.00000000e+00j
 2.95767519e-02+0.00000000e+00j  2.94831718e-02+0.00000000e+00j
 2.90450542e-02+0.00000000e+00j  2.87850471e-02+0.00000000e+00j
```

```
 2.83255038e-02+0.00000000e+00j  2.82995115e-02+0.00000000e+00j
 2.80096564e-02+0.00000000e+00j  2.78701650e-02+0.00000000e+00j
 2.75612676e-02+0.00000000e+00j  2.71509897e-02+0.00000000e+00j
 2.67814799e-02+0.00000000e+00j  2.64844559e-02+0.00000000e+00j
 2.60977362e-02+0.00000000e+00j  2.55195731e-02+0.00000000e+00j
 2.52779779e-02+0.00000000e+00j  2.51299811e-02+0.00000000e+00j
 2.49795567e-02+0.00000000e+00j  2.47936920e-02+0.00000000e+00j
 2.44013880e-02+0.00000000e+00j  2.42475534e-02+0.00000000e+00j
 2.38419825e-02+0.00000000e+00j  2.36467260e-02+0.00000000e+00j
 2.34438509e-02+0.00000000e+00j  2.33554132e-02+0.00000000e+00j
 2.27726040e-02+0.00000000e+00j  2.23200879e-02+0.00000000e+00j
 2.21150902e-02+0.00000000e+00j  2.18929748e-02+0.00000000e+00j
 2.17053413e-02+0.00000000e+00j  2.16048083e-02+0.00000000e+00j
 2.13231482e-02+0.00000000e+00j  2.10309118e-02+0.00000000e+00j
 2.07725193e-02+0.00000000e+00j  2.06441259e-02+0.00000000e+00j
 2.01144407e-02+0.00000000e+00j  1.99966930e-02+0.00000000e+00j
 1.98555313e-02+0.00000000e+00j  1.95097050e-02+0.00000000e+00j
 1.92775138e-02+0.00000000e+00j  1.91576328e-02+0.00000000e+00j
 1.89564005e-02+0.00000000e+00j  1.87412368e-02+0.00000000e+00j
 1.85392082e-02+0.00000000e+00j  1.84157050e-02+0.00000000e+00j
 1.82872903e-02+0.00000000e+00j  1.81549929e-02+0.00000000e+00j
 1.79251770e-02+0.00000000e+00j  1.76484506e-02+0.00000000e+00j
 1.75369985e-02+0.00000000e+00j  1.74524125e-02+0.00000000e+00j
 1.72267230e-02+0.00000000e+00j  1.69438391e-02+0.00000000e+00j
 1.66468766e-02+0.00000000e+00j  1.64184680e-02+0.00000000e+00j
 1.63000576e-02+0.00000000e+00j  1.62261348e-02+0.00000000e+00j
 1.58201230e-02+0.00000000e+00j  1.57015231e-02+0.00000000e+00j
 1.55705991e-02+0.00000000e+00j  1.55207632e-02+0.00000000e+00j
 1.53490986e-02+0.00000000e+00j  1.51697185e-02+0.00000000e+00j
 1.47618574e-02+0.00000000e+00j  1.46799873e-02+0.00000000e+00j
 1.44492714e-02+0.00000000e+00j  1.42934709e-02+0.00000000e+00j
 1.41603387e-02+0.00000000e+00j  1.39113269e-02+0.00000000e+00j
 1.38192154e-02+0.00000000e+00j  1.37401388e-02+0.00000000e+00j
 1.34631068e-02+0.00000000e+00j  1.32948977e-02+0.00000000e+00j
 1.31841704e-02+0.00000000e+00j  1.31336459e-02+0.00000000e+00j
 1.26962697e-02+0.00000000e+00j  1.25947186e-02+0.00000000e+00j
 1.24273114e-02+0.00000000e+00j  1.23331405e-02+0.00000000e+00j
 1.21670010e-02+0.00000000e+00j  1.21263340e-02+0.00000000e+00j
 1.18764862e-02+0.00000000e+00j  1.16909641e-02+0.00000000e+00j
 1.14974979e-02+0.00000000e+00j  1.13851244e-02+0.00000000e+00j
 1.13496541e-02+0.00000000e+00j  1.12151801e-02+0.00000000e+00j
 1.11201141e-02+0.00000000e+00j  1.09715037e-02+0.00000000e+00j
 1.09449121e-02+0.00000000e+00j  1.07502938e-02+0.00000000e+00j
 1.06029119e-02+0.00000000e+00j  1.04609037e-02+0.00000000e+00j
 1.03339367e-02+0.00000000e+00j  1.02149095e-02+0.00000000e+00j
 1.00317394e-02+0.00000000e+00j  9.91109289e-03+0.00000000e+00j
 9.76845587e-03+0.00000000e+00j  9.72942669e-03+0.00000000e+00j
 9.58936003e-03+0.00000000e+00j  9.50050501e-03+0.00000000e+00j
 9.39336505e-03+0.00000000e+00j  9.21854737e-03+0.00000000e+00j
```

