1	Carry out division or equivalent at least as far as two terms of quotient	M1	
	Obtain quotient $2x-4$	A1	
	Obtain remainder 8	A1	[3]
2	Obtain $1-x$ as first two terms of $(1+2x)^{-\frac{1}{2}}$	B1	
	Obtain $+\frac{3}{2}x^2$ or unsimplified equivalent as third term of $(1+2x)^{-\frac{1}{2}}$	B1	
	Multiply $1+3x$ by attempt at $(1+2x)^{-\frac{1}{2}}$, obtaining sufficient terms	M1	
	Obtain final answer $1+2x-\frac{3}{2}x^2$	A1	[4]
3	State or imply correct form $\dfrac{A}{x}+\dfrac{Bx+C}{x^2+1}$	B1	
	Use any relevant method to find at least one constant	M1	
	Obtain $A=2$	A1	
	Obtain $B=5$	A1	
	Obtain $C=-3$	A1	[5]
4	**(i)** Either State or imply non-modular equation $(4x-1)^2=(x-3)^2$ or pair of linear equations $4x-1=\pm(x-3)$	B1	
	Solve a three-term quadratic equation or two linear equations	M1	
	Obtain $-\dfrac{2}{3}$ and $\dfrac{4}{5}$	A1	
	Or Obtain value $-\dfrac{2}{3}$ from inspection or solving linear equation	B1	
	Obtain value $\dfrac{4}{5}$ similarly	B2	[3]
	(ii) State or imply at least $4^y=\dfrac{4}{5}$, following a positive answer from part (i)	B1√	
	Apply logarithms and use $\log a^b = b\log a$ property	M1	
	Obtain –0.161 and no other answer	A1	[3]
5	**(i)** Use correct quotient rule or equivalent	M1	
	Obtain $\dfrac{(1+e^{2x})2x-(1+x^2)2e^{2x}}{(1+e^{2x})^2}$ or equivalent	A1	
	Substitute $x=0$ and obtain $-\dfrac{1}{2}$ or equivalent	A1	[3]
	(ii) Differentiate y^3 and obtain $3y^2\dfrac{dy}{dx}$	B1	
	Differentiate $5xy$ and obtain $5y+5x\dfrac{dy}{dx}$	B1	
	Obtain $6x^2+5y+5x\dfrac{dy}{dx}+3y^2\dfrac{dy}{dx}=0$	B1	

		Substitute $x = 0, y = 2$ to obtain $-\frac{5}{6}$ or equivalent following correct work		B1	[4]
6	(i)	State or imply A is (1, 4, –2)		B1	
		State or imply $\overrightarrow{QP} = 12\mathbf{i} + 6\mathbf{j} - 6\mathbf{k}$ or equivalent		B1	
		Use QP as normal and A as mid-point to find equation of plane		M1	
		Obtain $12x + 6y - 6z = 48$ or equivalent		A1	[4]
	(ii)	Either	State equation of PB is $\mathbf{r} = 7\mathbf{i} + 7\mathbf{j} - 5\mathbf{k} + \lambda\mathbf{i}$	B1	
			Set up and solve a relevant equation for λ.	M1	
			Obtain $\lambda = -9$ and hence B is (–2, 7, –5)	A1	
			Use correct method to find distance between A and B.	M1	
			Obtain 5.20	A1	
		Or	Obtain 12 for result of scalar product of QP and $\mathbf{i}$ or equivalent	B1	
			Use correct method involving moduli, scalar product and cosine to find angle APB	M1	
			Obtain $35.26°$ or equivalent	A1	
			Use relevant trigonometry to find AB	M1	
			Obtain 5.20	A1	[5]
7	(a)	State or imply $3a + 3b\mathrm{i} + 2\mathrm{i}(a - b\mathrm{i}) = 17 + 8\mathrm{i}$		B1	
		Consider real and imaginary parts to obtain two linear equations in a and b		M1*	
		Solve two simultaneous linear equations for a or b		M1 (dep*)	
		Obtain $7 - 2\mathrm{i}$		A1	[4]
	(b)	Either	Show or imply a triangle with side 2	B1	
			State at least two of the angles $\frac{1}{4}\pi, \frac{2}{3}\pi$ and $\frac{1}{12}\pi$	B1	
			State or imply argument is $\frac{1}{4}\pi$	B1	
			Use sine rule or equivalent to find r	M1	
			Obtain $6.69\mathrm{e}^{\frac{1}{4}\pi\mathrm{i}}$	A1	
		Or	State $y = x$.	B1	
			State $y = \frac{1}{\sqrt{3}}x + 2$ or $\frac{\sqrt{3}}{2} = \frac{x}{\sqrt{x^2 + (y-2)^2}}$ or $\frac{1}{2} = \frac{y-2}{\sqrt{x^2 + (y-2)^2}}$	B1	
			State or imply argument is $\frac{\pi}{4}$	B1	
			Solve for x or y.	M1	
			Obtain $6.69e^{\frac{1}{4}\pi i}$	A1	[5]
8	(a)	Carry out integration by parts and reach $ax^2 \ln x + b\int \frac{1}{2}x^2 \mathrm{d}x$		M1*	
		Obtain $2x^2 \ln x - \int \frac{1}{x} . 2x^2 \, \mathrm{d}x$		A1	
		Obtain $2x^2 \ln x - x^2$		A1	
		Use limits, having integrated twice		M1 (dep*)	
		Confirm given result $56 \ln 2 - 12$		A1	[5]

	(b)	State or imply $\frac{\mathrm{d}u}{\mathrm{d}x} = 4\cos 4x$	B1	
		Carry out complete substitution except limits	M1	
		Obtain $\int(\frac{1}{4} - \frac{1}{4}u^2)\,\mathrm{d}u$ or equivalent	A1	
		Integrate to obtain form $k_1u + k_2u^3$ with non-zero constants k_1, k_2	M1	
		Use appropriate limits to obtain $\frac{11}{96}$	A1	[5]
9	**(i)**	State or imply $R = 5$	B1	
		Use relevant trigonometry to find α	M1	
		Obtain $\alpha = 0.6435$	A1	[3]
	(ii)	**(a)** Carry out appropriate method to find one value in given range	M1	
		Obtain 1.80	A1	
		Carry out appropriate method to find second value in given range	M1	
		Obtain 5.77 and no other value	A1	[4]
		(b) Express integrand as $k\sec^2(\theta - \text{their } \alpha)$ for any constant k	M1	
		Integrate to obtain result $k\tan(\theta - \text{their } \alpha)$	A1	
		Obtain correct answer $2\tan(\theta - 0.6435)$	A1	[3]
10	**(i)**	State $\frac{\mathrm{d}V}{\mathrm{d}t} = 80 - kV$	B1	
		Correctly separate variables and attempt integration of one side	M1	
		Obtain $a\ln(80 - kV) = t$ or equivalent	M1*	
		Obtain $-\frac{1}{k}\ln(80 - kV) = t$ or equivalent	A1	
		Use $t = 0$ and $V = 0$ to find constant of integration or as limits	M1 (dep*)	
		Obtain $-\frac{1}{k}\ln(80 - kV) = t - \frac{1}{k}\ln 80$ or equivalent	A1	
		Obtain given answer $V = \frac{1}{k}(80 - 80\mathrm{e}^{-kt})$ correctly	A1	[7]
	(ii)	Use iterative formula correctly at least once	M1	
		Obtain final answer 0.14	A1	
		Show sufficient iterations to 4 s.f. to justify answer to 2 s.f. or show a sign change in the interval (0.135, 0.145)	A1	[3]
	(iii)	State a value between 530 and 540 cm^3 inclusive	B1	
		State or imply that volume approaches 569 cm^3 (allowing any value between 567 and 571 inclusive)	B1	[2]

1	*EITHER*:	State or imply non-modular equation $(x-2)^2 = \left(\frac{1}{3}x\right)^2$, or pair of equations $x - 2 = \pm\frac{1}{3}x$	M1	
		Obtain answer $x = 3$	A1	
		Obtain answer $x = \frac{3}{2}$, or equivalent	A1	
	OR:	Obtain answer $x = 3$ by solving an equation or by inspection	B1	
		State or imply the equation $x - 2 = -\frac{1}{3}$, or equivalent	M1	
		Obtain answer $x = \frac{3}{2}$, or equivalent	A1	[3]
2	**(i)**	Use the iterative formula correctly at least once	M1	
		Obtain final answer 3.6840	A1	
		Show sufficient iterations to at least 6 d.p. to justify 3.6840, or show there is a sign change in the interval (3.68395, 3.68405)	A1	[3]
	(ii)	State a suitable equation, e.g. $x = \frac{x(x^3+100)}{2(x^3+25)}$	B1	
		State that the value of α is $3\sqrt{50}$, or exact equivalent	B1	[2]
3	*EITHER*:	State or imply $\ln y = \ln A - kx^2$	B1	
		Substitute values of $\ln y$ and x^2, and solve for k or $\ln A$	M1	
		Obtain $k = 0.42$ or $A = 2.80$	A1	
		Solve for $\ln A$ or k	M1	
		Obtain $A = 2.80$ or $k = 0.42$	A1	
	OR1:	State or imply $\ln y = \ln A - kx^2$	B1	
		Using values of $\ln y$ and x^2, equate gradient of line to $-k$ and solve for k	M1	
		Obtain $k = 0.42$	A1	
		Solve for $\ln A$	M1	
		Obtain $A = 2.80$	A1	
	OR2:	Obtain two correct equations in k and A and substituting y– and x^2 – values in $y = Ae^{-kx^2}$	B1	
		Solve for k	M1	
		Obtain $k = 0.42$	A1	
		Solve for A	M1	
		Obtain $A = 2.80$	A1	[5]
		[SR: If unsound substitutions are made, e.g. using $x = 0.;64$ and $y = 0.76$, give B1M0A0M1A0 in the *EITHER* and *OR1* schemes, and B0M1A0M1A0 in the *OR2* scheme.]		

4	**(i)**	Substitute $x = -\frac{1}{3}$, or divide by $3x+1$, and obtain a correct equation, e.g. $-\frac{1}{27}a - \frac{20}{9} - \frac{1}{3} + 3 = 0$	B1	
		Solve for a an equation obtained by a valid method	M1	
		Obtain $a = 12$	A1	[3]
	(ii)	Commence division by $3x+1$ reaching a partial quotient $\frac{1}{3}ax^2 + kx$	M1	
		Obtain quadratic factor $4x^2 - 8x + 3$	A1	
		Obtain factorisation $(3x+1)(2x-1)(2x-3)$	A1	[3]
		[The M1 is earned if inspection reaches an unknown factor $\frac{1}{3}ax^2 + Bx + C$ and an equation in B and/or C, or an unknown factor $Ax^2 + Bx + 3$ and an equation in A and/or B, or if two coefficients with the correct moduli are stated without working.]		
		[If linear factors are found by the factor theorem, give B1B1 for $(2x - 1)$ and $(2x - 3)$, and B1 for the complete factorisation.]		
		[Synthetic division giving $12x^2 - 24x + 9$ as quadratic factor earns M1A1, but the final factorisation needs $(x + \frac{1}{3})$, or equivalent, in order to earn the second A1.]		
		[SR: If $x = \frac{1}{3}$ is used in substitution or synthetic division, give the M1 in part **(i)** but give M0 in part **(ii)**.]		
5	*EITHER*:	State $2ay\frac{dy}{dx}$ as derivative of ay^2	B1	
		State $y^2 + 2xy\frac{dy}{dx}$ as derivative of xy^2	B1	
		Equate derivative of LHS to zero and set $\frac{dy}{dx}$ equal to zero	M1	
		Obtain $3x^2 + y^2 - 6ax = 0$, or horizontal equivalent	A1	
		Eliminate y and obtain an equation in x	M1	
		Solve for x and obtain answer $x = \sqrt{3}a$	A1	
	OR1:	Rearrange equation in the form $y^2 = \frac{3ax^2 - x^3}{x+a}$ and attempt differentiation of one side	B1	
		Use correct quotient or product rule to differentiate RHS	M1	
		Obtain correct derivative of RHS in any form	A1	
		Set $\frac{dy}{dx}$ equal to zero and obtain an equation in x	M1	
		Obtain a correct horizontal equation free of surds	A1	
		Solve for x and obtain answer $x = \sqrt{3a}$	A1	
	OR2:	Rearrange equation in the form $y = \left(\frac{3ax^2 - x^3}{x+a}\right)^{\frac{1}{2}}$ and differentiation of RHS	B1	
		Use correct quotient or product rule and chain rule	M1	
		Obtain correct derivative in any form	A1	

		Equate derivative to zero and obtain an equation in x	M1	
		Obtain a correct horizontal equation free of surds	A1	
		Solve for x and obtain answer $x = \sqrt{3}a$	A1	[6]
6	**(i)**	Use correct quotient or chain rule to differentiate sec x	M1	
		Obtain given derivative, sec x tan x, correctly	A1	
		Use chain rule to differentiate y	M1	
		Obtain the given answer	A1	[4]
	(ii)	Using $\mathrm{d}x\sqrt{3}\sec^2\theta\,\mathrm{d}\theta$, or equivalent, express integral in terms of θ and $\mathrm{d}\theta$	M1	
		Obtain $\int \sec\theta\,\mathrm{d}\theta$	A1	
		Use limits $\frac{1}{6}\pi$ and $\frac{1}{3}\pi$ correctly in an integral form of the form $k\ln(\sec\theta + \tan\theta)$	M1	
		Obtain a correct exact final answer in the given form, e.g. $\ln\left(\frac{2+\sqrt{3}}{\sqrt{3}}\right)$	A1	[4]
7	**(i)**	Use cos $(A + B)$ formula to express the given expression in terms of cos x and sin x	M1	
		Collect terms and reach $\frac{\cos x}{\sqrt{2}} - \frac{3}{\sqrt{2}}\sin x$, or equivalent	A1	
		Obtain $R = 2.236$	A1	
		Use trig formula to find α	M1	
		Obtain $\alpha = 71.57°$ with no errors seen	A1	[5]
	(ii)	Evaluate $\cos^{-1}$ (2/2.236) to at least 1 d.p. (26.56° to 2 d.p., use of $R = \sqrt{5}$ gives 26.57°)	B1$\sqrt{}$	
		Carry out an appropriate method to find a value of x in the interval $0° < x < 360°$	M1	
		Obtain answer, e.g. $x = 315°$ (315.0°)	A1	
		Obtain second answer, e.g. 261.9° and no others in the given interval	A1	[4]
		[Ignore answers outside the given range.]		
		[Treat answers in radians as a misread and deduct A1 from the answers for the angles.]		
		[SR: Conversion of the equation to a correct quadratic in sin x, cos x, or tan x earns B1, then M1 for solving a 3-term quadratic and obtaining a value of x in the given interval, and A1 + A1 for the two correct answers (candidates must reject spurious roots to earn the final A1).]		
8	**(i)**	Use any relevant method to determine a constant	M1	
		Obtain one of the values $A = 1$, $B = -2$, $C = 4$	A1	
		Obtain a second value	A1	
		Obtain the third value	A1	[4]
		[If A and C are found by the cover up rule, give B1 + B1 then M1A1 for finding B. If only one is found by the rule, give B1M1A1A1.]		
	(ii)	Separate variables and obtain one term by integrating $\frac{1}{y}$ or a partial fraction	M1	
		Obtain $\ln y = -\frac{1}{2} - 2\ln(2x + 1) + c$, or equivalent	A3$\sqrt{}$	

		Evaluate a constant, or use limits $x = 1$, $y = 1$, in a solution containing at least three terms of the form $k \ln y$, l/x, $m \ln x$ and $n \ln (2x + 1)$, or equivalent	M1	
		Obtain solution $\ln y = -\frac{1}{2} - 2\ln x + 2\ln(2x+1) + c$, or equivalent	A1	
		Substitute $x = 2$ and obtain $y = \frac{25}{36}e^{\frac{1}{2}}$, or exact equivalent free of logarithms	A1	[7]
		(The f.t. is on A, B, C. Give A2√ if there is only one error or omission in the integration; A1√ if two.)		
9	(a)	Substitute $w = x + iy$ and state a correct equation in x and y	B1	
		Use $i^2 = -1$ and equate real parts	M1	
		Obtain $y = -2$	A1	
		Equate imaginary parts and solve for x	M1	
		Obtain $x = 2\sqrt{2}$, or equivalent, only	A1	[5]
	(b)	Show a circle with centre 2i	B1	
		Show a circle with radius 2	B1	
		Show half line from –2 at $\frac{1}{4}\pi$ to real axis	B1	
		Shade the correct region	B1	
		Carry out a complete method for calculating the greatest value of $\lvert z \rvert$	M1	
		Obtain answer 3.70	A1	[6]
10	(i)	Carry out a correct method for finding a vector equation for AB	M1	
		Obtain $\mathbf{r} = 2\mathbf{i} - 3\mathbf{j} + 2\mathbf{k} + \lambda(3\mathbf{i} + \mathbf{j} - \mathbf{k})$ or $\mathbf{r} = \mu(2\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}) + (1 - \mu)(5\mathbf{i} - 2\mathbf{j} + \mathbf{k})$, or equivalent	A1	
		Substitute components in equation of p and solve for λ or for μ	M1	
		Obtain $\lambda = \frac{3}{2}$ or $\mu = -\frac{1}{2}$ and final answer $\frac{13}{2}\mathbf{i} - \frac{3}{2}\mathbf{j} + \frac{1}{2}\mathbf{k}$, or equivalent	A1	[4]
	(ii)	Either equate scalar product of direction vector of AB and normal to q to zero or substitute for A and B in the equation of q and subtract expressions	M1*	
		Obtain $3 + b - c = 0$, or equivalent	A1	
		Using the correct method for the moduli, divide the scalar product of the normals to p and q by the product of their moduli and equate to $\pm\frac{1}{2}$, or form horizontal equivalent	M1*	
		Obtain correct equation in any form, e.g. $\frac{1+b}{\sqrt{(1+b^2+c^2)}\sqrt{(1+1)}} = \pm\frac{1}{2}$	A1	
		Solve simultaneous equations for b or for c	M1 (dep*)	
		Obtain $b = -4$ and $c = -1$	A1	
		Use a relevant point and obtain final answer $x - 4y - z = 12$, or equivalent	A1√	[7]
		(The f.t. is on b and c.)		

1	*EITHER*:	State or imply non-modular inequality $(4x+3)^2 > x^2$, or corresponding equation or pair of equations $4x + 3 = \pm x$	M1	
		Obtain a critical value, e.g. -1	A1	
		Obtain a second critical value, e.g. $-\frac{3}{5}$	A1	
		State final answer $x < -1$, $x > -\frac{3}{5}$	A1	
	OR:	Obtain critical value $x = -1$, by solving a linear equation or inequality, or from a graphical method or by inspection	B1	
		Obtain the critical value $-\frac{3}{5}$ similarly	B2	
		State final answer $x < -1$, $x > -\frac{3}{5}$	B1	[4]
		[Do not condone $\le$ or $\ge$.]		

2	Use law for the logarithm of a product, quotient or power	M1	
	Use ln e = 1 or exp(l) = 3	M1	
	Obtain correct equation free of logarithms in any form, e.g. $\frac{y+1}{y} = \mathrm{e}x^3$	A1	
	Rearrange as $y = (\mathrm{e}x^3 - 1)^{-1}$, or equivalent	A1	[4]

3	Use correct tan $2A$ formula and cot x = 1/tan x to form an equation in tan x	M1	
	Obtain a correct horizontal equation in any form	A1	
	Solve an equation in $\tan^2 x$ for x	M1	
	Obtain answer, e.g. 40.2°	A1	
	Obtain second answer, e.g. 139.8°, and no other in the given interval	A1√	[5]
	[Ignore answers outside the given interval.]		
	[Treat answers in radians as a misread and deduct A1 from the marks for the angles.]		
	[SR: For the answer x = 90° give B1 and A1 for one of the other angles.]		

4	**(i)**	State $R = 2$	B1	
		Use trig formula to find α	M1	
		Obtain $\alpha = \frac{1}{6}\pi$ with no errors seen	A1	[3]
	(ii)	Substitute denominator of integrand and state integral k tan $(x - \alpha)$	M1*	
		State correct indefinite integral $\frac{1}{4}\tan\left(x - \frac{1}{6}\pi\right)$	A1√	
		Substitute limits	M1 (dep*)	
		Obtain the given answer correctly	A1	[4]

5	**(i)**	Substitute $x = -\frac{1}{2}$, or divide by $(2x + 1)$, and obtain a correct equation, e.g. $a - 2b + 8 = 0$	B1	
		Substitute $x = \frac{1}{2}$ and equate to 1, or divide by $(2x - 1)$ and equate constant remainder to 1	M1	
		Obtain a correct equation, e.g. $a + 2b + 12 = 0$	A1	
		Solve for a or for b	M1	
		Obtain $a = -10$ and $b = -1$	A1	[5]
	(ii)	Divide by $2x^2 - 1$ and reach a quotient of the form $4x + k$	M1	
		Obtain quotient $4x - 5$	A1	
		Obtain remainder $3x - 2$	A1	[3]
6	**(i)**	State the correct derivatives $2e^{2x-3}$ and $2/x$	B1	
		Equate derivatives and use a law of logarithms on an equation equivalent to $ke^{2x-3} = m/x$	M1	
		Obtain the given result correctly (or work *vice versa*)	A1	[3]
	(ii)	Consider the sign of $a - \frac{1}{2}(3 - \ln a)$ when $a = 1$ and $a = 2$, or equivalent	M1	
		Complete the argument with correct calculated values	A1	[2]
	(iii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.35	A1	
		Show sufficient iterations to 4 d.p. to justify 1.35 to 2 d.p., or show there is a sign change in the interval (1.345, 1.355)	A1	[3]
7	**(i)**	Show that $a^2 + b^2 = (a + ib)(a - ib)$	B1	
		Show that $(a + ib - ki)^* = a - ib + ki$	B1	[2]
	(ii)	Square both sides and express the given equation in terms of z and z^*	M1	
		Obtain a correct equation in any form, e.g. $(z - 10i)(z^* + 10i) = 4(z - 4i)(z^* + 4i)$	A1	
		Obtain the given equation	A1	
		Either express $\lvert z - 2i \rvert = 4$ in terms of z and z^* or reduce the given equation to the form $\lvert z - u \rvert = r$	M1	
		Obtain the given answer correctly	A1	[5]
	(iii)	State that the locus is a circle with centre 2i and radius 5	B1	[1]

8	**(i)**	Separate variables correctly and integrate at least one side	M1	
		Obtain term ln t, or equivalent	B1	
		Obtain term of the form $a\ \ln(k - x^3)$	M1	
		Obtain term $-\frac{2}{3}\ln(k-x^3)$, or equivalent	A1	
		EITHER: Evaluate a constant or use limits $t = 1$, $x = 1$ in a solution containing $a \ln t$ and $b \ln(k - x^3)$	M1*	
		Obtain correct answer in any form e.g. $\ln t = -\frac{2}{3}\ln(k-x^3)+\frac{2}{3}\ln(k-1)$	A1	
		Use limits $t = 4$, $x = 2$, and solve for k	M1(dep*)	
		Obtain $k = 9$	A1	
		OR: Using limits $t = 1$, $x = 1$ and $t = 4$, $x = 2$ in a solution containing $a \ln t$ and $b \ln (k - x^3)$ obtain an equation in k	M1*	
		Obtain a correct equation in any form, e.g. $\ln 4 = -\frac{2}{3}\ln(k-8)+\frac{2}{3}\ln(k-1)$	A1	
		Solve for k	M1(dep*)	
		Obtain $k = 9$	A1	
		Substitute $k = 9$ and obtain $x = (9-8t^{-\frac{3}{2}})^{\frac{1}{3}}$	A1	[9]
	(ii)	State that x approaches $9^{\frac{1}{3}}$, or equivalent	B1√	[1]
9	**(i)**	Use product rule	M1	
		Obtain correct derivative in any form, e.g. $4\sin 2x \cos 2x \cos x - \sin^2 2x \sin x$	A1	
		Equate derivative to zero and use a double angle formula	M1*	
		Reduce equation to one in a single trig function	M1(dep*)	
		Obtain a correct equation in any form, e.g. $10 \cos^3 x = 6 \cos x$, $4 = 6 \tan^2 x$ or $4 = 10 \sin^2 x$	A1	
		Solve and obtain $x = 0.685$	A1	[6]
	(ii)	Using $du = \pm \cos x\ dx$, or equivalent, express integral in terms of u and du	M1	
		Obtain $\int 4u^2(1-u^2)\,du$, or equivalent	A1	
		Use limits $u = 0$ and $u = 1$ in an integral of the form $au^3 + bu^5$	M1	
		Obtain answer $\frac{8}{15}$ (or 0.533)	A1	[4]
10	**(i)**	Equate scalar product of direction vector of l and p to zero	M1	
		Solve for a and obtain $a = -6$	A1	[2]
	(ii)	Express general point of l correctly in parametric form, e.g. $3\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \mu(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ or $(1 - \mu)(3\mathbf{i} + 2\mathbf{j} + \mathbf{k}) + \mu(\mathbf{i} + \mathbf{j} - \mathbf{k})$	B1	
		Equate at least two pairs of corresponding components of l and the second line and solve for λ or for μ	M1	
		Obtain either $\lambda = \frac{2}{3}$ or $\mu = \frac{1}{3}$; or $\lambda = \frac{2}{a-1}$ or $\mu = \frac{1}{a-1}$; or reach $\lambda(a-4)=0$ or $(1+\mu)(a-4)=0$	A1	
		Obtain $a = 4$ having ensured (if necessary) that all three component equations are satisfied	A1	[4]

(iii) Using the correct process for the moduli, divide scalar product of direction vector if l and normal to p by the product of their moduli and equate to the sine of the given angle, or form an equivalent horizontal equation — M1*

Use $\frac{2}{\sqrt{5}}$ as sine of the angle — A1

State equation in any form, e.g. $\dfrac{a+6}{\sqrt{(a^2+4+1)}\sqrt{(1+4+4)}} = \dfrac{2}{\sqrt{5}}$ — A1

Solve for a — M1 (dep*)

Obtain answers for $a = 0$ and $a = \dfrac{60}{31}$, or equivalent — A1 [5]

[Allow use of the cosine of the angle to score M1M1.]

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the May/June 2014 series

9709 MATHEMATICS

9709/31 Paper 3, maximum raw mark 75

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Penalties

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PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	**(i)** State $\sin 2\alpha = 2\sin\alpha\cos\alpha$ and $\sec\alpha = 1/\cos\alpha$	B1	
	Obtain $2\sin\alpha$	B1	[2]
	(ii) Use $\cos 2\beta = 2\cos^2\beta - 1$ or equivalent to produce correct equation in $\cos\beta$	B1	
	Solve three-term quadratic equation for $\cos\beta$	M1	
	Obtain $\cos\beta = \frac{1}{3}$ only	A1	[3]
2	State $\frac{du}{dx} = 3\sec^2 x$ or equivalent	B1	
	Express integral in terms of u and du (accept unsimplified and without limits)	M1	
	Obtain $\int \frac{1}{3}u^{\frac{1}{2}}\,du$	A1	
	Integrate $Cu^{\frac{1}{2}}$ to obtain $\frac{2C}{3}u^{\frac{3}{2}}$	M1	
	Obtain $\frac{14}{9}$	A1	[5]
3	Obtain $\frac{2}{2t+3}$ for derivative of x	B1	
	Use quotient of product rule, or equivalent, for derivative of y	M1	
	Obtain $\frac{5}{(2t+3)^2}$ or unsimplified equivalent	A1	
	Obtain $t = -1$	B1	
	Use $\frac{dy}{dx} = \frac{dy}{dt} / \frac{dx}{dt}$ in algebraic or numerical form	M1	
	Obtain gradient $\frac{5}{2}$	A1	[6]
4	Separate variables correctly and recognisable attempt at integration of at least one side	M1	
	Obtain $\ln y$, or equivalent	B1	
	Obtain $k\ln\left(2+e^{3x}\right)$	B1	
	Use $y(0) = 36$ to find constant in $y = A\left(2+e^{3x}\right)^k$ or $\ln y = k\ln\left(2+e^{3x}\right)+c$ or equivalent	M1*	
	Obtain equation correctly without logarithms from $\ln y = \ln\left(A\left(2+e^{3x}\right)^k\right)$	*M1	
	Obtain $y = 4\left(2+e^{3x}\right)^2$	A1	[6]

5	(i)	Either	Multiply numerator and denominator by $\sqrt{3}+\mathrm{i}$ and use $\mathrm{i}^2=-1$	M1	
			Obtain correct numerator $18+18\sqrt{3}\mathrm{i}$ or correct denominator 4	B1	
			Obtain $\frac{9}{2}+\frac{9}{2}\sqrt{3}\mathrm{i}$ or $\left(18+18\sqrt{3}\mathrm{i}\right)/4$	A1	
			Obtain modulus or argument	M1	
			Obtain $9\mathrm{e}^{\frac{1}{3}\pi\mathrm{i}}$	A1	[5]
		OR	Obtain modulus and argument of numerator or denominator, or both moduli or both arguments	M1	
			Obtain moduli and argument 18 and $\frac{1}{6}\pi$ or 2 and $-\frac{1}{6}\pi$		
			or moduli 18 and 2 or arguments $\frac{1}{6}\pi$ and $-\frac{1}{6}\pi$ (allow degrees)	B1	
			Obtain $18\mathrm{e}^{\frac{1}{6}\pi\mathrm{i}}\div 2\mathrm{e}^{-\frac{1}{6}\pi\mathrm{i}}$ or equivalent	A1	
			Divide moduli and subtract arguments	M1	
			Obtain $9\mathrm{e}^{\frac{1}{3}\pi\mathrm{i}}$	A1	[5]
	(ii)	State $3\mathrm{e}^{\frac{1}{6}\pi\mathrm{i}}$, following through their answer to part (i)		B1$\checkmark$	
		State $3\mathrm{e}^{\frac{1}{6}\pi\mathrm{i}\pm\frac{1}{2}\pi\mathrm{i}}$, following through their answer to part (i)		B1$\checkmark$	
		Obtain $3\mathrm{e}^{-\frac{5}{6}\pi\mathrm{i}}$		B1	[3]
6	(i)	Use law for the logarithm for a product or quotient or exponentiation AND for a power		M1	
		Obtain $(4x-5)^2(x+1)=27$		B1	
		Obtain given equation correctly $16x^3-24x^2-15x-2=0$		A1	[3]
	(ii)	Obtain $x=2$ is root or $(x-2)$ is a factor, or likewise with $x=-\frac{1}{4}$		B1	
		Divide by $(x-2)$ to reach a quotient of the form $16x^2+kx$		M1	
		Obtain quotient $16x^2+8x+1$		A1	
		Obtain $(x-2)(4x+1)^2$ or $(x-2)$, $(4x+1)$, $(4x+1)$		A1	[4]
	(iii)	State $x=2$ only		A1	[1]
7	(i)	Obtain $2x-3y+6z$ for LHS of equation		B1	
		Obtain $2x-3y+6z=23$		B1	[2]
	(ii)	Either	Use correct formula to find perpendicular distance	M1	
			Obtain unsimplified value $\dfrac{\pm 23}{\sqrt{2^2+(-3)^2+6^2}}$, following answer to (i)	A1$\checkmark$	
			Obtain $\frac{23}{7}$ or equivalent	A1	[3]

