UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2013**

1 hour 45 minutes

4064014490

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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JC13 06_9709_31/FP

UNIVERSITY *of* CAMBRIDGE
International Examinations

[Turn over

1 Find the quotient and remainder when $2x^2$ is divided by $x + 2$. [3]

2 Expand $\dfrac{1 + 3x}{\sqrt{(1 + 2x)}}$ in ascending powers of x up to and including the term in x^2, simplifying the coefficients. [4]

3 Express $\dfrac{7x^2 - 3x + 2}{x(x^2 + 1)}$ in partial fractions. [5]

4 **(i)** Solve the equation $|4x - 1| = |x - 3|$. [3]

(ii) Hence solve the equation $\left|4^{y+1} - 1\right| = |4^y - 3|$ correct to 3 significant figures. [3]

5 For each of the following curves, find the gradient at the point where the curve crosses the y-axis:

(i) $y = \dfrac{1 + x^2}{1 + e^{2x}}$; [3]

(ii) $2x^3 + 5xy + y^3 = 8$. [4]

6 The points P and Q have position vectors, relative to the origin O, given by

$$\overrightarrow{OP} = 7\mathbf{i} + 7\mathbf{j} - 5\mathbf{k} \quad \text{and} \quad \overrightarrow{OQ} = -5\mathbf{i} + \mathbf{j} + \mathbf{k}.$$

The mid-point of PQ is the point A. The plane Π is perpendicular to the line PQ and passes through A.

(i) Find the equation of Π, giving your answer in the form $ax + by + cz = d$. [4]

(ii) The straight line through P parallel to the x-axis meets Π at the point B. Find the distance AB, correct to 3 significant figures. [5]

7 **(a)** Without using a calculator, solve the equation

$$3w + 2\mathrm{i}w^* = 17 + 8\mathrm{i},$$

where w^* denotes the complex conjugate of w. Give your answer in the form $a + b\mathrm{i}$. [4]

(b) In an Argand diagram, the loci

$$\arg(z - 2\mathrm{i}) = \tfrac{1}{6}\pi \quad \text{and} \quad |z - 3| = |z - 3\mathrm{i}|$$

intersect at the point P. Express the complex number represented by P in the form $r\mathrm{e}^{\mathrm{i}\theta}$, giving the exact value of θ and the value of r correct to 3 significant figures. [5]

8 **(a)** Show that $\displaystyle\int_2^4 4x \ln x \, \mathrm{d}x = 56 \ln 2 - 12$. [5]

(b) Use the substitution $u = \sin 4x$ to find the exact value of $\displaystyle\int_0^{\frac{1}{24}\pi} \cos^3 4x \, \mathrm{d}x$. [5]

9 **(i)** Express $4\cos\theta + 3\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the value of α correct to 4 decimal places. [3]

(ii) Hence

(a) solve the equation $4\cos\theta + 3\sin\theta = 2$ for $0 < \theta < 2\pi$, [4]

(b) find $\displaystyle\int \frac{50}{(4\cos\theta + 3\sin\theta)^2}\,d\theta$. [3]

10 Liquid is flowing into a small tank which has a leak. Initially the tank is empty and, t minutes later, the volume of liquid in the tank is $V\,\text{cm}^3$. The liquid is flowing into the tank at a constant rate of $80\,\text{cm}^3$ per minute. Because of the leak, liquid is being lost from the tank at a rate which, at any instant, is equal to $kV\,\text{cm}^3$ per minute where k is a positive constant.

(i) Write down a differential equation describing this situation and solve it to show that

$$V = \frac{1}{k}(80 - 80e^{-kt}).$$ [7]

(ii) It is observed that $V = 500$ when $t = 15$, so that k satisfies the equation

$$k = \frac{4 - 4e^{-15k}}{25}.$$

Use an iterative formula, based on this equation, to find the value of k correct to 2 significant figures. Use an initial value of $k = 0.1$ and show the result of each iteration to 4 significant figures. [3]

(iii) Determine how much liquid there is in the tank 20 minutes after the liquid started flowing, and state what happens to the volume of liquid in the tank after a long time. [2]

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MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2013**

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1 Solve the equation $|x-2| = \left|\frac{1}{3}x\right|$. [3]

2 The sequence of values given by the iterative formula

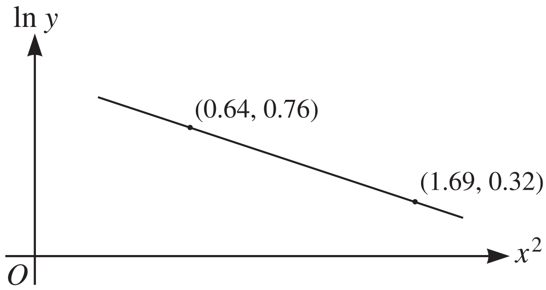
$$x_{n+1} = \frac{x_n(x_n^3 + 100)}{2(x_n^3 + 25)},$$

with initial value $x_1 = 3.5$, converges to α.

(i) Use this formula to calculate α correct to 4 decimal places, showing the result of each iteration to 6 decimal places. [3]

(ii) State an equation satisfied by α and hence find the exact value of α. [2]

3



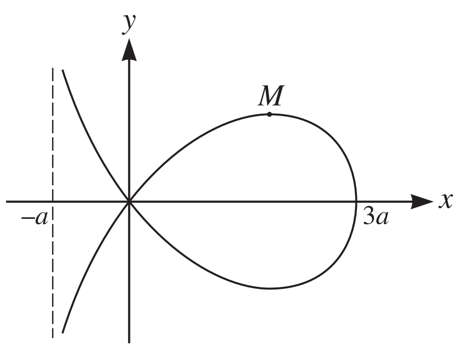
The variables x and y satisfy the equation $y = Ae^{-kx^2}$, where A and k are constants. The graph of $\ln y$ against x^2 is a straight line passing through the points (0.64, 0.76) and (1.69, 0.32), as shown in the diagram. Find the values of A and k correct to 2 decimal places. [5]

4 The polynomial $ax^3 - 20x^2 + x + 3$, where a is a constant, is denoted by $p(x)$. It is given that $(3x+1)$ is a factor of $p(x)$.