```
  9.11156308e-03+0.00000000e+00j  9.03195673e-03+0.00000000e+00j
  8.97664728e-03+0.00000000e+00j  8.82604348e-03+0.00000000e+00j
  8.75321539e-03+0.00000000e+00j  8.66288341e-03+0.00000000e+00j
  8.36153746e-03+0.00000000e+00j  8.32400081e-03+0.00000000e+00j
  8.16858746e-03+0.00000000e+00j  8.07461073e-03+0.00000000e+00j
  8.01248929e-03+0.00000000e+00j  7.88538453e-03+0.00000000e+00j
  7.66229249e-03+0.00000000e+00j  7.65038983e-03+0.00000000e+00j
  7.56302897e-03+0.00000000e+00j  7.39752005e-03+0.00000000e+00j
  7.18801067e-03+0.00000000e+00j  7.11917047e-03+0.00000000e+00j
  7.05821034e-03+0.00000000e+00j  6.93138290e-03+0.00000000e+00j
  6.82759047e-03+0.00000000e+00j  6.73393457e-03+0.00000000e+00j
  6.67328482e-03+0.00000000e+00j  6.64326292e-03+0.00000000e+00j
  6.55259807e-03+0.00000000e+00j  6.42689983e-03+0.00000000e+00j
  6.30144222e-03+0.00000000e+00j  6.21210755e-03+0.00000000e+00j
  6.13329675e-03+0.00000000e+00j  6.09132201e-03+0.00000000e+00j
  5.89697119e-03+0.00000000e+00j  5.80114692e-03+0.00000000e+00j
  5.76027142e-03+0.00000000e+00j  5.66084951e-03+0.00000000e+00j
  5.50449255e-03+0.00000000e+00j  5.49556345e-03+0.00000000e+00j
  5.40295862e-03+0.00000000e+00j  5.31728353e-03+0.00000000e+00j
  5.18645963e-03+0.00000000e+00j  5.09565048e-03+0.00000000e+00j
  5.08059167e-03+0.00000000e+00j  5.03323823e-03+0.00000000e+00j
  4.89338202e-03+0.00000000e+00j  4.85473238e-03+0.00000000e+00j
  4.81555612e-03+0.00000000e+00j  4.75816174e-03+0.00000000e+00j
  4.66718660e-03+0.00000000e+00j  4.63784011e-03+0.00000000e+00j
  4.51185632e-03+0.00000000e+00j  4.41847333e-03+0.00000000e+00j
  4.35605376e-03+0.00000000e+00j  4.27060835e-03+0.00000000e+00j
  4.22937032e-03+0.00000000e+00j  4.21064614e-03+0.00000000e+00j
  4.15514949e-03+0.00000000e+00j  4.11430767e-03+0.00000000e+00j
  4.07480929e-03+0.00000000e+00j  3.96287657e-03+0.00000000e+00j
  3.94087557e-03+0.00000000e+00j  3.89056188e-03+0.00000000e+00j
  3.84000664e-03+0.00000000e+00j  3.71956515e-03+0.00000000e+00j
  3.65366312e-03+0.00000000e+00j  3.60437849e-03+0.00000000e+00j
  3.53646219e-03+0.00000000e+00j  3.45380248e-03+0.00000000e+00j
  3.43778842e-03+0.00000000e+00j  3.37093325e-03+0.00000000e+00j
  3.33548649e-03+0.00000000e+00j  3.25448266e-03+0.00000000e+00j
  3.16273187e-03+0.00000000e+00j  3.08837912e-03+0.00000000e+00j
  3.03577997e-03+0.00000000e+00j  2.98553463e-03+0.00000000e+00j
  2.97401015e-03+0.00000000e+00j  2.91509555e-03+0.00000000e+00j
  2.80496771e-03+0.00000000e+00j  2.73651559e-03+0.00000000e+00j
  2.71947797e-03+0.00000000e+00j  2.68873271e-03+0.00000000e+00j
  2.66286104e-03+0.00000000e+00j  2.60450453e-03+0.00000000e+00j
  2.59961912e-03+0.00000000e+00j  2.53732934e-03+0.00000000e+00j
  2.49211797e-03+0.00000000e+00j  2.43900362e-03+0.00000000e+00j
  2.39401279e-03+0.00000000e+00j  2.38759176e-03+0.00000000e+00j
  2.33444886e-03+0.00000000e+00j  2.29235361e-03+0.00000000e+00j
  2.26611677e-03+0.00000000e+00j  2.20296760e-03+0.00000000e+00j
  2.16470380e-03+0.00000000e+00j  2.13886158e-03+0.00000000e+00j
  2.11896235e-03+0.00000000e+00j  2.06270316e-03+0.00000000e+00j
  2.04914713e-03+0.00000000e+00j  1.98318777e-03+0.00000000e+00j
```

