2024 Higher Education Press Cup
China Undergraduate Mathematical Contest in Modeling

(Please read the "China Undergraduate Mathematical Contest in Modeling Paper Formatting Specifications" first)

Problem C: Crop Planting Strategies

According to the actual situation of the countryside, fully utilizing limited arable land resources, adapting to local conditions, and developing the organic planting industry have important practical significance for the sustainable development of the rural economy. Choosing suitable crops and optimizing planting strategies are conducive to facilitating field management, improving production efficiency, and reducing planting risks caused by various uncertain factors.

A specific village is located in the mountainous area of North China. The temperature is generally low year-round, and most arable land can only grow one season of crops per year. The village currently has 1201 *mu* (Chinese acres) of open-air arable land, dispersed into 34 plots of different sizes, including four types: **Flat Dry Land**, **Terraced Fields**, **Hill Slopes**, and **Irrigated Land**. The Flat Dry Land, Terraced Fields, and Hill Slopes are suitable for planting one season of grain crops annually; the Irrigated Land is suitable for planting one season of rice or two seasons of vegetables annually.

The village also possesses 16 **Ordinary Greenhouses** and 4 **Smart Greenhouses**, with a cultivated area of 0.6 *mu* per greenhouse. Ordinary Greenhouses are suitable for planting one season of vegetables and one season of edible fungi annually. Smart Greenhouses are suitable for planting two seasons of vegetables annually. Different crops can be intercropped within the same plot in the same season. Details are provided in **Annex 1**.

According to the growth laws of crops, no crop can be planted continuously (continuous cropping) on the same plot (including greenhouses); otherwise, yield will decrease. Because soil containing root bacteria from leguminous crops is beneficial to the growth of other crops, starting from 2023, it is required that **every plot of land must plant leguminous crops at least once within any three-year period**. Simultaneously, the planting plan should consider the convenience of farming operations and field management. For example: the planting areas for each crop in each season should not be too dispersed, and the planting area for each crop in a single plot (including greenhouses) should not be too small, etc. The crop planting and related statistical data for 2023 are provided in **Annex 2**.

Please establish mathematical models to research the following problems:

Problem 1

Assume that the future expected sales volume, planting costs, yield per *mu*, and selling prices of various crops remain stable relative to 2023. Crops planted each season are sold in the current season. If the total yield of a certain crop in a season exceeds the corresponding

expected sales volume, the excess part cannot be sold normally. Please provide the optimal crop planting plan for this village for the years **2024–2030** for the following two cases, and fill the results into `result1_1.xlsx` and `result1_2.xlsx` respectively (template files are provided in Annex 3).

(1) The excess portion is unsold and becomes waste;

(2) The excess portion is sold at a reduced price of 50% of the 2023 selling price.

Problem 2

Based on experience, the expected sales volume of wheat and corn shows an increasing trend, with an average annual growth rate between 5% and 10%. The expected sales volume of other crops varies by approximately ±5% annually relative to 2023. Crop yields per *mu* are often affected by climate and other factors, with an annual variation of ±10%. Due to market conditions, crop planting costs increase by an average of 5% per year. The selling prices of grain crops are basically stable; the selling prices of vegetable crops show an increasing trend, with an average annual increase of about 5%. The selling prices of edible fungi are stable with a slight decline, decreasing by approximately 1% to 5% annually; in particular, the selling price of Morel mushrooms decreases by 5% annually.

Please comprehensively consider the uncertainty of expected sales volume, yield per *mu*, planting costs, and selling prices of various crops, as well as potential planting risks. Provide the optimal crop planting plan for this village for the years **2024–2030**, and fill the results into `result2.xlsx` (template file provided in Annex 3).

Problem 3

In real life, there may be certain substitutability and complementarity among various crops, and there is also a certain correlation between expected sales volume, selling price, and planting costs. Please comprehensively consider relevant factors based on Problem 2, provide the optimal crop planting strategy for this village for the years **2024–2030**, solve it through simulated data, and compare and analyze the results with those of Problem 2.

Annex 1: Basic situation of existing arable land and crops in the village
Annex 2: 2023 Village crop planting and related statistical data
Annex 3: Template files for submission of results (`result1_1.xlsx`, `result1_2.xlsx`, `result2.xlsx`)