	OR 1	Use scalar product of (4, –1, 2) and a vector normal to the plane	M1	
		Use unit normal to plane to obtain $\pm\dfrac{(8+3+12)}{\sqrt{49}}$	A1	
		Obtain $\dfrac{23}{7}$ or equivalent	A1	[3]
	OR 2	Find parameter intersection of p and $\mathbf{r} = \mu(2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k})$	M1	
		Obtain $\mu = \dfrac{23}{49}$ [and $\left(\dfrac{46}{49}, -\dfrac{69}{49}, \dfrac{138}{49}\right)$ as foot of perpendicular]	A1	
		Obtain distance $\dfrac{23}{7}$ or equivalent	A1	[3]
(iii)	Either	Recognise that plane is $2x - 3y + 6z = k$ and attempt use of formula for perpendicular distance to plane at least once	M1	
		Obtain $\dfrac{\lvert 23-k\rvert}{7} = 14$ or equivalent	A1	
		Obtain $2x - 3y + 6z = 121$ and $2x - 3y + 6z = -75$	A1	[3]
	OR	Recognise that plane is $2x - 3y + 6z = k$ and attempt to find at least one point on q using l with $\lambda = \pm 2$	M1	
		Obtain $2x - 3y + 6z = 121$	A1	
		Obtain $2x - 3y + 6z = -75$	A1	[3]

8	**(i)**	Sketch $y = \operatorname{cosec} x$ for at least $0 , x , \pi$	B1	
		Sketch $y = x(\pi - x)$ for at least $0 , x , \pi$	B1	
		Justify statement concerning two roots, with evidence of 1 and $\frac{1}{4}\pi^2$ for y-values on graph via scales	B1	[3]
	(ii)	Use $\operatorname{cosec} x = \dfrac{1}{\sin x}$ and commence rearrangement	M1	
		Obtain given equation correctly, showing sufficient detail	A1	[2]
	(iii)	**(a)** Use the iterative formula correctly at least once	M1	
		Obtain final answer 0.66	A1	
		Show sufficient iterations to 4 decimal places to justify answer or show a sign change in the interval (0.655, 0.665)	A1	[3]
		(b) Obtain 2.48	B1	[1]

9	(i)	Either	State or imply partial fractions are of form $\frac{A}{3-x}+\frac{B}{1+2x}+\frac{C}{(1+2x)^2}$	B1	
			Use any relevant method to obtain a constant	M1	
			Obtain $A = 1$	A1	
			Obtain $B=\frac{3}{2}$	A1	
			Obtain $C=-\frac{1}{2}$	A1	[5]
		Or	State or imply partial fractions are of form $\frac{A}{3-x}+\frac{Dx+E}{(1+2x)^2}$	B1	
			Use any relevant method to obtain a constant	M1	
			Obtain $A = 1$	A1	
			Obtain $D = 3$	A1	
			Obtain $E = 1$	A1	[5]
	(ii)	Obtain the first two terms of one of the expansion of $(3-x)^{-1}, \left(1-\frac{1}{3}x\right)^{-1}$ $(1+2x)^{-1}$ and $(1+2x)^{-2}$		M1	
		Obtain correct unsimplified expansion up to the term in x^2 of each partial fraction, following in each case the value of A, B, C		A1$\sqrt{}$ A1$\sqrt{}$ A1$\sqrt{}$	
		Obtain answer $\frac{4}{3}-\frac{8}{9}x+\frac{1}{27}x^2$		A1	[5]
		[If A, D, E approach used in part (i), give M1A1$\sqrt{}$A1$\sqrt{}$ for the expansions, M1 for multiplying out fully and A1 for final answer]			
10	(i)	Use of product or quotient rule		M1	
		Obtain $-5e^{-\frac{1}{2}x}\sin 4x+40e^{-\frac{1}{2}x}\cos 4x$		A1	
		Equate $\frac{dy}{dx}$ to zero and obtain $\tan 4z = k$ or $R\cos(4x \pm \alpha)$		M1	
		Obtain $\tan 4x = 8$ or $\sqrt{65}\cos\left(4x\pm\tan^{-1}\frac{1}{8}\right)$		A1	
		Obtain 0.362 or 20.7°		A1	
		Obtain 1.147 or 65.7°		A1	[6]
	(ii)	State or imply that x-coordinates of T_n are increasing by $\frac{1}{4}\pi$ or 45°		B1	
		Attempt solution of inequality (or equation) of form $x_1 + (n-1)k\pi$. 25		M1	
		Obtain $n>\frac{4}{\pi}(25-0.362)+1$, following through on their value of x_1		A1$\sqrt{}$	
		n = 33		A1	[4]

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

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9709/32 Paper 3, maximum raw mark 75

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1	*EITHER:* State or imply non-modular inequality $(x+2a)^2 > (3(x-a))^2$, or corresponding quadratic equation, or pair of linear equations $(x+2a) = \pm 3(x-a)$	B1	
	Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations for x	M1	
	Obtain critical values $x = \frac{1}{4}a$ and $x = \frac{5}{2}a$	A1	
	State answer $\frac{1}{4}a < x < \frac{5}{2}a$	A1	
	OR: Obtain critical value $x = \frac{5}{2}a$ from a graphical method, or by inspection, or by solving a linear equation or inequality	B1	
	Obtain critical value $x = \frac{1}{4}a$ similarly	B2	
	State answer $\frac{1}{4}a < x < \frac{5}{2}a$	B1	**4**
	[Do not condone $\leqslant$ for $<$.]		
2	Remove logarithms and obtain $5 - e^{-2x} = e^{\frac{1}{2}}$, or equivalent	B1	
	Obtain a correct value for e^{-2x}, e^{2x}, e^{-x} or e^{x}, e.g. $e^{2x} = 1/(5 - e^{\frac{1}{2}})$	B1	
	Use correct method to solve an equation of the form $e^{2x} = a$, $e^{-2x} = a$, $e^{x} = a$ or $e^{-x} = a$ where $a > 0$. [The M1 is dependent on the correct removal of logarithms.]	M1	
	Obtain answer $x = -0.605$ only.	A1	**4**
3	Use $\cos(A + B)$ formula to obtain an equation in $\cos x$ and $\sin x$	M1	
	Use trig formula to obtain an equation in $\tan x$ (or $\cos x$ or $\sin x$)	M1	
	Obtain $\tan x = \sqrt{3} - 4$, or equivalent (or find $\cos x$ or $\sin x$)	A1	
	Obtain answer $x = -66.2°$	A1	
	Obtain answer $x = 113.8°$ and no others in the given interval	A1	**5**
	[Ignore answers outside the given interval. Treat answers in radians as a misread $(-1.16, 1.99)$.]		
	[The other solution methods are *via* $\cos x = \pm 1/\sqrt{(1 + (\sqrt{3} - 4)^2)}$ and $\sin x = \pm(\sqrt{3} - 4)/\sqrt{(1 + (\sqrt{3} - 4)^2)}$.]		
4	**(i)** State $\dfrac{dx}{dt} = 1 - \sec^2 t$, or equivalent	B1	
	Use chain rule	M1	
	Obtain $\dfrac{dy}{dt} = -\dfrac{\sin t}{\cos t}$, or equivalent	A1	
	Use $\dfrac{dy}{dx} = \dfrac{dy}{dt} \div \dfrac{dx}{dt}$	M1	
	Obtain the given answer correctly.	A1	**5**
	(ii) State or imply $t = \tan^{-1}(\frac{1}{2})$	B1	
	Obtain answer $x = -0.0364$	B1	**2**

5	**(i)**	Differentiate f(x) and obtain $f'(x) = (x-2)^2 g'(x) + 2(x-2)g(x)$		B1	
		Conclude that $(x - 2)$ is a factor of $f'(x)$		B1	**2**
	(ii)	*EITHER:*	Substitute $x = 2$, equate to zero and state a correct equation, e.g. $32 + 16a + 24 + 4b + a = 0$	B1	
			Differentiate polynomial, substitute $x = 2$ and equate to zero or divide by $(x-2)$ and equate constant remainder to zero	M1*	
			Obtain a correct equation, e.g. $80 + 32a + 36 + 4b = 0$	A1	
		OR1:	Identify given polynomial with $(x-2)^2(x^3 + Ax^2 + Bx + C)$ and obtain an equation in a and/or b	M1*	
			Obtain a correct equation, e.g. $\frac{1}{4}a - 4(4+a) + 4 = 3$	A1	
			Obtain a second correct equation, e.g. $-\frac{3}{4}a + 4(4+a) = b$	A1	
		OR2:	Divide given polynomial by $(x-2)^2$ and obtain an equation in a and b	M1*	
			Obtain a correct equation, e.g. $29 + 8a + b + 0$	A1	
			Obtain a second correct equation, e.g. $176 + 47a + 4b = 0$	A1	
		Solve for a or for b		M1(dep*)	
		Obtain $a = -4$ and $b = 3$		A1	**5**
6	**(i)**	Use correct arc formula and form an equation in r and x		M1	
		Obtain a correct equation in any form		A1	
		Rearrange in the given form		A1	**3**
	(ii)	Consider sign of a relevant expression at $x = 1$ and $x = 1.5$, or compare values of relevant expressions at $x = 1$ and $x = 1.5$		M1	
		Complete the argument correctly with correct calculated values		A1	**2**
	(iii)	Use the iterative formula correctly at least once		M1	
		Obtain final answer 1.21		A1	
		Show sufficient iterations to 4 d.p. to justify 1.21 to 2 d.p., or show there is a sign change in the interval (1.205,1.215)		A1	**3**
7	**(a)**	*EITHER*:	Substitute and expand $(-1 + \sqrt{5}\,i)^3$ completely	M1	
			Use $i^2 = -1$ correctly at least once	M1	
			Obtain $a = -12$	A1	
			State that the other complex root is $-1 - \sqrt{5}\,i$	B1	
		OR1:	State that the other complex root is $-1 - \sqrt{5}\,i$	B1	
			State the quadratic factor $z^2 + 2z + 6$	B1	
			Divide the cubic by a 3-term quadratic, equate remainder to zero and solve for a or, using a 3-term quadratic, factorise the cubic and determine a	M1	
			Obtain $a = -12$	A1	
		OR2:	State that the other complex root is $-1 - \sqrt{5}i$	B1	
			State or show the third root is 2	B1	
			Use a valid method to determine a	M1	
			Obtain $a = -12$	A1	
		OR3:	Substitute and use De Moivre to cube $\sqrt{6}\text{cis}(114.1^\circ)$, or equivalent	M1	
			Find the real and imaginary parts of the expression	M1	
			Obtain $a = -12$	A1	
			State that the other complex root is $-1 - \sqrt{5}i$	B1	**4**

(b)	*EITHER:*	Substitute $w = \cos 2\theta + \mathrm{i}\sin 2\theta$ in the given expression	B1	
		Use double angle formulae throughout	M1	
		Express numerator and denominator in terms of $\cos\theta$ and $\sin\theta$ only	A1	
		Obtain given answer correctly	A1	
	OR:	Substitute $w = \mathrm{e}^{2\mathrm{i}\theta}$ in the given expression	B1	
		Divide numerator and denominator by $\mathrm{e}^{\mathrm{i}\theta}$, or equivalent	M1	
		Express numerator and denominator in terms of $\cos\theta$ and $\sin\theta$ only	A1	
		Obtain the given answer correctly	A1	**4**

8	**(i)**	Use product rule	M1	
		Obtain derivative in any correct form	A1	
		Differentiate first derivative using the product rule	M1	
		Obtain second derivative in any correct form, e.g. $-\frac{1}{2}\sin\frac{1}{2}x - \frac{1}{4}x\cos\frac{1}{2}x - \frac{1}{2}\sin\frac{1}{2}x$	A1	
		Verify the given statement	A1	**5**
	(ii)	Integrate and reach $kx\sin\frac{1}{2}x + l\int\sin\frac{1}{2}x\,\mathrm{d}x$	M1*	
		Obtain $2x\sin\frac{1}{2}x - 2\int\sin\frac{1}{2}x\,\mathrm{d}x$, or equivalent	A1	
		Obtain indefinite integral $2x\sin\frac{1}{2}x + 4\cos\frac{1}{2}x$	A1	
		Use correct limits $x = 0$, $x = \pi$ correctly	M1(dep*)	
		Obtain answer $2\pi - 4$, or exact equivalent	A1	**5**

9	**(i)**	State or imply $\frac{\mathrm{d}N}{\mathrm{d}t} = kN(1 - 0.01N)$ and obtain the given answer $k = 0.02$	B1	**1**
	(ii)	Separate variables and attempt integration of at least one side	M1	
		Integrate and obtain term $0.02t$, or equivalent	A1	
		Carry out a relevant method to obtain A or B such that $\frac{1}{N(1-0.01N)} \equiv \frac{A}{N} + \frac{B}{1-0.01N}$, or equivalent	M1*	
		Obtain $A = 1$ and $B = 0.01$, or equivalent	A1	
		Integrate and obtain terms $\ln N - \ln(1 - 0.01N)$, or equivalent	A1$\sqrt{}$	
		Evaluate a constant or use limits $t = 0$, $N = 20$ in a solution with terms $a\ln N$ and $b\ln(1 - 0.01N)$, $ab \neq 0$	M1(dep*)	
		Obtain correct answer in any form, e.g. $\ln N - \ln(1 - 0.01N) = 0.02t + \ln 25$	A1	
		Rearrange and obtain $t = 50\ln(4N/(100 - N))$, or equivalent	A1	**8**
	(iii)	Substitute $N = 40$ and obtain $t = 49.0$	B1	**1**

10	(**i**)	*EITHER:*	State or imply $\overrightarrow{AB}$ and $\overrightarrow{AC}$ correctly in component form	B1	
			Using the correct processes evaluate the scalar product $\overrightarrow{AB}.\overrightarrow{AC}$, or equivalent	M1	
			Using the correct process for the moduli divide the scalar product by the product of the moduli	M1	
			Obtain answer $\frac{20}{21}$	A1	
		OR:	Use correct method to find lengths of all sides of triangle *ABC*	M1	
			Apply cosine rule correctly to find the cosine of angle *BAC*	M1	
			Obtain answer $\frac{20}{21}$	A1	**4**
	(**ii**)		State an exact value for the sine of angle *BAC*, e.g. $\sqrt{41}/21$	B1$\sqrt{}$	
			Use correct area formula to find the area of triangle *ABC*	M1	
			Obtain answer $\frac{1}{2}\sqrt{41}$, or exact equivalent	A1	**3**
			[SR: Allow use of a vector product, e.g. $\overrightarrow{AB}\times\overrightarrow{AC} = -6\mathbf{i}+2\mathbf{j}-\mathbf{k}$ B1$\sqrt{}$. Using correct process for the modulus, divide the modulus by 2 M1. Obtain answer $\frac{1}{2}\sqrt{41}$ A1.]		
	(**iii**)	*EITHER*:	State or obtain $b = 0$	B1	
			Equate scalar product of normal vector and $\overrightarrow{BC}$ (or $\overrightarrow{CB}$) to zero	M1	
			Obtain $a + b - 4c = 0$ (or $a - 4c = 0$)	A1	
			Substitute a relevant point in $4x + z = d$ and evaluate d	M1	
			Obtain answer $4x + z = 9$, or equivalent	A1	
		*OR*1:	Attempt to calculate vector product of relevant vectors, e.g. $(\mathbf{j})\times(\mathbf{i}+\mathbf{j}-4\mathbf{k})$	M1	
			Obtain two correct components of the product	A1	
			Obtain correct product, e.g. $-4\mathbf{i}-\mathbf{k}$	A1	
			Substitute a relevant point in $4x + z = d$ and evaluate d	M1	
			Obtain $4x + z = 9$, or equivalent	A1	
		*OR*2:	Attempt to form 2-parameter equation for the plane with relevant vectors	M1	
			State a correct equation, e.g. $\mathbf{r} = 2\mathbf{i} + 4\mathbf{j} + \mathbf{k} + \lambda(\mathbf{j}) + \mu(\mathbf{i}+\mathbf{j}-4\mathbf{k})$	A1	
			State 3 equations in x, y, z, λ and μ	A1	
			Eliminate μ	M1	
			Obtain answer $4x + z = 9$, or equivalent	A1	
		*OR*3:	State or obtain $b = 0$	B1	
			Substitute for *B* and *C* in the plane equation and obtain $2a + c = d$ and $3a - 3c = d$ (or $2a + 4b + c = d$ and $3a + 5b - 3c = d$)	B1	
			Solve for one ratio, e.g. $a : d$	M1	
			Obtain $a : c : d$, or equivalent	M1	
			Obtain answer $4x + z = 9$, or equivalent	A1	
		OR4:	Attempt to form a determinant equation for the plane with relevant vectors	M1	
			State a correct equation, e.g. $\begin{vmatrix} x-2 & y-4 & z-1 \\ 0 & 1 & 0 \\ 1 & 1 & -4 \end{vmatrix} = 0$	A1	
			Attempt to use a correct method to expand the determinant	M1	
			Obtain two correct terms of a 3-term expansion, or equivalent	A1	
			Obtain answer $4x + z = 9$, or equivalent	A1	**5**

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the May/June 2014 series

9709 MATHEMATICS

9709/33	Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2014 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

Mark Scheme Notes

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M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a "fortuitous" answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1		Use law of the logarithm of a quotient or product or $2 = \log_{10} 100$	M1	
		Remove logarithms and obtain $x + 9 = 100x$, or equivalent	A1	
		Obtain answer $x = \frac{1}{11}$	A1	3
2		State a correct unsimplified version of the x or x^2 or x^3 term	M1	
		State correct first two terms $1 - x$	A1	
		Obtain the next two terms $2x^2 - \frac{14}{3}x^3$	A1 + A1	4
		[Symbolic binomial coefficients, e.g. $\begin{pmatrix} -\frac{1}{3} \\ 3 \end{pmatrix}$ are not sufficient for the M mark.]		
3	(i)	Use $\tan(A \pm B)$ formula and obtain an equation in $\tan x$	M1	
		Using $\tan 60° = \sqrt{3}$, obtain a horizontal equation in $\tan x$ in any correct form	A1	
		Reduce the equation to the given form	A1	3
	(ii)	Solve the given quadratic for $\tan x$	M1	
		Obtain a correct answer, e.g. $x = 21.6°$	A1	
		Obtain a second answer, e.g. $x = 128.4°$, and no others	A1	3
		[Ignore answers outside the given interval. Treat answers in radians as a misread (0.377, 2.24).]		
4	(i)	Consider sign of $x - 10/(e^{2x} - 1)$ at $x = 1$ and $x = 2$	M1	
		Complete the argument correctly with correct calculated values	A1	2
	(ii)	State or imply $\alpha = \frac{1}{2}\ln(1 + 10/\alpha)$	B1	
		Rearrange this as $\alpha = 10/(e^{2\alpha} - 1)$ or work *vice versa*	B1	2
	(iii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.14	A1	
		Show sufficient iterations to 4 d.p. to justify 1.14 to 2 d.p., or show there is a sign change in the interval (1.135, 1.145)	A1	3
5		Separate variables correctly and attempt integration of at least one side	B1	
		Obtain term in the form $a\sqrt{(2x+1)}$	M1	
		Express $1/(\cos^2\theta)$ as $\sec^2\theta$	B1	
		Obtain term of the form $k\tan\theta$	M1	
		Evaluate a constant, or use limits $x = 0, \theta = \frac{1}{4}\pi$ in a solution with terms $a\sqrt{(2x+1)}$ and $k \tan\theta$, $ak \neq 0$	M1	
		Obtain correct solution in any form, e.g. $\sqrt{(2x+1)} = \frac{1}{2}\tan\theta + \frac{1}{2}$	A1	
		Rearrange and obtain $x = \frac{1}{8}(\tan\theta + 1)^2 - \frac{1}{2}$, or equivalent	A1	7

6	Obtain correct derivative of RHS in any form	B1	
	Obtain correct derivative of LHS in any form	B1	
	Set $\frac{dy}{dx}$ equal to zero and obtain a horizontal equation	M1	
	Obtain a correct equation, e.g. $x^2 + y^2 = 1$, from correct work	A1	
	By substitution in the curve equation, or otherwise, obtain an equation in x^2 or y^2	M1	
	Obtain $x = \frac{1}{2}\sqrt{3}$	A1	
	Obtain $y = \frac{1}{2}$	A1	**7**
7	**(a)** *EITHER*: Multiply numerator and denominator by $1 - 4i$, or equivalent, and use $i^2 = -1$	M1	
	Simplify numerator to $-17 - 17i$, or denominator to 17	A1	
	Obtain final answer $-1 - i$	A1	
	OR: Using $i^2 = -1$, obtain two equations in x and y, and solve for x or for y	M1	
	Obtain $x = -1$ or $y = -1$, or equivalent	A1	
	Obtain final answer $-1 - i$	A1	**3**
	(b) (i) Show a point representing $2 + i$ in relatively correct position	B1	
	Show a circle with centre $2 + i$ and radius 1	B1√	
	Show the perpendicular bisector of the line segment joining i and 2	B1	
	Shade the correct region	B1	**4**
	(ii) State or imply that the angle between the tangents from the origin to the circle is required	M1	
	Obtain answer 0.927 radians (or 53.1°)	A1	**2**
8	**(i)** Use a correct method for finding a constant	M1	
	Obtain one of $A = 3$, $B = 3$, $C = 0$	A1	
	Obtain a second value	A1	
	Obtain a third value	A1	**4**
	(ii) Integrate and obtain term $-3\ln(2-x)$	B1√	
	Integrate and obtain term of the form $k\ln(2+x^2)$	M1	
	Obtain term $\frac{3}{2}\ln(2+x^2)$	A1√	
	Substitute limits correctly in an integral of the form $a\ln(2-x)+b\ln(2+x^2)$, where $ab \neq 0$	M1	
	Obtain given answer after full and correct working	A1	**5**
9	**(i)** Substitute for x and dx throughout using $u = \sin x$ and $du = \cos x\, dx$, or equivalent	M1	
	Obtain integrand e^{2u}	A1	
	Obtain indefinite integral $\frac{1}{2}e^{2u}$	A1	
	Use limits $u = 0$, $u = 1$ correctly, or equivalent	M1	
	Obtain answer $\frac{1}{2}(e^2 - 1)$, or exact equivalent	A1	**5**

	(ii)	Use chain rule or product rule	M1	
		Obtain correct terms of the derivative in any form, e.g. $2\cos x\, e^{2\sin x}\cos x - e^{2\sin x}\sin x$	A1 + A1	
		Equate derivative to zero and obtain a quadratic equation in sin x	M1	
		Solve a 3-term quadratic and obtain a value of x	M1	
		Obtain answer 0.896	A1	6
10	(i)	Express general point of l in component form, e.g. $(1+3\lambda, 2-2\lambda, -1+2\lambda)$	B1	
		Substitute in given equation of p and solve for λ	M1	
		Obtain final answer $-\frac{1}{2}\mathbf{i}+3\mathbf{j}-2\mathbf{k}$, or equivalent, from $\lambda=-\frac{1}{2}$	A1	3
	(ii)	State or imply a vector normal to the plane, e.g. $2\mathbf{i}+3\mathbf{j}-5\mathbf{k}$	B1	
		Using the correct process, evaluate the scalar product of a direction vector for l and a normal for p	M1	
		Using the correct process for the moduli, divide the scalar product by the product of the moduli and find the inverse sine or cosine of the result	M1	
		Obtain answer 23.2° (or 0.404 radians)	A1	4
	(iii)	*EITHER*: State $2a+3b-5c=0$ or $3a-2b+2c=0$	B1	
		Obtain two relevant equations and solve for one ratio, e.g. $a:b$	M1	
		Obtain $a:b:c = 4:19:13$, or equivalent	A1	
		Substitute coordinates of a relevant point in $4x+19y+13z=d$, and evaluate d	M1	
		Obtain answer $4x+19y+13z=29$, or equivalent	A1	
		*OR*1: Attempt to calculate vector product of relevant vectors, e.g. $(2\mathbf{i}+3\mathbf{j}-5\mathbf{k})\times(3\mathbf{i}-2\mathbf{j}+2\mathbf{k})$	M1	
		Obtain two correct components of the product	A1	
		Obtain correct product, e.g. $-4\mathbf{i}-19\mathbf{j}-13\mathbf{k}$	A1	
		Substitute coordinates of a relevant point in $4x+19y+13z=d$	M1	
		Obtain answer $4x+19y+13z=29$, or equivalent	A1	
		*OR*2: Attempt to form a 2-parameter equation with relevant vectors	M1	
		State a correct equation, e.g. $\mathbf{r}=\mathbf{i}+2\mathbf{j}-\mathbf{k}+\lambda(2\mathbf{i}+3\mathbf{j}-5\mathbf{k})+\mu(3\mathbf{i}-2\mathbf{j}+2\mathbf{k})$	A1	
		State 3 equations in x, y, z, λ and μ	A1	
		Eliminate λ and μ	M1	
		Obtain answer $4x+19y+13z=29$, or equivalent	A1	
		OR3: Using a relevant point and relevant direction vectors, form a determinant equation for the plane	M1	
		State a correct equation, e.g. $\begin{vmatrix} x-1 & y-2 & z+1 \\ 2 & 3 & -5 \\ 3 & -2 & 2 \end{vmatrix} = 0$	A1	
		Attempt to expand the determinant	M1	
		Obtain correct values of two cofactors	A1	
		Obtain answer $4x+19y+13z=29$, or equivalent	A1	5

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Level

MARK SCHEME for the May/June 2015 series

9709 MATHEMATICS

9709/31 Paper 3 (Paper 3), maximum raw mark 75

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CAMBRIDGE
International Examinations

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PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	Use law for the logarithm of a power at least once		*M1	
	Obtain correct linear equation, e.g. $5x\text{In}2 = (2x+1)\text{In}3$		A1	
	Solve a linear equation for x		M1 dep *M	
	Obtain $x = 0.866$		A1	[4]
2	Attempt calculation of at least 3 ordinates		M1	
	Obtain 9, 7, 1, 17		A1	
	Use trapezium rule with $h = 1$		M1	
	Obtain $\frac{1}{2}\left(9+14+2+17\right)$ or equivalent and hence 21		A1	[4]
3	<u>Either</u>	Obtain correct (unsimplified) version of x^2 or x^4 term in $(1-2x^2)^{-2}$	M1	
		Obtain $1+4x^2$	A1	
		Obtain … $+12x^4$	A1	
		Obtain correct (unsimplified) version of x^2 or x^4 term in $(1+6x^2)^{\frac{2}{3}}$	M1	
		Obtain $1+4x^2-4x^4$	A1	
		Combine expansions to obtain $k = 16$ with no error seen	A1	
	<u>Or</u>	Obtain correct (unsimplified) version of x^2 or x^4 term in $(1+6x^2)^{\frac{2}{3}}$	M1	
		Obtain $1+4x^2$	A1	
		Obtain … $-4x^4$	A1	
		Obtain correct (unsimplified) version of x^2 or x^4 term in $(1-2x^2)^{-2}$	M1	
		Obtain $1+4x^2+12x^4$	A1	
		Combine expansions to obtain $k = 16$ with no error seen	A1	[6]
4	Differentiate to obtain form $a\sin 2x + b\cos x$		M1	
	Obtain correct $-6\sin 2x + 7\cos x$		A1	
	Use identity $\sin 2x = 2\sin x\cos x$		B1	
	Solve equation of form $c\sin x\cos x + d\cos x = 0$ to find at least one value of x		M1	
	Obtain 0.623		A1	
	Obtain 2.52		A1	
	Obtain 1.57 or $\frac{1}{2}\pi$ from equation of form $c\sin x\cos x + d\cos x = 0$		A1	
	Treat answers in degrees as MR – 1 situation			[7]

5	(a)	Use identity $\tan^2 2x = \sec^2 2x - 1$	B1	
		Obtain integral of form $ax + b\tan 2x$	M1	
		Obtain correct $3x + \frac{1}{2}\tan 2x$, condoning absence of + c	A1	[3]
	(b)	State $\sin x \cos\frac{1}{2}\pi + \cos x \ \sin\frac{1}{6}\pi$	B1	
		Simplify integrand to $\cos\frac{1}{6}\pi + \frac{\cos x \sin\frac{1}{6}\pi}{\sin x}$ or equivalent	B1	
		Integrate to obtain at least term of form $a\ \mathrm{In}(\sin x)$	*M1	
		Apply limits and simplify to obtain two terms	M1 dep *M	
		Obtain $\frac{1}{8}\pi\sqrt{3} - \frac{1}{2}\ln(\frac{1}{\sqrt{2}})$ or equivalent	A1	[5]
6	(i)	Obtain $\pm\begin{pmatrix}2\\-3\\-4\end{pmatrix}$ as direction vector of l_1	B1	
		State that two direction vectors are not parallel	B1	
		Express general point of l_1 or l_2 in component form, e.g. $(2\lambda, 1-3\lambda, 5-4\lambda)$ or $(7+\mu, l+2\mu, 1+5\mu)$	B1	
		Equate at least two pairs of components and solve for λ or for μ	M1	
		Obtain correct answers for λ and μ	A1	
		Verify that all three component equations are not satisfied (with no errors seen)	A1	[6]
	(ii)	Carry out correct process for evaluating scalar product of $\begin{pmatrix}1\\2\\5\end{pmatrix}$ and $\begin{pmatrix}1\\0\\0\end{pmatrix}$	M1	
		Use correct process for finding modulus and evaluating inverse cosine	M1	
		Obtain $79.5°$ or 1.39 radians	A1	[3]
7		Separate variables and factorise to obtain $\frac{\mathrm{d}y}{(3y+1)(y+3)} = 4x\,\mathrm{d}x$ or equivalent	B1	
		State or imply the form $\frac{A}{3y+1} + \frac{B}{y+3}$ and use a relevant method to find A or B	M1	
		Obtain $A = \frac{3}{8}$ and $B = -\frac{1}{8}$	A1	
		Integrate to obtain form $k_1\ln(3y+1) + k_2\ln(y+3)$	M1	
		Obtain correct $\frac{1}{8}\ln(3y+1) - \frac{1}{8}\ln(y=3) = 2x^2$ or equivalent	A1	
		Substitute $x = 0$ and $y = 1$ in equation of form $k_1\ln(3y+1) + k_2\ln(y+3) = k_3x^2 + c$ to find a value of c	M1	
		Obtain $c = 0$	A1	
		Use correct process to obtain equation without natural logarithm present	M1	
		Obtain $y = \frac{3e^{16x^2} - 1}{3 - e^{16x^2}}$ or equivalent	A1	[9]

8	(i)	Either	Expand $(2-\mathrm{i})^2$ to obtain $3-4\mathrm{i}$ or unsimplified equivalent	B1	
			Multiply by $\dfrac{3+4\mathrm{i}}{3+4\mathrm{i}}$ and simplify to $x+\mathrm{i}y$ form or equivalent	M1	
			Confirm given answer $2+4\mathrm{i}$	A1	
		Or	Expand $(2-\mathrm{i})^2$ to obtain $3-4\mathrm{i}$ or unsimplified equivalent	B1	
			Obtain two equations in x and y and solve for x or y	M1	
			Confirm given answer $2+4\mathrm{i}$	A1	[3]
	(ii)		Identify $4+4$ or $-4+4\mathrm{i}$ as point at either end or state $p=2$ or state $p=-6$	B1	
			Use appropriate method to find both critical values of p	M1	
			State $-6 \leqslant p \leqslant 2$	A1	[3]
	(iii)		Identify equation as of form $\lvert z-a\rvert = a$ or equivalent	M1	
			Form correct equation for a not involving modulus, e.g. $(a-2)^2+4^2=a^2$	A1	
			State $\lvert z-5\rvert = 5$	A1	[3]
9	(i)		Use product rule to find first derivative	M1	
			Obtain $2x\mathrm{e}^{2-x} - x^2\mathrm{e}^{2-x}$	A1	
			Confirm $x=2$ at M	A1	[3]
	(ii)		Attempt integration by parts and reach $\pm x^2\mathrm{e}^{2-x} \pm \int 2x\mathrm{e}^{2-x}\,\mathrm{d}x$	*M1	
			Obtain $-x^2\mathrm{e}^{2-x} + \int 2x\mathrm{e}^{2-x}\,\mathrm{d}x$	A1	
			Attempt integration by parts and reach $\pm x^2\mathrm{e}^{2-x} \pm 2x\mathrm{e}^{2-x} \pm 2\mathrm{e}^{2-x}$	*M1	
			Obtain $-x^2\mathrm{e}^{2-x} - 2x\mathrm{e}^{2-x} - 2\mathrm{e}^{2-x}$	A1	
			Use limits 0 and 2 having integrated twice	M1 dep *M	
			Obtain $2\mathrm{e}^2 - 10$	A1	[6]
10	(i)		Obtain $\dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{2}{t+2}$ and $\dfrac{\mathrm{d}y}{\mathrm{d}t} = 3t^2+2$	B1	
			Use $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}y}{\mathrm{d}t} \div \dfrac{\mathrm{d}x}{\mathrm{d}t}$	M1	
			Obtain $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}(3t^2+2)(t+2)$	A1	
			Identify value of t at the origin as -1	B1	
			Substitute to obtain $\frac{5}{2}$ as gradient at the origin	A1	[5]
	(ii)	(a)	Equate derivative to $\frac{1}{2}$ and confirm $p = \dfrac{1}{3p^2+2} - 2$	B1	[1]
		(b)	Use the iterative formula correctly at least once	M1	
			Obtain value $p=-1.924$ or better $(-1.92367\ldots)$	A1	
			Show sufficient iterations to justify accuracy or show a sign change in appropriate interval	A1	
			Obtain coordinates $(-5.15, -7.97)$	A1	[4]

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Subsidiary and Advanced Level

MARK SCHEME for the May/June 2015 series

9709 MATHEMATICS

9709/32 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2015 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.

