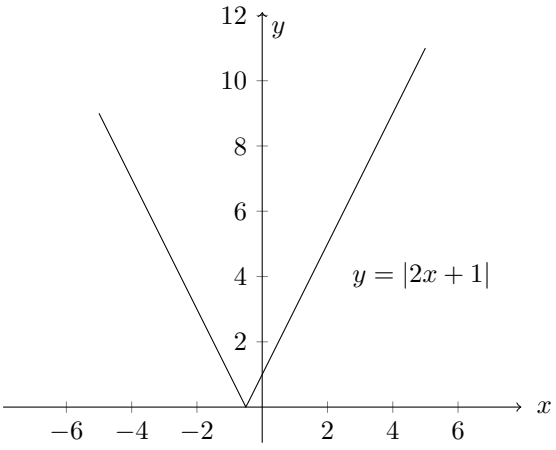
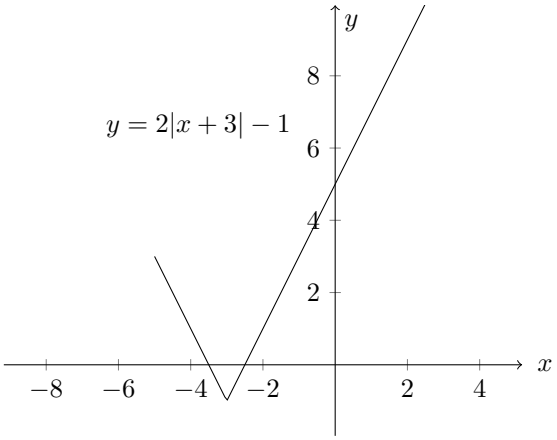
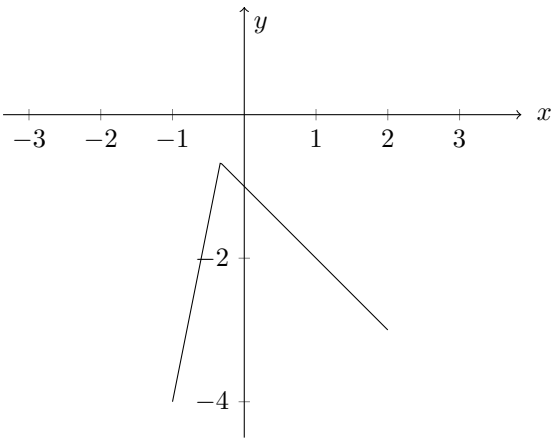
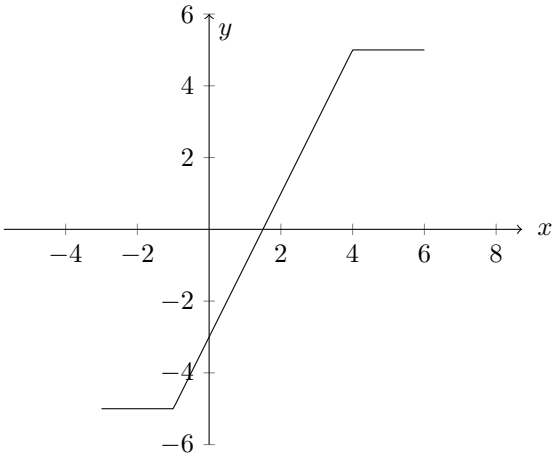
P3 Exercise Solution