(i) Find the value of a. [3]

(ii) When a has this value, factorise $p(x)$ completely. [3]

5



The diagram shows the curve with equation

$$x^3 + xy^2 + ay^2 - 3ax^2 = 0,$$

where a is a positive constant. The maximum point on the curve is M. Find the x-coordinate of M in terms of a. [6]

6 **(i)** By differentiating $\frac{1}{\cos x}$, show that the derivative of $\sec x$ is $\sec x \tan x$. Hence show that if $y = \ln(\sec x + \tan x)$ then $\frac{dy}{dx} = \sec x$. [4]

(ii) Using the substitution $x = (\sqrt{3}) \tan \theta$, find the exact value of

$$\int_1^3 \frac{1}{\sqrt{(3 + x^2)}} \, dx,$$

expressing your answer as a single logarithm. [4]

7 **(i)** By first expanding $\cos(x + 45°)$, express $\cos(x + 45°) - (\sqrt{2}) \sin x$ in the form $R \cos(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the value of R correct to 4 significant figures and the value of α correct to 2 decimal places. [5]

(ii) Hence solve the equation

$$\cos(x + 45°) - (\sqrt{2}) \sin x = 2,$$

for $0° < x < 360°$. [4]

8 **(i)** Express $\frac{1}{x^2(2x + 1)}$ in the form $\frac{A}{x^2} + \frac{B}{x} + \frac{C}{2x + 1}$. [4]

(ii) The variables x and y satisfy the differential equation

$$y = x^2(2x + 1)\frac{dy}{dx},$$

and $y = 1$ when $x = 1$. Solve the differential equation and find the exact value of y when $x = 2$. Give your value of y in a form not involving logarithms. [7]

9 **(a)** The complex number w is such that $\text{Re}\, w > 0$ and $w + 3w^* = iw^2$, where w^* denotes the complex conjugate of w. Find w, giving your answer in the form $x + iy$, where x and y are real. [5]

(b) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z which satisfy both the inequalities $|z - 2i| \leqslant 2$ and $0 \leqslant \arg(z + 2) \leqslant \frac{1}{4}\pi$. Calculate the greatest value of $|z|$ for points in this region, giving your answer correct to 2 decimal places. [6]

10 The points A and B have position vectors $2\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ and $5\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ respectively. The plane p has equation $x + y = 5$.

(i) Find the position vector of the point of intersection of the line through A and B and the plane p. [4]

(ii) A second plane q has an equation of the form $x + by + cz = d$, where b, c and d are constants. The plane q contains the line AB, and the acute angle between the planes p and q is $60°$. Find the equation of q. [7]

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[Turn over

1 Solve the inequality $|4x + 3| > |x|$. [4]

2 It is given that $\ln(y + 1) - \ln y = 1 + 3\ln x$. Express y in terms of x, in a form not involving logarithms. [4]

3 Solve the equation $\tan 2x = 5\cot x$, for $0° < x < 180°$. [5]

4 **(i)** Express $(\sqrt{3})\cos x + \sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$, giving the exact values of R and α. [3]

(ii) Hence show that

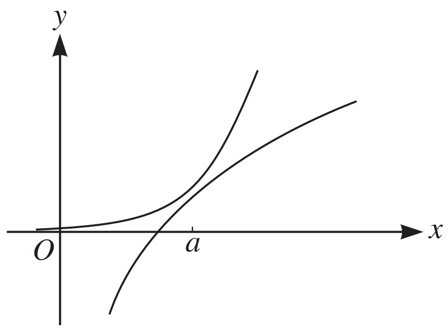
$$\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} \frac{1}{\left((\sqrt{3})\cos x + \sin x\right)^2}\,dx = \frac{1}{4}\sqrt{3}.$$ [4]

5 The polynomial $8x^3 + ax^2 + bx + 3$, where a and b are constants, is denoted by $p(x)$. It is given that $(2x + 1)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(2x - 1)$ the remainder is 1.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, find the remainder when $p(x)$ is divided by $2x^2 - 1$. [3]

6



The diagram shows the curves $y = e^{2x-3}$ and $y = 2\ln x$. When $x = a$ the tangents to the curves are parallel.

(i) Show that a satisfies the equation $a = \frac{1}{2}(3 - \ln a)$. [3]

(ii) Verify by calculation that this equation has a root between 1 and 2. [2]

(iii) Use the iterative formula $a_{n+1} = \frac{1}{2}(3 - \ln a_n)$ to calculate a correct to 2 decimal places, showing the result of each iteration to 4 decimal places. [3]

7 The complex number z is defined by $z = a + \mathrm{i}b$, where a and b are real. The complex conjugate of z is denoted by z^*.

(i) Show that $|z|^2 = zz^*$ and that $(z - k\mathrm{i})^* = z^* + k\mathrm{i}$, where k is real. [2]

In an Argand diagram a set of points representing complex numbers z is defined by the equation $|z - 10\mathrm{i}| = 2|z - 4\mathrm{i}|$.

