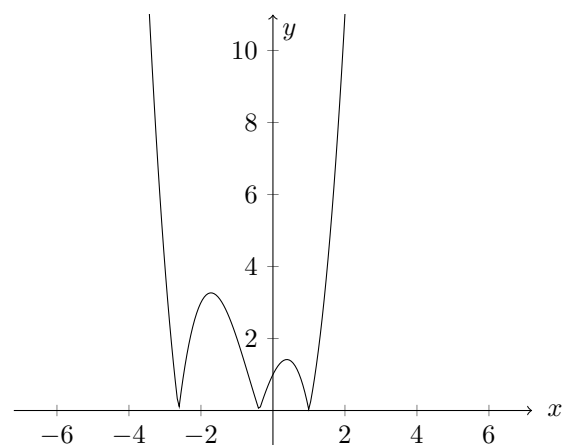
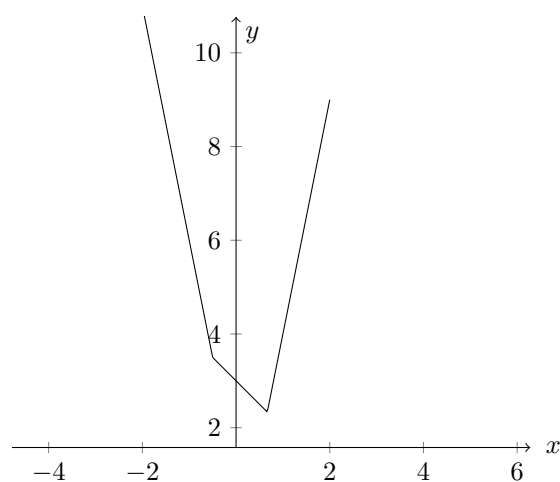
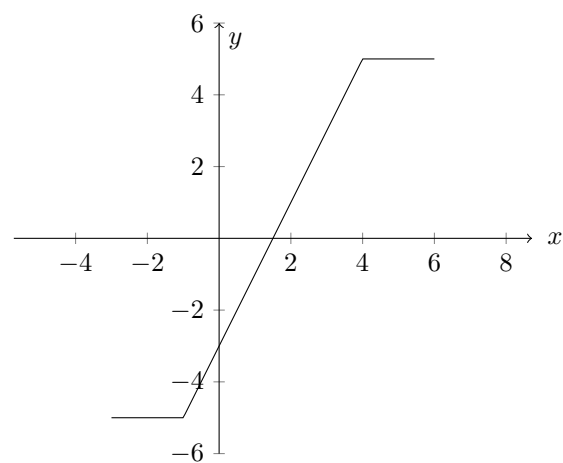
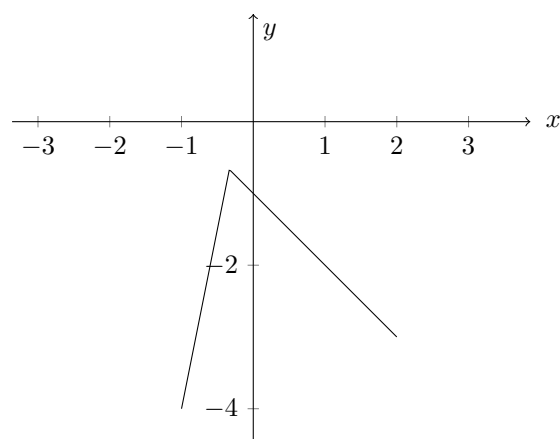
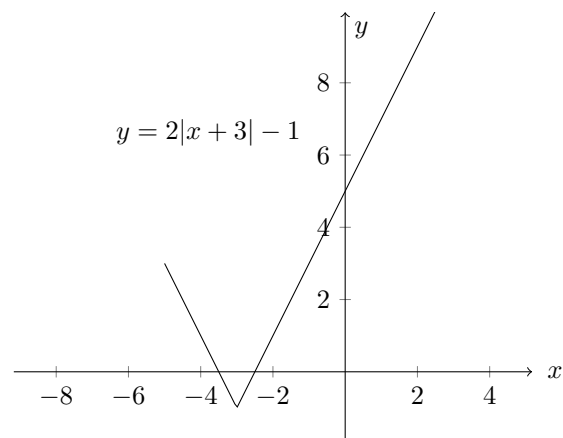
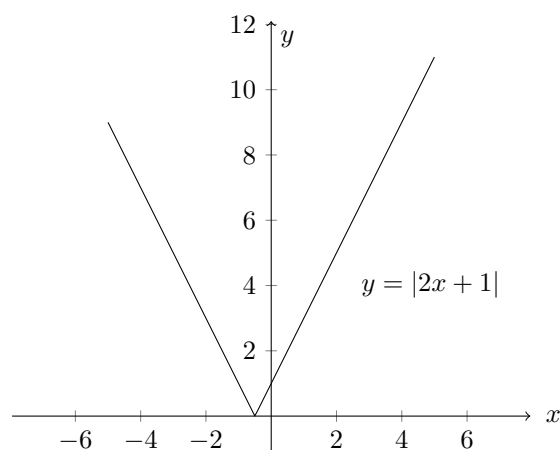
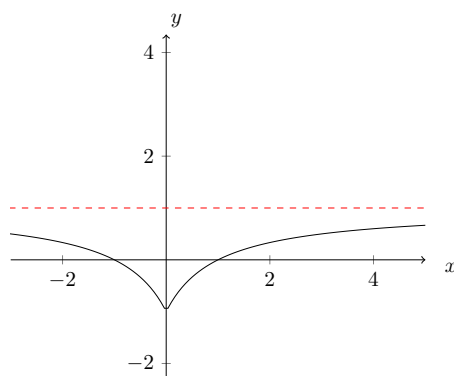
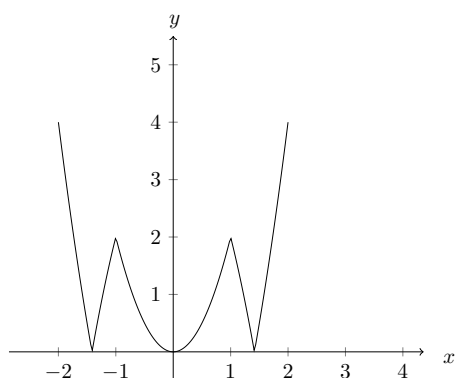
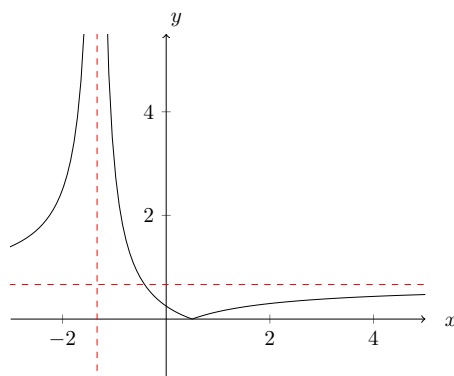
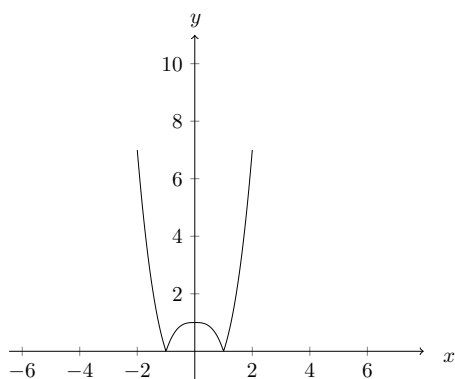