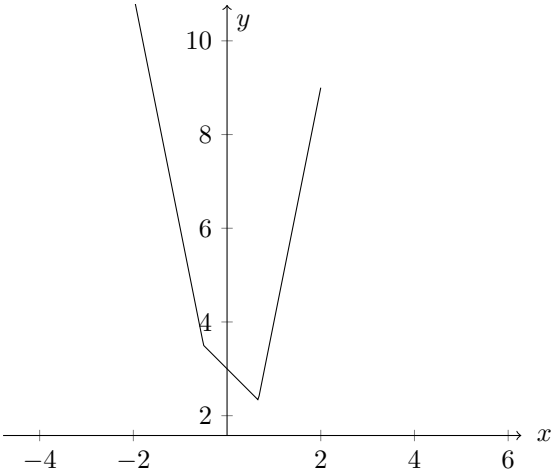
Exercise 1

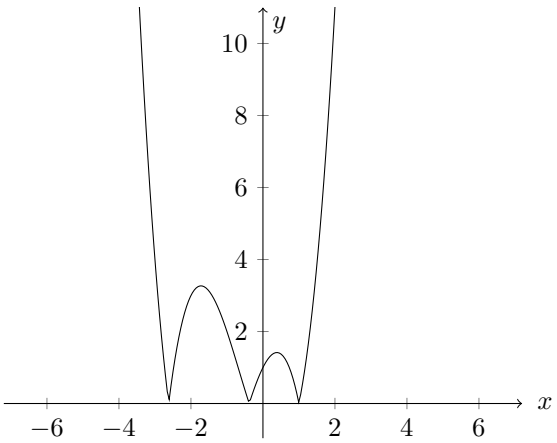


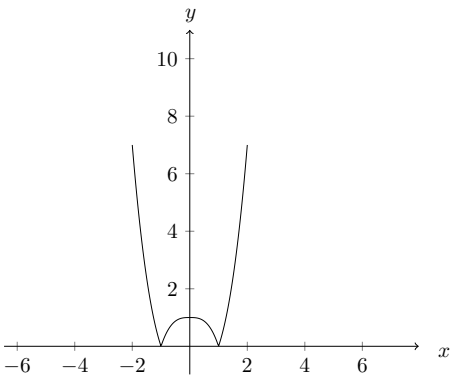


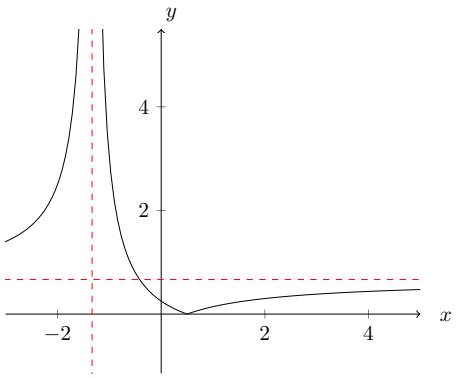


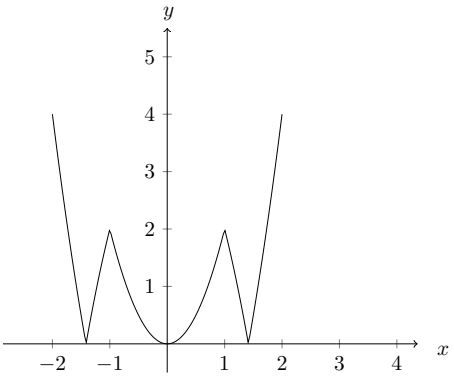


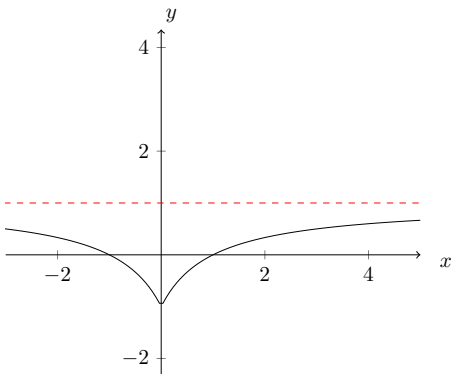












Exercise 2

1. 3 or -2
2. $\frac{2}{3}$ or -5
3. $\frac{3}{2}$
4. 2 or $-\frac{4}{3}$
5. 2
6. $-\frac{8}{7}$ or $\frac{12}{5}$
7. -2, $\frac{7}{3}$, 1, $-\frac{4}{3}$
8. -3, -1, 3,1
9. 1 or -1
10. 2 or -2