(ii) Show, by squaring both sides, that

$$zz^* - 2\mathrm{i}z^* + 2\mathrm{i}z - 12 = 0.$$

Hence show that $|z - 2\mathrm{i}| = 4$. [5]

(iii) Describe the set of points geometrically. [1]

8 The variables x and t satisfy the differential equation

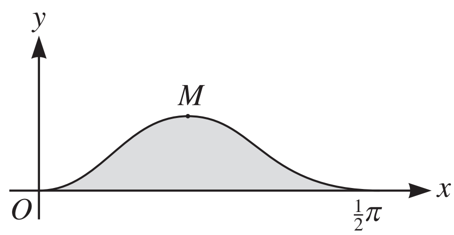
$$t\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{k - x^3}{2x^2},$$

for $t > 0$, where k is a constant. When $t = 1$, $x = 1$ and when $t = 4$, $x = 2$.

(i) Solve the differential equation, finding the value of k and obtaining an expression for x in terms of t. [9]

(ii) State what happens to the value of x as t becomes large. [1]

9



The diagram shows the curve $y = \sin^2 2x \cos x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M.

(i) Find the x-coordinate of M. [6]

(ii) Using the substitution $u = \sin x$, find by integration the area of the shaded region bounded by the curve and the x-axis. [4]

10 The line l has equation $\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + \lambda(a\mathbf{i} + 2\mathbf{j} + \mathbf{k})$, where a is a constant. The plane p has equation $x + 2y + 2z = 6$. Find the value or values of a in each of the following cases.

(i) The line l is parallel to the plane p. [2]

(ii) The line l intersects the line passing through the points with position vectors $3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and $\mathbf{i} + \mathbf{j} - \mathbf{k}$. [4]

(iii) The acute angle between the line l and the plane p is $\tan^{-1} 2$. [5]

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1 **(i)** Simplify $\sin 2\alpha \sec \alpha$. [2]

(ii) Given that $3\cos 2\beta + 7\cos\beta = 0$, find the exact value of $\cos\beta$. [3]

2 Use the substitution $u = 1 + 3\tan x$ to find the exact value of

$$\int_0^{\frac{1}{4}\pi} \frac{\sqrt{(1 + 3\tan x)}}{\cos^2 x}\,dx.$$ [5]

3 The parametric equations of a curve are

$$x = \ln(2t + 3), \quad y = \frac{3t + 2}{2t + 3}.$$

Find the gradient of the curve at the point where it crosses the y-axis. [6]

4 The variables x and y are related by the differential equation

$$\frac{dy}{dx} = \frac{6ye^{3x}}{2 + e^{3x}}.$$

Given that $y = 36$ when $x = 0$, find an expression for y in terms of x. [6]

5 The complex number z is defined by $z = \dfrac{9\sqrt{3} + 9i}{\sqrt{3} - i}$. Find, showing all your working,

(i) an expression for z in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$, [5]

(ii) the two square roots of z, giving your answers in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. [3]

6 It is given that $2\ln(4x - 5) + \ln(x + 1) = 3\ln 3$.

(i) Show that $16x^3 - 24x^2 - 15x - 2 = 0$. [3]

(ii) By first using the factor theorem, factorise $16x^3 - 24x^2 - 15x - 2$ completely. [4]

(iii) Hence solve the equation $2\ln(4x - 5) + \ln(x + 1) = 3\ln 3$. [1]

7 The straight line l has equation $\mathbf{r} = 4\mathbf{i} - \mathbf{j} + 2\mathbf{k} + \lambda(2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k})$. The plane p passes through the point $(4, -1, 2)$ and is perpendicular to l.

(i) Find the equation of p, giving your answer in the form $ax + by + cz = d$. [2]

(ii) Find the perpendicular distance from the origin to p. [3]

(iii) A second plane q is parallel to p and the perpendicular distance between p and q is 14 units. Find the possible equations of q. [3]

8 **(i)** By sketching each of the graphs $y = \operatorname{cosec} x$ and $y = x(\pi - x)$ for $0 < x < \pi$, show that the equation

$$\operatorname{cosec} x = x(\pi - x)$$

has exactly two real roots in the interval $0 < x < \pi$. [3]

(ii) Show that the equation $\operatorname{cosec} x = x(\pi - x)$ can be written in the form $x = \dfrac{1 + x^2 \sin x}{\pi \sin x}$. [2]

(iii) The two real roots of the equation $\operatorname{cosec} x = x(\pi - x)$ in the interval $0 < x < \pi$ are denoted by α and β, where $\alpha < \beta$.

(a) Use the iterative formula

$$x_{n+1} = \frac{1 + x_n^2 \sin x_n}{\pi \sin x_n}$$

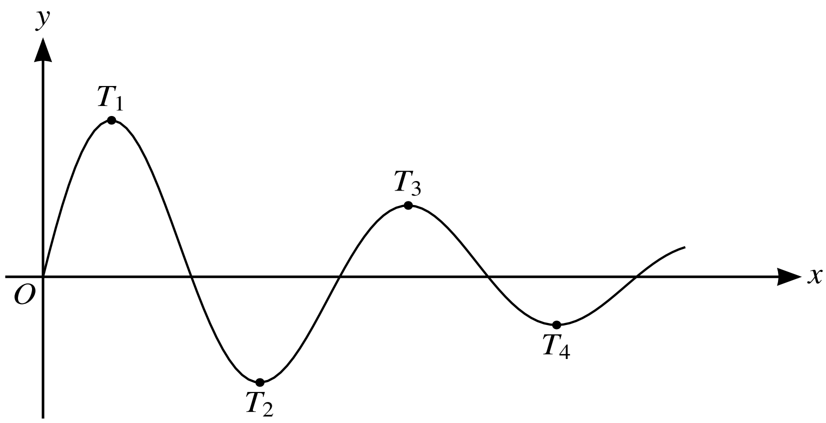
to find α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

(b) Deduce the value of β correct to 2 decimal places. [1]

9 **(i)** Express $\dfrac{4 + 12x + x^2}{(3 - x)(1 + 2x)^2}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\dfrac{4 + 12x + x^2}{(3 - x)(1 + 2x)^2}$ in ascending powers of x, up to and including the term in x^2. [5]

10



The diagram shows the curve $y = 10e^{-\frac{1}{2}x} \sin 4x$ for $x \geqslant 0$. The stationary points are labelled T_1, T_2, T_3, … as shown.

