Assignment 7

1.

$np = 270 \times 1/3 = 90$, $npq = 270 \times 1/3 \times 2/3 = 60$	**B1**	Correct unsimplified np and npq, SOI
$\mathrm{P}(x > 100) = \mathrm{P}\left(z > \dfrac{99.5-90}{\sqrt{60}}\right) = \mathrm{P}(z > 1.2264)$	**M1** **M1**	±Standardising using 100 need sq rt Continuity correction, 99.5 or 100.5 used
$= 1 - 0.8899$	**M1**	Correct area $1 - \Phi$ implied by final prob. < 0.5
$= 0.110$	**A1**	
Total:	**5**	

2.

(a)	$\mathrm{P}(x > 0) = \mathrm{P}\left(z > \pm\dfrac{0-\mu}{\sigma}\right)$ $= \mathrm{P}\left(z > \dfrac{-\mu}{\mu/1.5}\right)$ or $\mathrm{P}\left(z > \dfrac{-1.5\sigma}{\sigma}\right)$	**M1**	±Standardising, in terms of μ and/or σ with 0 - …. in numerator, no continuity correction, no √
	$= \mathrm{P}(z > -1.5)$	**A1**	Obtaining z value of ±1.5 by eliminating μ and σ, SOI
	$= 0.933$	**A1**	
	Total:	**3**	
(b)	$z = -1.151$	**B1**	± z value rounding to 1.1 or 1.2
	$-1.151 = \dfrac{70-120}{s}$	**M1**	± Standardising (using 70) equated to a z-value, no cc, no squaring, no √
	$\sigma = 43.4$ or 43.5	**A1**	
	Totals:	**3**	

3.

(a)	$z_1 = 2.4$	**B1**	± 2.4 seen accept 2.396
	$z_2 = -0.5$	**B1**	± 0.5 seen
	$2.4 = \dfrac{36800-\mu}{\sigma}$	**M1**	Either standardisation eqn with z value, not 0.5082, 0.7565, 0.0082, 0.6915, 0.3085, 0.6209, 0.0032 or any other probability
	$-0.5 = \dfrac{31000-\mu}{\sigma}$	**M1**	Sensible attempt to eliminate μ or σ by substitution or subtraction from their 2 equations (z-value not required), need at least 1 value stated
	$\sigma = 2000$ $\mu = 32000$	**A1**	Both correct answers
		5	
(b)	$\mathrm{P}(X < 3\mu) = \mathrm{P}\left(z < \dfrac{3\mu-\mu}{(4\mu/3)}\right)$ or $\mathrm{P} = \left(z < \dfrac{(9\sigma/4)-(3\sigma/4)}{\sigma}\right)$	**M1**	Standardise, in terms of one variable, accept σ^2 or $\sqrt{\sigma}$
	$\mathrm{P}(z < \frac{6}{4})$	**M1**	$\frac{6}{4}$ or $\frac{6}{4\sigma}$ seen
	$= 0.933$	**A1**	Correct final answer
		3	

4.

3(i)	$z = -1.282$	**B1**	±1.282 seen
	$-1.282 = \dfrac{440-\mu}{9}$	**M1**	±Standardisation equation with 440, 9 and μ, equated to a z-value, (not 1 – z-value or probability e.g. 0.1841, 0.5398, 0.6202, 0.8159)
	$\mu = 452$	**A1**	Correct answer rounding to 452, not dependent on B1
		3	
(ii)	$P(z > 1.8) = 1 - 0.9641 = 0.0359$	**B1**	
	Number $= 0.0359 \times 150$ $= 5.385$	**M1**	$p \times 150$, $0 < p < 1$
	(Number of cartons =) 5	**A1FT**	Accept either 5 or 6, not indicated as an approximation, e.g. ~, about **FT** *their* $p \times 150$, answer as an integer
		3	

5.

5(i)	$z_1 = \pm\dfrac{4.1-5.7}{0.8} = -2$ $\quad z_2 = \pm\dfrac{5-5.7}{0.8} = -0.875$	**M1**	At least one standardising no cc no sq rt no sq using 5.7 and 0.8 and either 4.1 or 5
	P(Toffee Apple) $= P(d < 5.0) - P(d < 4.1)$ $= P(z < -0.875) - P(z < -2)$ $= \Phi(-0.875) - \Phi(-2)$ $= \Phi(2) - \Phi(0.875)$	**M1**	Correct area Φ – Φ legitimately obtained – need 2 negative z-values or 2 positives – not one of each
	$= 0.9772 - 0.8092 = 0.168$ (or $0.1908 - 0.0228$)	**A1**	Correct final answer
	Total:	**3**	
(ii)	$np = 250 \times 0.168 = 42$, $\quad npq = 34.944$	**B1ft**	Correct unsimplified mean and var – ft their prob for **(i)** providing $(0 < p < 1)$ Implied by $\sigma = \sqrt{34.944} = 5.911$
	$P(< 50) = P\left(z < \dfrac{49.5-42}{\sqrt{34.944}}\right) = P(z < 1.2687)$	**M1**	± Standardising using 50, their mean and sd; must have sq rt.
		M1	49.5 or 50.5 seen as a cc
	$= \Phi(1.2687)$	**M1**	Correct area Φ(> 0.5 for + z and < 0.5 for –z)in their final answer
	$= 0.898$	**A1**	Correct final answer
	Total:	**5**	

6.