CAMBRIDGE
International Examinations

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol $\sqrt{}$ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- **Note:** B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a "fortuitous" answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR – 1 A penalty of MR – 1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR – 2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA – 1 This is deducted from A or B marks in the case of premature approximation. The PA – 1 penalty is usually discussed at the meeting.

1	State or imply ordinates 0, 0.405465…, 0.623810…, 0.693147…	B1	
	Use correct formula, or equivalent, with $h = \frac{1}{6}\pi$ and four ordinates	M1	
	Obtain answer 0.72	A1	**[3]**
2	Use laws of indices correctly and solve for u	M1	
	Obtain u in any correct form, e.g. $u = \dfrac{16}{16-1}$	A1	
	Use correct method for solving an equation of the form $4^x = a$, where $a > 0$	M1	
	Obtain answer $x = 0.0466$	A1	**[4]**
3	EITHER: Use correct product rule	M1	
	Obtain correct derivative in any form, e.g. $-\sin x\cos 2x - 2\cos x\sin 2x$	A1	
	Use the correct double angle formulae to express derivative in cos x and sin x, or cos 2x and sin x	M1	
	*OR*1: Use correct double angle formula to express y in terms of cos x and attempt differentiation	M1	
	Use chain rule correctly	M1	
	Obtain correct derivative in any form, e.g. $-6\cos^2 x\sin x + \sin x$	A1	
	*OR*2: Use correct factor formula and attempt differentiation	M1	
	Obtain correct derivative in any form, e.g. $-\frac{3}{2}\sin 3x - \frac{1}{2}\sin x$	A1	
	Use correct trig formulae to express derivative in terms of cos x and sin x, or sin x	M1	
	Equate derivative to zero and obtain an equation in one trig function	M1	
	Obtain $6\cos^2 x = 1$, $6\sin^2 x = 5$, $\tan^2 x = 5$ or $3\cos 2x = -2$	A1	
	Obtain answer x = 1.15 (or 65.9°) and no other in the given interval	A1	**[6]**
	[Ignore answers outside the given interval.]		
	[SR: Solution attempts following the *EITHER* scheme for the first two marks can earn the second and third method marks as follows:		
	Equate derivative to zero and obtain an equation in tan 2x and tan x	M1	
	Use correct double angle formula to obtain an equation in tan x	M1]	
4	**(i)** State $R = \sqrt{13}$	B1	
	Use trig formula to find α	M1	
	Obtain α = 33.69° with no errors seen	A1	**[3]**
	(ii) Evaluate $\sin^{-1}(1/\sqrt{13})$ to at least 1 d.p. (16.10° to 2 d.p)	B1√	
	Carry out an appropriate method to find a value of θ in the interval $0° < \theta < 180°$	M1	
	Obtain answer θ = 130.2° and no other in the given interval	A1	**[3]**
	[Ignore answers outside the given interval.]		
	[Treat answers in radians as a misread and deduct A1 from the marks for the angles.]		
5	**(i)** State or imply $AT = r\tan x$ or $BT = r\tan x$	B1	
	Use correct arc formula and form an equation in r and x	M1	
	Rearrange in the given form	A1	**[3]**
	(ii) Calculate values of a relevant expression or expressions at x = 1 and x = 1.3	M1	
	Complete the argument correctly with correct calculated values	A1	**[2]**

	(iii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.11	A1	
		Show sufficient iterations to 4 d.p. to justify 1.11 to 2 d.p., or show there is a sign change in the interval (1.105, 1.115)	A1	**[3]**
6	**(i)**	State or imply $\mathrm{d}u = -\dfrac{1}{2\sqrt{x}}\mathrm{d}x$, or equivalent	B1	
		Substitute for x and dx throughout	M1	
		Obtain integrand $\dfrac{\pm 2(2-u)^2}{u}$, or equivalent	A1	
		Show correct working to justify the change in limits and obtain the given answer with no errors seen	A1	**[4]**
	(ii)	Integrate and obtain at least two terms of the form $a\ln u, bu,$ and cu^2	M1*	
		Obtain indefinite integral $8\ln u - 8u + u^2$, or equivalent	A1	
		Substitute limits correctly	M1(dep*)	
		Obtain the given answer correctly having shown sufficient working	A1	**[4]**
7	**(i)**	Square $x + \mathrm{i}y$ and equate real and imaginary parts to -1 and $4\sqrt{3}$	M1	
		Obtain $x^2 - y^2 = -1$ and $2xy = 4\sqrt{3}$	A1	
		Eliminate one unknown and find an equation in the other	M1	
		Obtain $x^4 + x^2 - 12 = 0$ or $y^4 - y^2 - 12 = 0$, or three term equivalent	A1	
		Obtain answers $\pm(\sqrt{3} + 2\mathrm{i})$	A1	**[5]**
		[If the equations are solved by inspection, give B2 for the answers and B1 for justifying them]		
	(ii)	Show a circle with centre $-1 + 4\sqrt{3}$ in a relatively correct position	B1	
		Show a circle with radius 1 and centre not at the origin	B1	
		Carry out a complete method for calculating the greatest value of arg z	M1	
		Obtain answer 1.86 or 106.4°	A1	**[4]**
8	**(i)**	State or imply the form $\dfrac{A}{3-2x} + \dfrac{Bx+C}{x^2+4}$	B1	
		Use a relevant method to determine a constant	M1	
		Obtain one of the values $A = 3$, $B = -1$, $C = -2$	A1	
		Obtain a second value	A1	
		Obtain the third value	A1	**[5]**
	(ii)	Use correct method to find the first two terms of the expansion of $(3-2x)^{-1}$, $(1-\frac{2}{3}x)^{-1}$, $(4+x^2)^{-1}$ or $(1+\frac{1}{4}x^2)^{-1}$	M1	
		Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	A1√+A1√	
		Multiply out up to the term in x^2 by $Bx + C$, where $BC \neq 0$	M1	
		Obtain final answer $\frac{1}{2} + \frac{5}{12}x + \frac{41}{72}x^2$, or equivalent	A1	**[5]**

[Symbolic coefficients, e.g. $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ are not sufficient for the first M1.The f.t. is on A, B, C.]

[In the case of an attempt to expand $(5x^2 + x + 6)(3 - 2x)^{-1}(x^2 + 4)^{-1}$, give M1A1A1for the expansions, M1 for multiplying out fully, and A1 for the final answer.]

9	**(i)**	Separate variables correctly and attempt integration of one side	B1	
		Obtain term $\ln x$	B1	
		Obtain term of the form $a\ln(k + e^{-t})$	M1	
		Obtain term $-\ln(k + e^{-t})$	A1	
		Evaluate a constant or use limits $x = 10$, $t = 0$ in a solution containing terms $a\ln(k + e^{-t})$ and $b\ln x$	M1*	
		Obtain correct solution in any form, e.g. $\ln x - \ln 10 = -\ln(k + e^{-t}) + \ln(k + 1)$	A1	**[6]**
	(ii)	Substitute $x = 20$, $t = 1$ and solve for k	M1(dep*)	
		Obtain the given answer	A1	**[2]**
	(iii)	Using $e^{-t} \to 0$ and the given value of k, find the limiting value of x	M1	
		Justify the given answer	A1	**[2]**

10	**(i)**	Carry out a correct method for finding a vector equation for *AB*	M1	
		Obtain $\mathbf{r} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k} + \lambda(-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$, or equivalent	A1	
		Equate at least two pairs of components of general points on *AB* and *l* and solve for λ or for μ	M1	
		Obtain correct answer for λ or μ, e.g. $\lambda = 1$ or $\mu = 0$; $\lambda = -\frac{4}{5}$ or $\mu = \frac{3}{5}$; or $\lambda = \frac{1}{4}$ or $\mu = -\frac{3}{2}$	A1	
		Verify that not all three pairs of equations are satisfied and that the lines fail to intersect	A1	**[5]**
	(ii)	*EITHER*: Obtain a vector parallel to the plane and not parallel to *l*, e.g. $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$	B1	
		Use scalar product to obtain an equation in *a*, *b* and *c*, e.g. $3a + b - c = 0$	B1	
		Form a second relevant equation, e.g. $a - 2b + c = 0$ and solve for one ratio, e.g. $a : b$	M1	
		Obtain final answer $a : b : c = 1 : 4 : 7$ A1		
		Use coordinates of a relevant point and values of *a*, *b* and *c* in general equation and find *d*	M1	
		Obtain answer $x + 4y + 7z = 19$, or equivalent	A1	
		*OR*1: Obtain a vector parallel to the plane and not parallel to *l*, e.g. $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$	B1	
		Obtain a second relevant vector parallel to the plane and attempt to calculate their vector product, e.g. $(\mathbf{i} - 2\mathbf{j} + \mathbf{k}) \times (3\mathbf{i} + \mathbf{j} - \mathbf{k})$	M1	
		Obtain two correct components	A1	
		Obtain correct answer, e.g. $\mathbf{i} + 4\mathbf{j} + 7\mathbf{k}$	A1	
		Substitute coordinates of a relevant point in $x + 4y + 7z = d$, or equivalent, and find *d*	M1	
		Obtain answer $x + 4y + 7z = 19$, or equivalent	A1	
		*OR*2: Obtain a vector parallel to the plane and not parallel to *l*, e.g. $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$	B1	
		Using a relevant point and second relevant vector, form a 2-parameter equation for the plane	M1	
		State a correct equation, e.g. $\mathbf{r} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k} + s(\mathbf{i} - 2\mathbf{j} + \mathbf{k}) + t(3\mathbf{i} + \mathbf{j} - \mathbf{k})$	A1	
		State 3 correct equations in *x*, *y*, *z*, *s* and *t*	A1	
		Eliminate *s* and *t*	M1	
		Obtain answer $x + 4y + 7z = 19$, or equivalent	A1	
		*OR*3: Using the coordinates of *A* and two points on *l*, state three simultaneous equations in *a*, *b*, *c* and *d*, e.g. $a + b + 2c = d$, $2a - b + 3c = d$ and $4a + 2b + c = d$	B1	
		Solve and find one ratio, e.g. $a : b$	M1	
		State one correct ratio	A1	
		Obtain a correct ratio of three of the unknowns, e.g. $a : b : c = 1 : 4 : 7$, or equivalent	A1	
		Either use coordinates of a relevant point and the found ratio to find the fourth unknown, e.g. *d*, or find the ratio $a : b : c : d$	M1	
		Obtain answer $x + 4y + 7z = 19$, or equivalent	A1	
		*OR*4: Obtain a vector parallel to the plane and not parallel to *l*, e.g. $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$	B1	
		Using a relevant point and second relevant vector, form a determinant equation for the plane	M1	
		State a correct equation, e.g. $\begin{vmatrix} x-2 & y+1 & z-3 \\ 1 & -2 & 1 \\ 3 & 1 & -1 \end{vmatrix} = 0$	A1	
		Attempt to expand the determinant	M1	
		Obtain or imply two correct cofactors	A1	
		Obtain answer $x + 4y + 7z = 19$, or equivalent	A1	**[6]**

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Level

MARK SCHEME for the May/June 2015 series

9709 MATHEMATICS

9709/33 Paper 3, maximum raw mark 75

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CAMBRIDGE
International Examinations

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CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a "fortuitous" answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

<u>Penalties</u>

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	Use law for the logarithm of a product, quotient or power	M1	
	Obtain a correct equation free of logarithms, e.g. $\dfrac{x+4}{x^2}=4$	A1	
	Solve a 3-term quadratic obtaining at least one root	M1	
	Obtain final answer x = 1.13 only	A1	**4**
2	*EITHER*: State or imply non-modular inequality $(x-2)^2 > (2x-3)^2$, or corresponding equation	B1	
	Solve a 3-term quadratic, as in Q1.	M1	
	Obtain critical value $x=\frac{5}{3}$	A1	
	State final answer $x<\frac{5}{3}$ only	A1	
	*OR*1: State the relevant critical linear inequality $(2-x)>(2x-3)$, or corresponding equation	B1	
	Solve inequality or equation for x	M1	
	Obtain critical value $x=\frac{5}{3}$	A1	
	State final answer $x<\frac{5}{3}$ only	A1	
	*OR*2: Make recognisable sketches of $y = 2x - 3$ and $y=\lvert x-2\rvert$ on a single diagram	B1	
	Find x-coordinate of the intersection	M1	
	Obtain $x=\frac{5}{3}$	A1	
	State final answer $x<\frac{5}{3}$ only	A1	**4**
3	Use correct tan 2A and cot A formulae to form an equation in tan x	M1	
	Obtain a correct equation in any form	A1	
	Reduce equation to the form $\tan^2 x+6\tan x-3=0$, or equivalent	A1	
	Solve a three term quadratic in tan x for x, **as in Q1.**	M1	
	Obtain answer, e.g. 24.9° (24.896)	A1	
	Obtain second answer, e.g. 98.8 (98.794) and no others in the given interval	A1	**6**
	[Ignore outside the given interval. Treat answers in radians as a misread.]		
	Radian answers 0.43452, 1.7243		
4	Use correct quotient or product rule	M1	
	Obtain correct derivative in any form	A1	
	Equate derivative to zero and obtain a horizontal equation	M1	
	Carry out complete method for solving an equation of the form $ae^{3x}=b$, or $ae^{5x}=be^{2x}$	M1	
	Obtain $x=\ln 2$, or exact equivalent	A1	
	Obtain $y=\frac{1}{3}$, or exact equivalent	A1	**6**

5	(i)	State $\frac{dx}{dt} = -4a\cos^3 t\sin t$, or $\frac{dy}{dt} = 4a\sin^3 t\cos t$	B1	
		Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
		Obtain correct expression for $\frac{dy}{dx}$ in a simplified form	A1	3
	(ii)	Form the equation of the tangent	M1	
		Obtain a correct equation in any form	A1	
		Obtain the given answer	A1	3
	(iii)	State the x-coordinate of P or the y-coordinate of Q in any form	B1	
		Obtain the given result correctly	B1	2
6	(i)	Integrate and reach $\pm x\sin x \mp \int \sin x \, dx$	M1*	
		Obtain integral $x\sin x + \cos x$	A1	
		Substitute limits correctly, must be seen since AG, and equate result to 0.5	M1(dep*)	
		Obtain the given form of the equation	A1	4
	(ii)	*EITHER*: Consider the sign of a relevant expression at $a = 1$ and at another relevant value, e.g. $a = 1.5 \leqslant \frac{\pi}{2}$	M1	
		OR: Using limits correctly, consider the sign of $[x\sin x + \cos x]_0^a - 0.5$, or compare the value of $[x\sin x + \cos x]_0^a$ with 0.5, for a =1 AND for another relevant value, e.g $a = 1.5 \leqslant \frac{\pi}{2}$.	M1	
		Complete the argument, so change of sign, or above and below stated, both with correct calculated values	A1	2
	(iii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.2461	A1	
		Show sufficient iterations to 6 d.p. to justify 1.2461 to 4 d.p., or show there is a sign change in the interval (1.24605, 1.24615)	A1	3
7	(i)	Separate variables correctly and integrate one side	B1	
		Obtain term $2\sqrt{M}$, or equivalent	B1	
		Obtain term $50k\sin(0.02t)$, or equivalent	B1	
		Evaluate a constant of integration, or use limits M = 100, t =0 in a solution with terms of the form $a\sqrt{M}$ and $b\sin(0.02t)$	M1*	
		Obtain correct solution in any form, e.g. $2\sqrt{M} = 50k\sin(0.02t) + 20$	A1	5
	(ii)	Use values M =196, t =50 and calculate k	M1(dep*)	
		Obtain answer k = 0.190	A1	2
	(iii)	State an expression for M in terms of t, e.g. $M = (4.75\sin(0.02t) + 10)^2$	M1(dep*)	
		State that the least possible number of micro-organisms is 28 or 27.5 or 27.6 (27.5625)	A1	2

8	**(i)**	*EITHER*: Substitute for u in $\frac{\mathrm{i}}{u}$ and multiply numerator and denominator by $1 + \mathrm{i}$	M1	
		Obtain final answer $-\frac{1}{2}+\frac{1}{2}\mathrm{i}$, or equivalent	A1	
		OR: Substitute for u, obtain two equations in x and y and solve for x or for y	M1	
		Obtain final answer $-\frac{1}{2}+\frac{1}{2}\mathrm{i}$, or equivalent	A1	**2**
	(ii)	Show a point representing u in a relatively correct position	B1	
		Show the bisector of the line segment joining u to the origin	B1	
		Show a circle with centre at the point representing i	B1	
		Show a circle with radius 2	B1	**4**
	(iii)	State argument $-\frac{1}{2}\pi$, or equivalent, e.g. 270°	B1	
		State or imply the intersection in the first quadrant represents 2 + i	B1	
		State argument 0.464, (0.4636)or equivalent, e.g. 26.6° (26.5625)	B1	**3**
9	**(i)**	State or imply a correct normal vector to either plane, e.g. $\mathbf{i}+3\mathbf{j}-2\mathbf{k}$, or $2\mathbf{i}+\mathbf{j}+3\mathbf{k}$	B1	
		Carry out correct process for evaluating the scalar product of two normal vectors	M1	
		Using the correct process for the moduli, divide the scalar product of the two normals by the product of their moduli and evaluate the inverse cosine of the result	M1	
		Obtain answer 85.9° or 1.50 radians	A1	**4**

(ii) *EITHER*: Carry out a complete strategy for finding a point on l M1

Obtain such a point, e.g. (0, 2, 1) A1

EITHER: State two equations for a direction vector $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ for l,

e.g. $a + 3b - 2c = 0$

and $2a + b + 3c = 0$ B1

Solve for one ratio, e.g. $a : b$ M1

Obtain $a : b : c = 11 : -7 : -5$ A1

State a correct answer, e.g. $\mathbf{r} = 2\mathbf{j} + \mathbf{k} + \lambda(11\mathbf{i} - 7\mathbf{j} - 5\mathbf{k})$ A1√

*OR*1: Obtain a second point on l, e.g. $\left(\frac{22}{7}, 0, -\frac{3}{7}\right)$ B1

Subtract position vectors and obtain a direction vector for l M1

Obtain $22\mathbf{i} - 14\mathbf{j} - 10\mathbf{k}$, or equivalent A1

State a correct answer, e.g. $\mathbf{r} = 2\mathbf{j} + \mathbf{k} + \lambda(22\mathbf{i} - 14\mathbf{j} - 10\mathbf{k})$ A1√

*OR*2: Attempt to find the vector product of the two normal vectors M1

Obtain two correct components A1

Obtain $11\mathbf{i} - 7\mathbf{j} - 5\mathbf{k}$, or equivalent A1

State a correct answer, e.g. $\mathbf{r} = 2\mathbf{j} + \mathbf{k} + \lambda(11\mathbf{i} - 7\mathbf{j} - 5\mathbf{k})$ A1√

*OR*3: Express one variable in terms of a second M1

Obtain a correct simplified expression, e.g. $x = (22 - 11y)/7$ A1

Express the same variable in terms of the third M1

Obtain a correct simplified expression, e.g. $x = (11 - 11z)/5$ A1

Form a vector equation for the line M1

State a correct answer, e.g. $\mathbf{r} = 2\mathbf{j} + \mathbf{k} + \lambda\left(\mathbf{i} - \frac{7}{11}\mathbf{j} - \frac{5}{11}\mathbf{k}\right)$ A1√

*OR*4: Express one variable in terms of a second M1

Obtain a correct simplified expression, e.g. $y = (22 - 7x)/11$ A1

Express the third variable in terms of the second M1

Obtain a correct simplified expression, e.g. $z = (11 - 5x)/11$ A1

Form a vector equation for the line M1

State a correct answer, e.g. $\mathbf{r} = 2\mathbf{j} + \mathbf{k} + \lambda\left(\mathbf{i} - \frac{7}{11}\mathbf{j} - \frac{5}{11}\mathbf{k}\right)$ A1√ **6**

[The √ marks are dependent on all M marks being earned.]

10 (i) State or imply $\mathrm{f}(x) \equiv \frac{A}{2x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$ B1

Use a relevant method to determine a constant M1

Obtain one of the values $A = 2$, $B = -1$, $C = 3$ A1

Obtain the remaining values A1 + A1 **5**

[Apply an analogous scheme to the form $\frac{A}{2x-1} + \frac{Dx+E}{(x+2)^2}$; the values being $A = 2$, $D = -1$, $E = 1$.]

(ii) Integrate and obtain terms $\frac{1}{2}\cdot 2\ln(2x-1)-\ln(x+2)-\dfrac{3}{x+2}$ B1√ + B1√ + B1√

Use limits correctly, namely substitution must be seen in at least two of the partial fractions to obtain M1 Integrate all 3 partial fractions and substitute in all three partial fractions for A1 since AG. M1

Obtain the given answer following full and exact working A1 **5**

[The t marks are dependent on *A*, *B*, *C* etc.]

[SR: If *B*, *C* or *E* omitted, give B1M1 in part (**i**) and B1√B1√M1 in part (**ii**).]

[NB: Candidates who follow the *A*, *D*, *E* scheme in part (**i**) and then integrate $\dfrac{-x+1}{(x+2)^2}$ by parts should obtain $\frac{1}{2}\cdot 2\ln(2x-1)-\ln(x+2)+\dfrac{x-1}{x+2}$ (the third term is equivalent to $-\dfrac{3}{x+2}+1$).]

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/31**

Paper 3 **May/June 2016**

MARK SCHEME

Maximum Mark: 75

Published

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A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

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- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol $\sqrt{}$ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

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<u>Penalties</u>

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PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	**(i)**	*EITHER*: State or imply non-modular equation $(2(x-1))^2 = (3x)^2$, or pair of linear equations $2(x-1) = \pm 3x$	**B1**
		Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations	**M1**
		Obtain answers $x = -2$ and $x = \frac{2}{5}$	**A1**
		OR: Obtain answer $x = -2$ by inspection or by solving a linear equation	**(B1**
		Obtain answer $x = \frac{2}{5}$ similarly	**B2)**
			[3]
	(ii)	Use correct method for solving an equation of the form $5^x = a$ or $5^{x+1} = a$, where $a > 0$	**M1**
		Obtain answer $x = -0.569$ only	**A1**
			[2]
2		Integrate by parts and reach $axe^{-2x} + b\int e^{-2x}\,dx$	**M1**
		Obtain $-\frac{1}{2}xe^{-2x} + \frac{1}{2}\int e^{-2x}\,dx$, or equivalent	**A1**
		Complete the integration correctly, obtaining $-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}$, or equivalent	**A1**
		Use limits $x = 0$ and $x = \frac{1}{2}$ correctly, having integrated twice	**M1**
		Obtain answer $\frac{1}{4} - \frac{1}{2}e^{-1}$, or exact equivalent	**A1**
			[5]
3		Correctly restate the equation in terms of sin θ and cos θ	**B1**
		Using Pythagoras obtain a horizontal equation in cos θ	**M1**
		Reduce the equation to a correct quadratic in cos θ, e.g. $3\cos^2\theta - \cos\theta - 2 = 0$	**A1**
		Solve a 3-term quadratic for cos θ	**M1**
		Obtain answer $\theta = 131.8°$ only	**A1**
			[5]
		[Ignore answers outside the given interval.]	
4		Separate variables and attempt integration of at least one side	**M1***
		Obtain term $\ln y$	**A1**
		Obtain terms $\ln x - x^2$	**A1**
		Use $x = 1$ and $y = 2$ to evaluate a constant, or as limits	**DM1***
		Obtain correct solution in any form, e.g. $\ln y = \ln x - x^2 + \ln 2 + 1$	**A1**
		Obtain correct expression for y, free of logarithms, i.e. $y = 2x\exp(1 - x^2)$	**A1**
			[6]

5		Use product rule	**M1**
		Obtain correct derivative in any form, e.g. $\cos x\cos 2x - 2\sin x\sin 2x$	**A1**
		Equate derivative to zero and use double angle formulae	**M1**
		Remove factor of $\cos x$ and reduce equation to one in a single trig function	**M1**
		Obtain $6\sin^2 x = 1$, $6\cos^2 x = 5$ or $5\tan^2 x = 1$	**A1**
		Solve and obtain $x = 0.421$	**A1**
			[6]
		[Alternative: Use double angle formula M1.Use chain rule to differentiate M1. Obtain correct derivative e.g. $\cos\theta - 6\sin^2\theta\cos\theta$ A1, then as above.]	
6	**(i)**	Make recognizable sketch of a relevant graph	**B1**
		Sketch the other relevant graph and justify the given statement	**B1**
			[2]
	(ii)	State $x = \frac{1}{2}\ln(25/x)$	**B1**
		Rearrange this in the form $5e^{-x} = \sqrt{x}$	**B1**
			[2]
	(iii)	Use the iterative formula correctly at least once	**M1**
		Obtain final answer 1.43	**A1**
		Show sufficient iterations to 4 d.p. to justify 1.43 to 2 d.p., or show there is a sign change in the interval (1.425, 1.435)	**A1**
			[3]
7	**(i)**	State or imply $6xy + 3x^2\frac{dy}{dx}$ as derivative of $3x^2y$	**B1**
		State $3y^2\frac{dy}{dx}$ as derivative of y^3	**B1**
		Equate attempted derivative of the LHS to zero and solve for $\frac{dy}{dx}$	**M1**
		Obtain the given answer	**A1**
			[4]
	(ii)	Equate numerator to zero	**M1***
		Obtain $x = 2y$, or equivalent	**A1**
		Obtain an equation in x or y	**DM1***
		Obtain the point $(-2, -1)$	**A1**
		State the point $(0, 1.44)$	**B1**
			[5]

8	(i)	State or imply the form $\dfrac{A}{x+1}+\dfrac{B}{x-3}+\dfrac{C}{(x-3)^2}$	B1
		Use a correct method to determine a constant	M1
		Obtain one of the values $A = 1$, $B = 3$, $C = 12$	A1
		Obtain a second value	A1
		Obtain a third value	A1
			[5]
		[Mark the form $\dfrac{A}{x+1}+\dfrac{Dx+E}{(x-3)^2}$, where A =1, D = 3, E = 3, B1M1A1A1A1 as above.]	
	(ii)	Use correct method to find the first two terms of the expansion of $(x+1)^{-1}$, $(x-3)^{-1}$, $(1-\frac{1}{3}x)^{-1}$, $(x-3)^{-2}$, or $(1-\frac{1}{3}x)^{-2}$	M1
		Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	A1√ + A1√ + A1√
		Obtain final answer $\frac{4}{3}-\frac{4}{9}x+\frac{4}{3}x^2$, or equivalent	A1
			[5]

9	**(i)**	*EITHER*: Obtain a vector parallel to the plane, e.g. $\overrightarrow{AB} = \mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$	**B1**
		Use scalar product to obtain an equation in *a*, *b*, *c* e.g. $a - 2b - 3c = 0$, $a + b - c = 0$, or $3b + 2c = 0$	**M1**
		State two correct equations	**A1**
		Solve to obtain ratio $a : b : c$	**M1**
		Obtain $a : b : c = 5 : -2 : 3$	**A1**
		Obtain equation $5x - 2y + 3z = 5$, or equivalent	**A1**
		*OR*1: Substitute for two points, e.g. *A* and *B*, and obtain $a + 3b + 2c = d$ and $2a + b - c = d$	**(B1**
		Substitute for another point, e.g. *C*, to obtain a third equation and eliminate one unknown entirely from all three equations	**M1**
		Obtain two correct equations in three unknowns, e.g. in *a*, *b*, *c*	**A1**
		Solve to obtain their ratio	**M1**
		Obtain $a : b : c = 5 : -2 : 3$, $a : c : d = 5 : 3 : 5$, $a : b : d = 5 : -2 : 5$, or $b : c : d = -2 : 3 : 5$	**A1**
		Obtain equation $5x - 2y + 3z = 5$, or equivalent	**A1)**
		*OR*2: Obtain a vector parallel to the plane, e.g. $\overrightarrow{AC} = \mathbf{i} + \mathbf{j} - \mathbf{k}$	**(B1**
		Obtain a second such vector and calculate their vector product, e.g. $(\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}) \times (\mathbf{i} + \mathbf{j} - \mathbf{k})$	**M1**
		Obtain two correct components of the product	**A1**
		Obtain correct answer e.g. $5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$	**A1**
		Substitute in $5x - 2y + 3z = d$ to find *d*	**M1**
		Obtain equation $5x - 2y + 3z = 5$, or equivalent	**A1)**
		*OR*3: Obtain a vector parallel to the plane, e.g. $\overrightarrow{BC} = 3\mathbf{j} + 2\mathbf{k}$	**(B1**
		Obtain a second such vector and form correctly a 2-parameter equation for the plane	**M1**
		Obtain a correct equation, e.g. $\mathbf{r} = \mathbf{i} + 3\mathbf{j} + 2\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}) + \mu(3\mathbf{j} + 2\mathbf{k})$	**A1**
		State three correct equations in x, y, z, λ, μ	**A1**
		Eliminate λ and μ	**M1**
		Obtain equation $3x - 2y + 3z = 5$, or equivalent	**A1)**
			[6]
	(ii)	Correctly form an equation for the line through *D* parallel to *OA*	**M1**
		Obtain a correct equation e.g. $\mathbf{r} = -3\mathbf{i} + \mathbf{j} + 2\mathbf{k} + \lambda(\mathbf{i} + 3\mathbf{j} + 2\mathbf{k})$	**A1**
		Substitute components in the equation of the plane and solve for λ	**M1**
		Obtain $\lambda = 2$ and position vector $-\mathbf{i} + 7\mathbf{j} + 6\mathbf{k}$ for *P*	**A1**
		Obtain the given answer correctly	**A1**
			[5]
10	**(a)**	Square $x + iy$ and equate real and imaginary parts to 7 and $-6\sqrt{2}$ respectively	**M1**
		Obtain equations $x^2 - y^2 = 7$ and $2xy = -6\sqrt{2}$	**A1**
		Eliminate one variable and find an equation in the other	**M1**
		Obtain $x^4 - 7x^2 - 18 = 0$ or $y^4 + 7y^2 - 18 = 0$, or 3-term equivalent	**A1**
		Obtain answers $\pm(3 - i\sqrt{2})$	**A1**
			[5]

(b)	**(i)**	Show point representing 1 + 2i	**B1**
		Show circle with radius 1 and centre 1 + 2i	**B1**√
		Show a half line from the point representing 1	**B1**
		Show line making the correct angle with the real axis	**B1**
			[4]
	(ii)	State or imply the relevance of the perpendicular from 1 + 2i to the line	**M1**
		Obtain answer $\sqrt{2}-1$ (or 0.414)	**A1**
			[2]

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/32**

Paper 3 **May/June 2016**

MARK SCHEME

Maximum Mark: 75

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PA – 1 This is deducted from A or B marks in the case of premature approximation. The PA – 1 penalty is usually discussed at the meeting.