P3 Exercise Solution

updated: December 10, 2021

Exercise 1





Exercise 2

1. 3 or -2
2. $\frac{2}{3}$ or -5
3. $\frac{3}{2}$
4. 2 or $-\frac{4}{3}$
5. 2
6. $-\frac{8}{7}$ or $\frac{12}{5}$
7. $-2, \frac{7}{3}, 1, -\frac{4}{3}$
8. $-3, -1, 3, 1$
9. 1 or -1
10. 2 or -2

Exercise 3

1. $(-\infty, -\frac{1}{2}) \cup (1, \infty)$
2. $[-3, 1]$

3. $(-\frac{3}{2}, 4)$
4. $[1, \infty]$
5. $(-\infty, -2) \cup (\frac{1-\sqrt{17}}{2}, 1) \cup (\frac{1+\sqrt{17}}{2}, \infty)$
6. $(-\infty, \frac{5}{3}) \cup (5, \infty)$
7. $(-3, -1)$
8. $(-\infty, -\frac{3}{4}] \cup [\frac{7}{4}, \infty)$
9. $(-\infty, \frac{5-\sqrt{17}}{2}) \cup (2, 3) \cup (\frac{5+\sqrt{17}}{2}, \infty)$
10. $(-\infty, -\frac{7}{2}) \cup (-2, 0) \cup (0, 2) \cup (\frac{7}{2}, \infty)$

Exercise 4

1. Polynomial: $\frac{(x+1)(x+2)\cdots(x+n)}{n!}$, degree: n , leading term: $\frac{1}{n!}$, constant term 1.
2. $6x^4 + 3x^3 - 18x^2 - 9x - 2$
3. $A = 2, B = 3, C = -5$.
4. (a) m or n
 (b) m or n
 (c) $m + n$.

Exercise 5

1. quotient: $2x^2 - 3x + 4$, remainder: -13
2. quotient: $3x^2 + 4x + 1$, remainder: $9x - 7$
3. quotient: $x^4 - x^3 + x - 1$, remainder: 2
4. quotient: $x^{n-1} + kx^{n-2} + k^2x^{n-3} + \cdots + k^{n-1}x + k^n$, remainder: k^{n+1}

Exercise 6

1. (a) 302
 (b) $-\frac{1}{3}$
 (c) $2\frac{3}{4}$
2. $a = -\frac{5}{3}, b = \frac{1}{3}$
3. $k = -9, (x-2)(2x+1)(3x+1)$
4. $x > 1$
5. (a) $f(1) \neq 0$, and $f(-1) \neq 0$,
 (b) quotient: $x^3 + 3x^2 + 3x + 5$, remainder: $6x + 4$
 (c) omit