7(i)	$P(< 700) = P\left(z < \dfrac{700-830}{120}\right) = P(z < -1.083)$	**M1**	Using ± standardisation formula, no continuity correction, not σ^2 or $\sqrt{\sigma}$
	$= 1 - 0.8606$	**M1**	Appropriate area Φ from standardisation formula P(z<….) in final probability solution, (<0.5 if z is –ve, >0.5 if z is +ve)
	$= 0.1394$	**A1**	Correct final probability rounding to 0.139
	Expected number of female adults = 430 × *their* 0.1394 = 59.9 So 59 or 60	**B1**	**FT** *their* 3 or 4 SF probability, rounded or truncated to integer
		4	

7(ii)	P(giraffe < 830+w) = 95% so $z = 1.645$	**B1**	±1.645 seen (critical value)
	$\dfrac{(830+w)-830}{120} = \dfrac{w}{120} = 1.645$	**M1**	An equation using the standardisation formula with a z-value (not 1 – z), condone σ^2 or $\sqrt{\sigma}$ not 0.8519, 0.8289
	$w = 197$	**A1**	Correct answer
		3	
(iii)	P(male > 950) = 0.834, so $z = -0.97$	**B1**	± 0.97 seen
	$\dfrac{950-1190}{\sigma} = -0.97$	**M1**	Using ± standardisation formula, condone continuity correction, σ^2 or $\sqrt{\sigma}$, condone equating with non z-value not 0.834, 0.166
	$\sigma = 247$	**A1**	Condone $-\sigma = -247$. www.
		3	

7.

$z = 0.842 = \left(\dfrac{121-\mu}{\sigma}\right)$ so $0.842\sigma = 121 - \mu$	**B1**	$\pm$ 0.842 seen but B0 if 1 $\pm$ 0.842 oe seen
	M1	One appropriate standardisation equation with a z-value, μ, σ and 121 **or** 102, condone continuity correction. Not 0.158, 0.42,…
$z = -0.58 = \left(\dfrac{102-\mu}{\sigma}\right)$ so $-0.58\sigma = 102 - \mu$	**B1**	$\pm$ 0.58(0) seen but B0 if 1 $\pm$ 0.58 oe seen
Solving	**M1**	Correct algebraic elimination of μ or σ from *their* two simultaneous equations to form an equation in one variable, condone 1 numerical slip
$\sigma = 13.4$ $\mu = 110$	**A1**	If M0A0 scored (i.e. no algebraic elimination seen), **SC B1** can be awarded for both answers correct Consistent use of σ^2 or $\sqrt{\sigma}$ throughout apply **MR** penalty to A mark or SC B mark.
	5	

8.

(i)	$P(<570) = P\left(z < \dfrac{570-500}{91.5}\right) = P(z < 0.7650)$ $= 0.7779$	**M1**	Standardising for either 570 or 390, no cc, no sq, no $\sqrt{}$
	$P(<390) = P\left(z < \dfrac{390-500}{91.5}\right) = P(z < -1.202)$	**A1**	One correct z value
	$= 1 - 0.8853 = 0.1147$	**A1**	One correct Φ, final solution
	Large:0.222 (0.2221) Small: 0.115 (0.1147)	**A1**	Correct small and large
	Medium: 0.663 (0.6632)	**A1FT**	Correct Medium rounding to 0.66 or ft 1 – (their small + their large)
		5	

'(ii)	$1.645 = \left(\dfrac{x-500}{91.5}\right)$	**B1**	$\pm$ 1.645 seen (critical value)
		M1	Standardising accept cc, sq, sq rt
	$x = 651$	**A1**	$650 \leqslant \text{Ans} \leqslant 651$
		3	
(iii)	$P(x > 610) = 0.1147$ (symmetry)	**M1**	Attempt to find upper end prob $x > 610$ or $\Phi(x)$, ft their $P(< 390)$ from **(i)**
	$0.3 + 0.1147 = 0.4147 \Rightarrow \Phi(x) = 0.5853$	**M1**	Adding 0.3 to *their* $P(x > 610)$ or subt 0.5 from $\Phi(x)$ or $0.8853 - 0.3$
	$z = 0.215$ or 0.216	**M1**	Finding $z = \Phi^{-1}(0.5853)$
	$0.215 = \dfrac{k-500}{91.5}$	**M1**	Standardising and solving, accept cc, sq, sq rt
	$k = 520$	**A1**	
		5	