Exercise 3

1. $(-\infty, -\frac{1}{2}) \cup (1, \infty)$
2. $[-3, 1]$

3. $(-\frac{3}{2}, 4)$
4. $[1, \infty]$
5. $(-\infty, -2) \cup (\frac{1-\sqrt{17}}{2}, 1) \cup (\frac{1+\sqrt{17}}{2}, \infty)$
6. $(-\infty, \frac{5}{3}) \cup (5, \infty)$
7. $(-3, -1)$
8. $(-\infty, -\frac{3}{4}] \cup [\frac{7}{4}, \infty)$
9. $(-\infty, \frac{5-\sqrt{17}}{2}) \cup (2, 3) \cup (\frac{5+\sqrt{17}}{2}, \infty)$
10. $(-\infty, -\frac{7}{2}) \cup (-2, 0) \cup (0, 2) \cup (\frac{7}{2}, \infty)$

Exercise 4

1. Polynomial: $\frac{(x+1)(x+2)\cdot(x+n)}{n!}$, degree: n, leading term: $\frac{1}{n!}$, constant term 1.
2. $6x^4 + 3x^3 - 18x^2 - 9x - 2$
3. $A = 2$, $B = 3$, $C = -5$.
4. (a) m or n
   (b) m or n
   (c) $m + n$.

Exercise 5

1. quotient: $2x^2 - 3x + 4$, remainder: -13
2. quotient: $3x^2 + 4x + 1$, remainder: $9x - 7$
3. quotient: $x^4 - x^3 + x - 1$, remainder: 2
4. quotient: $x^{n-1} + kx^{n-2} + k^2x^{n-3} + \cdot + k^{n-1}x + k^n$, remainder: k^{n+1}

Exercise 6

1. (a) 302
   (b) $-\frac{1}{3}$
   (c) $2\frac{3}{4}$
2. $a = -\frac{5}{3}$, $b = \frac{1}{3}$
3. $k = -9$, $(x - 2)(2x + 1)(3x + 1)$
4. $x > 1$
5. (a) $f(1) \neq 0$, and $f(-1) \neq 0$,
   (b) quotient: $x^3 + 3x^2 + 3x + 5$, remainder: $6x + 4$
   (c) omit

(d) -1, $-1+\sqrt{2}$, $-1-\sqrt{2}$.

Exercise 7

1. (a) $1+6x+27x^2+108x^3$
   (b) $1+\frac{2}{3}x-\frac{4}{9}x^2+\frac{40}{81}x^3$
   (c) $2-\frac{1}{4}x-\frac{1}{64}-\frac{1}{512}x^3$
   (d) $\frac{1}{27}-\frac{2}{27}x+\frac{8}{81}x^2-\frac{80}{729}x^3$.
2. (a) $4+x-\frac{1}{16}x^2$
   (b) $2-\frac{1}{32}x-\frac{131}{4096}x^2$
   (c) $\frac{1}{2}+\frac{1}{4}x^2$
   (d) $\frac{1}{2}-\frac{1}{16}x-\frac{117}{256}x^2$.
3. $a=4$, $b=12$, $c=-192$
4. (a) $(-1)^n 2^n$
   (b) $\dfrac{(2n-1)!!}{2^n n!}$
   (c) $\binom{|k|+n-1}{n}$.
5. By part 4(*c*), $x^r : \frac{1}{2}(r+1)(r+2)$; $x^{r-1} : -\frac{1}{2}r(r+1)$; $x^{r-2} : r(r-1)$.
   Add them together to get r^2+1.
   (a) 8
   (b) 12.

Exercise 8

1. $-\frac{1}{x}-\frac{\frac{1}{2}}{x+1}+\frac{\frac{3}{2}}{x-1}$.
2. $x^2-x+3-\frac{\frac{16}{3}}{x+2}+\frac{\frac{1}{3}}{x-1}$.
3. $\frac{\frac{1}{2}}{x+1}-\frac{4}{x+2}+\frac{\frac{9}{2}}{x+3}$.