(d) $-1, -1 + \sqrt{2}, -1 - \sqrt{2}.$

Exercise 7

1. (a) $1 + 6x + 27x^2 + 108x^3$
 (b) $1 + \frac{2}{3}x - \frac{4}{9}x^2 + \frac{40}{81}x^3$
 (c) $2 - \frac{1}{4}x - \frac{1}{64} - \frac{1}{512}x^3$
 (d) $\frac{1}{27} - \frac{2}{27}x + \frac{8}{81}x^2 - \frac{80}{729}x^3.$
2. (a) $4 + x - \frac{1}{16}x^2$
 (b) $2 - \frac{1}{32}x - \frac{131}{4096}x^2$
 (c) $\frac{1}{2} + \frac{1}{4}x^2$
 (d) $\frac{1}{2} - \frac{1}{16}x - \frac{117}{256}x^2.$
3. $a = 4, b = 12, c = -192$
4. (a) $(-1)^n 2^n$
 (b) $\frac{(2n-1)!!}{2^n n!}$
 (c) $\binom{|k| + n - 1}{n}.$
5. By part 4(c), $x^r : \frac{1}{2}(r+1)(r+2) ; x^{r-1} : -\frac{1}{2}r(r+1); x^{r-2} : r(r-1).$
 Add them together to get $r^2 + 1.$
 (a) 8
 (b) 12.

Exercise 8

1. $-\frac{1}{x} - \frac{\frac{1}{2}}{x+1} + \frac{\frac{3}{2}}{x-1}.$
2. $x^2 - x + 3 - \frac{\frac{16}{3}}{x+2} + \frac{\frac{1}{3}}{x-1}.$
3. $\frac{\frac{1}{2}}{x+1} - \frac{4}{x+2} + \frac{\frac{9}{2}}{x+3}.$

Exercise 9

1. $\frac{1}{x} - \frac{1}{x-1} + \frac{2}{(x-1)^2}.$
2. $\frac{-\frac{7}{4}}{x} + \frac{\frac{11}{4}}{x-1} + \frac{\frac{3}{2}}{(x-1)^2}.$
3. $x + \frac{\frac{16}{9}}{x-2} + \frac{\frac{11}{9}}{x+2} + \frac{-\frac{1}{3}}{(x+1)^2}.$
4. $\frac{\frac{3}{4}}{(x-1)^2} + \frac{\frac{1}{4}}{(x+1)^2}.$

Exercise 10

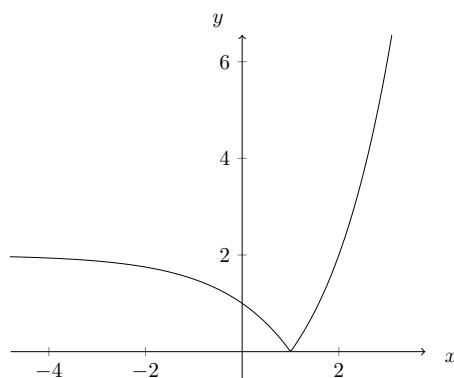
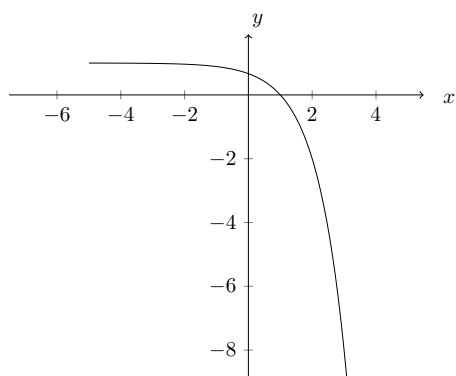
1. $\frac{1}{x} + \frac{-x+1}{x^2+1}$
2. $\frac{-1}{x+1} + \frac{1}{(x+1)^2} + \frac{x}{x^2+x+1}$
3. $x+1 + \frac{2}{x-2} + \frac{-3x+1}{2x^2+2x+1}$.
4. $\frac{1}{x-1} + \frac{2}{x+2} + \frac{-2x-3}{x^2-2x+4}$.
5. $\frac{2x-3}{x^2-2x+2} + \frac{2x+3}{x^2+2x+2}$.

Exercise 11

1. (a) $\frac{1}{x+1} - \frac{6}{x+2} + \frac{8}{x+4}$
 (b) $-\frac{1}{x+1} + \frac{1}{x+2} + \frac{2}{(x+2)^2}$.
 (c) $2 - \frac{2}{x} - \frac{3}{x^2} + \frac{4}{x-1}$
 (d) $\frac{1}{x} - \frac{2}{x-1} + \frac{3x-1}{x^2+1}$
 (e) $1 + \frac{\frac{1}{3}}{x-3} - \frac{\frac{1}{3}}{x+3} - \frac{2}{(x+3)^2}$.
2. (a) $\frac{1}{2} - \frac{3}{4}x + \frac{13}{24}x^2 - \frac{47}{144}x^3$.
 (b) $-1 - \frac{4}{3}x - x^2 - \frac{7}{9}x^3$
 (c) $\frac{5}{2} - \frac{3}{2}x + \frac{5}{4}x^2 - \frac{11}{4}x^3$.
3. $\frac{1}{x-5} - \frac{3}{x+5}$, $f'(x) = -(x-5)^{-2} + 3(x+5)^{-2}$.
4. $1 - \frac{1}{x-1} - \frac{2}{(x-1)^2} + \frac{2}{x-4}$, $f'(x) = (x-1)^{-2} + 4(x-1)^{-3} - 2(x-4)^{-2}$.