(i) Find the x-coordinates of T_1 and T_2, giving each x-coordinate correct to 3 decimal places. [6]

(ii) It is given that the x-coordinate of T_n is greater than 25. Find the least possible value of n. [4]

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1 Find the set of values of x satisfying the inequality

$$|x + 2a| > 3|x - a|,$$

where a is a positive constant. [4]

2 Solve the equation

$$2\ln(5 - e^{-2x}) = 1,$$

giving your answer correct to 3 significant figures. [4]

3 Solve the equation

$$\cos(x + 30°) = 2\cos x,$$

giving all solutions in the interval $-180° < x < 180°$. [5]

4 The parametric equations of a curve are

$$x = t - \tan t, \quad y = \ln(\cos t),$$

for $-\frac{1}{2}\pi < t < \frac{1}{2}\pi$.

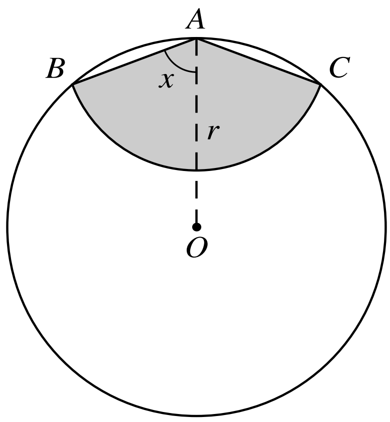
(i) Show that $\frac{dy}{dx} = \cot t$. [5]

(ii) Hence find the x-coordinate of the point on the curve at which the gradient is equal to 2. Give your answer correct to 3 significant figures. [2]

5 **(i)** The polynomial $f(x)$ is of the form $(x - 2)^2 g(x)$, where $g(x)$ is another polynomial. Show that $(x - 2)$ is a factor of $f'(x)$. [2]

(ii) The polynomial $x^5 + ax^4 + 3x^3 + bx^2 + a$, where a and b are constants, has a factor $(x - 2)^2$. Using the factor theorem and the result of part **(i)**, or otherwise, find the values of a and b. [5]

6



In the diagram, A is a point on the circumference of a circle with centre O and radius r. A circular arc with centre A meets the circumference at B and C. The angle OAB is equal to x radians. The shaded region is bounded by AB, AC and the circular arc with centre A joining B and C. The perimeter of the shaded region is equal to half the circumference of the circle.

(i) Show that $x = \cos^{-1}\left(\dfrac{\pi}{4 + 4x}\right)$. [3]

(ii) Verify by calculation that x lies between 1 and 1.5. [2]

(iii) Use the iterative formula

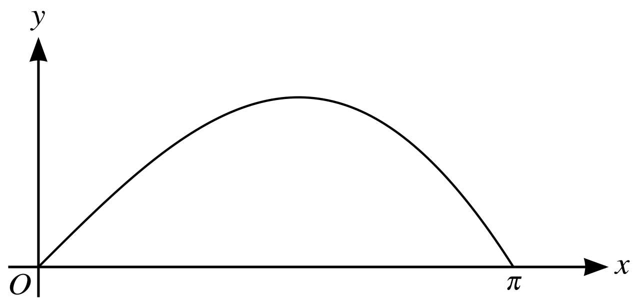
$$x_{n+1} = \cos^{-1}\left(\frac{\pi}{4 + 4x_n}\right)$$

to determine the value of x correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 **(a)** It is given that $-1 + (\sqrt{5})\mathrm{i}$ is a root of the equation $z^3 + 2z + a = 0$, where a is real. Showing your working, find the value of a, and write down the other complex root of this equation. [4]

(b) The complex number w has modulus 1 and argument 2θ radians. Show that $\dfrac{w-1}{w+1} = \mathrm{i}\tan\theta$. [4]

8



The diagram shows the curve $y = x\cos\frac{1}{2}x$ for $0 \leqslant x \leqslant \pi$.

(i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and show that $4\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} + y + 4\sin\frac{1}{2}x = 0$. [5]

(ii) Find the exact value of the area of the region enclosed by this part of the curve and the x-axis. [5]

9 The population of a country at time t years is N millions. At any time, N is assumed to increase at a rate proportional to the product of N and $(1 - 0.01N)$. When $t = 0$, $N = 20$ and $\dfrac{dN}{dt} = 0.32$.

(i) Treating N and t as continuous variables, show that they satisfy the differential equation

$$\frac{dN}{dt} = 0.02N(1 - 0.01N).$$ [1]

(ii) Solve the differential equation, obtaining an expression for t in terms of N. [8]

(iii) Find the time at which the population will be double its value at $t = 0$. [1]

10 Referred to the origin O, the points A, B and C have position vectors given by

$$\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}, \quad \overrightarrow{OB} = 2\mathbf{i} + 4\mathbf{j} + \mathbf{k} \quad \text{and} \quad \overrightarrow{OC} = 3\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}.$$

(i) Find the exact value of the cosine of angle BAC. [4]

(ii) Hence find the exact value of the area of triangle ABC. [3]

(iii) Find the equation of the plane which is parallel to the y-axis and contains the line through B and C. Give your answer in the form $ax + by + cz = d$. [5]

Cambridge International A Level

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MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2014**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

7019745292

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
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The total number of marks for this paper is 75.
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This document consists of **4** printed pages.