1	Use law of the logarithm of a product, power or quotient	**M1***	
	Obtain a correct linear equation, e.g. $(3x-1)\ln 4 = \ln 3 + x\ln 5$	**A1**	
	Solve a linear equation for x	**DM1***	
	Obtain answer $x = 0.975$	**A1**	[4]
2	State a correct un-simplified version of the x or x^2 or x^3 term	**M1**	
	State correct first two terms $1 + x$	**A1**	
	Obtain the next two terms $\frac{3}{2}x^2 + \frac{5}{2}x^3$	**A1 A1**	[4]
	[Symbolic binomial coefficients, e.g. $\begin{pmatrix} -\frac{1}{2} \\ 3 \end{pmatrix}$ are not sufficient for the M mark.]		
3	Integrate by parts and reach $ax^2\cos 2x + b\int x\cos 2x \, dx$	**M1***	
	Obtain $-\frac{1}{2}x^2\cos 2x + \int x\cos 2x$, or equivalent	**A1**	
	Complete the integration and obtain $-\frac{1}{2}x^2\cos 2x + \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x$, or equivalent	**A1**	
	Use limits correctly having integrated twice	**DM1***	
	Obtain answer $\frac{1}{8}(\pi^2 - 4)$, or exact equivalent, with no errors seen	**A1**	[5]
4	State or imply derivative of $(\ln x)^2$ is $\dfrac{2\ln x}{x}$	**B1**	
	Use correct quotient or product rule	**M1**	
	Obtain correct derivative in any form, e.g. $\dfrac{2\ln x}{x^2} - \dfrac{(\ln x)^2}{x^2}$	**A1**	
	Equate derivative (or its numerator) to zero and solve for $\ln x$	**M1**	
	Obtain the point (1, 0) with no errors seen	**A1**	
	Obtain the point $(e^2, \ 4e^{-2})$	**A1**	[6]
5 (i)	*EITHER*: Express $\cos 4\theta$ in terms of $\cos 2\theta$ and/or $\sin 2\theta$	**B1**	
	Use correct double angle formulae to express LHS in terms of $\sin\theta$ and/or $\cos\theta$	**M1**	
	Obtain a correct expression in terms of $\sin\theta$ alone	**A1**	
	Reduce correctly to the given form	**A1**	
	OR: Use correct double angle formula to express RHS in terms of $\cos 2\theta$	**M1**	
	Express $\cos^2 2\theta$ in terms of $\cos 4\theta$	**B1**	
	Obtain a correct expression in terms of $\cos 4\theta$ and $\cos 2\theta$	**A1**	
	Reduce correctly to the given form	**A1**	[4]
(ii)	Use the identity and carry out a method for finding a root	**M1**	
	Obtain answer 68.5°	**A1**	
	Obtain a second answer, e.g. 291.5°	**A1√**	
	Obtain the remaining answers, e.g. 111.5° and 248.5°, and no others in the given interval	**A1√**	[4]
	[Ignore answers outside the given interval. Treat answers in radians as a misread.]		

6	(i)	Separate variables correctly and attempt integration of at least one side	B1	
		Obtain term $\ln x$	B1	
		Obtain term of the form $k\ln(3+\cos 2\theta)$, or equivalent	M1	
		Obtain term $-\frac{1}{2}\ln(3+\cos 2\theta)$, or equivalent	A1	
		Use $x = 3$, $\theta = \frac{1}{4}\pi$ to evaluate a constant or as limits in a solution with terms $a\ln x$ and $b\ln(3+\cos 2\theta)$,where $ab \neq 0$	M1	
		State correct solution in any form, e.g. $\ln x = -\frac{1}{2}\ln(3+\cos 2\theta)+\frac{3}{2}\ln 3$	A1	
		Rearrange in a correct form, e.g. $x = \sqrt{\left(\dfrac{27}{3+\cos 2\theta}\right)}$	A1	[7]
	(ii)	State answer $x = 3\sqrt{3}/2$, or exact equivalent (accept decimal answer in [2.59, 2.60])	B1	[1]
7	(i)	State or imply the form $A+\dfrac{B}{2x+1}+\dfrac{C}{x+2}$	B1	
		State or obtain $A = 2$	B1	
		Use a correct method for finding a constant	M1	
		Obtain one of $B = 1$, $C = -2$	A1	
		Obtain the other value	A1	[5]
	(ii)	Integrate and obtain terms $2x+\frac{1}{2}\ln(2x+1)-2\ln(x+2)$	B3√	
		Substitute correct limits correctly in an integral with terms $a\ln(2x+1)$ and $b\ln(x+2)$, where $ab \neq 0$	M1	
		Obtain the given answer after full and correct working	A1	[5]
8	(i)	Use correct quotient or chain rule	M1	
		Obtain correct derivative in any form	A1	
		Obtain the given answer correctly	A1	[3]
	(ii)	State a correct equation, e.g. $-e^{-a} = -\operatorname{cosec} a \cot a$	B1	
		Rearrange it correctly in the given form	B1	[2]
	(iii)	Calculate values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.5$	M1	
		Complete the argument correctly with correct calculated values	A1	[2]
	(iv)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.317	A1	
		Show sufficient iterations to 5 d.p. to justify 1.317 to 3 d.p., or show there is a sign change in the interval (1.3165, 1,3175)	A1	[3]

9 **(i)** *Either* state or imply $\overrightarrow{AB}$ or $\overrightarrow{BC}$ in component form, *or* state position vector of midpoint of $\overrightarrow{AC}$ **B1**

Use a correct method for finding the position vector of D **M1**
Obtain answer $3\mathbf{i}+3\mathbf{j}+\mathbf{k}$, or equivalent **A1**

EITHER: Using the correct process for the moduli, compare lengths of a pair of adjacent sides,
e.g. AB and BC **M1**
Show that $ABCD$ has a pair of adjacent sides that are equal **A1**

OR: Calculate scalar product $\overrightarrow{AC}.\overrightarrow{BD}$ or equivalent **M1**
Show that $ABCD$ has perpendicular diagonals **A1** [5]

(ii) *EITHER*: State $a+2b+3c=0$ or $2a+b-2c=0$ **B1**
Obtain two relevant equations and solve for one ratio, e.g. $a : b$ **M1**
Obtain $a : b : c = -7 : 8 : -3$, or equivalent **A1**
Substitute coordinates of a relevant point in $-7x + 8y - 3z = d$, and evaluate **M1**
Obtain answer $-7x + 8y - 3z = 29$, or equivalent **A1**

*OR*1:Attempt to calculate vector product of relevant vectors,
e.g. $(\mathbf{i}+2\mathbf{j}+3\mathbf{k})\times(2\mathbf{i}+\mathbf{j}-2\mathbf{k})$ **M1**
Obtain two correct components of the product **A1**
Obtain correct product, e.g. $-7\mathbf{i}+8\mathbf{j}-3\mathbf{k}$ **A1**
Substitute coordinates of a relevant point in $-7x+8y-3z=d$ and evaluate d **M1**
Obtain answer $-7x+8y-3z=29$ or equivalent **A1**

*OR*2:Attempt to form a 2-parameter equation with relevant vectors **M1**
State a correct equation, e.g. $\mathbf{r}=2\mathbf{i}+5\mathbf{j}-\mathbf{k}+\lambda(\mathbf{i}+2\mathbf{j}+3\mathbf{k})+\mu(2\mathbf{i}+\mathbf{j}-2\mathbf{k})$ **A1**
State 3 equations in x, y, z, λ and μ **A1**
Eliminate λ and μ **M1**
Obtain answer $-7x+8y-3z=29$, or equivalent **A1**

*OR*3:Using a relevant point and relevant direction vectors, form a determinant equation for the plane **M1**

State a correct equation, e.g. $\begin{vmatrix} x-2 & y-5 & z+1 \\ 1 & 2 & 3 \\ 2 & 1 & -2 \end{vmatrix} = 0$ **A1**

Attempt to expand the determinant **M1**
Obtain correct values of two cofactors **A1**
Obtain answer $-7x+8y-3z=29$, or equivalent **A1** [5]

10	**(a)**	*EITHER*: Use quadratic formula to solve for z	**M1**	
		Use $\mathrm{i}^2 = -1$	**M1**	
		Obtain a correct answer in any form, simplified as far as $(-2 \pm \mathrm{i}\sqrt{8}) / 2\mathrm{i}$	**A1**	
		Multiply numerator and denominator by i, or equivalent	**M1**	
		Obtain final answers $\sqrt{2} + \mathrm{i}$ and $-\sqrt{2} + \mathrm{i}$	**A1**	
		OR: Substitute $x + \mathrm{i}y$ and equate real and imaginary parts to zero	**M1**	
		Use $\mathrm{i}^2 = -1$	**M1**	
		Obtain $-2xy + 2x = 0$ and $x^2 - y^2 + 2y - 3 = 0$, or equivalent	**A1**	
		Solve for x and y	**M1**	
		Obtain final answers $\sqrt{2} + \mathrm{i}$ and $-\sqrt{2} + \mathrm{i}$	**A1**	[5]
	(b) (i)	*EITHER*: Show the point representing 4 + 3i in relatively correct position	**B1**	
		Show the perpendicular bisector of the line segment joining this point to the origin	**B1**√	[2]
		OR: Obtain correct Cartesian equation of the locus in any form, e.g. $8x + 6y = 25$	**B1**	
		Show this line [This f.t. is dependent on using a correct method to determine the equation.]	**B1**√	
	(ii)	State or imply the relevant point is represented by 2 + 1.5i or is at (2, 1.5)	**B1**	
		Obtain modulus 2.5	**B1**√	
		Obtain argument 0.64 (or 36.9°) (allow decimals in [0.64, 0.65] or [36.8, 36.9])	**B1**√	[3]

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/33**

Paper 3 **May/June 2016**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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Cambridge is publishing the mark schemes for the May/June 2016 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol $\sqrt{}$ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a "fortuitous" answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

<u>Penalties</u>

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1		*EITHER*: State or imply non-modular inequality $(2(x-2))^2 > (3x+1)^2$, or corresponding quadratic equation, or pair of linear equations $2(x-2) = \pm(3x+1)$	**B1**
		Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations for x	**M1**
		Obtain critical values $x = -5$ and $x = \frac{3}{5}$	**A1**
		State final answer $-5 < x < \frac{3}{5}$	**A1**
		OR: Obtain critical value $x = -5$ from a graphical method, or by inspection, or by solving a linear equation or inequality	**(B1**
		Obtain critical value $x = \frac{3}{5}$ similarly	**B2**
		State final answer $-5 < x < \frac{3}{5}$	**B1)**
		[Do not condone ≤ for <.]	[4]
2	**(i)**	State or imply $y \ln 3 = (2-x)\ln 4$	**B1**
		State that this is of the form $ay = bx + c$ and thus a straight line, or equivalent	**B1**
		State gradient is $-\dfrac{\ln 4}{\ln 3}$, or exact equivalent	**B1**
			[3]
	(ii)	Substitute $y = 2x$ and solve for x, using a log law correctly at least once	**M1**
		Obtain answer $x = \ln 4 / \ln 6$, or exact equivalent	**A1**
			[2]
3	**(i)**	State answer $R = 3$	**B1**
		Use trig formula to find	**M1**
		Obtain $\alpha = 41.81°$ with no errors seen	**A1**
			[3]
	(ii)	Evaluate $\cos^{-1}(0.4)$ to at least 1 d.p. (66.42° to 2 d.p.)	**B1**$\sqrt{}$
		Carry out an appropriate method to find a value of x in the given range	**M1**
		Obtain answer 216.5° only	**A1**
		[Ignore answers outside the given interval.]	[3]
4	**(i)**	State $\dfrac{dx}{dt} = 1 - \sin t$	**B1**
		Use chain rule to find the derivative of y	**M1**
		Obtain $\dfrac{dy}{dt} = \dfrac{\cos t}{1+\sin t}$, or equivalent	**A1**
		Use $\dfrac{dy}{dx} = \dfrac{dy}{dt} \div \dfrac{dx}{dt}$	**M1**
		Obtain the given answer correctly	**A1**
			[5]
	(ii)	State or imply $t = \cos^{-1}(\frac{1}{3})$	**B1**
		Obtain answers $x = 1.56$ and $x = -0.898$	**B1 + B1**
			[3]

5		Separate variables and make reasonable attempt at integration of either integral	**M1**
		Obtain term $\frac{1}{2}e^{2y}$	**B1**
		Use Pythagoras	**M1**
		Obtain terms $\tan x - x$	**A1**
		Evaluate a constant or use $x = 0$, $y = 0$ as limits in a solution containing terms $ae^{\pm 2y}$ and $b\tan x$, ($ab \neq 0$)	**M1**
		Obtain correct solution in any form, e.g. $\frac{1}{2}e^{2y} = \tan x - x + \frac{1}{2}$	**A1**
		Set $x = \frac{1}{4}\pi$ and use correct method to solve an equation of the form $e^{\pm 2y} = a$ or $e^{\pm y} = a$, where $a > 0$	**M1**
		Obtain answer $y = 0.179$	**A1**
			[8]
6	**(i)**	Use the product rule	**M1**
		Obtain correct derivative in any form	**A1**
		Equate 2-term derivative to zero and obtain the given answer correctly	**A1**
			[3]
	(ii)	Use calculations to consider the sign of a relevant expression at $p = 2$ and $p = 2.5$, or compare values of relevant expressions at $p = 2$ and $p = 2.5$	**M1**
		Complete the argument correctly with correct calculated values	**A1**
			[2]
	(iii)	Use the iterative formula correctly at least once	**M1**
		Obtain final answer 2.15	**A1**
		Show sufficient iterations to 4 d.p. to justify 2.15 to 2 d.p., or show there is a sign change in the interval (2.145,2.155)	**A1**
			[3]
7	**(i)**	State or imply $du = 2x\,dx$, or equivalent	**B1**
		Substitute for x and dx throughout	**M1**
		Reduce to the given form and justify the change in limits	**A1**
			[3]
	(ii)	Convert integrand to a sum of integrable terms and attempt integration	**M1**
		Obtain integral $\frac{1}{2}\ln u + \frac{1}{u} - \frac{1}{4u^2}$, or equivalent	**A1 + A1**
		(deduct A1 for each error or omission)	
		Substitute limits in an integral containing two terms of the form $a\ln u$ and bu^{-2}	**M1**
		Obtain answer $\frac{1}{2}\ln 2 - \frac{5}{16}$, exact simplified equivalent	**A1**
			[5]

8	**(i)**	State a correct equation for *AB* in any form, e.g. $\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$, or equivalent	**B1**
		Equate at least two pairs of components of *AB* and *l* and solve for λ or for μ	**M1**
		Obtain correct answer for λ or for μ, e.g. $\lambda = -1$ or $\mu = 2$	**A1**
		Show that not all three equations are not satisfied and that the lines do not intersect	**A1**
			[4]
	(ii)	*EITHER*: Find $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) for a general point *P* on *l*, e.g. $(1-\mu)\mathbf{i} + (-3+2\mu)\mathbf{j} + (-2+\mu)\mathbf{k}$	**B1**
		Calculate the scalar product of $\overrightarrow{AP}$ and a direction vector for *l* and equate to zero	**M1**
		Solve and obtain $\mu = \frac{3}{2}$	**A1**
		Carry out a method to calculate *AP* when $\mu = \frac{3}{2}$	**M1**
		Obtain the given answer $\frac{1}{\sqrt{2}}$ correctly	**A1**
		OR 1:Find $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) for a general point *P* on *l*	***(*B1**
		Use correct method to express AP^2 (or *AP*) in terms of μ	**M1**
		Obtain a correct expression in any form, e.g. $(1-\mu)^2 + (-3+2\mu)^2 + (-2+\mu)^2$	**A1**
		Carry out a complete method for finding its minimum	**M1**
		Obtain the given answer correctly	**A1)**
		OR 2:Calling (2, –2, –1) *C*, state $\overrightarrow{AC}$ (or $\overrightarrow{CA}$) in component form, e.g. $\mathbf{i} - 3\mathbf{j} - 2\mathbf{k}$	**(B1**
		Use a scalar product to find the projection of $\overrightarrow{AC}$ (or $\overrightarrow{CA}$) on *l*	**M1**
		Obtain correct answer in any form, e.g. $\frac{9}{\sqrt{6}}$	**A1**
		Use Pythagoras to find the perpendicular	**M1**
		Obtain the given answer correctly	**A1)**
		OR 3:State $\overrightarrow{AC}$ (or $\overrightarrow{CA}$) in component form	**(B1**
		Calculate vector product of $\overrightarrow{AC}$ and a direction vector for *l*, e.g. $(\mathbf{i} - 3\mathbf{j} - 2\mathbf{k}) \times (-\mathbf{i} + 2\mathbf{j} + \mathbf{k})$	**M1**
		Obtain correct answer in any form, e.g. $\mathbf{i} + \mathbf{j} - \mathbf{k}$	**A1**
		Divide modulus of the product by that of the direction vector	**M1**
		Obtain the given answer correctly	**A1)**
			[5]
9	**(i)**	*EITHER*: Multiply numerator and denominator of $\frac{u}{v}$ by 2 + i, or equivalent	**M1**
		Simplify the numerator to –5 +5i or denominator to 5	**A1**
		Obtain final answer –1 + I	**A1**
		OR: Obtain two equations in *x* and *y* and solve for *x* or for *y*	***(*M1**
		Obtain $x = -1$ or $y = 1$	**A1**
		Obtain final answer –1 + I	**A1)**
			[3]
	(ii)	Obtain $u + v = 1 + 2i$	**B1**
		In an Argand diagram show points *A*, *B*, *C* representing *u*, *v* and *u* + *v* respectively	**B1**$\sqrt{}$
		State that *OB* and *AC* are parallel	**B1**
		State that *OB* = *AC*	**B1**
			[4]

(iii) Carry out an appropriate method for finding angle AOB, e.g. find $\arg(u / v)$ **M1**

Show sufficient working to justify the given answer $\frac{3}{4}\pi$ **A1**

[2]

10 **(i)** State or imply the form $\dfrac{A}{x+3}+\dfrac{B}{x-1}+\dfrac{C}{(x-1)^2}$ **B1**

Use a correct method to determine a constant **M1**
Obtain one of the values $A = -3$, $B = 1$, $C = 2$ **A1**
Obtain a second value **A1**
Obtain the third value **A1**

[Mark the form $\dfrac{A}{x+3}+\dfrac{Dx+E}{(x-1)^2}$, where $A = -3$, $D = 1$, $E = 1$, B1M1A1A1A1 as above.] [5]

(ii) Use a correct method to find the first two terms of the expansion of $(x+3)^{-1}$, $(1+\frac{1}{3}x)^{-1}$, $(x-1)^{-1}$, $(1-x)^{-1}$, $(x-1)^{-2}$, or $(1-x)^{-2}$ **M1**

Obtain correct unsimplified expressions up to the term in x^2 of each partial fraction **A1**$\sqrt{}$ + **A1**$\sqrt{}$ + **A1**$\sqrt{}$

Obtain final answer $\frac{10}{3}x+\frac{44}{9}x^2$, or equivalent **A1**

[5]

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS	**9709/31**
Paper 3	**May/June 2017**
MARK SCHEME	
Maximum Mark: 75	

Published

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CAO Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)

CWO Correct Working Only – often written by a ‘fortuitous’ answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

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Question	Answer	Marks
1	**EITHER:** State or imply non-modular inequality $(2x+1)^2 < (3(x-2))^2$, or corresponding quadratic equation, or pair of linear equations $(2x+1) = \pm 3(x-2)$	**(B1**
	Make reasonable solution attempt at a 3-term quadratic e.g. $5x^2 - 40x + 35 = 0$ or solve two linear equations for x	**M1**
	Obtain critical values $x = 1$ and $x = 7$	**A1**
	State final answer $x < 1$ and $x > 7$	**A1)**
	OR: Obtain critical value $x = 7$ from a graphical method, or by inspection, or by solving a linear equation or inequality	**(B1**
	Obtain critical value $x = 1$ similarly	**B2**
	State final answer $x < 1$ and $x > 7$	**B1)**
	Total:	**4**
2	**EITHER:** State a correct unsimplified version of the x or x^2 or x^3 term in the expansion of $(1+6x)^{-\frac{1}{3}}$	**(M1**
	State correct first two terms $1 - 2x$	**A1**
	Obtain term $8x^2$	**A1**
	Obtain term $-\frac{112}{3}x^3$ $\left(37\frac{1}{3}x^3\right)$ in final answer	**A1)**
	OR: Differentiate expression and evaluate $\mathrm{f}(0)$ and $\mathrm{f}'(0)$, where $\mathrm{f}'(x) = k(1+6x)^{-\frac{4}{3}}$	**(M1**
	Obtain correct first two terms $1 - 2x$	**A1**
	Obtain term $8x^2$	**A1**
	Obtain term $-\frac{112}{3}x^3$ in final answer	**A1)**
	Total:	**4**

Question	Answer	Marks
3(i)	Remove logarithms correctly and obtain $e^x = \frac{1-y}{y}$	B1
	Obtain the **given answer** $y = \frac{e^{-x}}{1+e^{-x}}$ following full working	B1
	Total:	2
3(ii)	State integral $k\ln(1+e^{-x})$ where $k = \pm 1$	*M1
	State correct integral $-\ln(1+e^{-x})$	A1
	Use limits correctly	DM1
	Obtain the **given answer** $\ln\left(\frac{2e}{e+1}\right)$ following full working	A1
	Total:	4
4(i)	Use chain rule to differentiate x $\left(\frac{dx}{d\theta} = -\frac{\sin\theta}{\cos\theta}\right)$	M1
	State $\frac{dy}{d\theta} = 3 - \sec^2\theta$	B1
	Use $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$	M1
	Obtain correct $\frac{dy}{dx}$ in any form e.g. $\frac{3-\sec^2\theta}{-\tan\theta}$	A1
	Obtain $\frac{dy}{dx} = \frac{\tan^2\theta - 2}{\tan\theta}$, or equivalent	A1
	Total:	5
4(ii)	Equate gradient to −1 and obtain an equation in $\tan\theta$	M1
	Solve a 3 term quadratic $\left(\tan^2\theta + \tan\theta - 2 = 0\right)$ in $\tan\theta$	M1
	Obtain $\theta = \frac{\pi}{4}$ and $y = \frac{3\pi}{4} - 1$ only	A1
	Total:	3

Question	Answer	Marks
5(i)	Use correct sector formula at least once and form an equation in r and x	**M1**
	Obtain a correct equation in any form	**A1**
	Rearrange in the given form	**A1**
	Total:	**3**
5(ii)	Calculate values of a relevant expression or expressions at $x = 1$ and $x = 1.5$	**M1**
	Complete the argument correctly with correct calculated values	**A1**
	Total:	**2**
5(iii)	Use the iterative formula correctly at least once	**M1**
	Obtain final answer 1.374	**A1**
	Show sufficient iterations to 5 d.p. to justify 1.374 to 3 d.p., or show there is a sign change in the interval (1.3745, 1.3755)	**A1**
	Total:	**3**
6(i)	State or obtain coordinates (1, 2, 1) for the mid-point of AB	**B1**
	Verify that the midpoint lies on m	**B1**
	State or imply a correct normal vector to the plane, e.g. $2\mathbf{i}+2\mathbf{j}-\mathbf{k}$	**B1**
	State or imply a direction vector for the segment AB, e.g. $-4\mathbf{i}-4\mathbf{j}+2\mathbf{k}$	**B1**
	Confirm that m is perpendicular to AB	**B1**
	Total:	**5**
6(ii)	State or imply that the perpendicular distance of m from the origin is $\frac{5}{3}$, or unsimplified equivalent	**B1**
	State or imply that n has an equation of the form $2x+2y-z=k$	**B1**
	Obtain answer $2x+2y-z=2$	**B1**
	Total:	**3**

Question	Answer	Marks
7(i)	State that $u - 2w = -7 - \mathrm{i}$	B1
	EITHER: Multiply numerator and denominator of $\frac{u}{w}$ by $3 - 4\mathrm{i}$, or equivalent	(M1
	Simplify the numerator to $25 + 25\mathrm{i}$ or denominator to 25	A1
	Obtain final answer $1 + \mathrm{i}$	A1)
	OR: Obtain two equations in x and y and solve for x or for y	(M1
	Obtain $x = 1$ or $y = 1$	A1
	Obtain final answer $1 + \mathrm{i}$	A1)
	Total:	4
7(ii)	Find the argument of $\frac{u}{w}$	M1
	Obtain the given answer	A1
	Total:	2
7(iii)	State that OB and CA are parallel	B1
	State that $CA = 2OB$, or equivalent	B1
	Total:	2
8(i)	Use $\sin(A - B)$ formula and obtain an expression in terms of $\sin x$ and $\cos x$	M1
	Collect terms and reach $\sqrt{3}\sin x - 2\cos x$, or equivalent	A1
	Obtain $R = \sqrt{7}$	A1
	Use trig formula to find α	M1
	Obtain $\alpha = 49.11°$ with no errors seen	A1
	Total:	5

Question	Answer	Marks
8(ii)	Evaluate $\sin^{-1}(1/\sqrt{7})$ to at least 1 d.p. (22.21° to 2 d.p.)	**B1 FT**
	Use a correct method to find a value of x in the interval $0° < x < 180°$	**M1**
	Obtain answer 71.3°	**A1**
	[ignore answers outside given range.]	
	Total:	**3**
9(i)	Carry out a relevant method to obtain A and B such that $\frac{1}{x(2x+3)} \equiv \frac{A}{x} + \frac{B}{2x+3}$, or equivalent	**M1**
	Obtain $A = \frac{1}{3}$ and $B = -\frac{2}{3}$, or equivalent	**A1**
	Total:	**2**
9(ii)	Separate variables and integrate one side	**B1**
	Obtain term $\ln y$	**B1**
	Integrate and obtain terms $\frac{1}{3}\ln x - \frac{1}{3}\ln(2x+3)$, or equivalent	**B2 FT**
	Use $x = 1$ and $y = 1$ to evaluate a constant, or as limits, in a solution containing $a\ln y,\ b\ln x,\ c\ln(2x+3)$	**M1**
	Obtain correct solution in any form, e.g. $\ln y = \frac{1}{3}\ln x - \frac{1}{3}\ln(2x+3) + \frac{1}{3}\ln 5$	**A1**
	Obtain answer $y = 1.29$ (3s.f. only)	**A1**
	Total:	**7**
10(i)	State or imply $\mathrm{d}u = -\sin x\ \mathrm{d}x$	**B1**
	Using correct double angle formula, express the integral in terms of u and du	**M1**
	Obtain integrand $\pm(2u^2 - 1)^2$	**A1**
	Change limits and obtain correct integral $\int_{\frac{1}{\sqrt{2}}}^{1} (2u^2 - 1)^2\,\mathrm{d}u$ with no errors seen	**A1**
	Substitute limits in an integral of the form $au^5 + bu^3 + cu$	**M1**
	Obtain answer $\frac{1}{15}(7 - 4\sqrt{2})$, or exact simplified equivalent	**A1**
	Total:	**6**

Question	Answer	Marks
10(ii)	Use product rule and chain rule at least once	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and use trig formulae to obtain an equation in $\cos x$ and $\sin x$	**M1**
	Use correct methods to obtain an equation in cos x or sin x only	**M1**
	Obtain $10\cos^2 x = 9$ or $10\sin^2 x = 1$, or equivalent	**A1**
	Obtain answer 0.32	**A1**
	Total:	**6**

Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS	**9709/32**
Paper 3	**May/June 2017**
MARK SCHEME	
Maximum Mark: 75	

Published

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Cambridge is publishing the mark schemes for the May/June 2017 series for most Cambridge IGCSE®, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

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This document consists of **12** printed pages.