Exercise 12

1. $x \in (-\infty, 0) \cup (0, \infty)$, $y \in (0, \infty)$
2. The graphs are as following:



3. $>, <, <, <, <$
4. $m < n$
5. $a > 1$, then $x < -6$
 $a < 1$, then $x > -6$

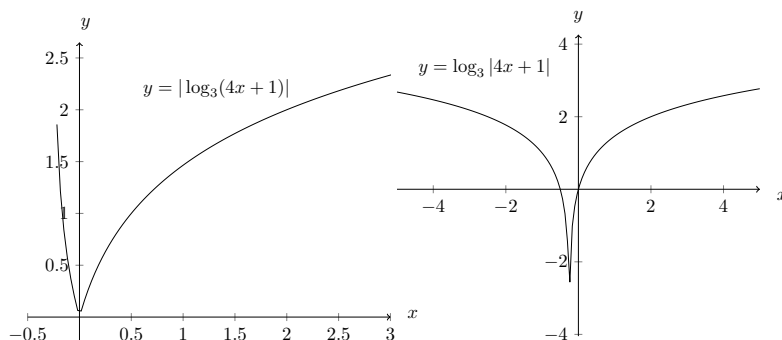
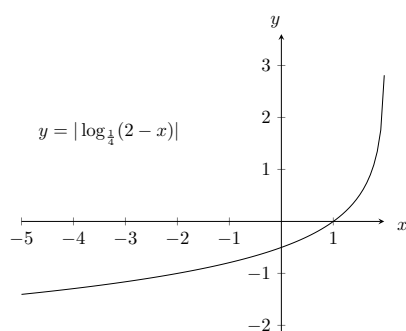
6. (a) $\sqrt{5}$
 (b) 7
 (c) $8\sqrt{5}$ or $-8\sqrt{5}$
7. no
8. 3 or $-\frac{1}{3}$
9. $f(0) = 1$, $f(\frac{1}{p} + \dots + \frac{1}{p}) = \left[f(\frac{1}{p})\right]^p = f(1) = a$, then $f(\frac{1}{p}) = a^{\frac{1}{p}}$, which is $f(x) = a^x$.

Exercise 13

1. 3, -3, $-\frac{7}{4}$, $\frac{37}{2}$, $\frac{5}{4}$
2. $\frac{121}{3}$
3. $\frac{5 \ln 2 - \ln 3}{2 \ln 3 - 3 \ln 2}$
4. $\frac{\ln 2}{\ln 3 + 2 \ln 2} = 0.279$
5. $x = \frac{\ln 5}{\ln 15}$, $y = \frac{\ln 3}{\ln 15}$
6. $\frac{5}{6} + \log_{10} \frac{3}{2}$
7. (a) $a^{\ln b} = e^{\ln a \ln b} = b^{\ln a}$, then $(a^{\ln b})^{\frac{1}{\ln c}} = (b^{\ln a})^{\frac{1}{\ln c}}$
 (b) omit
 (c) omit
8. 2
9. 25

Exercise 14

1. $<, >, >, >$
2. The graph are as following:



3. $m > n$
4. (a) $(-1, 1)$
 (b) even.

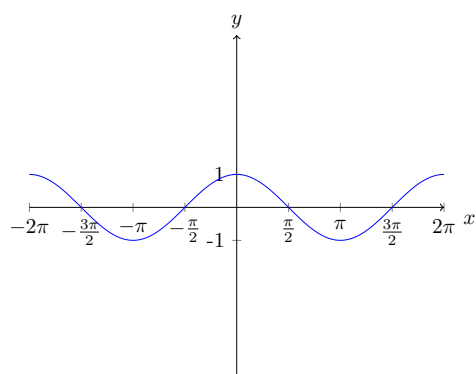
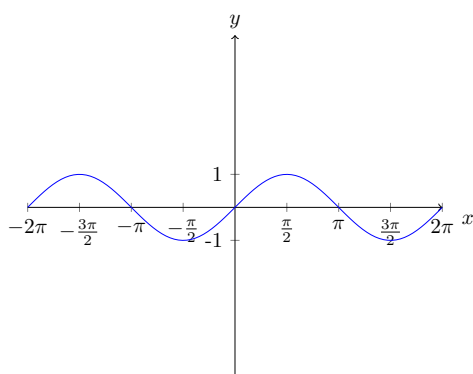
5. $y = a^x - 1$
6. $0 < a < 1, 0 < x < \log_a(\sqrt{3} - 1), a > 1, \log_a(\sqrt{3} - 1) < x < 0$
7. $k = 1, \mathbb{R}$
8. $m = n = 5$

Exercise 15

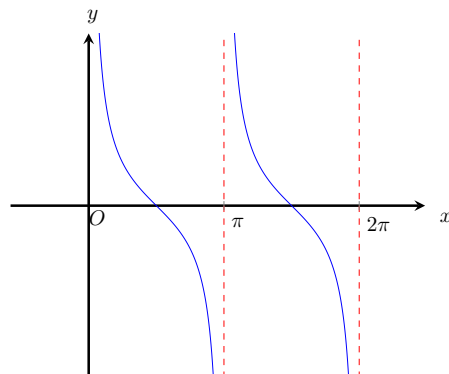
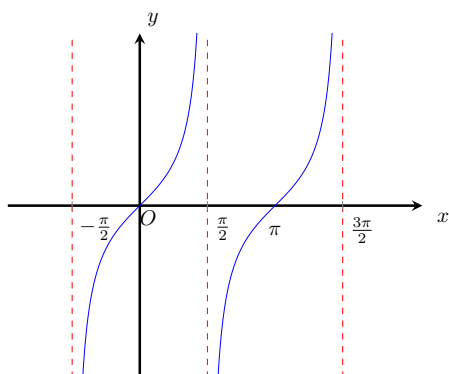
1. (a) 0.323
(b) 130
2. $a = \frac{3}{5 \lg 2}, b = \frac{5}{2} - \frac{3}{5 \lg 2}$
3. $A = 0.4, c = 1.01, x = 504$
4. (a) 5
(b) 1,1,9
5. Consider $f(1) = 0$