Exercise 9

1. $\frac{1}{x}-\frac{1}{x-1}+\frac{2}{(x-1)^2}$.
2. $\frac{-\frac{7}{4}}{x}+\frac{\frac{11}{4}}{x-1}+\frac{\frac{3}{2}}{(x-1)^2}$.
3. $x+\frac{\frac{16}{9}}{x-2}+\frac{\frac{11}{9}}{x+2}+\frac{-\frac{1}{3}}{(x+1)^2}$.
4. $\frac{\frac{3}{4}}{(x-1)^2}+\frac{\frac{1}{4}}{(x+1)^2}$.

Exercise 10

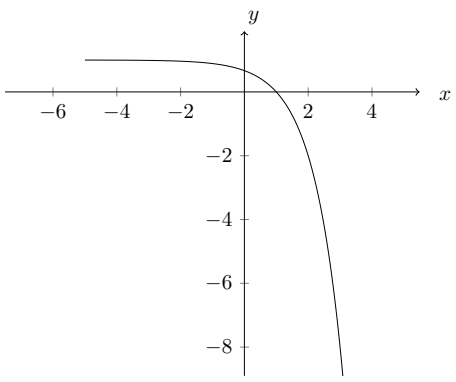
1. $\frac{1}{x} + \frac{-x+1}{x^2+1}$
2. $\frac{-1}{x+1} + \frac{1}{(x+1)^2} + \frac{x}{x^2+x+1}$
3. $x + 1 + \frac{2}{x-2} + \frac{-3x+1}{2x^2+2x+1}$.
4. $\frac{1}{x-1} + \frac{2}{x+2} + \frac{-2x-3}{x^2-2x+4}$.
5. $\frac{2x-3}{x^2-2x+2} + \frac{2x+3}{x^2+2x+2}$.

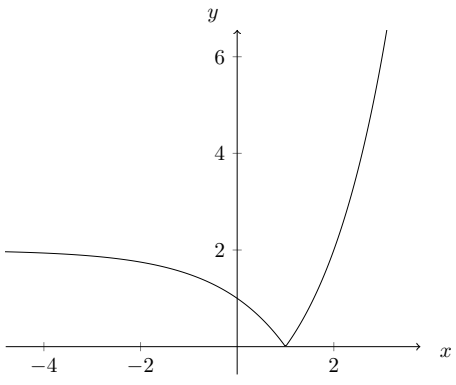
Exercise 11

1. (a) $\frac{1}{x+1} - \frac{6}{x+2} + \frac{8}{x+4}$
   (b) $-\frac{1}{x+1} + \frac{1}{x+2} + \frac{2}{(x+2)^2}$.
   (c) $2 - \frac{2}{x} - \frac{3}{x^2} + \frac{4}{x-1}$
   (d) $\frac{1}{x} - \frac{2}{x-1} + \frac{3x-1}{x^2+1}$
   (e) $1 + \frac{\frac{1}{3}}{x-3} - \frac{\frac{1}{3}}{x+3} - \frac{2}{(x+3)^2}$.
2. (a) $\frac{1}{2} - \frac{3}{4}x + \frac{13}{24}x^2 - \frac{47}{144}x^3$.
   (b) $-1 - \frac{4}{3}x - x^2 - \frac{7}{9}x^3$
   (c) $\frac{5}{2} - \frac{3}{2}x + \frac{5}{4}x^2 - \frac{11}{4}x^3$.
3. $\frac{1}{x-5} - \frac{3}{x+5}$, $f'(x) = -(x-5)^{-2} + 3(x+5)^{-2}$.
4. $1 - \frac{1}{x-1} - \frac{2}{(x-1)^2} + \frac{2}{x-4}$, $f'(x) = (x-1)^{-2} + 4(x-1)^{-3} - 2(x-4)^{-2}$.