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous’ answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks
1	Use law of the logarithm of a power or a quotient	**M1**
	Remove logarithms and obtain a correct equation in x. e.g. $x^2+1=ex^2$	**A1**
	Obtain answer 0.763 and no other	**A1**
	Total:	**3**
2	*EITHER*: State or imply non-modular inequality $(x-3)^2<(3x-4)^2$, or corresponding equation	**(B1**
	Make reasonable attempt at solving a three term quadratic	**M1**
	Obtain critical value $x=\frac{7}{4}$	**A1**
	State final answer $x>\frac{7}{4}$ only	**A1)**
	*OR*1: State the relevant critical inequality $3-x<3x-4$, or corresponding equation	**(B1**
	Solve for x	**M1**
	Obtain critical value $x=\frac{7}{4}$	**A1**
	State final answer $x>\frac{7}{4}$ only	**A1)**
	*OR*2: Make recognizable sketches of $y=\lvert x-3\rvert$ and $y=3x-4$ on a single diagram	**(B1**
	Find x-coordinate of the intersection	**M1**
	Obtain $x=\frac{7}{4}$	**A1**
	State final answer $x>\frac{7}{4}$ only	**A1)**
	Total:	**4**

Question	Answer	Marks
3(i)	Use correct formulae to express the equation in terms of cos θ and sin θ	**M1**
	Use Pythagoras and express the equation in terms of cos θ only	**M1**
	Obtain correct 3-term equation, e.g. $2\cos^4\theta + \cos^2\theta - 2 = 0$	**A1**
	Total:	**3**
3(ii)	Solve a 3-term quadratic in $\cos^2\theta$ for cos θ	**M1**
	Obtain answer $\theta = 152.1°$ only	**A1**
	Total:	**2**
4(i)	State $\dfrac{dy}{dt} = 4 + \dfrac{2}{2t-1}$	**B1**
	Use $\dfrac{dy}{dx} = \dfrac{dy}{dt} \div \dfrac{dx}{dt}$	**M1**
	Obtain answer $\dfrac{dy}{dx} = \dfrac{8t-2}{2t(2t-1)}$, or equivalent e.g. $\dfrac{2}{t} + \dfrac{2}{4t^2-2t}$	**A1**
	Total:	**3**
4(ii)	Use correct method to find the gradient of the normal at $t = 1$	**M1**
	Use a correct method to form an equation for the normal at $t = 1$	**M1**
	Obtain final answer $x + 3y - 14 = 0$, or horizontal equivalent	**A1**
	Total:	**3**

Question	Answer	Marks
5(i)	State $\dfrac{dy}{dt} = -\dfrac{2y}{(1+t)^2}$, or equivalent	B1
	Separate variables correctly and attempt integration of one side	M1
	Obtain term $\ln y$, or equivalent	A1
	Obtain term $\dfrac{2}{(1+t)}$, or equivalent	A1
	Use $y = 100$ and $t = 0$ to evaluate a constant, or as limits in an expression containing terms of the form $a\ln y$ and $\dfrac{b}{1+t}$	M1
	Obtain correct solution in any form, e.g. $\ln y = \dfrac{2}{1+t} - 2 + \ln 100$	A1
	Total:	6
5(ii)	State that the mass of B approaches $\dfrac{100}{e^2}$, or **exact** equivalent	B1
	State or imply that the mass of A tends to zero	B1
	Total:	2

Question	Answer	Marks
6(i)	*EITHER:* Substitute $x = 2 - \mathrm{i}$ (or $x = 2 + \mathrm{i}$) in the equation and attempt expansions of x^2 and x^3	**(M1**
	Equate real and/or imaginary parts to zero	**M1**
	Obtain $a = -2$	**A1**
	Obtain $b = 10$	**A1)**
	*OR*1: Substitute $x = 2 - \mathrm{i}$ in the equation and attempt expansions of x^2 and x^3	**(M1**
	Substitute $x = 2 + \mathrm{i}$ in the equation and add/subtract the two equations	**M1**
	Obtain $a = -2$	**A1**
	Obtain $b = 10$	**A1)**
	*OR*2: Factorise to obtain $(x-2+\mathrm{i})(x-2-\mathrm{i})(x-p)$ $\left(=\left(x^2-4x+5\right)(x-p)\right)$	**(M1**
	Compare coefficients	**M1**
	Obtain $a = -2$	**A1**
	Obtain $b = 10$	**A1)**
	*OR*3: Obtain the quadratic factor (x^2-4x+5)	**(M1**
	Use algebraic division to obtain a real linear factor of the form $x - p$ and set the remainder equal to zero	**M1**
	Obtain $a = -2$	**A1**
	Obtain $b = 10$	**A1)**
	*OR*4: Use $\alpha\beta = 5$ and $\alpha + \beta = 4$ in $\alpha\beta + \beta\gamma + \gamma\alpha = -3$	**(M1**
	Solve for γ and use in $\alpha\beta\gamma = -b$ and/or $\alpha + \beta + \gamma = -a$	**M1**
	Obtain $a = -2$	**A1**
	Obtain $b = 10$	**A1)**

Question	Answer	Marks
	OR5: Factorise as $(x–- (2\text{-i}))(x^2 + ex + g)$ and compare coefficients to form an equation in a and b	**(M1**
	Equate real and/or imaginary parts to zero	**M1**
	Obtain $a = -2$	**A1**
	Obtain $b = 10$	**A1)**
	Total:	**4**
6(ii)	Show a circle with centre $2-\text{ i}$ in a relatively correct position	**B1**
	Show a circle with radius 1 and centre not at the origin	**B1**
	Show the perpendicular bisector of the line segment joining 0 to – i	**B1**
	Shade the correct region	**B1**
	Total:	**4**
7(i)	Use quotient or chain rule	**M1**
	Obtain given answer correctly	**A1**
	Total:	**2**
7(ii)	*EITHER*: Multiply numerator and denominator of LHS by $1+\sin\theta$	**(M1**
	Use Pythagoras and express LHS in terms of sec θ and $\tan\theta$	**M1**
	Complete the proof	**A1)**
	OR1: Express RHS in terms of cos θ and sin θ	**(M1**
	Use Pythagoras and express RHS in terms of sin θ	**M1**
	Complete the proof	**A1)**
	OR2: Express LHS in terms of $\sec\theta$ and $\tan\theta$	**(M1**
	Multiply numerator and denominator by $\sec\theta + \tan\theta$ and use Pythagoras	**M1**
	Complete the proof	**A1)**
	Total:	**3**

Question	Answer	Marks
7(iii)	Use the identity and obtain integral $2\tan\theta + 2\sec\theta - \theta$	**B2**
	Use correct limits correctly in an integral containing terms $a\tan\theta$ and $b\sec\theta$	**M1**
	Obtain answer $2\sqrt{2} - \frac{1}{4}\pi$	**A1**
	Total:	**4**
8(i)	State or imply the form $\dfrac{A}{3x+2} + \dfrac{Bx+C}{x^2+5}$	**B1**
	Use a relevant method to determine a constant	**M1**
	Obtain one of the values $A = 2$, $B = 1$, $C = -3$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
	Total:	**5**
8(ii)	Use correct method to find the first two terms of the expansion of $(3x+2)^{-1}$, $(1+\frac{3}{2}x)^{-1}$, $(5+x^2)^{-1}$ or $(1+\frac{1}{5}x^2)^{-1}$ [Symbolic coefficients, e.g. $\begin{pmatrix}-1\\2\end{pmatrix}$ are not sufficient]	**M1**
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction. The FT is on A, B, C. from part (i)	**A1FT + A1FT**
	Multiply out up to the term in x^2 by $Bx + C$, where $BC \neq 0$	**M1**
	Obtain **final answer** $\frac{2}{5} - \frac{13}{10}x + \frac{237}{100}x^2$, or equivalent	**A1**
	Total:	**5**
9(i)	*EITHER*: Find $\overrightarrow{AP}$ for a general point P on l with parameter λ, e.g.$(8 + 3\lambda, -3 - \lambda, 4 + 2\lambda)$	**(B1**
	Equate scalar product of $\overrightarrow{AP}$ and direction vector of l to zero and solve for λ	**M1**
	Obtain $\lambda = -\frac{5}{2}$ and foot of perpendicular $\frac{3}{2}\mathbf{i} + \frac{3}{2}\mathbf{j} + 3\mathbf{k}$	**A1**
	Carry out a complete method for finding the position vector of the reflection of A in l	**M1**
	Obtain answer $2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$	**A1)**

Question	Answer	Marks
	OR: Find $\overrightarrow{AP}$ for a general point P on l with parameter λ, e.g.$(8 + 3\lambda, -3 - \lambda, 4 + 2\lambda)$	(B1
	Differentiate $\lvert AP\rvert^2$ and solve for λ at minimum	M1
	Obtain $\lambda = -\frac{5}{2}$ and foot of perpendicular $\frac{3}{2}\mathbf{i} + \frac{3}{2}\mathbf{j} + 3\mathbf{k}$	A1
	Carry out a complete method for finding the position vector of the reflection of A in l	M1
	Obtain answer $2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$	A1)
	Total:	**5**
9(ii)	*EITHER:* Use scalar product to obtain an equation in a, b and c, e.g. $3a - b + 2c = 0$	(B1
	Form a second relevant equation, e.g. $9a - b + 8c = 0$ and solve for one ratio, e.g. $a : b$	M1
	Obtain final answer $a : b : c = 1 : 1 : -1$ and state plane equation $x + y - z = 0$	A1)
	*OR*1: Attempt to calculate vector product of two relevant vectors, e.g. $(3\mathbf{i} - \mathbf{j} + 2\mathbf{k}) \times (9\mathbf{i} - \mathbf{j} + 8\mathbf{k})$	(M1
	Obtain two correct components	A1
	Obtain correct answer, e.g. $-6\mathbf{i} - 6\mathbf{j} + 6\mathbf{k}$, and state plane equation $-x - y + z = 0$	A1)
	*OR*2: Using a relevant point and relevant vectors, attempt to form a 2-parameter equation for the plane, e.g. $\mathbf{r} = 6\mathbf{i} + 6\mathbf{k} + s(3\mathbf{i} - \mathbf{j} + 2\mathbf{k}) + t(9\mathbf{i} - \mathbf{j} + 8\mathbf{k})$	(M1
	State 3 correct equations in x, y, z, s and t	A1
	Eliminate s and t and state plane equation $x + y - z = 0$, or equivalent	A1)
	*OR*3: Using a relevant point and relevant vectors, attempt to form a determinant equation for the plane, e.g. $\begin{vmatrix} x-3 & y-1 & z-4 \\ 3 & -1 & 2 \\ 9 & -1 & 8 \end{vmatrix} = 0$	(M1
	Expand a correct determinant and obtain two correct cofactors	A1
	Obtain answer $-6x - 6y + 6z = 0$, or equivalent	A1)
	Total:	**3**

Question	Answer	Marks
9(iii)	*EITHER*: Using the correct processes, divide the scalar product of $\overrightarrow{OA}$ and a normal to the plane by the modulus of the normal or make a recognisable attempt to apply the perpendicular formula	**(M1**
	Obtain a correct expression in any form, e.g. $\dfrac{1+2-4}{\sqrt{(1^2+1^2+(-1)^2)}}$, or equivalent	**A1 FT**
	Obtain answer $1/\sqrt{3}$, or exact equivalent	**A1)**
	OR1: Obtain equation of the parallel plane through A, e.g. $x+y-z=-1$ [The f.t. is on the plane found in part (**ii**).]	**(B1 FT**
	Use correct method to find its distance from the origin	**M1**
	Obtain answer $1/\sqrt{3}$, or exact equivalent	**A1)**
	OR2: Form equation for the intersection of the perpendicular through A and the plane [FT on their **n**]	**(B1 FT**
	Solve for λ	**M1**
	$\lvert\lambda\mathbf{n}\rvert=\dfrac{1}{\sqrt{3}}$	**A1)**
	Total:	**3**
10(i)	Use correct product rule	**M1**
	Obtain correct derivative in any form $\left(y'=2x\cos 2x-2x^2\sin 2x\right)$	**A1**
	Equate to zero and derive the given equation	**A1**
	Total:	**3**
10(ii)	Use the iterative formula correctly at least once e.g. $0.5\to 0.55357\to 0.53261\to 0.54070\to 0.53755$	**M1**
	Obtain final answer 0.54	**A1**
	Show sufficient iterations to 4 d.p. to justify 0.54 to 2 d.p., or show there is a sign change in the interval (0.535, 0.545)	**A1**
	Total:	**3**

Question	Answer	Marks
10(iii)	Integrate by parts and reach $ax^2 \sin 2x + b\int x \sin 2x \, dx$	*M1
	Obtain $\frac{1}{2}x^2 \sin 2x - \int 2x.\frac{1}{2}\sin 2x \, dx$	A1
	Complete integration and obtain $\frac{1}{2}x^2 \sin 2x + \frac{1}{2}x \cos 2x - \frac{1}{4}\sin 2x$, or equivalent	A1
	Substitute limits $x = 0$, $x = \frac{1}{4}\pi$, having integrated twice	DM1
	Obtain answer $\frac{1}{32}(\pi^2 - 8)$, or exact equivalent	A1
	Total:	**5**

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/33**

Paper 3 **May/June 2017**

MARK SCHEME

Maximum Mark: 75

Published

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    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous’ answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

<u>Penalties</u>

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks
1	Express the LHS in terms of either cos x and sin x or in terms of tan x	**B1**
	Use Pythagoras	**M1**
	Obtain the given answer	**A1**
	Total:	**3**
2	*EITHER:* State a correct unsimplified version of the x or x^2 term in the expansion of $\left(1+\frac{2}{3}x\right)^{-3}$ or $\left(3+2x\right)^{-3}$ [Symbolic binomial coefficients, e.g. $\begin{pmatrix}-3\\2\end{pmatrix}$, are not sufficient for **M1**.]	**(M1**
	State correct first term $\frac{1}{27}$	**B1**
	Obtain term $-\frac{2}{27}x$	**A1**
	Obtain term $\frac{8}{81}x^2$	**A1)**
	OR: Differentiate expression and evaluate $\mathrm{f}(0)$ and $\mathrm{f}'(0)$, where $\mathrm{f}'(x)=k(3+2x)^{-4}$	**(M1**
	State correct first term $\frac{1}{27}$	**B1**
	Obtain term $-\frac{2}{27}x$	**A1**
	Obtain term $\frac{8}{81}x^2$	**A1)**
	Total:	**4**
3	Rearrange as $3u^2+4u-4=0$, or $3\mathrm{e}^{2x}+4\mathrm{e}^{x}-4=0$, or equivalent	**B1**
	Solve a 3-term quadratic for e^x or for u	**M1**
	Obtain $\mathrm{e}^x=\frac{2}{3}$ or $u=\frac{2}{3}$	**A1**
	Obtain answer $x = –0.405$ and no other	**A1**
	Total:	**4**

Question	Answer	Marks
4	Integrate by parts and reach $a\theta\cos\frac{1}{2}\theta + b\int\cos\frac{1}{2}\theta\ \mathrm{d}\theta$	*M1
	Complete integration and obtain indefinite integral $-2\theta\cos\frac{1}{2}\theta + 4\sin\frac{1}{2}\theta$	A1
	Substitute limits correctly, having integrated twice	DM1
	Obtain final answer $(4-\pi)/\sqrt{2}$, or exact equivalent	A1
	Total:	4
5(i)	Use the chain rule	M1
	Obtain correct derivative in any form	A1
	Use correct trigonometry to express derivative in terms of $\tan x$	M1
	Obtain $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{4\tan x}{4+\tan^2 x}$, or equivalent	A1
	Total:	4
5(ii)	Equate derivative to –1 and solve a 3–term quadratic for tan x	M1
	Obtain answer x=1.11 and no other in the given interval	A1
	Total:	2
6(i)	Calculate the value of a relevant expression or expressions at x = 2.5 and at another relevant value, e.g. x = 3	M1
	Complete the argument correctly with correct calculated values	A1
	Total:	2
6(ii)	State a suitable equation, e.g. $x = \pi + \tan^{-1}\left(1/(1-x)\right)$ without suffices	B1
	Rearrange this as $\cot x = 1 - x$, or commence working *vice versa*	B1
	Total:	2
6(iii)	Use the iterative formula correctly at least once	M1
	Obtain final answer 2.576 only	A1
	Show sufficient iterations to 5 d.p. to justify 2.576 to 3 d.p., or show there is a sign change in the interval (2.5755, 2.5765)	A1
	Total:	3

Question	Answer	Marks
7(i)	Use correct quotient rule or product rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and solve for x	**M1**
	Obtain $x = 2$	**A1**
	Total:	**4**
7(ii)	State or imply ordinates 1.6487…, 1.3591…, 1.4938…	**B1**
	Use correct formula, or equivalent, with $h = 1$ and three ordinates	**M1**
	Obtain answer 2.93 only	**A1**
	Total:	**3**
7(iii)	Explain why the estimate would be less than E	**B1**
	Total:	**1**
8(i)	Justify the given differential equation	**B1**
	Total:	**1**
8(ii)	Separate variables correctly and attempt to integrate one side	**B1**
	Obtain term kt, or equivalent	**B1**
	Obtain term $-\ln(50-x)$, or equivalent	**B1**
	Evaluate a constant, or use limits $x = 0$, $t = 0$ in a solution containing terms $a\ln(50-x)$ and bt	**M1***
	Obtain solution $-\ln(50-x) = kt - \ln 50$, or equivalent	**A1**
	Use $x = 25$, $t = 10$ to determine k	**DM1**
	Obtain correct solution in any form, e.g. $\ln 50 - \ln(50-x) = \frac{1}{10}(\ln 2)t$	**A1**
	Obtain answer $x = 50(1-\exp(-0.0693t))$, or equivalent	**A1**
	Total:	**8**

Question	Answer	Marks
9(i)	State or imply the form $\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{3x+2}$	**B1**
	Use a relevant method to determine a constant	**M1**
	Obtain one of the values $A=3$, $B=-2$, $C=-6$	**A1**
	Obtain a second value	**A1**
	Obtain the third value [Mark the form $\dfrac{Ax+B}{x^2}+\dfrac{C}{3x+2}$ using same pattern of marks.]	**A1**
	Total:	**5**
9(ii)	Integrate and obtain terms $3\ln x = \dfrac{2}{x} - 2\ln(3x+2)$ [The FT is on A, B and C] **Note:** Candidates who integrate the partial fraction $\dfrac{3x-2}{x^2}$ by parts should obtain $3\ln x+\dfrac{2}{x}-3$ or equivalent	**B3 FT**
	Use limits correctly, having integrated all the partial fractions, in a solution containing terms $a\ln x+\frac{b}{x}+c\ln(3x+2)$	**M1**
	Obtain the given answer following full and exact working	**A1**
	Total:	**5**
10(i)	Carry out a correct method for finding a vector equation for AB	**M1**
	Obtain $\mathbf{r}=\mathbf{i}-2\mathbf{j}+2\mathbf{k}+\lambda(2\mathbf{i}+3\mathbf{j}-\mathbf{k})$, or equivalent	**A1**
	Equate two pairs of components of general points on AB and l and solve for λ or for μ	**M1**
	Obtain correct answer for λ or μ, e.g. $\lambda=\frac{5}{7}$ or $\mu=\frac{3}{7}$	**A1**
	Obtain $m=3$	**A1**
	Total:	**5**

Question	Answer	Marks
10(ii)	*EITHER:* Use scalar product to obtain an equation in a, b and c, e.g. $a-2b-4c=0$	**(B1**
	Form a second relevant equation, e.g. $2a+3b-c=0$ and solve for one ratio, e.g. $a : b$	**M1**
	Obtain final answer $a : b : c = 14 : -7 : 7$	**A1**
	Use coordinates of a relevant point and values of a, b and c and find d	**M1**
	Obtain answer $14x-7y+7z=42$, or equivalent	**A1)**
	OR 1: Attempt to calculate the vector product of relevant vectors, e.g. $(\mathbf{i}-2\mathbf{j}-4\mathbf{k})\times(2\mathbf{i}+3\mathbf{j}-\mathbf{k})$	**(M1**
	Obtain two correct components	**A1**
	Obtain correct answer, e.g. $14\mathbf{i}-7\mathbf{j}+7\mathbf{k}$	**A1**
	Substitute coordinates of a relevant point in $14x-7y+7z=d$, or equivalent, and find d	**M1**
	Obtain answer $14x-7y+7z=42$, or equivalent	**A1)**
	OR 2: Using a relevant point and relevant vectors, form a 2–parameter equation for the plane	**(M1**
	State a correct equation, e.g. $\mathbf{r}=\mathbf{i}-2\mathbf{j}+2\mathbf{k}+s(\mathbf{i}-2\mathbf{j}-4\mathbf{k})+t(2\mathbf{i}+3\mathbf{j}-\mathbf{k})$	**A1**
	State 3 correct equations in x, y, z, s and t	**A1**
	Eliminate s and t	**M1**
	Obtain answer $2x-y+z=6$, or equivalent	**A1)**
	OR 3: Using a relevant point and relevant vectors, form a determinant equation for the plane	**(M1**
	State a correct equation, e.g. $\begin{vmatrix} x-1 & y+2 & z-1 \\ 1 & -2 & -4 \\ 2 & 3 & -1 \end{vmatrix}=0$	**A1**
	Attempt to expand the determinant	**M1**
	Obtain or imply two correct cofactors	**A1**
	Obtain answer $14x-7y+7z=42$, or equivalent	**A1)**
	Total:	**5**

Question	Answer	Marks
11(a)	Solve for z or for w	**M1**
	Use $i^2 = -1$	**M1**
	Obtain $w = \dfrac{i}{2-i}$ or $z = \dfrac{2+i}{2-i}$	**A1**
	Multiply numerator and denominator by the conjugate of the denominator	**M1**
	Obtain $w = -\frac{1}{5} + \frac{2}{5}i$	**A1**
	Obtain $z = \frac{3}{5} + \frac{4}{5}i$	**A1**
	Total:	**6**
11(b)	*EITHER:* Find $\pm\left[2 + \left(2 - 2\sqrt{3}\right)i\right]$	**(B1**
	Multiply by 2i (or –2i)	**M1***
	Add result to v	**DM1**
	Obtain answer $4\sqrt{3} - 1 + 6i$	**A1)**
	OR: State $\dfrac{z-v}{v-u} = ki$, or equivalent	**(M1**
	State $k = 2$	**A1**
	Substitute and solve for z even if i omitted	**M1**
	Obtain answer $4\sqrt{3} - 1 + 6i$	**A1)**
	Total:	**4**

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/31**

Paper 3 **May/June 2018**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2018 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously 'correct' answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

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Question	Answer	Marks
1	Use law for the logarithm of a product, quotient or power	**M1**
	Obtain a correct equation free of logarithms, e.g. $4(x^4 - 4) = x^4$	**A1**
	Solve for x	**M1**
	Obtain answer x = 1.52 only	**A1**
		4

Question	Answer	Marks
2(i)	Use trig formulae and obtain an equation in sin x and cos x	**M1***
	Obtain a correct equation in any form	**A1**
	Substitute exact trig ratios and obtain an expression for tan x	**M1(dep*)**
	Obtain answer $\tan x = \dfrac{-\left(6+\sqrt{6}\right)}{\left(6-\sqrt{2}\right)}$ or equivalent	**A1**
		4
2(ii)	State answer, e.g. 118.5°	**B1**
	State second answer, e.g. 298.5°	**B1ft**
		2

Question	Answer	Marks
3	Use quotient or product rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and obtain a quadratic in $\tan\frac{1}{2}x$ or an equation of the form $a \sin x = b$	**M1 ***
	Solve for x	**M1(dep*)**
	Obtain answer 0.340	**A1**
	Obtain second answer 2.802 and no other in the given interval	**A1**
		6

Question	Answer	Marks
4	*EITHER:* Commence division by $x^2 - x + 1$ and reach a partial quotient of the form $x^2 + kx$	**M1**
	Obtain quotient $x^2 + 3x + 2$	**A1**
	Either Set remainder identically equal to zero and solve for *a* or for *b*, *or* multiply given divisor and found quotient and obtain *a* or *b*	**M1**
	Obtain $a = 1$	**A1**
	Obtain $b = 2$	**A1**
	OR: Assume an unknown factor $x^2 + Bx + C$ and obtain an equation in *B* and/or *C*	**M1**
	Obtain $B = 3$ and $A = 2$	**A1**
	Either Use equations to obtain *a* or *b* *or* multiply given divisor and found factor to obtain *a* or *b*	**M1**
	Obtain $a = 1$	**A1**
	Obtain $b = 2$	**A1**
		5

Question	Answer	Marks
5(i)	State or imply $dx = -2\cos\theta\sin\theta\,d\theta$, or equivalent	**B1**
	Substitute for *x* and d*x*, and use Pythagoras	**M1**
	Obtain integrand $\pm 2\cos^2\theta$	**A1**
	Justify change of limits and obtain given answer correctly	**A1**
		4
5(ii)	Obtain indefinite integral of the form $a\theta + b\sin 2\theta$	**M1***
	Obtain $\theta + \frac{1}{2}\sin 2\theta$	**A1**
	Use correct limits correctly	**M1(dep*)**
	Obtain answer $\frac{1}{6}\pi$ with no errors seen	**A1**
		4

Question	Answer	Marks
6(i)	Separate variables correctly and integrate at least one side	**B1**
	Obtain term ln x	**B1**
	Obtain term $-\frac{2}{3}kt\sqrt{t}$, or equivalent	**B1**
	Evaluate a constant, or use limits $x = 100$ and $t = 0$, in a solution containing terms a ln x and $b\,t\sqrt{t}$	**M1**
	Obtain correct solution in any form, e.g. $1\text{n}\, x = -\frac{2}{3}kt\sqrt{t} + \ln 100$	**A1**
		5
6(ii)	Substitute $x = 80$ and $t = 25$ to form equation in k	**M1**
	Substitute $x = 40$ and eliminate k	**M1**
	Obtain answer $t = 64.1$	**A1**
		3

Question	Answer	Marks
7(i)	Use quadratic formula, or completing the square, or the substitution $z = x + iy$ to find a root, using $i^2 = -1$	**M1**
	Obtain a root, e.g. $-\sqrt{6} - \sqrt{2}i$	**A1**
	Obtain the other root, e.g. $-\sqrt{6} - \sqrt{2}i$	**A1**
		3
7(ii)	Represent both roots in relatively correct positions	**B1ft**
		1
7(iii)	State or imply correct value of a relevant length or angle, e.g. OA, OB, AB, angle between OA or OB and the real axis	**B1ft**
	Carry out a complete method for finding angle OAB	**M1**
	Obtain $AOB = 60°$ correctly	**A1**
		3
7(iv)	Give a complete justification of the given statement	**B1**
		1

Question	Answer	Marks
8(i)	Integrate by parts and reach $lxe^{-\frac{1}{2}x} + m\int e^{-\frac{1}{2}x}\,dx$	M1*
	Obtain $-2xe^{-\frac{1}{2}x} + 2\int e^{-\frac{1}{2}x}\,dx$	A1
	Complete the integration and obtain $-2xe^{-\frac{1}{2}x} - 4e^{-\frac{1}{2}x}$, or equivalent	A1
	Having integrated twice, use limits and equate result to 2	M1(dep*)
	Obtain the given equation correctly	A1
		5
8(ii)	Calculate values of a relevant expression or pair of expressions at $a = 3$ and $a = 3.5$	M1
	Complete the argument correctly with correct calculated values	A1
		2
8(iii)	Use the iterative formula $a_{n+1} = 2\ln(a_n + 2)$ correctly at least once	M1
	Obtain final answer 3.36	A1
	Show sufficient iterations to 4 d.p. to justify 3.36 to 2 d.p., or show there is a sign change in the interval (3.355, 3.365)	A1
		3

Question	Answer	Marks
9(i)	State or imply the form $A+\dfrac{B}{x-1}+\dfrac{C}{3x+2}$	**B1**
	State or obtain $A = 4$	**B1**
	Use a correct method to obtain a constant	**M1**
	Obtain one of $B = 3$, $C = -1$	**A1**
	Obtain the other value	**A1**
		5
9(ii)	Use correct method to find the first two terms of the expansion of $(x-1)^{-1}$ or $(3x+2)^{-1}$, or equivalent	**M1**
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A1ft + A1ft**
	Add the value of A to the sum of the expansions	**M1**
	Obtain final answer $\dfrac{1}{2}-\dfrac{9}{4}x-\dfrac{33}{8}x^2$	**A1**
		5

Question	Answer	Marks
10(a)	*EITHER*: Find $\overrightarrow{PQ}$ (or $\overrightarrow{QP}$) for a general point Q on l, e.g. $(1+\mu)\,\mathbf{i}+(4+2\mu)\mathbf{j}+(4+3\mu)\mathbf{k}$	**B1**
	Calculate the scalar product of $\overrightarrow{PQ}$ and a direction vector for l and equate to zero	**M1**
	Solve and obtain correct solution e.g. $\mu=-\dfrac{3}{2}$	**A1**
	Carry out method to calculate PQ	**M1**
	Obtain answer 1.22	**A1**
	*OR*1: Find $\overrightarrow{PQ}$ (or $\overrightarrow{QP}$) for a general point Q on l	**B1**
	Use a correct method to express PQ^2 (or PQ) in terms of μ	**M1**
	Obtain a correct expression in any form	**A1**
	Carry out a complete method for finding its minimum	**M1**
	Obtain answer 1.22	**A1**
	*OR*2: Calling (4, 2, 5) A, state $\overrightarrow{PA}$ (or $\overrightarrow{AP}$) in component form, e.g. $\mathbf{i}+4\mathbf{j}+4\mathbf{k}$	**B1**
	Use a scalar product to find the projection of $\overrightarrow{PA}$ (or $\overrightarrow{AP}$) on l	**M1**
	Obtain correct answer $21/\sqrt{14}$, or equivalent	**A1**
	Use Pythagoras to find the perpendicular	**M1**
	Obtain answer 1.22	**A1**
	*OR*3: State $\overrightarrow{PA}$ (or $\overrightarrow{AP}$) in component form	**B1**
	Calculate vector product of $\overrightarrow{PA}$ and a direction vector for l	**M1**
	Obtain correct answer, e.g. $4\mathbf{i}+\mathbf{j}-2\mathbf{k}$	**A1**
	Divide modulus of the product by that of the direction vector	**M1**
	Obtain answer 1.22	**A1**
		5

Question	Answer	Marks
10(ii)	*EITHER*: Use scalar product to obtain a relevant equation in a, b and c, e.g. $a + 2b + 3c = 0$	**B1**
	Obtain a second relevant equation, e.g. using $\overrightarrow{PA}$ $a + 4b + 4c = 0$, and solve for one ratio	**M1**
	Obtain $a : b : c = 4 : 1 : -2$, or equivalent	**A1**
	Substitute a relevant point and values of a, b, c in general equation and find d	**M1**
	Obtain correct answer, $4x + y - 2z = 8$, or equivalent	**A1**
	*OR*1: Attempt to calculate vector product of relevant vectors, e.g. $(\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}) \times (\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$	**M1**
	Obtain two correct components	**A1**
	Obtain correct answer, e.g. $4\mathbf{i} + \mathbf{j} - 2\mathrm{k}$	**A1**
	Substitute a relevant point and find d	**M1**
	Obtain correct answer, $4x + y - 2z = 8$, or equivalent	**A1**
	*OR*2: Using a relevant point and relevant vectors form a 2-parameter equation for the plane	**M1**
	State a correct equation, e.g. $\mathbf{r} = 4\mathbf{i} + 2\mathbf{j} + 5\mathbf{k} + \lambda(\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}) + \mu(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$	**A1**
	State three correct equations in x, y, z, λ and μ	**A1**
	Eliminate λ and μ	**M1**
	Obtain correct answer $4x + y - 2z = 8$, or equivalent	**A1**
		5