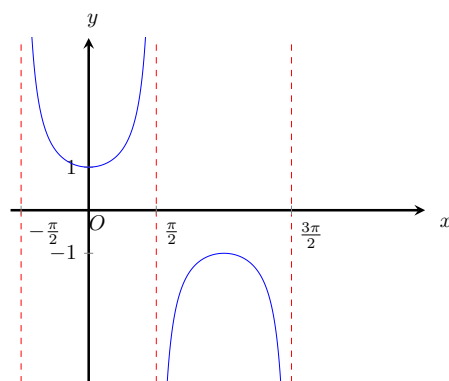
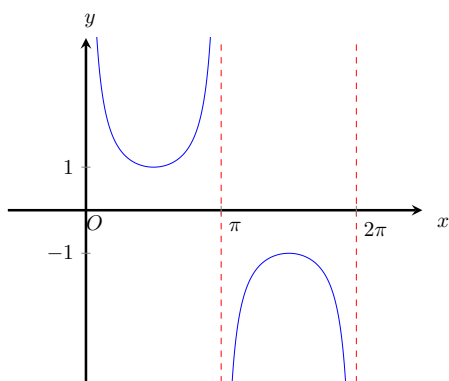
Exercise 16

1. The graph of $y = \sin x$ and $y = \cos x$:



2. The graph of $y = \tan x$ and $y = \cot x$:





3. The graph of $y = \sec x$ and $y = \csc x$:

Exercise 17

1. (a) 0.5
(b) $\frac{131}{65}$
(c) $\frac{457}{289}$
(d) $\frac{3}{5}$
2. omit
3. (a) $73.9^\circ, 286.1^\circ$
(b) $37.2^\circ, 217.2^\circ, 327.8^\circ, 142.8^\circ$
(c) $30^\circ, 210^\circ, 150^\circ, 330^\circ$
(d) $60^\circ, 240^\circ, 120^\circ, 300^\circ$

Exercise 18

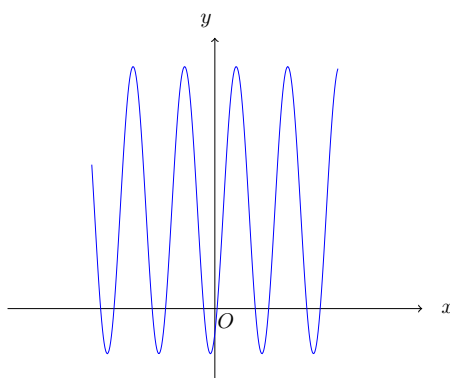
1. $\frac{\sqrt{2}}{4}(\sqrt{3} - 1)$
2. $\frac{\sqrt{3} \tan x + 1}{\tan x - \sqrt{3}}$
3. $-\frac{56}{65}, \frac{63}{65}, -\frac{16}{63}$
4. $\pm \frac{\pi}{4}, \pm \frac{3\pi}{4}, \pm \frac{5\pi}{4}, \pm \frac{7\pi}{4}$
5. 0.12, 3.26, -3.02, -6.16
6. $-\frac{59}{72}$
7. $-\frac{4}{3}, -\frac{3}{4}$
8. 0
9. Use $\sin 2\xi = 2 \sin \xi \cos \xi$, $\cos \xi = 1 - 2 \sin^2 \xi$.

Exercise 19

1. $\pm \frac{11}{6}, \pm \frac{5}{6}$
2. $-1 \pm \sqrt{2}, -1 + \sqrt{2}$
3. $\frac{\pi}{2}, 5.44, 2.29$.
4. $x = k\pi, 0.869 + k\pi, -0.869 + k\pi$.
5. $\sin^2 \frac{1}{2}A = \pm \frac{\sqrt{2}}{2} \sqrt{1 - \cos A}, \cos^2 \frac{1}{2}A = \pm \frac{\sqrt{2}}{2} \sqrt{1 + \cos A}$.
6. $\sin 3A = 3 \sin A - 4 \sin^3 A, \cos 3A = 4 \cos^3 A - 3 \cos A, \tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$.
7. Omit.
8. Omit.
9. (a) $\tan \theta$
 (b) $\frac{1}{2}$
 (c) 1
10. $1, -1 + \sqrt{2}$
11. $\frac{7\sqrt{65}}{65}$.