JC14 06_9709_33/FP

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[Turn over

1 Solve the equation $\log_{10}(x+9) = 2 + \log_{10} x$. [3]

2 Expand $(1 + 3x)^{-\frac{1}{3}}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients. [4]

3 (i) Show that the equation

$$\tan(x - 60°) + \cot x = \sqrt{3}$$

can be written in the form

$$2\tan^2 x + (\sqrt{3})\tan x - 1 = 0.$$ [3]

(ii) Hence solve the equation

$$\tan(x - 60°) + \cot x = \sqrt{3},$$

for $0° < x < 180°$. [3]

4 The equation $x = \dfrac{10}{e^{2x} - 1}$ has one positive real root, denoted by α.

(i) Show that α lies between $x = 1$ and $x = 2$. [2]

(ii) Show that if a sequence of positive values given by the iterative formula

$$x_{n+1} = \tfrac{1}{2}\ln\left(1 + \frac{10}{x_n}\right)$$

converges, then it converges to α. [2]

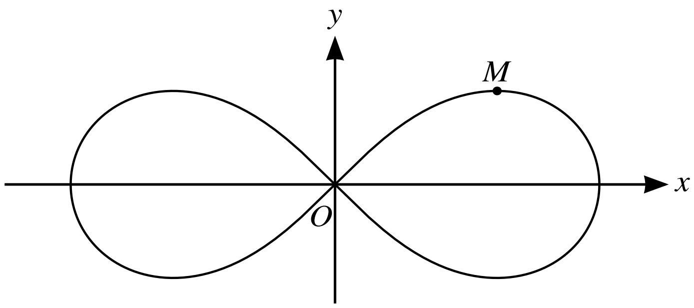
(iii) Use this iterative formula to determine α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

5 The variables x and θ satisfy the differential equation

$$2\cos^2\theta \frac{dx}{d\theta} = \sqrt{(2x + 1)},$$

and $x = 0$ when $\theta = \frac{1}{4}\pi$. Solve the differential equation and obtain an expression for x in terms of θ. [7]

6



The diagram shows the curve $\left(x^2 + y^2\right)^2 = 2\left(x^2 - y^2\right)$ and one of its maximum points M. Find the coordinates of M. [7]

7 **(a)** The complex number $\dfrac{3 - 5\mathrm{i}}{1 + 4\mathrm{i}}$ is denoted by u. Showing your working, express u in the form $x + \mathrm{i}y$, where x and y are real. [3]

(b) **(i)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers satisfying the inequalities $|z - 2 - \mathrm{i}| \leqslant 1$ and $|z - \mathrm{i}| \leqslant |z - 2|$. [4]

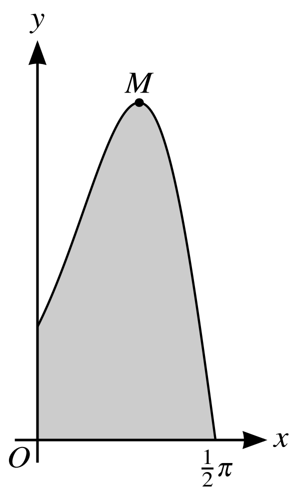
(ii) Calculate the maximum value of $\arg z$ for points lying in the shaded region. [2]

8 Let $\mathrm{f}(x) = \dfrac{6 + 6x}{(2 - x)(2 + x^2)}$.

(i) Express $\mathrm{f}(x)$ in the form $\dfrac{A}{2 - x} + \dfrac{Bx + C}{2 + x^2}$. [4]

(ii) Show that $\displaystyle\int_{-1}^{1} \mathrm{f}(x)\,\mathrm{d}x = 3\ln 3$. [5]

9



The diagram shows the curve $y = \mathrm{e}^{2\sin x}\cos x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M.

(i) Using the substitution $u = \sin x$, find the exact value of the area of the shaded region bounded by the curve and the axes. [5]

(ii) Find the x-coordinate of M, giving your answer correct to 3 decimal places. [6]

10 The line l has equation $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - \mathbf{k} + \lambda(3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$ and the plane p has equation $2x + 3y - 5z = 18$.

(i) Find the position vector of the point of intersection of l and p. [3]

(ii) Find the acute angle between l and p. [4]

(iii) A second plane q is perpendicular to the plane p and contains the line l. Find the equation of q, giving your answer in the form $ax + by + cz = d$. [5]

Cambridge International Examinations
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MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2015**

1 hour 45 minutes

8254149865

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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JC15 06_9709_31/2R

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[Turn over

1 Use logarithms to solve the equation $2^{5x} = 3^{2x+1}$, giving the answer correct to 3 significant figures. [4]

2 Use the trapezium rule with three intervals to find an approximation to

$$\int_0^3 |3^x - 10| \, dx. \qquad [4]$$

3 Show that, for small values of x^2,

$$(1 - 2x^2)^{-2} - (1 + 6x^2)^{\frac{2}{3}} \approx kx^4,$$

where the value of the constant k is to be determined. [6]

4 The equation of a curve is

$$y = 3\cos 2x + 7\sin x + 2.$$

Find the x-coordinates of the stationary points in the interval $0 \leqslant x \leqslant \pi$. Give each answer correct to 3 significant figures. [7]

5 **(a)** Find $\int (4 + \tan^2 2x) \, dx$. [3]

(b) Find the exact value of $\int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} \frac{\sin(x + \frac{1}{6}\pi)}{\sin x} \, dx$. [5]

6 The straight line l_1 passes through the points $(0, 1, 5)$ and $(2, -2, 1)$. The straight line l_2 has equation $\mathbf{r} = 7\mathbf{i} + \mathbf{j} + \mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} + 5\mathbf{k})$.