```
 1.93201115e-03+0.00000000e+00j  1.90309434e-03+0.00000000e+00j
 1.86721365e-03+0.00000000e+00j  1.81567454e-03+0.00000000e+00j
 1.77554540e-03+0.00000000e+00j  1.75900826e-03+0.00000000e+00j
 1.71614966e-03+0.00000000e+00j  1.62492910e-03+0.00000000e+00j
 1.58999518e-03+0.00000000e+00j  1.58867436e-03+0.00000000e+00j
 1.56348906e-03+0.00000000e+00j  1.53749979e-03+0.00000000e+00j
 1.47719429e-03+0.00000000e+00j  1.47261533e-03+0.00000000e+00j
 1.40465301e-03+0.00000000e+00j  1.39601753e-03+0.00000000e+00j
 1.36897306e-03+0.00000000e+00j  1.28907961e-03+0.00000000e+00j
 1.27324578e-03+0.00000000e+00j  1.25774234e-03+0.00000000e+00j
 1.23771131e-03+0.00000000e+00j  1.22432532e-03+0.00000000e+00j
 1.21453232e-03+0.00000000e+00j  1.18691896e-03+0.00000000e+00j
 1.17930198e-03+0.00000000e+00j  1.17017826e-03+0.00000000e+00j
 1.10792379e-03+0.00000000e+00j  1.08106782e-03+0.00000000e+00j
 1.04400210e-03+0.00000000e+00j  1.03764897e-03+0.00000000e+00j
 1.00476550e-03+0.00000000e+00j  9.86494433e-04+0.00000000e+00j
 9.65215909e-04+0.00000000e+00j  9.23866847e-04+0.00000000e+00j
 9.10573081e-04+0.00000000e+00j  8.78740889e-04+0.00000000e+00j
 8.64138342e-04+0.00000000e+00j  8.45084805e-04+0.00000000e+00j
 8.25277688e-04+0.00000000e+00j  8.04940354e-04+0.00000000e+00j
 7.78973518e-04+0.00000000e+00j  7.73594329e-04+0.00000000e+00j
 7.59557552e-04+0.00000000e+00j  7.38553199e-04+0.00000000e+00j
 7.16388988e-04+0.00000000e+00j  7.11577442e-04+0.00000000e+00j
 6.93262832e-04+0.00000000e+00j  6.61825332e-04+0.00000000e+00j
 6.39109374e-04+0.00000000e+00j  6.08873018e-04+0.00000000e+00j
 6.00058419e-04+0.00000000e+00j  5.89904718e-04+0.00000000e+00j
 5.61125403e-04+0.00000000e+00j  5.53345106e-04+0.00000000e+00j
 5.47219524e-04+0.00000000e+00j  5.25896978e-04+0.00000000e+00j
 5.20389972e-04+0.00000000e+00j  5.03290485e-04+0.00000000e+00j
 4.69214302e-04+0.00000000e+00j  4.65427197e-04+0.00000000e+00j
 4.57191371e-04+0.00000000e+00j  4.49752824e-04+0.00000000e+00j
 4.34373080e-04+0.00000000e+00j  4.20453140e-04+0.00000000e+00j
 4.01785323e-04+0.00000000e+00j  3.98557906e-04+0.00000000e+00j
 3.79632979e-04+0.00000000e+00j  3.65156122e-04+0.00000000e+00j
 3.55784376e-04+0.00000000e+00j  3.46768531e-04+0.00000000e+00j
 3.40672048e-04+0.00000000e+00j  3.15953983e-04+0.00000000e+00j
 3.01742009e-04+0.00000000e+00j  2.94082148e-04+0.00000000e+00j
 2.84048261e-04+0.00000000e+00j  2.82781853e-04+0.00000000e+00j
 2.63041350e-04+0.00000000e+00j  2.53576670e-04+0.00000000e+00j
 2.51187076e-04+0.00000000e+00j  2.48158138e-04+0.00000000e+00j
 2.26676702e-04+0.00000000e+00j  2.22631106e-04+0.00000000e+00j
 2.08597729e-04+0.00000000e+00j  2.01752195e-04+0.00000000e+00j
 1.98797743e-04+0.00000000e+00j  1.95356840e-04+0.00000000e+00j
 1.89180246e-04+0.00000000e+00j  1.78127118e-04+0.00000000e+00j
 1.72063941e-04+0.00000000e+00j  1.63457266e-04+0.00000000e+00j
 1.53674083e-04+0.00000000e+00j  1.51434655e-04+0.00000000e+00j
 1.48233717e-04+0.00000000e+00j  1.42255680e-04+0.00000000e+00j
 1.32505540e-04+0.00000000e+00j  1.25328694e-04+0.00000000e+00j
 1.15552958e-04+0.00000000e+00j  1.10607363e-04+0.00000000e+00j
```