Exercise 20

1. (a) $\sqrt{53}, 15.9^\circ, 74.1^\circ$
 (b) $\sqrt{41}, 51.3^\circ$
 (c) $\sqrt{34}, 59.0^\circ$
2. 5.57
3. $-2 \sin \alpha$
4. $[\frac{2}{37}, \frac{2}{3}]$
5. 9, -17, 0.588, 0.983
6. the graph is as follows: period $\frac{2\pi}{3}$



7. π

Exercise 21

- (a) 1
(b) $-\frac{3}{4}$
- $k = -1, m = -1$
- $a_n = \cos \frac{1}{2}x \cos \frac{1}{4}x \cdots \cos^2 \frac{1}{2^{n-1}}x, b_n = \cos \frac{1}{2}x \cos \frac{1}{4}x \cdots \cos \frac{1}{2^{n-1}}x$
- omit
- prove $\tan \frac{q+p}{2} = \tan \frac{r+s}{2}$

Exercise 22

- (a) $3e^{3x+1}$
(b) $\frac{1}{2} \cdot 2^{\sqrt{x}}(\ln 2) \cdot (x^{-\frac{1}{2}})$
(c) $3(2 + e^{1-x})^{-2}e^{1-x}$
(d) $\frac{1}{3}e^{3x-x^2}(3-2x) - \frac{2}{3}e^{1-x-x^2}(-1-2x)$.
(e) omit
- Maximum, $(\frac{\ln 2}{2}, 4\sqrt{2})$
- Minimum, $(-1, e^{-3})$
- $x = 0$.
- Omit.

Exercise 23

- (a) $\frac{2x}{x^2+1}$
(b) $\frac{2}{(2x-5)\ln 3}$
(c) $\frac{3}{x} + \frac{8x}{x^2-1}$
(d) $\frac{4}{4x+3} - \frac{3x^2}{x^3-1}$
(e) $-\frac{1}{(x+1)(\ln(x+1))^2}$
(f) $-\frac{2}{x(\ln x)^2}$.
- $(1, \ln 2)$
- $y = -x + 4$
- $y - 2e = -2x$
- $x = -3$. $x < -3$, then $\frac{dy}{dx} < 0$, $x > -3$, then $\frac{dy}{dx} > 0$.
- (a) $(-\infty, \infty)$
(b) $f^{-1}(x) = \frac{x+\sqrt{x^2+4}}{2}$.
(c) $y - 1 = \frac{1}{2}(x - 0)$.

Exercise 24

1. (a) $2x \cos(x^2)$.
 (b) $-\frac{1}{2} \sin x (1 + \cos x)^{-\frac{1}{2}}$.
 (c) $6 \cos 2x e^{3 \sin 2x}$.
 (d) $\tan x$.
 (e) $2e^{2x} \cos(e^{2x})$.
 (f) $-\sin 2x$.
 (g) $\sin 2x$.
2. $y - \ln \sqrt{2} = x - \frac{\pi}{4}$
3. $y + \frac{1}{3} = -\frac{2}{3\sqrt{3}}(x - \frac{\pi}{6})$
4. $x = \frac{\pi}{6}$, max; $x = \frac{5}{6}\pi$, min.
5. $\cos 2x(1 - \sin 2x)$.
6. omit, $-(\csc x - \sin x)^{-2}(-\csc x \cot x - \cos x)$.
7. Omit.
8. (a) $\frac{1}{2} \cos \frac{1}{2}x + \frac{1}{3} \sin \frac{1}{3}x$.
 (b) -1 and $\frac{1}{2}$
 (c) 12π
 (d) 12π
9. $\frac{1}{\sqrt{1-x^2}}, -\frac{1}{\sqrt{1-x^2}}$
10. (a) $-\frac{1}{2}x^{-\frac{1}{2}} \sin \sqrt{x}$
 (b) $-\frac{1}{2} \sin x (\cos x)^{-\frac{1}{2}}$
 (c) $2x \cos x^2$
 (d) $(-x^2) \cos \frac{1}{x}$.

Exercise 25

1. (a) $\sec^2(x)$.
 (b) $-\csc^2 x$
 (c) $x + 2x \ln x$.
 (d) $2x \sec^2(x^2 + 1)$
 (e) $\frac{2xe^{2x} - (e^{2x} + 1)}{x^2}$.
 (f) $(2x) \sin x^2 + (x^2 - 2)(\cos x^2)(2x)$
2. Max $(-3, 6e^{-3})$, Min $(1, -2e)$
3. omit
4. $(e, \frac{1}{3})$
5. $(1, \frac{1}{2})$
6. (a) $x = \frac{\pi}{4}$
 (b) max

7. $u'_1 u_2 u_3 \cdots u_k + u_1 u'_2 \cdots u_k + \cdots + u_1 u_2 \cdots u'_k$
8. $\sum_{k=0}^n \binom{n}{k} u^{(n-k)} v^{(k)}$

Exercise 26

Omit

Exercise 27

1. $y - 4 = 3(x + 8)$
2. $x = a \cos t, y = a \sin t. AB = a.$
3. $\frac{dy}{dx} = \frac{e^{-t}}{1-e^{-t}}. t = \ln 3 - \ln 2.$
4. $t = 3, (\frac{4}{3}, \frac{27}{e^3}).$
5. Omit.
6. (a) $\frac{5}{3}$
(b) $-\frac{32}{27}.$
7. Omit.

Exercise 28

1. $-\frac{1}{\pi+2}$
2. $y - 2 = \frac{1}{3}(x - 1)$
3. Horizontal: $y = \pm 3\sqrt{3}$, vertical: $x = \pm 3$
4. $(-1, -2)$, max; $(0, 0)$, min.
5. $(\frac{8}{3}, -4)$
6. $-1, 4.$
7. Omit
8. $(2e, e^{\frac{1}{2}}). \ln y \neq 0.$
9. $(1, -4)$, min; $(-1, 4)$, max.
10. Omit.