Cambridge International A Level

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Cambridge International Advanced Level

MATHEMATICS	**9709/32**
Paper 3	**May/June 2018**
MARK SCHEME	
Maximum Mark: 75	

Published

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GENERIC MARKING PRINCIPLE 1:

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1	*EITHER*: State or imply non-modular equation $3^2\left(2^x-1\right)^2=\left(2^x\right)^2$, or pair of equations $3\left(2^x-1\right)=\pm 2^x$	**M1**	$8\left(2^x\right)^2-18\left(2^x\right)+9=0$
	Obtain $2^x=\dfrac{3}{2}$ and $2^x=\dfrac{3}{4}$ or equivalent	**A1**	
	OR: Obtain $2^x=\dfrac{3}{2}$ by solving an equation	**B1**	
	Obtain $2^x=\dfrac{3}{4}$ by solving an equation	**B1**	
	Use correct method for solving an equation of the form $2^x=a$, where $a>0$	**M1**	
	Obtain **final** answers $x = 0.585$ and $x = – 0.415$ only	**A1**	The question requires 3 s.f. Do not ISW if they go on to reject one value
		4	

Question	Answer	Marks	Guidance
2	Use correct tan $(A \pm B)$ formula and obtain an equation in tan θ	**M1**	$\frac{1}{\tan\theta}+\frac{1-\tan\theta\tan 45}{\tan\theta+\tan 45}=2$ Allow M1 with $\tan 45°$ $=\frac{1}{\tan\theta}+\frac{1-\tan\theta}{\tan\theta+1}$
	Obtain a correct equation in any form	**A1**	With values substituted
	Reduce to $3\tan^2\theta = 1$, or equivalent	**A1**	
	Obtain answer $x = 30°$	**A1**	One correct solution
	Obtain answer $x = 150°$	**A1**	Second correct solution and no others in range
	OR: use correct $\sin(A \pm B)$ and $\cos(A \pm B)$ to form equation in $\sin\theta$ and $\cos\theta$ M1A1		
	Reduce to $\tan^2\theta=\frac{1}{3}, \sin^2\theta=\frac{1}{4}, \cos^2\theta=\frac{3}{4}$ or $\cot^2\theta=3$ A1 etc.		
		5	

Question	Answer	Marks	Guidance
3(i)	Fully justify the given statement	**B1**	Some indication of use of gradient of curve = gradient of tangent (*PT*) and no errors seen /no incorrect statements
		1	
3(ii)	Separate variables and attempt integration of at least one side Obtain terms ln y and $\frac{1}{2}x$	**B1 B1**	Must be working from $\int \frac{1}{y}\,dy = \int k\,dx$ B marks are not available for fortuitously correct answers
	Use $x = 4$, $y = 3$ to evaluate a constant or as limits in a solution with terms $a \ln y$ and bx, where $ab \neq 0$	**M1**	
	Obtain correct solution in any form	**A1**	$\ln y = \frac{1}{2}x + \ln 3 - 2$
	Obtain answer $y = 3e^{\frac{1}{2}x-2}$, or equivalent	**A1**	Accept $y = e^{\frac{1}{2}x+\ln 3-2}$, $y = e^{\frac{x-1.80}{2}}$, $y = 3\sqrt{e^{x-4}}$ $\lvert y \rvert = \ldots$ scores A0
		5	

Question	Answer	Marks	Guidance
4(i)	Use correct double angle formulae and express LHS in terms of $\cos x$ and $\sin x$	**M1**	$\dfrac{2\sin x - 2\sin x\cos x}{1-\left(2\cos^2 x - 1\right)}$
	Obtain a correct expression	**A1**	
	Complete method to get correct denominator e.g. by factorising to remove a factor of $1-\cos x$	**M1**	
	Obtain the given RHS correctly *OR (working R to L)*:	**A1**	
	$\dfrac{\sin x}{1+\cos x}\times\dfrac{1-\cos x}{1-\cos x}=\dfrac{\sin x-\sin x\cos x}{1-\cos^2 x}$ M1A1 $=\dfrac{2\sin x-2\sin x\cos x}{2-2\cos^2 x}$		Given answer so check working carefully
	$=\dfrac{2\sin x-\sin 2x}{1-\cos 2x}$ M1A1		
		4	
4(ii)	State integral of the form $a\ln(1+\cos x)$	**M1***	If they use the substitution $u = 1+\cos x$ allow M1A1 for $-\ln u$
	Obtain integral $-\ln(1+\cos x)$	**A1**	
	Substitute correct limits in correct order	**M1(dep)***	
	Obtain answer $\ln\left(\frac{3}{2}\right)$, or equivalent	**A1**	
		4	

Question	Answer	Marks	Guidance
5(i)	State or imply $3y^2\dfrac{dy}{dx}$ as derivative of y^3	**B1**	
	State or imply $6xy+3x^2\dfrac{dy}{dx}$ as derivative of $3x^2y$ *OR* State or imply $2x(x+3y)+x^2\left(1+3\dfrac{dy}{dx}\right)$ as derivative of $x^2(x+3y)$	**B1**	$3x^2+6xy+3x^2\dfrac{dy}{dx}-3y^2\dfrac{dy}{dx}=0$
	Equate derivative of the LHS to zero and solve for $\dfrac{dy}{dx}$	**M1**	Given answer so check working carefully
	Obtain the given answer	**A1**	
		4	
5(ii)	Equate derivative to – 1 and solve for y	**M1***	
	Use their $y = -2x$ or equivalent to obtain an equation in x or y	**M1(dep*)**	
	Obtain answer $(1, -2)$	**A1**	
	Obtain answer $(\sqrt[3]{3}, 0)$	**B1**	Must be exact e.g. $e^{\frac{1}{3}\ln 3}$ but ISW if decimals after exact value seen
		4	

Question	Answer	Marks	Guidance
6(i)	Use correct method for finding the area of a segment and area of semicircle and form an equation in θ	**M1**	e.g. $\frac{\pi a^2}{4} = \frac{1}{2}a^2\theta - \frac{1}{2}a^2 \sin\theta$
	State a correct equation in any form	**A1**	Given answer so check working carefully
	Obtain the given answer correctly	**A1**	
		3	
6(ii)	Calculate values of a relevant expression or pair of expressions at $\theta = 2.2$ and $\theta = 2.4$	**M1**	e.g. $\mathrm{f}(\theta) = \frac{\pi}{2} + \sin\theta$ $\begin{cases} \mathrm{f}(2.2) = 2.37... > 2.2 \\ \mathrm{f}(2.4) = 2.24... < 2.4 \end{cases}$ or $\mathrm{f}(\theta) = \theta - \frac{\pi}{2} - \sin\theta$ $\begin{cases} \mathrm{f}(2.2) = -0.17... < 0 \\ \mathrm{f}(2.4) = +0.15... > 0 \end{cases}$
	Complete the argument correctly with correct calculated values	**A1**	
		2	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="4">6(iii)</td><td>Use $\theta_{n+1} = \frac{1}{2}\pi + \sin\theta_n$ correctly at least once</td><td>M1</td><td rowspan="3">e.g.
<table>
<tr><td>2.2</td><td>2.3</td><td>2.4</td></tr>
<tr><td>2.3793</td><td>2.3165</td><td>2.2463</td></tr>
<tr><td>2.2614</td><td>2.3054</td><td>2.3512</td></tr>
<tr><td>2.3417</td><td>2.3129</td><td>2.2814</td></tr>
<tr><td>2.2881</td><td>2.3079</td><td>2.3288</td></tr>
<tr><td>2.3244</td><td></td><td>2.2970</td></tr>
<tr><td>2.3000</td><td></td><td>2.3185</td></tr>
<tr><td>2.3165</td><td></td><td>2.3041</td></tr>
<tr><td>2.3054</td><td></td><td>2.3138</td></tr>
<tr><td>2.3129</td><td></td><td>2.3072</td></tr>
</table></td></tr>
<tr><td>Obtain final answer 2.31</td><td>A1</td></tr>
<tr><td>Show sufficient iterations to 4 d.p. to justify 2.31 to 2 d.p. or show there is a sign change in the interval (2.305, 2.315)</td><td>A1</td></tr>
<tr><td></td><td>3</td><td></td></tr>
</table>

Question	Answer		Marks	Guidance
7(i)	Substitute in uv, expand the product and use $\mathrm{i}^2 = -1$		M1	
	Obtain answer $uv = -11 - 5\sqrt{3}\mathrm{i}$		A1	
	EITHER:	Substitute in u/v and multiply numerator and denominator by the conjugate of v, or equivalent	M1	
		Obtain numerator $-7 + 7\sqrt{3}\mathrm{i}$ or denominator 7	A1	
		Obtain final answer $-1 + \sqrt{3}\mathrm{i}$	A1	
	OR:	Substitute in u/v , equate to $x + \mathrm{i}y$ and solve for x or for y	M1	$\begin{cases} -3\sqrt{3} = \sqrt{3}x - 2y \\ 1 = 2x + \sqrt{3}y \end{cases}$
	Obtain $x = -1$ or $y = \sqrt{3}$		A1	
	Obtain final answer $-1 + \sqrt{3}\,\mathrm{i}$		A1	
			5	

Question	Answer	Marks	Guidance
7(ii)	Show the points A and B representing u and v in relatively correct positions	B1	
	Carry out a complete method for finding angle AOB, e.g. calculate arg(u/v) If using $\theta = \tan^{-1}\left(-\sqrt{3}\right)$ must refer to $\arg\left(\frac{u}{v}\right)$	M1	*OR*: $\tan a = \frac{-1}{3\sqrt{3}}, \tan b = \frac{2}{\sqrt{3}} \Rightarrow \tan\left(a-b\right) = \frac{\frac{-1}{3\sqrt{3}} - \frac{2}{\sqrt{3}}}{1-\frac{2}{9}}$ $= -\sqrt{3}$ $\Rightarrow \theta = \frac{2\pi}{3}$ *OR*: $\cos\theta = \frac{\begin{pmatrix}-3\sqrt{3}\\1\end{pmatrix}\begin{pmatrix}\sqrt{3}\\2\end{pmatrix}}{\sqrt{7}\sqrt{28}} = \frac{-9+2}{14} = \frac{-1}{2}$ $\Rightarrow \theta = \frac{2\pi}{3}$ *OR*: $\cos\theta = \frac{28+7-49}{2\sqrt{28}\sqrt{7}} = -\frac{1}{2} \Rightarrow \theta = \frac{2\pi}{3}$
	Prove the given statement	A1	Given answer so check working carefully
		3	

Question	Answer	Marks	Guidance
8(i)	Use correct product or quotient rule	**M1**	$\frac{dy}{dx} = -\frac{1}{3}(x+1)e^{-\frac{1}{3}x} + e^{-\frac{1}{3}x}$ or $\frac{dy}{dx} = \frac{e^{\frac{1}{3}x} - (x+1)\frac{1}{3}e^{\frac{1}{3}x}}{e^{\frac{2}{3}x}}$
	Obtain complete correct derivative in any form	**A1**	
	Equate derivative to zero and solve for x	**M1**	
	Obtain answer $x = 2$ with no errors seen	**A1**	
		4	
8(ii)	Integrate by parts and reach $a(x+1)e^{-\frac{1}{3}x} + b\int e^{-\frac{1}{3}x}\,dx$	**M1***	
	Obtain $-3(x+1)e^{-\frac{1}{3}x} + 3\int e^{-\frac{1}{3}x}\,dx$, or equivalent	**A1**	$-3xe^{-\frac{1}{3}x} + \int 3e^{-\frac{1}{3}x}dx - 3e^{-\frac{1}{3}x}$
	Complete integration and obtain $-3(x+1)e^{-\frac{1}{3}x} - 9e^{-\frac{1}{3}x}$, or equivalent	**A1**	
	Use **correct** limits $x = -1$ and $x = 0$ **in the correct order**, having integrated twice	**M1(dep*)**	
	Obtain answer $9e^{\frac{1}{3}} - 12$, or equivalent	**A1**	
		5	

Question	Answer	Marks	Guidance
9(i)	Use a correct method to find a constant	**M1**	
	Obtain one of the values $A = -3$, $B = 1$, $C = 2$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
		4	
9(ii)	Use a correct method to find the first two terms of the expansion of $(3-x)^{-1}$, $\left(1-\frac{1}{3}x\right)^{-1}$, $\left(2+x^2\right)^{-1}$ or $\left(1+\frac{1}{2}x^2\right)^{-1}$	**M1**	Symbolic binomial coefficients are not sufficient for the M1.
	Obtain correct unsimplified expansions up to the term in x^3 of each partial fraction	**A1Ft + A1Ft**	The ft is on A, B and C. $-1\left(1+\frac{x}{3}+\frac{x^2}{9}+\frac{x^3}{27}...\right) \quad + \quad \frac{x+2}{2}\left(1-\frac{x^2}{2}...\right)$ $-1-\frac{x}{3}-\frac{x^2}{9}-\frac{x^3}{27}+1-\frac{x^2}{2}+\frac{x}{2}-\frac{x^3}{4}$
	Multiply out their expansion, up to the terms in x^3, by $Bx + C$, where $BC \neq 0$	**M1**	
	Obtain **final** answer $\frac{1}{6}x-\frac{11}{18}x^2-\frac{31}{108}x^3$, or equivalent	**A1**	
		5	

Question	Answer		Marks	Guidance
10(i)	Equate at least two pairs of components and solve for s or for t		**M1**	$\begin{cases} s = \frac{-4}{3} \\ t = \frac{-5}{3} \\ -5 \neq \frac{-1}{3} \end{cases}$ or $\begin{cases} s = -6 \\ t = -11 \\ 7 \neq -7 \end{cases}$ or $\begin{cases} s = \frac{-2}{5} \\ t = \frac{-13}{5} \\ \frac{6}{5} \neq \frac{-8}{5} \end{cases}$
	Obtain correct answer for s or t, e.g. $s = -6$, $t = -11$		**A1**	
	Verify that all three equations are not satisfied and the lines fail to intersect		**A1**	
	State that the lines are not parallel		**B1**	
			4	
10(ii)	*EITHER*:	Use scalar product to obtain a relevant equation in a, b and c, e.g. $2a + 3b - c = 0$	**B1**	
		Obtain a second equation, e.g. $a + 2b + c = 0$, and solve for one ratio, e.g. $a : b$	**M1**	
		Obtain $a : b : c$ and state correct answer, e.g. $5\mathbf{i} - 3\mathbf{j} + \mathbf{k}$, or equivalent	**A1**	
	OR:	Attempt to calculate vector product of relevant vectors, e.g. $(2\mathbf{i} + 3\mathbf{j} - \mathbf{k}) \times (\mathbf{i} + 2\mathbf{j} + \mathbf{k})$	**M1**	
		Obtain two correct components	**A1**	
		Obtain correct answer, e.g. $5\mathbf{i} - 3\mathbf{j} + \mathbf{k}$	**A1**	
			3	

Question	Answer		Marks	Guidance
10(iii)	*EITHER*:	State position vector or coordinates of the mid-point of a line segment joining points on l and m, e.g. $\frac{3}{2}\mathbf{i}+\mathbf{j}+\frac{5}{2}\mathbf{k}$	**B1**	*OR*: Use the result of **(ii)** to form equations of planes containing l and m B1
		Use the result of (**ii**) and the mid-point to find d	**M1**	Use average of distances to find equation of p. M1
		Obtain answer $5x - 3y + z = 7$, or equivalent	**A1**	Obtain answer $5x - 3y + z = 7$, or equivalent A1
	OR:	Using the result of part **(ii)**, form an equation in d by equating perpendicular distances to the plane of a point on l and a point on m	**M1**	
		State a correct equation, e.g. $\left\|\frac{14-d}{\sqrt{35}}\right\| = \left\|\frac{-d}{\sqrt{35}}\right\|$	**A1**	
		Solve for d and obtain answer $5x - 3y + z = 7$, or equivalent	**A1**	
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1	Obtain a correct unsimplified version of the x or x^2 term of the expansion of $(4-3x)^{-\frac{1}{2}}$ or $\left(1-\frac{3}{4}x\right)^{-\frac{1}{2}}$	**M1**
	State correct first term 2	**B1**
	Obtain the next two terms $\frac{3}{4}x+\frac{27}{64}x^2$	**A1 + A1**
	Total:	**4**

Question	Answer	Marks
2	State or imply $u^2=u+5$, or equivalent in 5^x	**B1**
	Solve for u, or 5^x	**M1**
	Obtain root $\frac{1}{2}(1+\sqrt{21})$, or decimal in [2.79, 2.80]	**A1**
	Use correct method for finding x from a positive root	**M1**
	Obtain answer x = 0.638 and no other answer	**A1**
	Total:	**5**

Question	Answer	Marks
3	Integrate by parts and reach $ax\sin 3x+b\int\sin 3x\,\mathrm{d}x$	**M1***
	Obtain $\frac{1}{3}x\sin 3x-\frac{1}{3}\int\sin 3x\,\mathrm{d}x$, or equivalent	**A1**
	Complete the integration and obtain $\frac{1}{3}x\sin 3x+\frac{1}{9}\cos 3x$, or equivalent	**A1**
	Substitute limits correctly having integrated twice and obtained ax sin $3x$ + b cos $3x$	**M1(dep*)**
	Obtain answer $\frac{1}{18}(\pi-2)$ OE	**A1**
	Total:	**5**

Question	Answer	Marks
4(i)	Use the quotient or product rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and obtain the given equation	**A1**
	Total:	**3**
4(ii)	Sketch a relevant graph, e.g. $y = \ln x$	**B1**
	Sketch a second relevant graph, e.g. $y = 1 + \dfrac{3}{x}$, and justify the given statement	**B1**
	Total:	**2**
4(iii)	Use iterative formula $x_{n+1} = \dfrac{3 + x}{\ln x_n}$ correctly at least once	**M1**
	Obtain final answer 4.97	**A1**
	Show sufficient iterations to 4 d.p.to justify 4.97 to 2 d.p. or show there is a sign change in the interval (4.965, 4.975)	**A1**
	Total:	**3**

Question	Answer	Marks
5(i)	Attempt cubic expansion and equate to 1	**M1**
	Obtain a correct equation	**A1**
	Use Pythagoras and double angle formula in the expansion	**M1**
	Obtain the given result correctly	**A1**
	Total:	**4**
5(ii)	Use the identity and carry out a method for finding a root	**M1**
	Obtain answer 20.9°	**A1**
	Obtain a second answer, e.g. 69.1°	**A1FT**
	Obtain the remaining answers, e.g. 110.9° and 159.1°, and no others in the given interval	**A1FT**
	Total:	**4**

Question	Answer	Marks
6(i)	Carry out relevant method to find A and B such that $\frac{1}{4-y^2} \equiv \frac{A}{2+y} + \frac{B}{2-y}$	**M1**
	Obtain $A = B = \frac{1}{4}$	**A1**
	Total:	**2**
6(ii)	Separate variables correctly and integrate at least one side to obtain one of the terms $a \ln x$, $b \ln (2 + y)$ or $c \ln (2 - y)$	**M1**
	Obtain term $\ln x$	**B1**
	Integrate and obtain terms $\frac{1}{4}\ln(2+y) - \frac{1}{4}\ln(2-y)$	**A1FT**
	Use $x = 1$ and $y = 1$ to evaluate a constant, or as limits, in a solution containing at least two terms of the form $a \ln x$, $b \ln (2 + y)$ and $c \ln (2 - y)$	**M1**
	Obtain a correct solution in any form, e.g. $\ln x = \frac{1}{4}\ln(2+y) - \frac{1}{4}\ln(2-y) - \frac{1}{4}\ln 3$	**A1**
	Rearrange as $\frac{2(3x^4-1)}{(3x^4+1)}$, or equivalent	**A1**
	Total:	**6**

Question	Answer	Marks
7(i)	State answer $R = \sqrt{5}$	**B1**
	Use trig formulae to find $\tan \alpha$	**M1**
	Obtain $\tan \alpha = 2$	**A1**
	Total:	**3**
7(ii)	State that the integrand is $3\sec^2(\theta - \alpha)$	**B1FT**
	State correct indefinite integral $3\tan(\theta - \alpha)$	**B1FT**
	Substitute limits correctly	**M1**
	Use $\tan(A \pm B)$ formula	**M1**
	Obtain the given exact answer correctly	**A1**
	Total:	**5**

Question	Answer	Marks
8(i)	State or imply $3y^2 \dfrac{dy}{dx}$ as derivative of y^3	**B1**
	State or imply $3y^2 + 6xy \dfrac{dy}{dx}$ as derivative of $3xy^2$	**B1**
	Equate derivative of LHS to zero and solve for $\dfrac{dy}{dx}$	**M1**
	Obtain the given answer	**A1**
	Total:	**4**
8(ii)	Equate denominator to zero and solve for y	**M1***
	Obtain $y = 0$ and $x = a$	**A1**
	Obtain $y = \alpha x$ and substitute in curve equation to find x or to find y	**M1(dep*)**
	Obtain $x = -a$	**A1**
	Obtain $y = 2a$	**A1**
	Total:	**5**

Question	Answer	Marks
9(a)	Substitute and obtain a correct equation in x and y	**B1**
	Use $i^2 = -1$ and equate real and imaginary parts	**M1**
	Obtain two correct equations in x and y, e.g. $3x - y = 1$ and $3y - x = 5$	**A1**
	Solve and obtain answer $z = 1 +$ **2(i)**	**A1**
	Total:	**4**
9(b)	Show a circle with radius 3	**B1**
	Show the line $y = 2$ extending in both quadrants	**B1**
	Shade the correct region	**B1**
	Carry out a complete method for finding the greatest value of arg z	**M1**
	Obtain answer 2.41	**A1**
	Total:	**5**

Question	Answer	Marks
10(i)	Carry out a correct method for finding a vector equation for AB	**M1**
	Obtain $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k} + \lambda(2\mathbf{i} - 2\mathbf{k})$, or equivalent	**A1**
	Equate pair(s) of components AB and l and solve for λ or μ	**M1(dep*)**
	Obtain correct answer for λ or μ	**A1**
	Verify that all three component equations are not satisfied	**A1**
	Total:	**5**
10(ii)	State or imply a direction vector for AP has components $(2 + t, 5 + 2t, -3 - 2t)$	**B1**
	State or imply that cos 120° equals the scalar product of $\overrightarrow{AP}$ and $\overrightarrow{AB}$ divided by the product of their moduli	**M1**
	Carry out the correct processes for finding the scalar product and the product of the moduli in terms of t, and obtain an equation in terms of t	**M1**
	Obtain the given equation correctly	**A1**
	Solve the quadratic and use a root to find a position vector for P	**M1**
	Obtain position vector $2\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$ from $t = -2$, having rejected the root $t = -\frac{2}{3}$	**A1**
	Total:	**6**

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/31**

Paper 3 **May/June 2019**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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Cambridge International is publishing the mark schemes for the May/June 2019 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	State or imply ordinates 3, 2, 0, 4	**B1**	These and no more Accept in unsimplified form $\left\|2^0 - 4\right\|$ etc.
	Use correct formula, or equivalent, with h = 1 and four ordinates	**M1**	
	Obtain answer 5.5	**A1**	
		3	

Question	Answer	Marks	Guidance
2	Use law for the logarithm of a product, quotient or power	**M1**	Condone $\ln\dfrac{x}{x-1}$ for M1
	Obtain a correct equation free of logarithms	**A1**	e.g. $(2x-3)(x-1)=x^2$ or $x^2-5x+3=0$
	Solve a 3-term quadratic obtaining at least one root	**M1**	Must see working if using an incorrect quadratic $\left(\dfrac{5\pm\sqrt{13}}{2}\right)$
	Obtain answer x = 4.30 only	**A1**	Q asks for 2 dp. Do not ISW. Overspecified answers score A0 Overspecified and no working can score M1A0
		4	

Question	Answer	Marks	Guidance
3	State or imply $3y^2 + 6xy\dfrac{dy}{dx}$ as derivative of $3xy^2$	B1	
	State or imply $3y^2\dfrac{dy}{dx}$ as derivative of y^3	B1	
	Equate derivative of LHS to zero, substitute (1, 3) and find the gradient	M1	$\left(\dfrac{dy}{dx} = \dfrac{x^2 + y^2}{y^2 - 2xy}\right)$ For incorrect derivative need to see the substitution
	Obtain final answer $\dfrac{10}{3}$ or equivalent	A1	3.33 or better. Allow $\dfrac{30}{9}$ ISW after correct answer seen
		4	

Question	Answer	Marks	Guidance
4	Use correct trig formula and obtain an equation in $\tan\theta$	**M1**	Allow with $45°$ e.g. $\dfrac{1}{\tan\theta}-\dfrac{1}{\dfrac{\tan\theta+\tan 45°}{1-\tan\theta\tan 45°}}=3$
	Obtain a correct horizontal equation in any form	**A1**	e.g. $1+\tan\theta-\tan\theta(1-\tan\theta)=3\tan\theta(1+\tan\theta)$
	Reduce to $2\tan^2\theta+3\tan\theta-1=0$	**A1**	or 3-term equivalent
	Solve 3-term quadratic and find a value of θ	**M1**	Must see working if using an incorrect quadratic
	Obtain answer $15.7°$	**A1**	One correct solution (degrees to at least 3 sf)
	Obtain answer $119.(3)°$	**A1**	Second correct solution and no others in range (degrees to at least 3 sf) Mark 0.274, 2.082 as MR: A0A1
		6	

Question	Answer	Marks	Guidance
5(i)	Use chain rule	**M1**	$k\cos\theta\sin^{-3}\theta\left(=-k\operatorname{cosec}^2\theta\cot\theta\right)$ Allow M1 for $-2\cos\theta\sin^{-1}\theta$
	Obtain correct answer in any form	**A1**	e.g. $-2\operatorname{cosec}^2\theta\cot\theta$, $\dfrac{-2\cos\theta}{\sin^3\theta}$ Accept $\dfrac{-2\sin\theta\cos\theta}{\sin^4\theta}$
		2	
5(ii)	Separate variables correctly and integrate at least one side	**B1**	$\int x\,dx=\int -\operatorname{cosec}^2\theta\cot\theta\,d\theta$
	Obtain term $\frac{1}{2}x^2$	**B1**	
	Obtain term of the form $\dfrac{k}{\sin^2\theta}$	**M1***	or equivalent
	Obtain term $\dfrac{1}{2\sin^2\theta}$	**A1**	or equivalent
	Use $x=4$, $\theta=\frac{1}{6}\pi$ to evaluate a constant, or as limits, in a solution with terms ax^2 and $\dfrac{b}{\sin^2\theta}$, where $ab\neq 0$	**DM1**	Dependent on the preceding M1
	Obtain solution $x=\sqrt{\left(\operatorname{cosec}^2\theta+12\right)}$	**A1**	or equivalent
		6	

Question	Answer	Marks	Guidance
6(i)	State correct expansion of $\sin(2x+x)$	B1	
	Use trig formulae and Pythagoras to express $\sin 3x$ in terms of $\sin x$	M1	
	Obtain a correct expression in any form	A1	e.g. $2\sin x(1-\sin^2 x)+\sin x(1-2\sin^2 x)$
	Obtain $\sin 3x \equiv 3\sin x - 4\sin^3 x$ correctly **AG**	A1	Accept $=$ for $\equiv$
		4	
6(ii)	Use identity, integrate and obtain $-\frac{3}{4}\cos x+\frac{1}{12}\cos 3x$	B1 B1	One mark for each term correct
	Use limits correctly in an integral of the form $a\cos x + b\cos 3x$, where $ab \neq 0$	M1	$\left(-\frac{3}{8}-\frac{1}{12}+\frac{3}{4}-\frac{1}{12}=-\frac{11}{24}+\frac{2}{3}\right)$
	Obtain answer $\frac{5}{24}$	A1	Must be exact. Accept simplified equivalent e.g. $\frac{15}{72}$ Answer only with no working is 0/4
		4	

Question	Answer	Marks	Guidance
7(i)	State at least one correct derivative	**B1**	$-2\sin\frac{1}{2}x$, $\frac{1}{(4-x)^2}$
	Equate product of derivatives to – 1	**M1**	or equivalent
	Obtain a correct equation, e.g. $2\sin\frac{1}{2}x=(4-x)^2$	**A1**	
	Rearrange correctly to obtain $a=4-\sqrt{2\sin\frac{a}{2}}$ **AG**	**A1**	
		4	
7(ii)	Calculate values of a relevant expression or pair of expressions at a = 2 and a = 3	**M1**	e.g. $a=2$ $2<2.7027..$ $a=3$ $3>2.587..$ $\begin{pmatrix}0.703\\-0.412\end{pmatrix}$ $\begin{pmatrix}2.317\\-0.995\end{pmatrix}$ Values correct to at least 2 dp
	Complete the argument correctly with correct calculated values	**A1**	
		2	
7(iii)	Use the iterative formula $a_{n+1}=4-\sqrt{(2\sin\frac{1}{2}a_n)}$ correctly at least once	**M1**	
	Obtain final answer 2.611	**A1**	
	Show sufficient iterations to 5 d.p. to justify 2.611 to 3 d.p., or show there is a sign change in the interval (2.6105, 2.6115)	**A1**	2, 2.70272, 2.60285, 2.61152, 2.61070, 2.61077 2.5, 2.62233, 2.60969, 2.61087, 2.61076 3, 2.58756, 2.61301, 2.61056, 2.61079 Condone truncation. Accept more than 5 dp
		3	

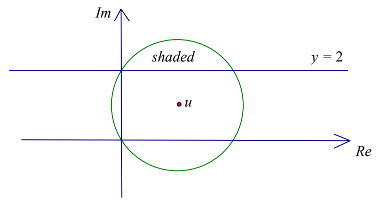
Question	Answer	Marks	Guidance
8(i)	State or imply the form $\dfrac{A}{2+x}+\dfrac{B}{3-x}+\dfrac{C}{(3-x)^2}$	**B1**	
	Use a correct method to obtain a constant	**M1**	
	Obtain one of $A=2$, $B=2$, $C=-7$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	[Mark the form $\dfrac{A}{2+x}+\dfrac{Dx+E}{(3-x)^2}$, where $A=2$, $D=-2$ and $E=-1$, B1M1A1A1A1.]
		5	
8(ii)	Use a correct method to find the first two terms of the expansion of $(2+x)^{-1}$, $(3-x)^{-1}$ or $(3-x)^{-2}$, or equivalent, e.g. $\left(1+\dfrac{1}{2}x\right)^{-1}$	**M1**	
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A1** **A1** **A1**	**FT** on A, B and C $1-\dfrac{x}{2}+\dfrac{x^2}{4}$ $\dfrac{2}{3}\left(1+\dfrac{x}{3}+\dfrac{x^2}{9}\right)$ $-\dfrac{7}{9}\left(1+\dfrac{2x}{3}+\dfrac{3x^2}{9}\right)$
	Obtain final answer $\dfrac{8}{9}-\dfrac{43}{54}x+\dfrac{7}{108}x^2$	**A1**	
			For the A, D, E form of fractions give M1A1**ft**A1**ft** for the expanded partial fractions, then, if $D\neq 0$, M1 for multiplying out fully, and A1 for the final answer.
		5	