Exercise 12

1. $x \in (-\infty, 0) \cup (0, \infty)$, $y \in (0, \infty)$
2. The graphs are as following:





3. $>, <, <, <, <$
4. $m < n$
5. $a > 1$, then $x < -6$
   $a < 1$, then $x > -6$

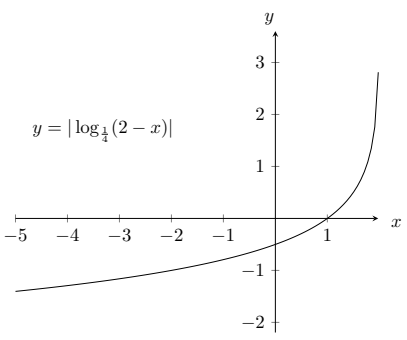
6. (a) $\sqrt{5}$
   (b) 7
   (c) $8\sqrt{5}$ or $-8\sqrt{5}$
7. no
8. 3 or $-\frac{1}{3}$
9. $f(0) = 1$, $f(\frac{1}{p} + \cdots + \frac{1}{p}) = \left[f(\frac{1}{p})\right]^p = f(1) = a$, then $f(\frac{1}{p}) = a^{\frac{1}{p}}$, which is $f(x) = a^x$.

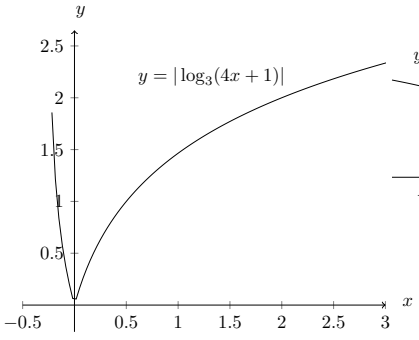
Exercise 13

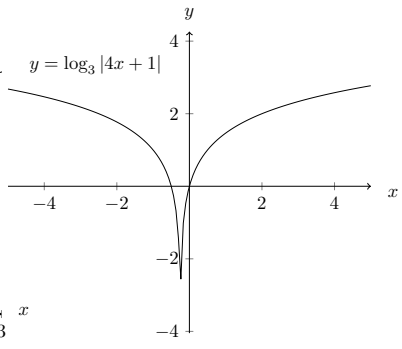
1. 3, -3, $-\frac{7}{4}$, $\frac{37}{2}$, $\frac{5}{4}$
2. $\frac{121}{3}$
3. $\frac{5\ln 2 - \ln 3}{2\ln 3 - 3\ln 2}$.
4. $\frac{\ln 2}{\ln 3 + 2\ln 2} = 0.279$
5. $x = \frac{\ln 5}{\ln 15}$, $y = \frac{\ln 3}{\ln 15}$.
6. $\frac{5}{6} + \log_{10}\frac{3}{2}$
7. (a) $a^{\ln b} = e^{\ln a \ln b} = b^{\ln a}$, then $\left(a^{\ln b}\right)^{\frac{1}{\ln c}} = \left(b^{\ln a}\right)^{\frac{1}{\ln c}}$
   (b) omit
   (c) omit
8. 2
9. 25

Exercise 14

1. $<, >, >, >$
2. The graph are as following:







3. $m > n$
4. (a) $(-1, 1)$
   (b) even.