(i) Show that the lines l_1 and l_2 are skew. [6]

(ii) Find the acute angle between the direction of the line l_2 and the direction of the x-axis. [3]

7 Given that $y = 1$ when $x = 0$, solve the differential equation

$$\frac{dy}{dx} = 4x(3y^2 + 10y + 3),$$

obtaining an expression for y in terms of x. [9]

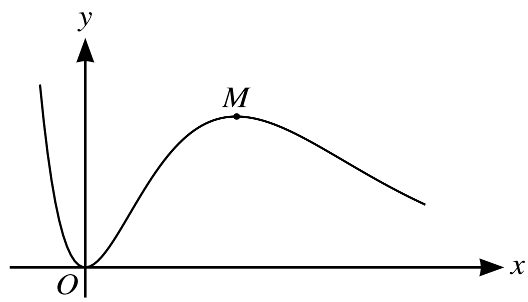
8 The complex number w is defined by $w = \frac{22 + 4i}{(2 - i)^2}$.

(i) Without using a calculator, show that $w = 2 + 4i$. [3]

(ii) It is given that p is a real number such that $\frac{1}{4}\pi \leqslant \arg(w + p) \leqslant \frac{3}{4}\pi$. Find the set of possible values of p. [3]

(iii) The complex conjugate of w is denoted by w^*. The complex numbers w and w^* are represented in an Argand diagram by the points S and T respectively. Find, in the form $|z - a| = k$, the equation of the circle passing through S, T and the origin. [3]

9

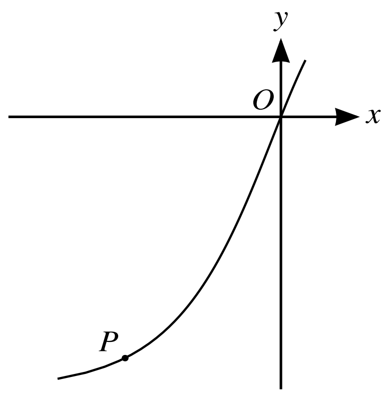


The diagram shows the curve $y = x^2 e^{2-x}$ and its maximum point M.

(i) Show that the x-coordinate of M is 2. [3]

(ii) Find the exact value of $\int_0^2 x^2 e^{2-x}\, dx$. [6]

10



The diagram shows part of the curve with parametric equations

$$x = 2\ln(t+2), \qquad y = t^3 + 2t + 3.$$

(i) Find the gradient of the curve at the origin. [5]

(ii) At the point P on the curve, the value of the parameter is p. It is given that the gradient of the curve at P is $\frac{1}{2}$.

(a) Show that $p = \dfrac{1}{3p^2 + 2} - 2$. [1]

(b) By first using an iterative formula based on the equation in part **(a)**, determine the coordinates of the point P. Give the result of each iteration to 5 decimal places and each coordinate of P correct to 2 decimal places. [4]

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MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2015**

1 hour 45 minutes

* 2 3 9 9 5 0 7 0 3 3 *

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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JC15 06_9709_32/RP

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[Turn over

1 Use the trapezium rule with three intervals to estimate the value of

$$\int_0^{\frac{1}{2}\pi} \ln(1 + \sin x)\,dx,$$

giving your answer correct to 2 decimal places. [3]

2 Using the substitution $u = 4^x$, solve the equation $4^x + 4^2 = 4^{x+2}$, giving your answer correct to 3 significant figures. [4]

3 A curve has equation $y = \cos x \cos 2x$. Find the x-coordinate of the stationary point on the curve in the interval $0 < x < \frac{1}{2}\pi$, giving your answer correct to 3 significant figures. [6]

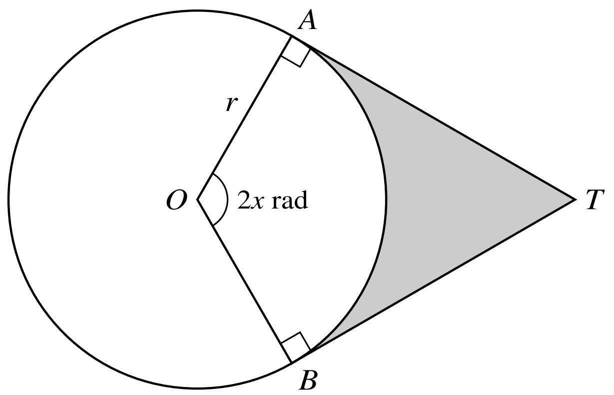
4 **(i)** Express $3\sin\theta + 2\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, stating the exact value of R and giving the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$3\sin\theta + 2\cos\theta = 1,$$

for $0° < \theta < 180°$. [3]

5



The diagram shows a circle with centre O and radius r. The tangents to the circle at the points A and B meet at T, and the angle AOB is $2x$ radians. The shaded region is bounded by the tangents AT and BT, and by the minor arc AB. The perimeter of the shaded region is equal to the circumference of the circle.

(i) Show that x satisfies the equation

$$\tan x = \pi - x.$$ [3]

(ii) This equation has one root in the interval $0 < x < \frac{1}{2}\pi$. Verify by calculation that this root lies between 1 and 1.3. [2]

(iii) Use the iterative formula

$$x_{n+1} = \tan^{-1}(\pi - x_n)$$

to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

6 Let $I = \int_0^1 \frac{\sqrt{x}}{2 - \sqrt{x}}\, dx$.

(i) Using the substitution $u = 2 - \sqrt{x}$, show that $I = \int_1^2 \frac{2(2-u)^2}{u}\, du$. [4]

(ii) Hence show that $I = 8 \ln 2 - 5$. [4]

7 The complex number u is given by $u = -1 + (4\sqrt{3})i$.

(i) Without using a calculator and showing all your working, find the two square roots of u. Give your answers in the form $a + ib$, where the real numbers a and b are exact. [5]

(ii) On an Argand diagram, sketch the locus of points representing complex numbers z satisfying the relation $|z - u| = 1$. Determine the greatest value of $\arg z$ for points on this locus. [4]

8 Let $f(x) = \frac{5x^2 + x + 6}{(3 - 2x)(x^2 + 4)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

9 The number of organisms in a population at time t is denoted by x. Treating x as a continuous variable, the differential equation satisfied by x and t is

$$\frac{dx}{dt} = \frac{xe^{-t}}{k + e^{-t}},$$

where k is a positive constant.