Question	Answer	Marks	Guidance
9(i)	Obtain a vector parallel to the plane, e.g. $\overrightarrow{CB} = 2\mathbf{i} + \mathbf{j}$	**B1**	
	Use scalar product to obtain an equation in a, b, c,	**M1**	e.g. $2a + b = 0$, $a + 5c = 0$, $a + b - 5c = 0$
	Obtain two correct equations in a, b, c	**A1**	
	Solve to obtain $a : b : c$,	**M1**	or equivalent
	Obtain $a : b : c = 5 : -10 : -1$,	**A1**	or equivalent
	Obtain equation $5x - 10y - z = -25$,	**A1**	or equivalent
	Alternative method 1		
	Obtain a vector parallel to the plane, e.g. $\overrightarrow{CD} = \mathbf{i} + 5\mathbf{k}$	**B1**	$\overrightarrow{BD} = -\mathbf{i} - \mathbf{j} + 5\mathbf{k}$
	Obtain a second such vector and calculate their vector product, e.g. $(2\mathbf{i} + \mathbf{j}) \times (\mathbf{i} + 5\mathbf{k})$	**M1**	
	Obtain two correct components	**A1**	
	Obtain correct answer, e.g. $5\mathbf{i} - 10\mathbf{j} - \mathbf{k}$	**A1**	
	Substitute to find d	**M1**	
	Obtain equation $5x - 10y - z = -25$,	**A1**	or equivalent

Question	Answer	Marks	Guidance
9(i)	**Alternative method 2**		
	Obtain a vector parallel to the plane, e.g. $\overrightarrow{DB} = \mathbf{i} + \mathbf{j} - 5\mathbf{k}$	**B1**	
	Obtain a second such vector and form correctly a 2-parameter equation for the plane	**M1**	
	State a correct equation, e.g. $\mathbf{r} = 3\mathbf{i} + 4\mathbf{j} + \lambda(\mathbf{i} + 5\mathbf{k}) + \mu(\mathbf{i} + \mathbf{j} - 5\mathbf{k})$	**A1**	
	State three equations in x, y, z, λ and μ	**A1**	
	Eliminate λ and μ	**M1**	
	Obtain equation $5x - 10y - z = -25$	**A1**	or equivalent
	Alternative method 3		
	Substitute for B and C and obtain $3a + 4b = d$ and $a + 3b = d$	**B1**	
	Substitute for D to obtain a third equation and eliminate one unknown (a, b, or d) entirely	**M1**	
	Obtain two correct equations in two unknowns, e.g. a, b, c	**A1**	
	Solve to obtain their ratio, e.g. $a : b : c$	**M1**	
	Obtain $a : b : c = 5 : -10 : -1$, $a : c : d = 5 : -1 : -25$, or $b : c : d = 10 : 1 : 25$	**A1**	or equivalent
	Obtain equation $5x - 10y - z = -25$	**A1**	or equivalent

Question	Answer	Marks	Guidance
9(i)	**Alternative method 4**		
	Substitute for B and C and obtain $3a + 4b = d$ and $a + 3b = d$	**B1**	
	Solve to obtain $a : b : d$	**M2**	or equivalent
	Obtain $a : b : d = 1 : -2 : -5$	**A1**	or equivalent
	Substitute for C to obtain c	**M1**	
	Obtain equation $5x - 10y - z = -25$	**A1**	or equivalent
		6	
9(ii)	State or imply a normal vector for the plane $OABC$ is **k**	**B1**	
	Carry out correct process for evaluating a scalar product of two relevant vectors, e.g. (5**i** – 10**j** – **k**).(**k**)	**M1**	i.e. correct process using **k** and their normal
	Using the correct process for calculating the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	**M1**	Allow M1M1 for clear use of an incorrect vector that has been stated to be the normal to $OABC$
	Obtain answer 84.9° or 1.48 radians	**A1**	
		4	

Question	Answer	Marks	Guidance
10(i)	State or imply $r = 2$	**B1**	Accept $\sqrt{4}$
	State or imply $\theta = \frac{1}{6}\pi$	**B1**	
	Use a correct method for finding the modulus or the argument of u^4	**M1**	Allow correct answers from correct u with minimal working shown
	Obtain modulus 16	**A1**	
	Obtain argument $\frac{2}{3}\pi$	**A1**	Accept $16e^{i\frac{2\pi}{3}}$
		5	
10(ii)	Substitute u and carry out a correct method for finding u^3	**M1**	$\left(u^3 = 8i\right)$ Follow *their* u^3 if found in part **(i)**
	Verify u is a root of the given equation	**A1**	
	State that the other root is $\sqrt{3} - \mathrm{i}$	**B1**	
	Alternative method		
	State that the other root is $\sqrt{3} - \mathrm{i}$	**B1**	
	Form quadratic factor and divide cubic by quadratic	**M1**	$\left(z - \sqrt{3} - i\right)\left(z - \sqrt{3} + i\right)\left(= z^2 - 2\sqrt{3}z + 4\right)$
	Verify that remainder is zero and hence that u is a root of the given equation	**A1**	
		3	

Question	Answer	Marks	Guidance
10(iii)	Show the point representing u in a relatively correct position	**B1**	
	Show a circle with centre u and radius 2	**B1**	**FT** on the point representing u. Condone near miss of origin
	Show the line $y = 2$	**B1**	
	Shade the correct region	**B1**	
	Show that the line and circle intersect on $x = 0$	**B1**	Condone near miss
		5	

Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS	**9709/32**
Paper 3	**May/June 2019**
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Question	Answer	Marks	Guidance
1	State unsimplified term in x^2, or its coefficient in the expansion of $(1+3x)^{\frac{1}{3}}$ $\left(\dfrac{\frac{1}{3}\times\frac{-2}{3}}{2}(3x)^2\right)$	**B1**	Symbolic binomial coefficients are not sufficient for the B marks
	State unsimplified term in x^3, or its coefficient in the expansion of $(1+3x)^{\frac{1}{3}}$ $\left(\dfrac{\frac{1}{3}\times\frac{-2}{3}\times\frac{-5}{3}}{6}(3x)^3\right)$	**B1**	
	Multiply by $(3-x)$ to give 2 terms in x^3, or their coefficients	**M1**	$\left(3\times\dfrac{10}{6}+1\right)$ Ignore errors in terms other than x^3 $3\times x^3\text{coeff} - x^2\text{coeff}$ and no other term in x^3
	Obtain answer 6	**A1**	Not $6x^3$
		4	

Question	Answer	Marks	Guidance
2	State or imply $u^2 - u - 12(=0)$, or equivalent in 3^x	**B1**	Need to be convinced they know $3^{2x} = \left(3^x\right)^2$
	Solve for *u*, or for 3^x, and obtain root 4	**B1**	
	Use a correct method to solve an equation of the form $3^x = a$ where a >0	**M1**	Need to see evidence of method. Do not penalise an attempt to use the negative root as well. e.g. $x\ln 3 = \ln a$, $x = \log_3 a$ If seen, accept solution of straight forward cases such as 3^x = 3 , x = 1 without working
	Obtain final answer x = 1.26 **only**	**A1**	The Q asks for 2 dp
		4	

Question	Answer	Marks	Guidance
3	Use correct trig formulae to obtain an equation in tan θ or equivalent (e.g all in sin θ or all in cos θ)	*M1	$\frac{1-\tan^2\theta}{2\tan\theta}=2\tan\theta$. Allow $\frac{\cot^2\theta-1}{2\cot\theta}=\frac{2}{\cot\theta}$
	Obtain a correct simplified equation	A1	$5\tan^2\theta=1$ or $\sin^2\theta=\frac{1}{6}$ or $\cos^2\theta=\frac{5}{6}$
	Solve for θ	DM1	Dependent on the first M1
	Obtain answer 24.1° (or 155.9°)	A1	One correct in range to at least 3 sf
	Obtain second answer	A1	**FT** $180°-\textit{their}\ 24.1°$ and no others in range. Correct to at least 3 sf. Accept $156°$ but not 156.0 Ignore values outside range If working in $\tan\theta$ or $\cos\theta$ need to be considering both square roots to score the second A1 Mark 0.421, 2.72 as a MR, so A0A1
		5	

Question	Answer	Marks	Guidance
4	Use correct quotient rule	M1	Allow use of correct product rule on $x\times(1+\ln x)^{-1}$
	Obtain correct derivative in any form	A1	$\frac{dy}{dx}=\frac{(1+\ln x)-x\times\frac{1}{x}}{(1+\ln x)^2}=\left(\frac{1}{1+\ln x}-\frac{1}{(1+\ln x)^2}\right)$
	Equate derivative to $\frac{1}{4}$ and obtain a quadratic in ln x or (1+ ln x)	M1	Horizontal form. Accept $\ln x=\frac{1}{4}(1+\ln x)^2$
	Reduce to $(\ln x)^2-2\ln x+1=0$	A1	or 3-term equivalent. Condone $\ln x^2$ if later used correctly
	Solve a 3-term quadratic in ln x for x	M1	Must see working if solving incorrect quadratic
	Obtain answer x = e	A1	Accept e^1
	Obtain answer $y=\frac{1}{2}$ e	A1	Exact only with no decimals seen before the exact value. Accept $\frac{e^1}{2}$ but not $\frac{e}{1+\ln e}$
		7	

Question	Answer	Marks	Guidance
5(i)	State answer $-1-\sqrt{3}i$	B1	If $-\frac{1}{2}$ given as well at this point, still just B1
		1	

Question	Answer	Marks	Guidance
5(ii)	Substitute $x = -1+\sqrt{3}\mathrm{i}$ in the equation and attempt expansions of x^2 and x^3	M1	Need to see sufficient working to be convinced that a calculator has not been used.
	Use $\mathrm{i}^2 = -1$ correctly at least once	M1	Allow for relevant use at any point in the solution
	Obtain $k = 2$	A1	
	Carry out a complete method for finding a quadratic factor with zeros $-1+\sqrt{3}\mathrm{i}$ and $-1-\sqrt{3}\mathrm{i}$	M1	Could use factor theorem from this point. Need to see working. M1 for correct testing of correct root or allow M1 for three unsuccessful valid attempts.
	Obtain x^2+2x+4	A1	Using factor theorem, obtain $\mathrm{f}\left(-\frac{1}{2}\right)=0$
	Obtain root $x = -\frac{1}{2}$, or equivalent, *via* division or inspection	A1	Final answer

Question	Answer	Marks	Guidance
5(ii)	**Alternative method 1**		
	Carry out a complete method for finding a quadratic factor with zeros $-1+\sqrt{3}\mathrm{i}$ and $-1-\sqrt{3}\mathrm{i}$ (multiplying two linear factors or using sum and product of roots)	**M1**	Need to see sufficient working to be convinced that a calculator has not been used.
	Use $\mathrm{i}^2 = -1$ correctly at least once	**M1**	Allow for relevant use at any point in the solution
	Obtain $x^2 + 2x + 4$	**A1**	Allow M1A0 for $x^2 + 2x + 3$
	Obtain linear factor $kx + 1$ and compare coefficients of x or x^2 and solve for k	**M1**	Can find the factor by inspection or by long division Must get to zero remainder
	Obtain $k = 2$	**A1**	
	Obtain root $x = -\frac{1}{2}$	**A1**	Final answer
			Note: Verification that $x = -\frac{1}{2}$ is a root is worth no marks without a clear demonstration of how the root was obtained

Question	Answer	Marks	Guidance
5(ii)	**Alternative method 2**		
	Use equation for sum of roots of cubic and use equation for product of roots of cubic	**M1**	
	Use $i^2 = -1$ correctly at least once	**M1**	Allow for relevant use at any point in the solution
	Obtain $-\frac{5}{k} = -2 + \gamma, \quad -\frac{4}{k} = 4\gamma$	**A1**	
	Solve simultaneous equations for k and γ	**M1**	
	Obtain $k = 2$	**A1**	
	Obtain root $\gamma = -\frac{1}{2}$	**A1**	Final answer
		6	

Question	Answer	Marks	Guidance
6(i)	Correct use of trigonometry to obtain $AB = 2r\cos x$	**B1**	**AG**
		1	

Question	Answer	Marks	Guidance
6(ii)	Use correct method for finding the area of the sector and the semicircle and form an equation in x	M1	$\frac{1}{2}\times\frac{1}{2}\pi r^2=\frac{1}{2}(2r\cos x)^2\,2x$
	Obtain $x=\cos^{-1}\sqrt{\frac{\pi}{16x}}$ correctly **AG**	A1	Via correct simplification e.g. from $\cos^2 x=\frac{\pi}{16x}$
		2	
6(iii)	Calculate values of a relevant expression or pair of expressions at x = 1 and x = 1.5 Must be working in radians	M1	e.g. $\begin{matrix} x=1 & 1\to 1.11 \\ x=1.5 & 1.5\to 1.20\end{matrix}$ Accept $\begin{matrix}\mathrm{f}(1)=1.11\\ \mathrm{f}(1.5)=1.20\end{matrix}$ $\mathrm{f}(x)=x-\cos^{-1}\sqrt{\frac{\pi}{16x}}$: $\mathrm{f}(1)=-0.111.,\ \mathrm{f}(1.5)=0.3..$ $\mathrm{f}(x)=\cos x-\sqrt{\frac{\pi}{16x}}$: $\mathrm{f}(1)=0.097.,\mathrm{f}(1.5)=-0.291.$ For $16x\cos^2 x-\pi$ $\mathrm{f}(1)=1.529..,\mathrm{f}(1.5)=-3.02..$ Must find values. M1 if at least one value correct
	Correct values and complete the argument correctly	A1	
		2	

Question	Answer	Marks	Guidance
6(iv)	Use $x_{n+1} = \cos^{-1}\sqrt{\left(\frac{\pi}{16x_n}\right)}$ correctly at least twice Must be working in radians	M1	$1, 1.11173, 1.13707, 1.14225, 1.14329, 1.14349,$ $1.14354, 1.14354$ $1.25, 1.16328, 1.14742, 1.14432, 1.14370$ $1.5, 1.20060, 1.15447, 1.14570, 1.14397, 1.14363$
	Obtain final answer 1.144	A1	
	Show sufficient iterations to at least 5 d.p. to justify 1.144 to 3 d.p. or show there is a sign change in the interval (1.1435, 1.1445)	A1	
		3	

Question	Answer	Marks	Guidance
7(i)	Separate variables correctly and attempt integration of at least one side	**B1**	$\int e^{-y}\, dy = \int xe^{x}\, dx$
	Obtain term $-e^{-y}$	**B1**	B0B1 is possible
	Commence integration by parts and reach $xe^{x} \pm \int e^{x}\, dx$	**M1**	B0B0M1A1 is possible
	Obtain $xe^{x} - e^{x}$	**A1**	or equivalent
			B1B1M1A1 is available if there is no constant of integration
	Use $x = 0$, $y = 0$ to evaluate a constant, or as limits in a definite integral, in a solution with terms ae^{-y}, bxe^{x} and ce^{x}, where $abc \neq 0$	**M1**	Must see this step
	Obtain correct solution in any form	**A1**	e.g. $e^{-y} = e^{x} - xe^{x}$
	Rearrange as $y = -\ln(1-x) - x$	**A1**	or equivalent e.g. $y = \ln \dfrac{1}{e^{x}(1-x)}$ ISW
		7	
7(ii)	Justify the given statement	**B1**	e.g. require $1 - x > 0$ for the ln term to exist, hence $x < 1$ Must be considering the range of values of *x*, and must be relevant to *their y* involving $\ln(1-x)$
		1	

Question	Answer	Marks	Guidance
8(i)	State or imply the form $\dfrac{A}{2x+1}+\dfrac{B}{2x+3}+\dfrac{C}{(2x+3)^2}$	**B1**	
	Use a correct method to find a constant	**M1**	
	Obtain the values $A = 1$, $B = -1$, $C = 3$	**A1 A1 A1**	
	[Mark the form $\dfrac{A}{2x+1}+\dfrac{Dx+E}{(2x+3)^2}$, where $A = 1$, $D = -2$ and $E = 0$, B1M1A1A1A1 as above.]		Full marks for the three correct constants – do not actually need to see the partial fractions
		5	
8(ii)	Integrate and obtain terms $\dfrac{1}{2}\ln(2x+1)-\dfrac{1}{2}\ln(2x+3)-\dfrac{3}{2(2x+3)}$ [Correct integration of the A, D, E form of fractions gives $\dfrac{1}{2}\ln(2x+1)+\dfrac{x}{2x+3}-\dfrac{1}{2}\ln(2x+3)$ if integration by parts is used for the second partial fraction.]	**B1 B1 B1**	**FT** on A, B and C.
	Substitute limits correctly in an integral with terms $a\ln(2x+1)$, $b\ln(2x+3)$ and $c/(2x+3)$, where $abc \neq 0$ If using alternative form: $cx/(2x+3)$	**M1**	value for upper limit – value for lower limit 1 slip in substituting can still score M1 Condone omission of $\ln(1)$
	Obtain the **given answer** following full and correct working	**A1**	Need to see at least one interim step of valid log work. **AG**
		5	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="6">9(i)</td><td>Carry out correct method for finding a vector equation for AB</td><td>M1</td><td></td></tr>
<tr><td>Obtain $(\mathbf{r} =)\mathbf{i} + 2\mathbf{j} - \mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} + 2\mathbf{k})$, or equivalent</td><td>A1</td><td></td></tr>
<tr><td>Equate two pairs of components of general points on their AB and l and solve for λ or for μ</td><td>M1</td><td>$\begin{pmatrix} 1+2\lambda \\ 2-\lambda \\ -1+2\lambda \end{pmatrix} = \begin{pmatrix} 2+\mu \\ 1+\mu \\ 1+2\mu \end{pmatrix}$</td></tr>
<tr><td>Obtain correct answer for λ or μ, e.g. $\lambda = 0$, $\mu = -1$</td><td>A1</td><td></td></tr>
<tr><td>Verify that all three equations are not satisfied and the lines fail to intersect ($\neq$ is sufficient justification e.g. $2 \neq 0$)
Conclusion needs to follow correct values</td><td>A1</td><td>Alternatives
<table>
<tr><th>A</th><th>λ</th><th>μ</th><th></th><th>B</th><th>λ</th><th>μ</th><th></th></tr>
<tr><td>ij</td><td>$2/3$</td><td>$1/3$</td><td>$\frac{1}{3} \neq \frac{5}{3}$</td><td></td><td>$-1/3$</td><td>$1/3$</td><td>$\frac{1}{3} \neq \frac{5}{3}$</td></tr>
<tr><td>ik</td><td>0</td><td>–1</td><td>$2 \neq 0$</td><td></td><td>–1</td><td>–1</td><td>$2 \neq 0$</td></tr>
<tr><td>jk</td><td>1</td><td>0</td><td>$3 \neq 2$</td><td></td><td>0</td><td>0</td><td>$3 \neq 2$</td></tr>
</table></td></tr>
<tr><td></td><td>5</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
9(ii)	State or imply midpoint has position vector $2\mathbf{i}+\frac{3}{2}\mathbf{j}$	**B1**	
	Substitute in $2x-y+2z=d$ and find d	**M1**	Correct use of *their* direction for AB and *their* midpoint
	Obtain plane equation $4x-2y+4z=5$	**A1**	or equivalent e.g. $\mathbf{r}.\begin{pmatrix}2\\-1\\2\end{pmatrix}=\frac{5}{2}$
	Substitute components of l in plane equation and solve for μ	**M1**	Correct use of their plane equation.
	Obtain $\mu=-\frac{1}{2}$ and position vector $\frac{3}{2}\mathbf{i}+\frac{1}{2}\mathbf{j}$ for the point P	**A1**	Final answer Accept coordinates in place of position vector
		5	

Question	Answer	Marks	Guidance
10(i)	State correct expansion of $\sin(3x+x)$ or $\sin(3x-x)$	**B1**	B0 If their formula retains ± in the middle
	Substitute expansions in $\frac{1}{2}(\sin 4x + \sin 2x)$	**M1**	
	Obtain $\sin 3x \cos x = \frac{1}{2}(\sin 4x + \sin 2x)$ correctly	**A1**	Must see the $\sin 4x$ and $\sin 2x$ or reference to LHS and RHS for A1 **AG**
		3	
10(ii)	Integrate and obtain $-\frac{1}{8}\cos 4x - \frac{1}{4}\cos 2x$	**B1 B1**	
	Substitute limits $x = 0$ and $x = \frac{1}{3}\pi$ correctly	**M1**	In their expression
	Obtain answer $\frac{9}{16}$	**A1**	From correct working seen.
		4	

Question	Answer	Marks	Guidance
10(iii)	State correct derivative $2\cos 4x + \cos 2x$	**B1**	
	Using correct double angle formula, express derivative in terms of $\cos 2x$ and equate the result to zero	**M1**	
	Obtain $4\cos^2 2x + \cos 2x - 2 = 0$	**A1**	
	Solve for x or $2x$ (could be labelled x) $\left(\cos 2x = \dfrac{-1 \pm \sqrt{33}}{8}\right)$	**M1**	Must see working if solving an incorrect quadratic The roots of the correct quadratic are -0.843 and 0.593 Need to get as far as $x = \ldots$ The wrong value of x is 0.468 and can imply M1 if correct quadratic seen Could be working from a quartic in $\cos x$: $16\cos^4 x - 14\cos^2 x + 1 = 0$
	Obtain answer $x = 1.29$ only	**A1**	
		5	

Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/33**

Paper 3 **May/June 2019**

MARK SCHEME

Maximum Mark: 75

Published

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	Use law of the logarithm of a product or quotient	**M1**	
	Use law of the logarithm of power **twice**	**M1**	
	Obtain a correct linear equation in x, e.g. $(3-2x)\ln 5 = \ln 4 + x\ln 7$	**A1**	
	Obtain answer $x = 0.666$	**A1**	
		4	

Question	Answer	Marks	Guidance
2	Commence integration and reach $ax^2 \sin 2x + b\int x \sin 2x \, dx$	**M1***	
	Obtain $\frac{1}{2}x^2 \sin 2x - \int x \sin 2x \, dx$, or equivalent	**A1**	
	Complete the integration and obtain $\frac{1}{2}x^2 \sin 2x + \frac{1}{2}x \cos 2x - \frac{1}{4}\sin 2x$, or equivalent	**A1**	
	Use limits correctly, having integrated twice	**DM1**	
	Obtain given answer correctly	**A1**	
		5	

Question	Answer	Marks	Guidance
3(i)	Use double angle formulae and express entire fraction in terms of $\sin\theta$ and $\cos\theta$	M1	
	Obtain a correct expression	A1	
	Obtain the given answer	A1	
		3	
3(ii)	State integral of the form $\pm\ln\cos\theta$	M1*	
	Use correct limits correctly and insert exact values for the trig ratios	DM1	
	Obtain a correct expression, e.g. $-\ln\frac{1}{\sqrt{2}} + \ln\frac{\sqrt{3}}{2}$	A1	
	Obtain the given answer following full and exact working	A1	
		4	

Question	Answer	Marks	Guidance
4(i)	Use the quotient or product rule	M1	
	Obtain correct derivative in any form	A1	
	Reduce to $-\frac{2e^{-x}}{\left(1-e^{-x}\right)^2}$, or equivalent, and explain why this is always negative	A1	
		3	

Question	Answer	Marks	Guidance
4(ii)	Equate derivative to – 1 and obtain the given equation	**B1**	
	State or imply $u^2 - 4u + 1 = 0$, or equivalent in e^a	**B1**	
	Solve for a	**M1**	
	Obtain answer $a = \ln\left(2 + \sqrt{3}\right)$ and no other	**A1**	
		4	

Question	Answer	Marks	Guidance
5	Separate variables correctly and integrate at least one side	**B1**	
	Obtain term $\ln(x+1)$	**B1**	
	Obtain term of the form $a\ln\left(y^2 + 5\right)$	**M1**	
	Obtain term $\frac{1}{2}\ln(y^2 + 5)$	**A1**	
	Use $y = 2$, $x = 0$ to determine a constant, or as limits, in a solution containing terms $a\ln(y^2 + 5)$ and $b\ln(x+1)$, where $ab \neq 0$	**M1**	
	Obtain correct solution in any form	**A1**	
	Obtain final answer $y^2 = 9(x+1)^2 - 5$	**A1**	
		7	

Question	Answer	Marks	Guidance
6(i)	State $b = 3$	**B1**	
		1	
6(ii)	Commence division by $x - b$ and reach partial quotient $x^3 + kx^2$	**M1**	
	Obtain quotient $x^3 + x^2 + 3x + 2$	**A1**	There being no remainder
	Equate quotient to zero and rearrange to make the subject a	**M1**	
	Obtain the given equation	**A1**	
		4	
6(iii)	Use the iterative formula $a_{n+1} = -\frac{1}{3}\left(2 + a_n^{\ 2} + a_n^{\ 3}\right)$ correctly at least once	**M1**	
	Obtain final answer –0.715	**A1**	
	Show sufficient iterations to 5 d.p. to justify –0.715 to 3 d.p., or show there is a sign change in the interval (–0.7145, –0.7155)	**A1**	
		3	

Question	Answer	Marks	Guidance
7(i)	Use product rule	**M1**	
	Obtain correct derivative in any form	**A1**	
		2	
7(ii)	Equate derivative to zero and use correct $\cos(A + B)$ formula	**M1**	
	Obtain the given equation	**A1**	
		2	
7(iii)	Use correct method to solve for x	**M1**	
	Obtain answer, e.g. $x = \frac{1}{12}\pi$	**A1**	
	Obtain second answer, e.g. $\frac{7}{12}\pi$, and no other	**A1**	
		3	

Question	Answer	Marks	Guidance
8(i)	Multiply numerator and denominator by $1 + \sqrt{3}\mathrm{i}$, or equivalent	**M1**	
	$4\mathrm{i} - 4\sqrt{3}$ and $3 + 1$	**A1**	
	Obtain final answer $-\sqrt{3} + \mathrm{i}$	**A1**	
		3	

Question	Answer	Marks	Guidance
8(ii)	State that the modulus of u is 2	**B1**	
	State that the argument of u is $\frac{5}{6}\pi$ (or 150°)	**B1**	
		2	
8(iii)	Show a circle with centre the origin and radius 2	**B1**	
	Show u in a relatively correct position	**B1**	**FT**
	Show the perpendicular bisector of the line joining u and the origin	**B1**	**FT**
	Shade the correct region	**B1**	
		4	

Question	Answer	Marks	Guidance
9(i)	State or imply the form $\frac{A}{3+x}+\frac{B}{1-x}+\frac{C}{(1-x)^2}$	B1	
	Use a correct method for finding a constant	M1	
	Obtain one of $A=-3$, $B=-1$, $C=2$	A1	
	Obtain a second value	A1	
	Obtain the third value	A1	Mark the form $\frac{A}{3+x}+\frac{Dx+E}{(1-x)^2}$, where $A=-3$, $D=1$ and $E=1$, B1M1A1A1A1 as above.
		5	
9(ii)	Use a correct method to find the first two terms of the expansion of $(3+x)^{-1}$, $(1+\frac{1}{3}x)^{-1}$, $(1-x)^{-1}$ or $(1-x)^{-2}$	M1	
	Obtain correct unsimplified expansions up to the term in x^3 of each partial fraction	A1	**FT** on A
		A1	**FT** on B
		A1	**FT** on C
	Obtain final answer $\frac{10}{3}x+\frac{44}{9}x^2+\frac{190}{27}x^3$	A1	For the A, D, E form of fractions give M1A1ftA1ft for the expanded partial fractions, then, if $D\neq 0$, M1 for multiplying out fully, and A1 for the final answer.
		5	

Question	Answer	Marks	Guidance
10(i)	Find $\overrightarrow{PQ}$ for a general point Q on l, e.g. $-3\mathbf{i}+6\mathbf{k}+\mu(2\mathbf{i}-\mathbf{j}-2\mathbf{k})$	**B1**	
	Calculate scalar product of $\overrightarrow{PQ}$ and a direction vector for l and equate the result to zero	**M1**	
	Solve for μ and obtain $\mu = 2$	**A1**	
	Carry out a complete method for finding the length of $\overrightarrow{PQ}$	**M1**	
	Obtain answer 3	**A1**	
	Alternative method for question 10(i)		
	Calling the point (1, 2, 3) A, state $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) in component form, e.g. $3\mathbf{i} - 6\mathbf{k}$	**B1**	
	Use a scalar product with a direction vector for l to find the projection of $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) on l	**M1**	
	Obtain correct answer in any form, e.g. $\frac{18}{\sqrt{9}}$	**A1**	
	Use Pythagoras to find the perpendicular	**M1**	
	Obtain answer 3	**A1**	

Question	Answer	Marks	Guidance
10(i)	**Alternative method for question 10(i)**		
	State $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) in component form	**B1**	
	Calculate a vector product with a direction vector for l	**M1**	
	Obtain correct answer, e.g. $6\mathbf{i} - 6\mathbf{j} - 3\mathbf{k}$	**A1**	
	Divide modulus of the product by that of the direction vector	**M1**	
	Obtain answer 3	**A1**	
		5	

Question	Answer	Marks	Guidance
10(ii)	Substitute coordinates of a general point of l in the plane equation and equate constant terms	**M1**	
	Obtain a correct equation, e.g. $a + 2b + 6 = 13$	**A1**	
	Equate the coefficient of μ to zero	**M1**	
	Obtain a correct equation, e.g. $2a - b - 4 = 0$	**A1**	
	Substitute (1, 2, 3) in the plane equation	**M1**	
	Obtain a correct equation, e.g. $a + 2b + 6 = 13$	**A1**	
	Alternative method for question 10(ii)		
	Find a second point on l and obtain an equation in a and/or b	**M1**	
	Obtain a correct equation, e.g. $5a - 2 = 13$	**A1**	
	Equate scalar product of a direction vector for l and a vector normal for the plane to zero	**M1**	
	Obtain a correct equation, e.g. $2a - b - 4 = 0$	**A1**	
	Solve for a or for b	**M1**	
	Obtain $a = 3$ and $b = 2$	**A1**	
		6	

Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **May/June 2020**

MARK SCHEME

Maximum Mark: 75

Published

Students did not sit exam papers in the June 2020 series due to the Covid-19 global pandemic.