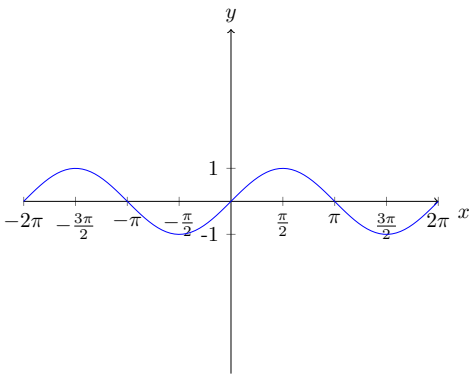
5. $y = a^x - 1$
6. $0 < a < 1$, $0 < x < \log_a(\sqrt{3} - 1)$, $a > 1$, $\log_a(\sqrt{3} - 1) < x < 0$
7. $k = 1$, $\mathbb{R}$
8. $m = n = 5$

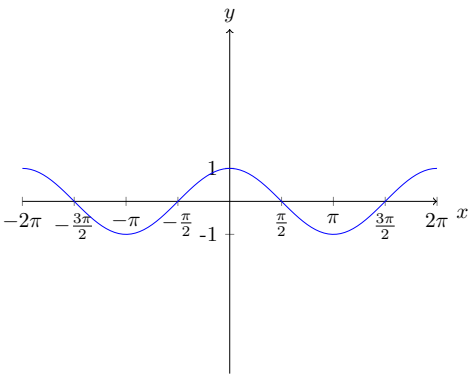
Exercise 15

1. (a) 0.323
   (b) 130
2. $a = \frac{3}{5 \lg 2}$, $b = \frac{5}{2} - \frac{3}{5 \lg 2}$
3. $A = 0.4$, $c = 1.01$, $x = 504$
4. (a) 5
   (b) 1,1,9
5. Consider $f(1) = 0$

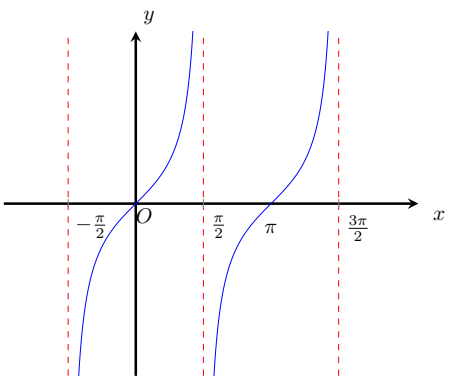
Exercise 16

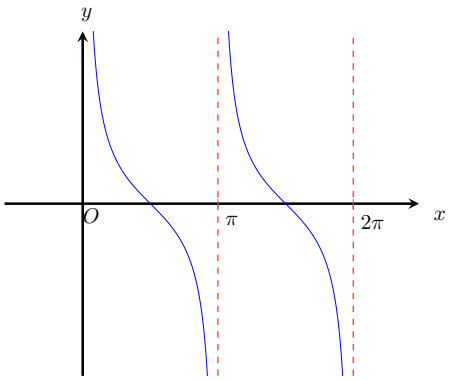
1. The graph of $y = \sin x$ and $y = \cos x$:

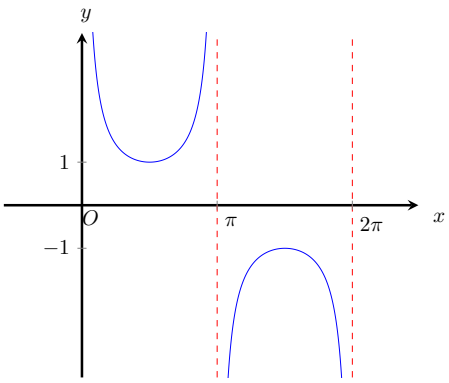


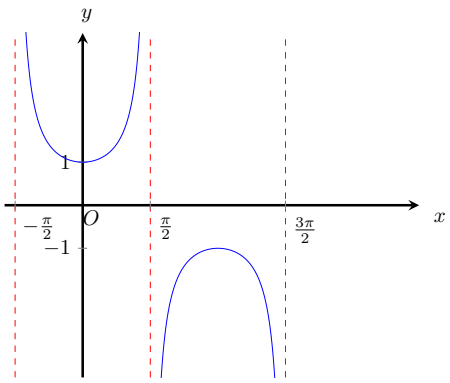


2. The graph of $y = \tan x$ and $y = \cot x$:









3. The graph of $y = \sec x$ and $y = \csc x$:

Exercise 17

1. (a) 0.5
   (b) $\frac{131}{65}$
   (c) $\frac{457}{289}$
   (d) $\frac{3}{5}$
2. omit
3. (a) $73.9°, 286.1°$
   (b) $37.2°, 217.2°, 327.8°, 142.8°$
   (c) $30°, 210°, 150°, 330°$
   (d) $60°, 240°, 120°, 300°$