```
 1.06588645e-04+0.00000000e+00j  1.04311633e-04+0.00000000e+00j
 1.01245902e-04+0.00000000e+00j  9.35552389e-05+0.00000000e+00j
 8.88235296e-05+0.00000000e+00j  8.36549927e-05+0.00000000e+00j
 7.91487001e-05+0.00000000e+00j  7.50760397e-05+0.00000000e+00j
 6.85985165e-05+0.00000000e+00j  6.70935520e-05+0.00000000e+00j
 6.41538408e-05+0.00000000e+00j  6.26501617e-05+0.00000000e+00j
 5.71910545e-05+0.00000000e+00j  5.58694493e-05+0.00000000e+00j
 5.35146877e-05+0.00000000e+00j  4.81543331e-05+0.00000000e+00j
 4.57440119e-05+0.00000000e+00j  3.94017406e-05+0.00000000e+00j
 3.75232625e-05+0.00000000e+00j  3.61105493e-05+0.00000000e+00j
 3.37293235e-05+0.00000000e+00j  3.18162115e-05+0.00000000e+00j
 2.62088829e-05+0.00000000e+00j  2.22371364e-05+0.00000000e+00j
 8.18684603e-15+0.00000000e+00j  6.51131216e-15+0.00000000e+00j
 5.93729814e-15+0.00000000e+00j  5.58448457e-15+0.00000000e+00j
 5.48858627e-15+0.00000000e+00j  5.23926955e-15+0.00000000e+00j
 5.18691896e-15+5.02196642e-16j  5.18691896e-15-5.02196642e-16j
 5.08855232e-15+0.00000000e+00j  4.96560645e-15+0.00000000e+00j
 4.84873257e-15+0.00000000e+00j  4.73320794e-15+6.56247790e-16j
 4.73320794e-15-6.56247790e-16j  4.72403534e-15+3.72865862e-16j
 4.72403534e-15-3.72865862e-16j  4.66507471e-15+1.28549752e-15j
 4.66507471e-15-1.28549752e-15j  4.64893926e-15+0.00000000e+00j
 4.37972007e-15+1.09794838e-16j  4.37972007e-15-1.09794838e-16j
 4.32110338e-15+0.00000000e+00j  4.17789635e-15+5.74097699e-16j
 4.17789635e-15-5.74097699e-16j  3.95517006e-15+2.91654149e-16j
 3.95517006e-15-2.91654149e-16j  3.86930101e-15+5.79040306e-16j
 3.86930101e-15-5.79040306e-16j  3.75856180e-15+1.01008157e-16j
 3.75856180e-15-1.01008157e-16j  3.63736416e-15+8.56134366e-16j
 3.63736416e-15-8.56134366e-16j  3.59393505e-15+0.00000000e+00j
 3.43550842e-15+0.00000000e+00j  3.36155064e-15+1.58921125e-16j
 3.36155064e-15-1.58921125e-16j  3.27903022e-15+6.87577546e-16j
 3.27903022e-15-6.87577546e-16j  3.14991245e-15+3.22845769e-16j
 3.14991245e-15-3.22845769e-16j  3.14864542e-15+4.98207945e-16j
 3.14864542e-15-4.98207945e-16j  3.09566076e-15+9.16933380e-17j
 3.09566076e-15-9.16933380e-17j  2.98366889e-15+9.16067047e-16j
 2.98366889e-15-9.16067047e-16j  2.75217840e-15+5.88450736e-16j
 2.75217840e-15-5.88450736e-16j  2.70985012e-15+1.54811901e-16j
 2.70985012e-15-1.54811901e-16j  2.66342577e-15+9.53790274e-16j
 2.66342577e-15-9.53790274e-16j  2.65786444e-15+2.99880544e-17j
 2.65786444e-15-2.99880544e-17j  2.62521435e-15+1.32634045e-16j
 2.62521435e-15-1.32634045e-16j  2.58500495e-15+6.79945295e-16j
 2.58500495e-15-6.79945295e-16j  2.37642987e-15+5.32566221e-16j
 2.37642987e-15-5.32566221e-16j  2.29362085e-15+8.13255684e-16j
 2.29362085e-15-8.13255684e-16j  2.27476547e-15+1.13586589e-16j
 2.27476547e-15-1.13586589e-16j  2.12364878e-15+9.95018525e-16j
 2.12364878e-15-9.95018525e-16j  2.10692245e-15+2.77979553e-16j
 2.10692245e-15-2.77979553e-16j  2.10499753e-15+4.89636014e-16j
 2.10499753e-15-4.89636014e-16j  2.01141159e-15+1.69023917e-16j
 2.01141159e-15-1.69023917e-16j  1.98135587e-15+7.57712132e-17j
 1.98135587e-15-7.57712132e-17j  1.97585299e-15+7.26862783e-16j
```