(i) Given that $x = 10$ when $t = 0$, solve the differential equation, obtaining a relation between x, k and t. [6]

(ii) Given also that $x = 20$ when $t = 1$, show that $k = 1 - \frac{2}{e}$. [2]

(iii) Show that the number of organisms never reaches 48, however large t becomes. [2]

10 The points A and B have position vectors given by $\overrightarrow{OA} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and $\overrightarrow{OB} = \mathbf{i} + \mathbf{j} + 5\mathbf{k}$. The line l has equation $\mathbf{r} = \mathbf{i} + \mathbf{j} + 2\mathbf{k} + \mu(3\mathbf{i} + \mathbf{j} - \mathbf{k})$.

(i) Show that l does not intersect the line passing through A and B. [5]

(ii) Find the equation of the plane containing the line l and the point A. Give your answer in the form $ax + by + cz = d$. [6]

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Cambridge International A Level

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Cambridge International Advanced Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2015**

1 hour 45 minutes

* 9 6 8 4 6 1 9 6 5 3 *

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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JC15 06_9709_33/RP

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1 Solve the equation $\ln(x+4) = 2\ln x + \ln 4$, giving your answer correct to 3 significant figures. [4]

2 Solve the inequality $|x-2| > 2x-3$. [4]

3 Solve the equation $\cot 2x + \cot x = 3$ for $0° < x < 180°$. [6]

4 The curve with equation $y = \dfrac{e^{2x}}{4+e^{3x}}$ has one stationary point. Find the exact values of the coordinates of this point. [6]

5 The parametric equations of a curve are

$$x = a\cos^4 t, \quad y = a\sin^4 t,$$

where a is a positive constant.

(i) Express $\dfrac{dy}{dx}$ in terms of t. [3]

(ii) Show that the equation of the tangent to the curve at the point with parameter t is

$$x\sin^2 t + y\cos^2 t = a\sin^2 t\cos^2 t.$$ [3]

(iii) Hence show that if the tangent meets the x-axis at P and the y-axis at Q, then

$$OP + OQ = a,$$

where O is the origin. [2]

6 It is given that $\int_0^a x\cos x\,dx = 0.5$, where $0 < a < \frac{1}{2}\pi$.

(i) Show that a satisfies the equation $\sin a = \dfrac{1.5 - \cos a}{a}$. [4]

(ii) Verify by calculation that a is greater than 1. [2]

(iii) Use the iterative formula

$$a_{n+1} = \sin^{-1}\left(\frac{1.5 - \cos a_n}{a_n}\right)$$

to determine the value of a correct to 4 decimal places, giving the result of each iteration to 6 decimal places. [3]

7 The number of micro-organisms in a population at time t is denoted by M. At any time the variation in M is assumed to satisfy the differential equation

$$\frac{dM}{dt} = k(\sqrt{M})\cos(0.02t),$$

where k is a constant and M is taken to be a continuous variable. It is given that when $t = 0$, $M = 100$.

(i) Solve the differential equation, obtaining a relation between M, k and t. [5]

(ii) Given also that $M = 196$ when $t = 50$, find the value of k. [2]

(iii) Obtain an expression for M in terms of t and find the least possible number of micro-organisms. [2]

8 The complex number $1 - i$ is denoted by u.

(i) Showing your working and without using a calculator, express

$$\frac{i}{u}$$

in the form $x + iy$, where x and y are real. [2]

(ii) On an Argand diagram, sketch the loci representing complex numbers z satisfying the equations $|z - u| = |z|$ and $|z - i| = 2$. [4]

(iii) Find the argument of each of the complex numbers represented by the points of intersection of the two loci in part **(ii)**. [3]

9 Two planes have equations $x + 3y - 2z = 4$ and $2x + y + 3z = 5$. The planes intersect in the straight line l.

(i) Calculate the acute angle between the two planes. [4]

(ii) Find a vector equation for the line l. [6]

10 Let $f(x) = \dfrac{11x + 7}{(2x - 1)(x + 2)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Show that $\displaystyle\int_1^2 f(x)\,dx = \tfrac{1}{4} + \ln\left(\tfrac{9}{4}\right)$. [5]

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MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2016**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

* 4 1 4 6 7 0 4 9 0 2 *

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JC16 06_9709_31/RP

1 **(i)** Solve the equation $2|x - 1| = 3|x|$. [3]

(ii) Hence solve the equation $2|5^x - 1| = 3|5^x|$, giving your answer correct to 3 significant figures. [2]

2 Find the exact value of $\int_0^{\frac{1}{2}} xe^{-2x}\, dx$. [5]

3 By expressing the equation $\operatorname{cosec}\theta = 3\sin\theta + \cot\theta$ in terms of $\cos\theta$ only, solve the equation for $0° < \theta < 180°$. [5]

4 The variables x and y satisfy the differential equation

$$x\frac{dy}{dx} = y(1 - 2x^2),$$

and it is given that $y = 2$ when $x = 1$. Solve the differential equation and obtain an expression for y in terms of x in a form not involving logarithms. [6]

5 The curve with equation $y = \sin x \cos 2x$ has one stationary point in the interval $0 < x < \frac{1}{2}\pi$. Find the x-coordinate of this point, giving your answer correct to 3 significant figures. [6]

6 **(i)** By sketching a suitable pair of graphs, show that the equation

$$5e^{-x} = \sqrt{x}$$

has one root. [2]

(ii) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = \frac{1}{2}\ln\left(\frac{25}{x_n}\right)$$

converges, then it converges to the root of the equation in part **(i)**. [2]

(iii) Use this iterative formula, with initial value $x_1 = 1$, to calculate the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 The equation of a curve is $x^3 - 3x^2y + y^3 = 3$.