Exercise 29

1. (a) $\frac{2}{3} \ln |3x - 1| + C$
(b) $\frac{1}{2} e^{2x+3} + C$
(c) $\frac{\sqrt{3}}{3} \tan^{-1}(\frac{2}{\sqrt{3}}x) + C$

- (d) $-\frac{1}{2} \cos 2x + C$
 - (e) $\frac{1}{2}x + \frac{1}{4} \sin 2x + C$
2. $x^4 - 3x^2 + x + 7$

Exercise 30

Omit

Exercise 31

- 1. $8\frac{5}{8} + 3 \ln 2$
- 2. $\frac{1}{2} - \frac{1}{2e}$
- 3. 2
- 4. $\frac{\pi}{8} - \frac{1}{12}$
- 5. $\frac{\pi}{8}$
- 6. 0

Exercise 32

- 1. $1 - \frac{\pi}{4}$
- 2. $2\frac{11}{12} - \ln 6$
- 3. Omit
- 4. (a) $4e^{\frac{x}{2}} - 2x - 1$
(b) min.
- 5. Omit
- 6. Omit
- 7. Omit
- 8. Omit
- 9. Omit

Exercise 33

- 1. If $p = 1$, infinite;
if $p > 1$, $\int_0^1 \frac{1}{x^p} dx$ infinite, $\int_1^\infty \frac{1}{x^p} dx = \frac{1}{1-p}$;
if $p < 1$, $\int_0^1 \frac{1}{x^p} dx = \frac{1}{1-p}$, $\int_1^\infty \frac{1}{x^p} dx$ infinite.
- 2. $\frac{1}{2}e$

3. Infinite
4. Omit

Exercise 34

1. (a) $\frac{287}{192}\pi + \pi \ln 4$
 (b) $\frac{\pi}{4}(e^2 - e^{-2})$
 (c) $\frac{\pi^2}{2}$
 (d) $\pi(\sqrt{3} - 1 - \frac{\pi}{12})$.
2. $\frac{1}{2}$
3. $2\pi^2 r a^2$
4. (a) $\pi(\sqrt{3} - \frac{\pi}{3})$
 (b) Omit.

Exercise 35

1. (a) $\int 1 + \frac{-2}{x-2} + \frac{-3}{x+3} dx = x - 2 \ln |x-2| - 3 \ln |x+3| + C$.
 (b) $\int \frac{\frac{5}{3}}{x+4} + \frac{\frac{-3}{4}}{x+2} + \frac{\frac{1}{12}}{x-2} dx = \frac{5}{3} \ln |x+4| - \frac{3}{4} \ln |x+2| + \frac{1}{12} \ln |x-2| + C$.
 (c) $\int \frac{1}{4} \frac{1}{1-x} + \frac{1}{4} \frac{1}{1+x} + \frac{1}{2} \frac{1}{1+x^2} dx = -\frac{1}{4} \ln |1-x| + \frac{1}{4} \ln |1+x| + \frac{1}{2} \tan^{-1} x + C$.
 (d) $\int_{-\frac{1}{2}}^1 \frac{1}{2} \frac{1}{x+1} - \frac{1}{x+2} + \frac{1}{2} \frac{1}{x+3} dx = \frac{1}{2}(\ln 8 - \ln 5)$.
 (e) $\frac{3}{2} \ln 3 - \ln 4$.
2. $10\frac{5}{12}\pi$; 20.8333π

Exercise 36

1. $dt = 2x dx$
2. $dx = t dt$
3. $dt = \cos x dx$
4. $dx = e^t dt$
5. $dx = \frac{1}{t+1} dt$
6. $dt = \frac{1}{2} \sec^2 \frac{x}{2} dx$
7. $dt = \frac{2x}{1+x^4} dx$
8. $dt = -\tan x dx$

Exercise 37

1. $e^{\sin x} + C$
2. $-\frac{1}{6}(3-2x^2)^{\frac{3}{2}} + C$
3. $\frac{1}{3}\ln|x^3-1| + C$
4. $\frac{\pi}{8}$
5. $\frac{7}{24}$
6. $\frac{3}{14}(x^2-1)^7 + \frac{1}{4}(x^2-1)^6 + C$
7. $x + 4\sqrt{x} + 4\ln|\sqrt{x}-1| + C$
8. $\frac{27}{140}$
9. $\frac{1696}{2835}$
10. $\frac{3}{8}$
11. $\frac{2}{3}\pi$
12. π
13. $2x^{\frac{1}{2}} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\ln|1+x^{\frac{1}{6}}| + C$
14. $\ln(\sec x + \tan x) + C$
15. $\frac{1}{3}\ln|1+x| - \frac{1}{3}\left(\frac{1}{2}\ln|x^2-x+1| - \sqrt{3}\tan^{-1}\left(\frac{2}{\sqrt{3}}(x-\frac{1}{2})\right)\right) + C$