```
  1.97585299e-15-7.26862783e-16j  1.80390556e-15+4.07083374e-16j
  1.80390556e-15-4.07083374e-16j  1.79201002e-15+7.31983272e-16j
  1.79201002e-15-7.31983272e-16j  1.68203782e-15+3.11638253e-16j
  1.68203782e-15-3.11638253e-16j  1.63586456e-15+6.16062534e-17j
  1.63586456e-15-6.16062534e-17j  1.59614193e-15+1.06354839e-15j
  1.59614193e-15-1.06354839e-15j  1.53875208e-15+0.00000000e+00j
  1.53220721e-15+2.97903295e-16j  1.53220721e-15-2.97903295e-16j
  1.43262889e-15+8.13471858e-16j  1.43262889e-15-8.13471858e-16j
  1.42849070e-15+4.89169378e-16j  1.42849070e-15-4.89169378e-16j
  1.36183110e-15+1.41293639e-15j  1.36183110e-15-1.41293639e-15j
  1.34091034e-15+4.06722262e-16j  1.34091034e-15-4.06722262e-16j
  1.28284933e-15+0.00000000e+00j  1.22603910e-15+7.47161169e-16j
  1.22603910e-15-7.47161169e-16j  1.11648261e-15+1.40361901e-16j
  1.11648261e-15-1.40361901e-16j  1.10476056e-15+3.99025051e-16j
  1.10476056e-15-3.99025051e-16j  1.09890362e-15+0.00000000e+00j
  1.07915970e-15+2.26471293e-16j  1.07915970e-15-2.26471293e-16j
  8.79342166e-16+0.00000000e+00j  8.59737526e-16+2.13862177e-16j
  8.59737526e-16-2.13862177e-16j  7.59779044e-16+5.44929291e-16j
  7.59779044e-16-5.44929291e-16j  7.59335704e-16+1.29700288e-15j
  7.59335704e-16-1.29700288e-15j  7.09536747e-16+0.00000000e+00j
  6.53006584e-16+6.71497257e-16j  6.53006584e-16-6.71497257e-16j
  6.48931796e-16+9.47271837e-16j  6.48931796e-16-9.47271837e-16j
  5.91323747e-16+3.60682667e-16j  5.91323747e-16-3.60682667e-16j
  5.60271082e-16+1.31628869e-16j  5.60271082e-16-1.31628869e-16j
  3.86625967e-16+9.30603756e-16j  3.86625967e-16-9.30603756e-16j
  3.79945372e-16+3.43271518e-16j  3.79945372e-16-3.43271518e-16j
  3.13250133e-16+7.09691988e-16j  3.13250133e-16-7.09691988e-16j
  3.07987066e-16+0.00000000e+00j  3.03727784e-16+8.34238183e-17j
  3.03727784e-16-8.34238183e-17j  2.98259939e-16+0.00000000e+00j
  2.48601628e-16+5.55362637e-16j  2.48601628e-16-5.55362637e-16j
  1.29845359e-16+3.96619982e-16j  1.29845359e-16-3.96619982e-16j
  1.10158605e-16+1.10272077e-15j  1.10158605e-16-1.10272077e-15j
  9.70514820e-17+6.47755955e-16j  9.70514820e-17-6.47755955e-16j
 -3.71690555e-17+1.42385252e-16j -3.71690555e-17-1.42385252e-16j
 -8.47199563e-17+8.51939762e-16j -8.47199563e-17-8.51939762e-16j
 -1.11400046e-16+0.00000000e+00j -1.19363349e-16+0.00000000e+00j
 -1.46332897e-16+4.85766392e-16j -1.46332897e-16-4.85766392e-16j
 -2.11636041e-16+1.10182485e-15j -2.11636041e-16-1.10182485e-15j
 -3.32096470e-16+2.68153069e-16j -3.32096470e-16-2.68153069e-16j
 -3.61340096e-16+1.32219498e-16j -3.61340096e-16-1.32219498e-16j
 -3.90794825e-16+9.01329730e-16j -3.90794825e-16-9.01329730e-16j
 -3.94564948e-16+0.00000000e+00j -3.98876582e-16+7.82314798e-16j
 -3.98876582e-16-7.82314798e-16j -4.10795391e-16+0.00000000e+00j
 -4.31723638e-16+3.90084720e-16j -4.31723638e-16-3.90084720e-16j
 -4.50154557e-16+5.91146011e-16j -4.50154557e-16-5.91146011e-16j
 -6.04149563e-16+1.38925155e-15j -6.04149563e-16-1.38925155e-15j
 -6.38413471e-16+2.56707793e-16j -6.38413471e-16-2.56707793e-16j
 -7.31955468e-16+0.00000000e+00j -7.51641710e-16+3.43939956e-16j
 -7.51641710e-16-3.43939956e-16j -7.61558522e-16+6.29352918e-16j
```