Exercise 18

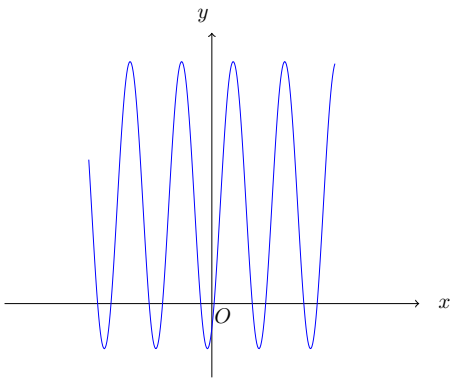
1. $\frac{\sqrt{2}}{4}(\sqrt{3}-1)$
2. $\frac{\sqrt{3}\tan x+1}{\tan x-\sqrt{3}}$
3. $-\frac{56}{65}, \frac{63}{65}, -\frac{16}{63}$
4. $\pm\frac{\pi}{4}, \pm\frac{3\pi}{4}, \pm\frac{5\pi}{4}, \pm\frac{7\pi}{4}$
5. $0.12, 3.26, -3.02, -6.16$
6. $-\frac{59}{72}$
7. $-\frac{4}{3}, -\frac{3}{4}$
8. 0
9. Use $\sin 2\xi = 2\sin\xi\cos\xi$, $\cos\xi = 1 - 2\sin^2\xi$.

Exercise 19

1. $\pm\frac{11}{6}$, $\pm\frac{5}{6}$
2. $-1\pm\sqrt{2}$, $-1+\sqrt{2}$
3. $\frac{\pi}{2}$, 5.44, 2.29.
4. $x=k\pi$, $0.869+k\pi$, $-0.869+k\pi$.
5. $\sin^2\frac{1}{2}A=\pm\frac{\sqrt{2}}{2}\sqrt{1-\cos A}$, $\cos^2\frac{1}{2}A=\pm\frac{\sqrt{2}}{2}\sqrt{1+\cos A}$.
6. $\sin 3A=3\sin A-4\sin^3 a$, $\cos 3A=4\cos^3 A-3\cos A$, $\tan 3A=\frac{3\tan A-\tan^3 A}{1-3\tan^2 A}$.
7. Omit.
8. Omit.
9. (a) $\tan\theta$
   (b) $\frac{1}{2}$
   (c) 1
10. 1, $-1+\sqrt{2}$
11. $\frac{7\sqrt{65}}{65}$.

Exercise 20

1. (a) $\sqrt{53}, 15.9°, 74.1°$
   (b) $\sqrt{41}, 51.3°$
   (c) $\sqrt{34}, 59.0°$
2. 5.57
3. $-2\sin\alpha$
4. $[\frac{2}{37},\frac{2}{3}]$
5. 9, -17, 0.588, 0.983
6. the graph is as follows: period $\frac{2\pi}{3}$



7. π

Exercise 21

1. (a) 1
   (b) $-\frac{3}{4}$
2. $k=-1$, $m=-1$
3. $a_n=\cos\frac{1}{2}x\cos\frac{1}{4}x\cdots\cos^2\frac{1}{2^{n-1}}x$, $b_n=\cos\frac{1}{2}x\cos\frac{1}{4}x\cdots\cos\frac{1}{2^{n-1}}x$
4. omit
5. prove $\tan\frac{q+p}{2}=\tan\frac{r+s}{2}$

Exercise 22

1. (a) $3e^{3x+1}$
   (b) $\frac{1}{2}\cdot 2^{\sqrt{x}}(\ln 2)\cdot(x^{-\frac{1}{2}})$
   (c) $3(2+e^{1-x})^{-2}e^{1-x}$
   (d) $\frac{1}{3}e^{3x-x^2}(3-2x)-\frac{2}{3}e^{1-x-x^2}(-1-2x)$.
   (e) omit
2. Maximum, $(\frac{\ln 2}{2},4\sqrt{2})$
3. Minimum, $(-1,e^{-3})$
4. $x=0$.
5. Omit.