Exercise 38

1. $\frac{1}{3}xe^{3x} - \frac{1}{9}e^x + C$
2. $\frac{1}{5}x^5\ln 3x - \frac{1}{25}x^5 + C$
3. $\frac{-x\ln x-1}{x} + C$
4. $\frac{\sqrt{2}}{2}(4-\pi)$
5. $\frac{\pi^2}{4}$
6. $-\frac{1}{2}x^2\cos 2x + \frac{x}{2}\sin 2x + \frac{1}{4}\cos 2x + C$
7. $\frac{4}{25}e^{3x}\sin 4x + \frac{3}{25}e^{3x}\cos 4x + C$
8. $\frac{3}{10}(e^{-\pi} - e^{\pi})$
9. $\frac{\pi}{4} - \frac{1}{2}\ln 2$
10. $\frac{1}{3}x^3\ln^2 x - \frac{2}{9}x^3\ln x + \frac{2}{27}x^3 + C$
11. $\frac{1}{2}e^{x^2}(x^2-1) + C$
12. $2e^{\sin x}(\sin x - 1) + C$
13. $4(\sqrt{2}-1)$

Exercise 39

1. $-\frac{\sqrt{1+x^2}}{x} + C$
2. $\frac{1}{11} \tan^{11} x + C$
3. $-2 \cos \sqrt{x} + C$
4. $\frac{1}{2} \ln \left| \frac{e^x - 1}{e^x + 1} \right| + C$
5. $\frac{3}{2} (\sin x - \cos x)^{\frac{3}{2}} + C$
6. $-(\arcsin x)^{-1} + C$
7. $-\ln |\cos \sqrt{1+x^2}| + C$
8. $\frac{1}{2} (\ln |\sqrt{1+e^{2x}} - 1| - \ln |\sqrt{1+e^{2x}} + 1|) + C$
9. $-\frac{1}{3} \left(\frac{x}{\sqrt{1+x^2}} \right)^{-3} + \left(\frac{x}{\sqrt{1+x^2}} \right)^{-1} + C$
10. $-\ln \left| 1 + \sqrt{\frac{x-1}{x+1}} \right| + \ln \left| 1 - \sqrt{\frac{x-1}{x+1}} \right| - \left(1 + \sqrt{\frac{x-1}{x+1}} \right)^{-1} + \left(1 - \sqrt{\frac{x-1}{x+1}} \right)^{-1} + C$
11. $\frac{x}{\sqrt{1-x^2}} + C$
12. $\frac{1}{2} \arcsin x + \frac{1}{2} \ln |x + \sqrt{1-x^2}| + C$
13. $\sqrt{2x} - \ln |1 + \sqrt{2x}| + C$
14. $\operatorname{arcsec} x + C$
15. $(\arctan \sqrt{x})^2 + C$
16. $\frac{1}{4} \ln \left| \frac{\sqrt[4]{1+x^4}-1}{\sqrt[4]{1+x^4}+1} \right| + \frac{1}{2} \arctan \sqrt[4]{1+x^4} + C$
17. $-(x \ln x)^{-1} + C$
18. $2 \left(\sqrt{x+1} e^{\sqrt{x+1}} - e^{\sqrt{x+1}} \right) + C$
19. $x(-\cot x) + \ln |\sin x| + C$
20. $x \arcsin x + \sqrt{1-x^2} + C$
21. $\arctan \left(\frac{1}{3} x^3 \right) - \frac{1}{3} \left(\frac{1}{2} x^2 - \frac{1}{2} \ln |1+x^2| + C \right)$
22. $x \ln^2 x - 2x \ln x + 2x + C$
23. $x \tan x + \ln |\cos x| - \frac{x^2}{2} + C$
24. $-x^{-1} \ln^3 x - 3x^{-1} \ln^2 x - 6x^{-1} \ln x - 6x^{-1} + C$
25. $\frac{1}{2} (\sin(\ln x) + \cos(\ln x)) + C$
26. $x (\arcsin x)^2 + 2 (\arcsin x) (1-x^2)^{\frac{1}{2}} + 2x + C$
27. $e^{\sqrt{x}} (2x - 4\sqrt{x} + 4) + C$
28. $x \ln (x + \sqrt{1+x^2}) - (1+x^2)^{\frac{1}{2}} + C$
29. *Omit*
30. *Omit*
31. *Omit*

32. *Omit*
33. *Omit*
34. *Omit*
35. *Omit*
36. *Omit*
37. *Omit*
38. *Omit*

Exercise 40

1. *Omit*
2. *Omit*
3. *Omit*
4. *Omit*
5. *Omit*
6. *Omit*
7. *Omit*
8. *Omit*
9. *Omit*
10. *Omit*
11. *Omit*
12. *Omit*
13. *Omit*

Exercise 41

1. Omit
2. $-2, 1, 16$
3. 1.466
4. 7.725

Exercise 42

1. 1.17
2. 1.37

3. 2.38
4. 0.739
5. 0.79

Exercise 43

1. (a) Omit
 (b) $3x^2 > 1$ for $x \in (1, 2)$
 (c) $|\frac{1}{3}(x+1)^{-\frac{2}{3}}| < 1$ for $x \in (1, 2)$
2. (a) 3
 (b) 3.25
 (c) $x = -\sqrt{e^x - 5x + 1}$, -5.19 ; $x = \frac{e^x - x^2 + 1}{5}$, 0.48 .

Exercise 44

1. 6.2417.
2. 1.95.
3. 0.7828, 0.7842, 0.7847. 0.7854
4. 7.500, 7.306.