```
 -7.61558522e-16-6.29352918e-16j -8.41610725e-16+2.22972477e-15j
 -8.41610725e-16-2.22972477e-15j -8.80840699e-16+0.00000000e+00j
 -8.92835498e-16+5.60883641e-16j -8.92835498e-16-5.60883641e-16j
 -9.80915301e-16+1.03010482e-15j -9.80915301e-16-1.03010482e-15j
 -9.86687344e-16+2.70132014e-16j -9.86687344e-16-2.70132014e-16j
 -1.00014093e-15+7.17696293e-17j -1.00014093e-15-7.17696293e-17j
 -1.12005165e-15+4.28808637e-16j -1.12005165e-15-4.28808637e-16j
 -1.18351126e-15+6.74634302e-16j -1.18351126e-15-6.74634302e-16j
 -1.24437767e-15+2.38580010e-16j -1.24437767e-15-2.38580010e-16j
 -1.31942567e-15+8.13473108e-17j -1.31942567e-15-8.13473108e-17j
 -1.36398414e-15+2.97112733e-16j -1.36398414e-15-2.97112733e-16j
 -1.36433626e-15+1.15204760e-15j -1.36433626e-15-1.15204760e-15j
 -1.38166895e-15+0.00000000e+00j -1.49483626e-15+3.97785627e-16j
 -1.49483626e-15-3.97785627e-16j -1.55647982e-15+6.73038183e-16j
 -1.55647982e-15-6.73038183e-16j -1.62514770e-15+5.56973447e-16j
 -1.62514770e-15-5.56973447e-16j -1.72316633e-15+0.00000000e+00j
 -1.76067889e-15+1.18497011e-16j -1.76067889e-15-1.18497011e-16j
 -1.88753989e-15+3.46982801e-16j -1.88753989e-15-3.46982801e-16j
 -1.96513673e-15+1.13759872e-15j -1.96513673e-15-1.13759872e-15j
 -1.96643371e-15+4.22896440e-16j -1.96643371e-15-4.22896440e-16j
 -2.05777110e-15+2.01740528e-16j -2.05777110e-15-2.01740528e-16j
 -2.12334717e-15+0.00000000e+00j -2.13219558e-15+1.38089803e-15j
 -2.13219558e-15-1.38089803e-15j -2.19156760e-15+0.00000000e+00j
 -2.21311563e-15+4.81213786e-16j -2.21311563e-15-4.81213786e-16j
 -2.21567521e-15+7.79161798e-16j -2.21567521e-15-7.79161798e-16j
 -2.31410760e-15+2.48396117e-16j -2.31410760e-15-2.48396117e-16j
 -2.42512104e-15+0.00000000e+00j -2.45126606e-15+3.29722172e-16j
 -2.45126606e-15-3.29722172e-16j -2.51105802e-15+6.18113333e-16j
 -2.51105802e-15-6.18113333e-16j -2.53786171e-15+0.00000000e+00j
 -2.58141602e-15+6.66797084e-16j -2.58141602e-15-6.66797084e-16j
 -2.59565163e-15+9.69513393e-16j -2.59565163e-15-9.69513393e-16j
 -2.63545404e-15+3.10820022e-16j -2.63545404e-15-3.10820022e-16j
 -2.83975426e-15+0.00000000e+00j -2.98176270e-15+1.98594429e-16j
 -2.98176270e-15-1.98594429e-16j -3.01531930e-15+8.43850172e-16j
 -3.01531930e-15-8.43850172e-16j -3.03346193e-15+0.00000000e+00j
 -3.10840029e-15+5.44234038e-16j -3.10840029e-15-5.44234038e-16j
 -3.32663259e-15+0.00000000e+00j -3.45357441e-15+1.05150320e-15j
 -3.45357441e-15-1.05150320e-15j -3.47459240e-15+3.25562142e-16j
 -3.47459240e-15-3.25562142e-16j -3.48108405e-15+1.26868448e-16j
 -3.48108405e-15-1.26868448e-16j -3.48112578e-15+5.04564252e-16j
 -3.48112578e-15-5.04564252e-16j -3.55850534e-15+7.38060557e-16j
 -3.55850534e-15-7.38060557e-16j -3.74409681e-15+5.82889445e-17j
 -3.74409681e-15-5.82889445e-17j -3.77626733e-15+4.31871204e-16j
 -3.77626733e-15-4.31871204e-16j -3.91029793e-15+1.28846802e-15j
 -3.91029793e-15-1.28846802e-15j -3.93214490e-15+0.00000000e+00j
 -4.17973872e-15+0.00000000e+00j -4.34463261e-15+4.49167980e-16j
 -4.34463261e-15-4.49167980e-16j -4.40880939e-15+2.19102629e-16j
 -4.40880939e-15-2.19102629e-16j -4.47748120e-15+7.53040943e-16j
 -4.47748120e-15-7.53040943e-16j -4.62644349e-15+0.00000000e+00j
 -4.71247943e-15+4.10095913e-16j -4.71247943e-15-4.10095913e-16j
```