(i) Show that $\frac{dy}{dx} = \frac{x^2 - 2xy}{x^2 - y^2}$. [4]

(ii) Find the coordinates of the points on the curve where the tangent is parallel to the x-axis. [5]

8 Let $\mathrm{f}(x) = \dfrac{4x^2 + 12}{(x+1)(x-3)^2}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\mathrm{f}(x)$ in ascending powers of x, up to and including the term in x^2. [5]

9 With respect to the origin O, the points A, B, C, D have position vectors given by

$$\overrightarrow{OA} = \mathbf{i} + 3\mathbf{j} + 2\mathbf{k}, \quad \overrightarrow{OB} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}, \quad \overrightarrow{OC} = 2\mathbf{i} + 4\mathbf{j} + \mathbf{k}, \quad \overrightarrow{OD} = -3\mathbf{i} + \mathbf{j} + 2\mathbf{k}.$$

(i) Find the equation of the plane containing A, B and C, giving your answer in the form $ax + by + cz = d$. [6]

(ii) The line through D parallel to OA meets the plane with equation $x + 2y - z = 7$ at the point P. Find the position vector of P and show that the length of DP is $2\sqrt{(14)}$. [5]

10 **(a)** Showing all your working and without the use of a calculator, find the square roots of the complex number $7 - (6\sqrt{2})\mathrm{i}$. Give your answers in the form $x + \mathrm{i}y$, where x and y are real and exact. [5]

(b) **(i)** On an Argand diagram, sketch the loci of points representing complex numbers w and z such that $|w - 1 - 2\mathrm{i}| = 1$ and $\arg(z - 1) = \frac{3}{4}\pi$. [4]

(ii) Calculate the least value of $|w - z|$ for points on these loci. [2]

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MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2016**

1 hour 45 minutes

5402648439

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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[Turn over

1 Use logarithms to solve the equation $4^{3x-1} = 3(5^x)$, giving your answer correct to 3 decimal places. [4]

2 Expand $\dfrac{1}{\sqrt{(1-2x)}}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients. [4]

3 Find the exact value of $\displaystyle\int_0^{\frac{1}{2}\pi} x^2 \sin 2x \, dx$. [5]

4 The curve with equation $y = \dfrac{(\ln x)^2}{x}$ has two stationary points. Find the exact values of the coordinates of these points. [6]

5 **(i)** Prove the identity $\cos 4\theta - 4\cos 2\theta \equiv 8\sin^4\theta - 3$. [4]

(ii) Hence solve the equation

$$\cos 4\theta = 4\cos 2\theta + 3,$$

for $0° \leqslant \theta \leqslant 360°$. [4]

6 The variables x and θ satisfy the differential equation

$$(3 + \cos 2\theta)\frac{dx}{d\theta} = x \sin 2\theta,$$

and it is given that $x = 3$ when $\theta = \frac{1}{4}\pi$.

(i) Solve the differential equation and obtain an expression for x in terms of θ. [7]

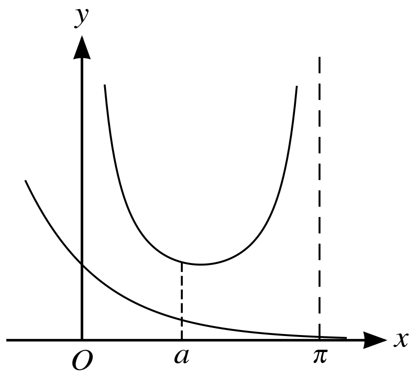
(ii) State the least value taken by x. [1]

7 Let $f(x) = \dfrac{4x^2 + 7x + 4}{(2x+1)(x+2)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Show that $\displaystyle\int_0^4 f(x)\, dx = 8 - \ln 3$. [5]

8



The diagram shows the curve $y = \operatorname{cosec} x$ for $0 < x < \pi$ and part of the curve $y = e^{-x}$. When $x = a$, the tangents to the curves are parallel.

(i) By differentiating $\frac{1}{\sin x}$, show that if $y = \operatorname{cosec} x$ then $\frac{dy}{dx} = -\operatorname{cosec} x \cot x$. [3]

(ii) By equating the gradients of the curves at $x = a$, show that

$$a = \tan^{-1}\left(\frac{e^a}{\sin a}\right).$$ [2]

(iii) Verify by calculation that a lies between 1 and 1.5. [2]

(iv) Use an iterative formula based on the equation in part **(ii)** to determine a correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

9 The points A, B and C have position vectors, relative to the origin O, given by $\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$, $\overrightarrow{OB} = 4\mathbf{j} + \mathbf{k}$ and $\overrightarrow{OC} = 2\mathbf{i} + 5\mathbf{j} - \mathbf{k}$. A fourth point D is such that the quadrilateral $ABCD$ is a parallelogram.

(i) Find the position vector of D and verify that the parallelogram is a rhombus. [5]

(ii) The plane p is parallel to OA and the line BC lies in p. Find the equation of p, giving your answer in the form $ax + by + cz = d$. [5]

10 **(a)** Showing all necessary working, solve the equation $iz^2 + 2z - 3i = 0$, giving your answers in the form $x + iy$, where x and y are real and exact. [5]

(b) **(i)** On a sketch of an Argand diagram, show the locus representing complex numbers satisfying the equation $|z| = |z - 4 - 3i|$. [2]

(ii) Find the complex number represented by the point on the locus where $|z|$ is least. Find the modulus and argument of this complex number, giving the argument correct to 2 decimal places. [3]

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