Exercise 23

1. (a) $\frac{2x}{x^2+1}$
   (b) $\frac{2}{(2x-5)\ln 3}$
   (c) $\frac{3}{x}+\frac{8x}{x^2-1}$
   (d) $\frac{4}{4x+3}-\frac{3x^2}{x^3-1}$
   (e) $-\frac{1}{(x+1)(\ln(x+1))^2}$
   (f) $-\frac{2}{x(\ln x)^2}$.
2. $(1,\ln 2)$
3. $y=-x+4$
4. $y-2e=-2x$
5. $x=-3$. $x<-3$, then $\frac{\mathrm{d}y}{\mathrm{d}x}<0$, $x>-3$, then $\frac{\mathrm{d}y}{\mathrm{d}x}>0$.
6. (a) $(-\infty,\infty)$
   (b) $f^{-1}(x)=\frac{x+\sqrt{x^2+4}}{2}$.
   (c) $y-1=\frac{1}{2}(x-0)$.

Exercise 24

1. (a) $2x\cos\left(x^2\right)$.
   (b) $-\frac{1}{2}\sin x(1+\cos x)^{-\frac{1}{2}}$.
   (c) $6\cos 2xe^{3\sin 2x}$.
   (d) $\tan x$.
   (e) $2e^{2x}\cos\left(e^{2x}\right)$.
   (f) $-\sin 2x$.
   (g) $\sin 2x$.
2. $y-\ln\sqrt{2}=x-\frac{\pi}{4}$
3. $y+\frac{1}{3}=-\frac{2}{3\sqrt{3}}(x-\frac{\pi}{6})$
4. $x=\frac{\pi}{6}$, max; $x=\frac{5}{6}\pi$, min.
5. $\cos 2x(1-\sin 2x)$.
6. omit, $-(\csc x-\sin x)^{-2}(-\csc x\cot x-\cos x)$.
7. Omit.
8. (a) $\frac{1}{2}\cos\frac{1}{2}x+\frac{1}{3}\sin\frac{1}{3}x$.
   (b) -1 and $\frac{1}{2}$
   (c) 12π
   (d) 12π
9. $\frac{1}{\sqrt{1-x^2}}$, $-\frac{1}{\sqrt{1-x^2}}$
10. (a) $-\frac{1}{2}x^{-\frac{1}{2}}\sin\sqrt{x}$
    (b) -$\frac{1}{2}\sin x(\cos x)^{-\frac{1}{2}}$
    (c) $2x\cos x^2$
    (d) $(-x^2)\cos\frac{1}{x}$.

Exercise 25

1. (a) $\sec^2(x)$.
   (b) $-\csc^2 x$
   (c) $x+2x\ln x$.
   (d) $2x\sec^2(x^2+1)$
   (e) $\dfrac{2xe^{2x}-(e^{2x}+1)}{x^2}$.
   (f) $(2x)\sin x^2+(x^2-2)(\cos x^2)(2x)$
2. Max $(-3,6e^{-3})$, Min $(1,-2e)$
3. omit
4. $(e,\frac{1}{3})$
5. $(1,\frac{1}{2})$
6. (a) $x=\frac{\pi}{4}$
   (b) max
7. $u_1'u_2u_3\cdots u_k+u_1u_2'\cdots u_k+u_1u_2\cdots u_k'$
8. $\sum_{k=0}^{n}nCku^{(n-k)}v^{(k)}+$