```
 -5.09568925e-15+0.00000000e+00j -5.26471762e-15+8.13867195e-17j
 -5.26471762e-15-8.13867195e-17j -5.49829250e-15+0.00000000e+00j
 -5.64044956e-15+1.64958376e-16j -5.64044956e-15-1.64958376e-16j
 -5.75482927e-15+0.00000000e+00j -6.05479343e-15+3.12412891e-16j
 -6.05479343e-15-3.12412891e-16j -6.46864303e-15+0.00000000e+00j]

Top 5 Eigenvectors:
 [[ 0.00188655+0.j   0.02183558+0.j   0.02098779+0.j   0.02896678+0.j
  -0.04197795+0.j]
 [-0.00897735+0.j   0.02043084+0.j   0.0311767 +0.j   0.01295214+0.j
  -0.03380168+0.j]
 [-0.02158904+0.j   0.01456826+0.j   0.03692057+0.j   0.00274835+0.j
  -0.01384473+0.j]
 ...
 [-0.02033322+0.j -0.05212493+0.j   0.01663545+0.j   0.02190505+0.j
   0.0301225 +0.j]
 [-0.01829304+0.j -0.0542988 +0.j   0.0099443 +0.j   0.02614967+0.j
   0.02492793+0.j]
 [-0.01712649+0.j -0.05304859+0.j   0.01362308+0.j   0.01861034+0.j
   0.02102942+0.j]]
Number of Components for 95% Variance: 120
```

1. Reconstruction and Experiment:

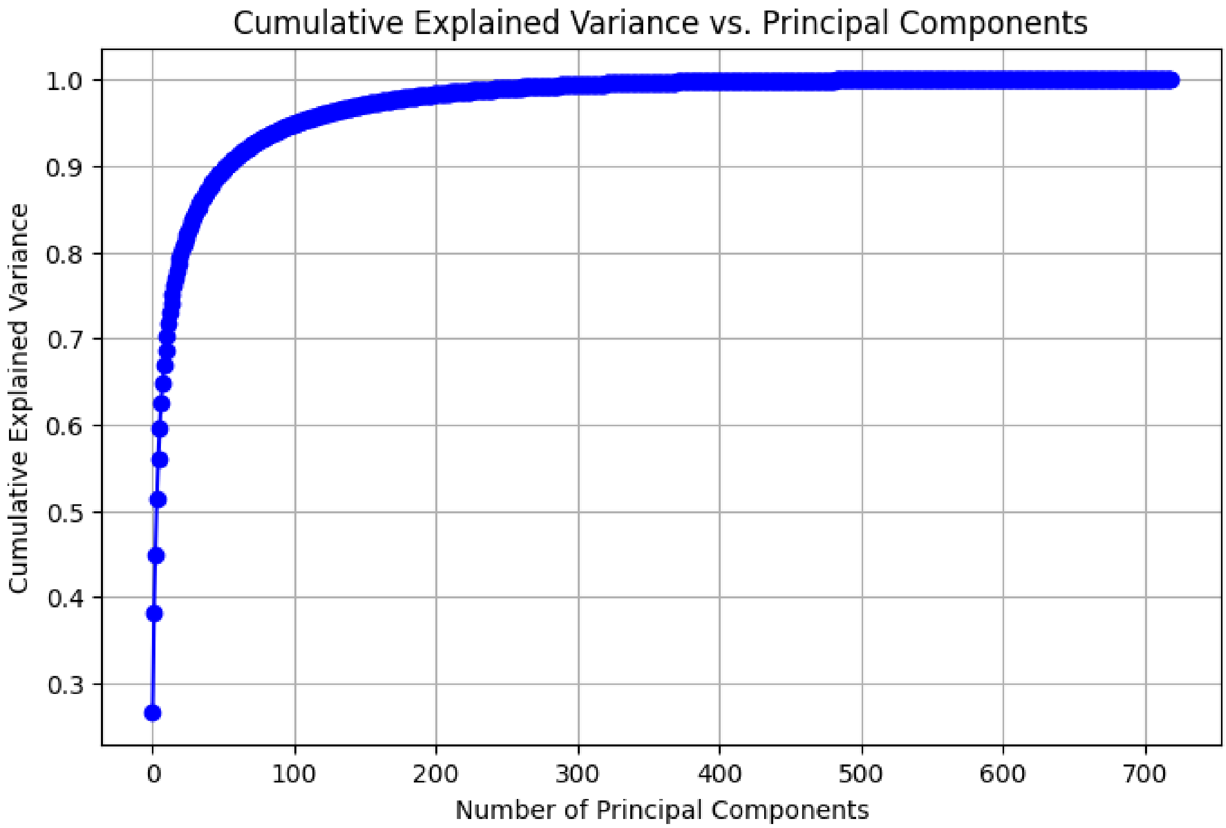
• Reconstruction: Transform the original data by multiplying it with the selected eigenvectors(PCs) to obtain a lower-dimensional representation.

• Experiments: Pick Four different combination of principal components with various ex- plained variance value and compare the result.

• Display the Results and Evaluate.

```
from PIL import Image
import numpy as np
import matplotlib.pyplot as plt
from sklearn.decomposition import PCA

# Step 1: Load and Prepare Data
image = Image.open("PeakyBlinders.jpg").convert("L")  # Convert to
grayscale
image_array = np.array(image)  # Convert image to NumPy array

# Step 2: Apply PCA and Compute Explained Variance
pca_full = PCA()  # PCA with all components
pca_full.fit(image_array)  # Fit PCA

# Cumulative explained variance
cumulative_explained_variance =
np.cumsum(pca_full.explained_variance_ratio_)
```

```
# Plot cumulative explained variance
plt.figure(figsize=(8, 5))
plt.plot(cumulative_explained_variance, marker="o", linestyle="-",
color="b")
plt.xlabel("Number of Principal Components")
plt.ylabel("Cumulative Explained Variance")
plt.title("Cumulative Explained Variance vs. Principal Components")
plt.grid()
plt.show()

# Step 3: Reconstruct and Display Images using Different Principal
Components
num_components_list = [5, 20, 50, 100]  # Different numbers of
principal components
fig, axes = plt.subplots(1, len(num_components_list) + 1, figsize=(12,
6))

# Display the original image
axes[0].imshow(image_array, cmap='gray')
axes[0].set_title("Original Image")
axes[0].axis("off")

for i, num_components in enumerate(num_components_list):
    pca = PCA(n_components=num_components)  # Apply PCA
    transformed = pca.fit_transform(image_array)  # Transform image
using PCA
    reconstructed = pca.inverse_transform(transformed)  # Reconstruct
image

    # Display reconstructed image
    axes[i + 1].imshow(reconstructed, cmap='gray')
    axes[i + 1].set_title(f"PCs: {num_components}")
    axes[i + 1].axis("off")

plt.show()
```


Cumulative Explained Variance vs. Principal Components
Cumulative Explained Variance
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0
100
200
300
400
500
600
700
Number of Principal Components


Original Image
PCs: 5
PCs: 20
PCs: 50
PCs: 100