This mark scheme is published to support teachers and students and should be read together with the question paper. It shows the requirements of the exam. The answer column of the mark scheme shows the proposed basis on which Examiners would award marks for this exam. Where appropriate, this column also provides the most likely acceptable alternative responses expected from students. Examiners usually review the mark scheme after they have seen student responses and update the mark scheme if appropriate. In the June series, Examiners were unable to consider the acceptability of alternative responses, as there were no student responses to consider.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
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- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

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Mathematics-Specific Marking Principles	
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5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
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CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks
1	Use law of the logarithm of a product or power	**M1**
	Obtain a correct linear inequality in any form, e.g. $\ln 2 + (1 - 2x)\ln 3 < x \ln 5$	**A1**
	Solve for x	**M1**
	Obtain $x > \frac{\ln 6}{\ln 45}$	**A1**
		4

Question	Answer	Marks
2(a)	State a correct unsimplified version of the x or x^2 term of the expansion of $(2-3x)^{-2}$ or $\left(1-\frac{3}{2}x\right)^{-2}$	**M1**
	State correct first term $\frac{1}{4}$	**B1**
	Obtain the next two terms $\frac{3}{4}x + \frac{27}{16}x^2$	**A1 + A1**
		4
2(b)	State answer $\lvert x\rvert < \frac{2}{3}$, or equivalent	**B1**
		1

Question	Answer	Marks
3	Use tan $(A \pm B)$ formula and obtain an equation in tan θ	**M1**
	Using $\tan 60° = \sqrt{3}$, obtain a horizontal equation in tan θ in any correct form	**A1**
	Reduce the equation to $3\tan^2\theta + 4\tan\theta - 1 = 0$, or equivalent	**A1**
	Solve a 3-term quadratic for tan θ	**M1**
	Obtain a correct answer, e.g. 12.1°	**A1**
	Obtain a second correct answer, e.g. 122.9°, and no others in the given interval	**A1**
		6

Question	Answer	Marks
4(a)	Use product rule	**M1**
	Obtain derivative in any correct form e.g. $2e^{2x}(\sin x + 3\cos x) + e^{2x}(\cos x - 3\sin x)$	**A1**
	Equate derivative to zero and obtain an equation in one trigonometric ratio	**M1**
	Obtain $x = 1.43$ only	**A1**
		4
4(b)	Use a correct method to determine the nature of the stationary point e.g. $x = 1.42,\ y' = 0.06e^{2.84} > 0$ $x = 1.44,\ y' = -0.07e^{2.88} < 0$	**M1**
	Show that it is a maximum point	**A1**
		2

Question	Answer	Marks
5(a)	Commence division and reach quotient of the form $2x+k$	**M1**
	Obtain quotient $2x-1$	**A1**
	Obtain remainder 6	**A1**
		3
5(b)	Obtain terms x^2-x (**FT** on quotient of the form $2x+k$)	**B1FT**
	Obtain term of the form $a\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)$	**M1**
	Obtain term $\frac{6}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)$ (**FT** on a constant remainder)	**A1FT**
	Use $x=1$ and $x=3$ as limits in a solution containing a term of the form $a\tan^{-1}(bx)$	**M1**
	Obtain final answer $\frac{1}{\sqrt{3}}\pi+6$, or exact equivalent	**A1**
		5

Question	Answer	Marks
6(a)	State or imply $AT = r\tan x$ or $BT = r\tan x$	B1
	Use correct area formula and form an equation in r and x	M1
	Rearrange in the given form	A1
		3
6(b)	Calculate the values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.4$	M1
	Complete the argument correctly with correct calculated values	A1
		2
6(c)	Use the iterative formula correctly at least once	M1
	Obtain final answer 1.35	A1
	Show sufficient iterations to 4 d.p. to justify 1.35 to 2 d.p. or show there is a sign change in the interval (1.345, 1.355)	A1
		3

Question	Answer	Marks
7(a)	Use quotient or product rule	M1
	Obtain derivative in any correct form e.g. $\dfrac{-\sin x(1+\sin x)-\cos x(\cos x)}{(1+\sin x)^2}$	A1
	Use Pythagoras to simplify the derivative	M1
	Justify the given statement	A1
		4

Question	Answer	Marks
7(b)	State integral of the form $a \ln(1 + \sin x)$	*M1
	State correct integral $\ln(1 + \sin x)$	A1
	Use limits correctly	DM1
	Obtain answer $\ln\dfrac{4}{3}$	A1
		4
8(a)	State $\dfrac{dy}{dx} = k\dfrac{y}{x\sqrt{x}}$, or equivalent	B1
	Separate variables correctly and attempt integration of at least one side	M1
	Obtain term $\ln y$, or equivalent	A1
	Obtain term $-2k\dfrac{1}{\sqrt{x}}$, or equivalent	A1
	Use given coordinates to find k or a constant of integration c in a solution containing terms of the form $a \ln y$ and $\dfrac{b}{\sqrt{x}}$, where $ab \neq 0$	M1
	Obtain $k = 1$ and $c = 2$	A1 + A1
	Obtain final answer $y = \exp\left(-\dfrac{2}{\sqrt{x}} + 2\right)$, or equivalent	A1
		8

Question	Answer	Marks
8(b)	State that y approaches e^2 (FT *their* c in part **(a)** of the correct form)	**B1FT**
		1
9(a)	State $\overrightarrow{AB}\left(\text{or } \overrightarrow{BA}\right)$ and $\overrightarrow{BC}\left(\text{or } \overrightarrow{CB}\right)$ in vector form	**B1**
	Calculate their scalar product	**M1**
	Show product is zero and confirm angle ABC is a right angle	**A1**
		3
9(b)	Use correct method to calculate the lengths of AB and BC	**M1**
	Show that $AB = BC$ and the triangle is isosceles	**A1**
		2
9(c)	State a correct equation for the line through B and C, e.g. $\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ or $\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \mu(-2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$	**B1**
	Taking a general point of BC to be P, form an equation in λ by **either** equating the scalar product of $\overrightarrow{OP}$ and $\overrightarrow{BC}$ to zero, **or** applying Pythagoras to triangle OBP (or OCP), *or* setting the derivative of $\left\|\overrightarrow{OP}\right\|$ to zero	**M1**
	Solve and obtain $\lambda = -\dfrac{5}{9}$	**A1**
	Obtain answer $\dfrac{1}{3}\sqrt{2}$, or equivalent	**A1**

Question	Answer	Marks
	Alternative method for question 9(c)	
	Use a scalar product to find the projection *CN* (or *BN*) of *OC* (or *OB*) on *BC*	**M1**
	Obtain answer $CN = \frac{5}{3}\left(\text{or } BN = \frac{14}{3}\right)$	**A1**
	Use Pythagoras to find *ON*	**M1**
	Obtain answer $\frac{1}{3}\sqrt{2}$, or equivalent	**A1**
		4

Question	Answer	Marks
10(a)(i)	Multiply numerator and denominator by $a - 2\text{i}$, or equivalent	**M1**
	Use $\text{i}^2 = -1$ at least once	**A1**
	Obtain answer $\frac{6}{a^2+4} + \frac{3ai}{a^2+4}$	**A1**
		3
10(a)(ii)	Either state that $\arg u = -\frac{1}{3}\pi$ or express u^* in terms of a (FT on u)	**B1**
	Use correct method to form an equation in a	**M1**
	Obtain answer $a = -2\sqrt{3}$	**A1**
		3

Question	Answer	Marks
10(b)(i)	Show the perpendicular bisector of points representing 2i and 1 + i	**B1**
	Show the point representing 2 + i	**B1**
	Show a circle with radius 2 and centre 2 + i (FT on the position of the point for 2 + i)	**B1FT**
	Shade the correct region	**B1**
		4
10(b)(ii)	State or imply the critical point 2 + 3i	**B1**
	Obtain answer 56.3° or 0.983 radians	**B1**
		2

Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **May/June 2020**

MARK SCHEME

Maximum Mark: 75

Published

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WWW	Without Wrong Working
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Question	Answer	Marks
1	Commence division and reach partial quotient $3x^2 + kx$	**M1**
	Obtain quotient $3x^2 + 2x - 1$	**A1**
	Obtain remainder $2x - 5$	**A1**
		4

Question	Answer	Marks
2	State or imply $2 \ln y = \ln A + kx$	**B1**
	Substitute values of $\ln y$ and x, or equate gradient of line to k, and solve for k	**M1**
	Obtain $k = 0.80$	**A1**
	Solve for $\ln A$	**M1**
	Obtain $A = 3.31$	**A1**
	Alternative method for question 2	
	Obtain two correct equations in y and x by substituting y- and x- values in the given equation	**B1**
	Solve for k	**M1**
	Obtain $k = 0.80$	**A1**
	Solve for A	**M1**
	Obtain $A = 3.31$	**A1**
		5

Question	Answer	Marks
3	Commence integration and reach $ax^{\frac{5}{2}}\ln x + b\int x^{\frac{5}{2}}.\frac{1}{x}\,dx$	M1*
	Obtain $\frac{2}{5}x^{\frac{5}{2}}\ln x - \frac{2}{5}\int x^{\frac{5}{2}}.\frac{1}{x}\,dx$	A1
	Complete the integration and obtain $\frac{2}{5}x^{\frac{5}{2}}\ln x - \frac{4}{25}x^{\frac{5}{2}}$, or equivalent	A1
	Use limits correctly, having integrated twice e.g $\frac{2}{5}\times 32\ln 4 - \frac{4}{25}\times 32 - \left(\frac{2}{5}\times 0\right) + \frac{4}{25}$	DM1
	Obtain answer $\frac{128}{5}\ln 2 - \frac{124}{25}$, or exact equivalent	A1
		5

Question	Answer	Marks
4	Use correct product rule	M1
	Obtain correct derivative in any form, e.g. $-\sin x \sin 2x + 2\cos x \cos 2x$	A1
	Use double angle formula to express derivative in terms of $\sin x$ and $\cos x$	M1
	Equate derivative to zero and obtain an equation in one trig function	M1
	Obtain $3\sin 2x = 1$, or $3\cos 2x = 2$ or $2\tan 2x = 1$	A1
	Solve and obtain $x = 0.615$	A1
		6

Question	Answer	Marks
5(a)	State $R = \sqrt{7}$	**B1**
	Use trig formulae to find α	**M1**
	Obtain $\alpha = 57.688°$	**A1**
		3
5(b)	Evaluate $\cos{-1}\left(\frac{1}{\sqrt{7}}\right)$ to at least 3 d.p. (67.792°) (**FT** is on *their* R)	**B1 FT**
	Use correct method to find a value of θ in the interval	**M1**
	Obtain answer, e.g. 5.1°	**A1**
	Obtain second answer, e.g.117.3°, only	**A1**
		4

6(a)	Use quotient or product rule	**M1**
	Obtain correct derivative in any form e.g. $\frac{\left(1+3x^4\right)-x\times12x^3}{\left(1+3x^4\right)^2}$	**A1**
	Equate derivative to zero and solve for x	**M1**
	Obtain answer 0.577	**A1**
		4

Question	Answer	Marks
6(b)	State or imply $\mathrm{d}u = 2\sqrt{3}x\,\mathrm{d}x$, or equivalent	B1
	Substitute for x and $\mathrm{d}x$	M1
	Obtain integrand $\dfrac{1}{2\sqrt{3}\left(1+u^2\right)}$, or equivalent	A1
	State integral of the form $a\tan^{-1}u$ and use limits $u = 0$ and $u = \sqrt{3}$ (or $x = 0$ and $x = 1$) correctly	M1
	Obtain answer $\dfrac{\sqrt{3}}{18}\pi$, or exact equivalent	A1
		5

Question	Answer	Marks
7	Separate variables correctly and integrate at least one side	**B1**
	Obtain term $\ln(y-1)$	**B1**
	Carry out a relevant method to determine A and B such that $\dfrac{1}{(x+1)(x+3)} \equiv \dfrac{A}{x+1} + \dfrac{B}{x+3}$	**M1**
	Obtain $A = \dfrac{1}{2}$ and $B = -\dfrac{1}{2}$	**A1**
	Integrate and obtain terms $\dfrac{1}{2}\ln(x+1) - \dfrac{1}{2}\ln(x+3)$ $\frac{1}{2}\ln(x+1) - \frac{1}{2}\ln(x+3)$, or equivalent (**FT** is on A and B)	**A1 FT** **+ A1 FT**
	Use $x = 0$, $y = 2$ to evaluate a constant, or as limits in a solution containing terms of the form $a\ln(y-1)$, $b\ln(x+1)$ and $c\ln(x+3)$, where $abc \neq 0$	**M1**
	Obtain correct answer in any form	**A1**
	Obtain final answer $y = 1 + \sqrt{\left(\dfrac{3x+3}{x+3}\right)}$, or equivalent	**A1**
		9

Question	Answer	Marks
8(a)	Substitute and obtain a correct equation in x and y	**B1**
	Use $i^2 = -1$ and equate real and imaginary parts	**M1**
	Obtain two correct equations in x and y, e.g. $x - y = 3$ and $3x + y = 5$	**A1**
	Solve and obtain answer $z = 2 - i$	**A1**
		4
8(b)(i)	Show a point representing $2 + 2i$	**B1**
	Show a circle with radius 1 and centre not at the origin (**FT** is on the point representing the centre)	**B1 FT**
	Show the correct half line from $4i$	**B1**
	Shade the correct region	**B1**
		4
8(b)(ii)	Carry out a complete method for finding the least value of $\operatorname{Im} z$	**M1**
	Obtain answer $2 - \frac{1}{2}\sqrt{2}$, or exact equivalent	**A1**
		2

Question	Answer	Marks
9(a)	State $\cos p = \frac{k}{1+p}$	**B1**
	Differentiate both equations and equate derivatives at $x = p$	**M1**
	Obtain a correct equation in any form, e.g. $-\sin p = -\frac{k}{(1+p)^2}$	**A1**
	Eliminate k	**M1**
	Obtain the **given answer** showing sufficient working	**A1**
		5
9(b)	Use the iterative formula correctly at least once	**M1**
	Obtain final answer $p = 0.568$	**A1**
	Show sufficient iterations to justify 0.568 to 3 d.p., or show there is a sign change in the interval (0.5675, 0.5685)	**A1**
		3
9(c)	Use a correct method to find k	**M1**
	Obtain answer $k = 1.32$	**A1**
		2

Question	Answer	Marks
10(a)	State that the position vector of M is $3\mathbf{i} + \mathbf{j}$	B1
	Use a correct method to find the position vector of N	M1
	Obtain answer $\frac{10}{3}\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$	A1
	Use a correct method to form an equation for MN	M1
	Obtain correct answer in any form, e.g. $\mathbf{r} = 3\mathbf{i} + \mathbf{j} + \lambda\left(\frac{1}{3}\mathbf{i} + \mathbf{j} + 2\mathbf{k}\right)$	A1
		5
10(b)	State or imply $\mathbf{r} = \mu(2\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$ as equation for OB	B1
	Equate sufficient components of MN and OB and solve for λ or for μ	M1
	Obtain $\lambda = 3$ or $\mu = 2$ and position vector $4\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$ for P	A1
		3
10(c)	Carry out correct process for evaluating the scalar product of direction vectors for OP and MP, or equivalent	M1
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	M1
	Obtain answer 21.6°	A1
		3

Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS	**9709/33**
Paper 3 Pure Mathematics 3	**May/June 2020**
MARK SCHEME	
Maximum Mark: 75	

Published

Students did not sit exam papers in the June 2020 series due to the Covid-19 global pandemic.

This mark scheme is published to support teachers and students and should be read together with the question paper. It shows the requirements of the exam. The answer column of the mark scheme shows the proposed basis on which Examiners would award marks for this exam. Where appropriate, this column also provides the most likely acceptable alternative responses expected from students. Examiners usually review the mark scheme after they have seen student responses and update the mark scheme if appropriate. In the June series, Examiners were unable to consider the acceptability of alternative responses, as there were no student responses to consider.

Mark schemes should usually be read together with the Principal Examiner Report for Teachers. However, because students did not sit exam papers, there is no Principal Examiner Report for Teachers for the June 2020 series.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the June 2020 series for most Cambridge IGCSE™ and Cambridge International A & AS Level components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks
1	State or imply non-modular inequality $(2x-1)^2 > 3^2(x+2)^2$, or corresponding quadratic equation, or pair of linear equations	**B1**
	Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	**M1**
	Obtain critical values $x = -7$ and $x = -1$	**A1**
	State final answer $-7 < x < -1$	**A1**
	Alternative method for question 1	
	Obtain critical value $x = -1$ from a graphical method, or by solving a linear equation or linear inequality	**B1**
	Obtain critical value $x = -7$ similarly	**B2**
	State final answer $-7 < x < -1$ [Do not condone $\leqslant$ for $<$ in the final answer.]	**B1**
		4

Question	Answer	Marks
2	Commence integration and reach $a(2-x)e^{-2x}+b\int e^{-2x}\,\mathrm{d}x$, or equivalent	**M1***
	Obtain $-\frac{1}{2}(2-x)e^{-2x}-\frac{1}{2}\int e^{-2x}\,\mathrm{d}x$, or equivalent	**A1**
	Complete integration and obtain $-\frac{1}{2}(2-x)e^{-2x}+\frac{1}{4}e^{-2x}$, or equivalent	**A1**
	Use limits correctly, having integrated twice	**DM1**
	Obtain answer $\frac{1}{4}\left(3-e^{-2}\right)$, or exact equivalent	**A1**
		5

Question	Answer	Marks
3(a)	Remove logarithms correctly and state $1 + e^{-x} = e^{-2x}$, or equivalent	B1
	Show equation is $u^2 + u - 1 = 0$, where $u = e^x$, or equivalent	B1
		2
3(b)	Solve a 3-term quadratic for u	M1
	Obtain root $\frac{1}{2}\left(-1+\sqrt{5}\right)$, or decimal in [0.61, 0.62]	A1
	Use correct method for finding x from a positive root	M1
	Obtain answer $x = -0.481$ only	A1
		4

Question	Answer	Marks
4(a)	Use the product rule	**M1**
	State or imply derivative of $\tan^{-1}(\frac{1}{2}x)$ is of the form $k/(4+x^2)$, where $k = 2$ or 4, or equivalent	**M1**
	Obtain correct derivative in any form, e.g. $\tan^{-1}\left(\frac{1}{2}x\right)+\frac{2x}{x^2+4}$, or equivalent	**A1**
		3
4(b)	State or imply y-coordinate is $\frac{1}{2}\pi$	**B1**
	Carry out a complete method for finding p, e.g. by obtaining the equation of the tangent and setting $x = 0$, or by equating the gradient at $x = 2$ to $\frac{\frac{1}{2}\pi - p}{2}$	**M1**
	Obtain answer $p = -1$	**A1**
		3

Question	Answer	Marks
5	Use tan $2A$ formula to express RHS in terms of tan θ	**M1**
	Use tan $(A \pm B)$ formula to express LHS in terms of tan θ	**M1**
	Using tan 45° = 1, obtain a correct horizontal equation in any form	**A1**
	Reduce equation to $2\tan^2\theta + \tan\theta - 1 = 0$	**A1**
	Solve a 3-term quadratic and find a value of θ	**M1**
	Obtain answer $\theta = 26.6°$ and no other	**A1**
		6

Question	Answer	Marks
6(a)	Sketch a relevant graph, e.g. $y = x^5$	**B1**
	Sketch a second relevant graph, e.g. $y = x + 2$ and justify the given statement	**B1**
		2
6(b)	State a suitable equation, e.g. $x = \dfrac{4x^5 + 2}{5x^4 - 1}$	**B1**
	Rearrange this as $x^5 = 2 + x$ or commence working *vice versa*	**B1**
		2
6(c)	Use the iterative formula correctly at least once	**M1**
	Obtain final answer 1.267	**A1**
	Show sufficient iterations to 5 d.p. to justify 1.267 to 3 d.p., or show there is a sign change in the interval (1.2665, 1.2675)	**A1**
		3

Question	Answer	Marks
7(a)	State or imply the form $\dfrac{A}{2x-1}+\dfrac{B}{2x+1}$ and use a relevant method to find A or B	**M1**
	Obtain $A=1$, $B=-1$	**A1**
		2
7(b)	Square the result of part (**a**) and substitute the fractions of part (**a**)	**M1**
	Obtain the given answer correctly	**A1**
		2
7(c)	Integrate and obtain $-\dfrac{1}{2(2x-1)}-\dfrac{1}{2}\ln(2x-1)+\dfrac{1}{2}\ln(2x+1)-\dfrac{1}{2(2x+1)}$, or equivalent	**B3, 2, 1, 0**
	Substitute limits correctly	**M1**
	Obtain the given answer correctly	**A1**
		5

Question	Answer	Marks
8(a)	State or imply $\overrightarrow{AB}$ or $\overrightarrow{AD}$ in component form	**B1**
	Use a correct method for finding the position vector of C	**M1**
	Obtain answer $4\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$, or equivalent	**A1**
	Using the correct process for the moduli, compare lengths of a pair of adjacent sides, e.g. AB and AD	**M1**
	Show that $ABCD$ has a pair of unequal adjacent sides	**A1**
	Alternative method for question 8(a)	
	State or imply $\overrightarrow{AB}$ or $\overrightarrow{AD}$ in component form	**B1**
	Use a correct method for finding the position vector of C	**M1**
	Obtain answer $4\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$, or equivalent	**A1**
	Use the correct process to calculate the scalar product of $\overrightarrow{AC}$ and $\overrightarrow{BD}$, or equivalent	**M1**
	Show that the diagonals of $ABCD$ are not perpendicular	**A1**
		5
8(b)	Use the correct process to calculate the scalar product of a pair of relevant vectors, e.g. $\overrightarrow{AB}$ and $\overrightarrow{AD}$	**M1**
	Using the correct process for the moduli, divide the scalar product by the product of the moduli of the two vectors and evaluate the inverse cosine of the result	**M1**
	Obtain answer $100.3°$	**A1**
		3

Question	Answer	Marks
8(c)	Use a correct method to calculate the area, e.g. calculate *AB.AC* sin *BAD*	**M1**
	Obtain answer 11.0 (**FT** on angle BAD)	**A1 FT**
		2

Question	Answer	Marks
9(a)	Eliminate u or w and obtain an equation w or u	**M1**
	Obtain a quadratic in u or w, e.g. $u^2 - 2iu - 6 = 0$ or $w^2 + 2iw - 6 = 0$	**A1**
	Solve a 3-term quadratic for u or for w	**M1**
	Obtain answer $u = \sqrt{5} + \mathrm{i}$, $w = \sqrt{5} - \mathrm{i}$	**A1**
	Obtain answer $u = -\sqrt{5} + \mathrm{i}$, $w = -\sqrt{5} - \mathrm{i}$	**A1**
		5
9(b)	Show the point representing $2 + 2\mathrm{i}$	**B1**
	Show a circle with centre $2 + 2\mathrm{i}$ and radius 2 (**FT** is on the position of $2 + 2\mathrm{i}$)	**B1 FT**
	Show half-line from origin at 45° to the positive x-axis	**B1**
	Show line for Re $z = 3$	**B1**
	Shade the correct region	**B1**
		5

Question	Answer	Marks
10(a)	State or imply $\frac{dV}{dt} = -k\sqrt{h}$	B1
	State or imply $\frac{dV}{dh} = 2\pi rh - \pi h^2$, or equivalent	B1
	Use $\frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt}$	M1
	Obtain the **given answer** correctly	A1
		4
10(b)	Separate variables and attempt integration of at least one side	M1
	Obtain terms $\frac{4}{3}rh^{\frac{3}{2}} - \frac{2}{5}h^{\frac{5}{2}}$ and $-Bt$	A3, 2, 1, 0
	Use $t = 0$, $h = r$ to find a constant of integration c	M1
	Use $t = 14$, $h = 0$ to find B	M1
	Obtain correct c and B, e.g. $c = \frac{14}{15}r^{\frac{5}{2}}$, $B = \frac{1}{15}r^{\frac{5}{2}}$	A1
	Obtain final answer $t = 14 - 20\left(\frac{h}{r}\right)^{\frac{3}{2}} + 6\left(\frac{h}{r}\right)^{\frac{5}{2}}$, or equivalent	A1
		8

1	Use correct quotient or product rule	M1	
	Obtain correct derivative in any form	A1	
	Justify the given statement	A1	**[3]**
2	*EITHER:* State or imply non-modular equation $2^2\left(3^x-1\right)^2=\left(3^x\right)^2$, or pair of equations $2\left(3^x-1\right)=\pm 3^x$	M1	
	Obtain $3^x = 2$ and $3^x=\dfrac{2}{3}$ (or $3^{x+1} = 2$)	A1	
	OR: Obtain $3^x = 2$ by solving an equation or by inspection	B1	
	Obtain $3^x=\dfrac{2}{3}$ (or $3^{x+1} = 2$) by solving an equation or by inspection	B1	
	Use correct method for solving an equation of the form $3^x = a$ (or $3^{x+1} = a$), where $a > 0$	M1	
	Obtain final answers 0.631 and –0.369	A1	**[4]**
3	*EITHER:* Integrate by parts and reach $kx^{\frac{1}{2}}\ln x - m\int x^{\frac{1}{2}}.\dfrac{1}{x}\,dx$	M1*	
	Obtain $2x^{\frac{1}{2}}\ln x-2\int\dfrac{1}{x^{\frac{1}{2}}}\,dx$, or equivalent	A1	
	Integrate again and obtain $2x^{\frac{1}{2}}\ln x-4x^{\frac{1}{2}}$, or equivalent	A1	
	Substitute limits $x = 1$ and $x = 4$, having integrated twice	M1(dep*)	
	Obtain answer $4(\ln 4-1)$, or exact equivalent	A1	
	OR1: Using $u = \ln x$, or equivalent, integrate by parts and reach $kue^{\frac{1}{2}u}-m\int e^{\frac{1}{2}u}\,du$	M1*	
	Obtain $2ue^{\frac{1}{2}u}-2\int e^{\frac{1}{2}u}\,du$, or equivalent	A1	
	Integrate again and obtain $2ue^{\frac{1}{2}u}-4e^{\frac{1}{2}u}$, or equivalent	A1	
	Substitute limits $u = 0$ and $u = \ln 4$, having integrated twice	M1(dep*)	
	Obtain answer $4\ln 4-4$, or exact equivalent	A1	
	OR2: Using $u=\sqrt{x}$, or equivalent, integrate and obtain $ku\ln u-m\int u.\dfrac{1}{u}\,du$	M1*	
	Obtain $4u\ln u-4\int 1\,du$, or equivalent	A1	
	Integrate again and obtain $4u\ln u-4u$, or equivalent	A1	
	Substitute limits $u = 1$ and $u = 2$, having integrated twice or quoted $\int \ln u\,du$ as $u\ln u\pm u$	M1(dep*)	
	Obtain answer $8\ln 2-4$, or exact equivalent	A1	
	OR3: Integrate by parts and reach $I=\dfrac{x\ln x\pm x}{\sqrt{x}}+k\int\dfrac{x\ln x\pm x}{x\sqrt{x}}\,dx$	M1*	
	Obtain $I=\dfrac{x\ln x-x}{\sqrt{x}}+\dfrac{1}{2}I-\dfrac{1}{2}\int\dfrac{1}{\sqrt{x}}\,dx$	A1	
	Integrate and obtain $I=2\sqrt{x}\ln x-4\sqrt{x}$, or equivalent	A1	
	Substitute limits $x = 1$ and $x = 4$, having integrated twice	M1(dep*)	
	Obtain answer $4\ln 4-4$, or exact equivalent	A1	**[5]**

4		Use correct product or quotient rule at least once	M1*	
		Obtain $\frac{dx}{dt} = e^{-t}\sin t - e^{-t}\cos t$ or $\frac{dy}{dt} = e^{-t}\cos t - e^{-t}\sin t$, or equivalent	A1	
		Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
		Obtain $\frac{dy}{dx} = \frac{\sin t - \cos t}{\sin t + \cos t}$, or equivalent	A1	
		EITHER: Express $\frac{dy}{dx}$ in terms of tan t only	M1(dep*)	
		Show expression is identical to $\tan\left(t - \frac{1}{4}\pi\right)$	A1	
		OR: Express $\tan\left(t - \frac{1}{4}\pi\right)$ in terms of tan t	M1	
		Show expression is identical to $\frac{dy}{dx}$	A1	**[6]**
5	**(i)**	Use Pythagoras	M1	
		Use the sin2A formula	M1	
		Obtain the given result	A1	**[3]**
	(ii)	Integrate and obtain a $k\ln\sin\theta$ or $m\ln\cos\theta$ term, or obtain integral of the form $p\ln\tan\theta$	M1*	
		Obtain indefinite integral $\frac{1}{2}\ln\sin\theta - \frac{1}{2}\ln\cos\theta$, or equivalent, or $\frac{1}{2}\ln\tan\theta$	A1	
		Substitute limits correctly	M1(dep)*	
		Obtain the given answer correctly having shown appropriate working	A1	**[4]**
6	**(i)**	State or imply $AB = 2r\cos\theta$ or $AB^2 = 2r^2 - 2r^2\cos(\pi - 2\theta)$	B1	
		Use correct formula to express the area of sector ABC in terms of r and θ	M1	
		Use correct area formulae to express the area of a segment in terms of r and θ	M1	
		State a correct equation in r and θ in any form	A1	
		Obtain the given answer	A1	**[5]**
		[SR: If the complete equation is approached by adding two sectors to the shaded area above BO and OC give the first M1 as on the scheme, and the second M1 for using correct area formulae for a triangle AOB or AOC, and a sector AOB or AOC.]		
	(ii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 0.95	A1	
		Show sufficient iterations to 4 d.p. to justify 0.95 to 2 d.p., or show there is a sign change in the interval (0.945, 0.955)	A1	**[3]**

7	**(i)**	State or imply partial fractions are of the form $\dfrac{A}{x-2}+\dfrac{Bx+C}{x^2+3}$	B1	
		Use a relevant method to determine a constant	M1	
		Obtain one of the values $A=-1$, $B=3$, $C=-1$	A1	
		Obtain a second value	A1	
		Obtain the third value	A1	**[5]**
	(ii)	Use correct method to obtain the first two terms of the expansions of $(x-2)^{-1}$, $\left(1-\dfrac{1}{2}x\right)^{-1}$, $(x^2+3)^{-1}$ or $\left(1+\dfrac{1}{3}x^2\right)^{-1}$	M1	
		Substitute correct unsimplified expansions up to the term in x^2 into each partial fraction	A1√+A1√	
		Multiply out fully by $Bx + C$, where $BC \neq 0$	M1	
		Obtain final answer $\dfrac{1}{6}+\dfrac{5}{4}x+\dfrac{17}{72}x^2$, or equivalent	A1	**[5]**
		[Symbolic binomial coefficients, e.g. $\begin{pmatrix}-1\\1\end{pmatrix}$ are not sufficient for the M1. The f.t. is on *A, B, C*.]		
		[In the case of an attempt to expand $(2x^2-7x-1)(x-2)^{-1}(x^2+3)^{-1}$, give M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for the final answer.]		
		[If *B* or *C* omitted from the form of partial fractions, give B0M1A0A0A0 in **(i)**; M1A1√A1√ in **(ii)**]		
8	**(a)**	*EITHER:* Solve for u or for v	M1	
		Obtain $u=\dfrac{2\mathrm{i}-6}{1-2\mathrm{i}}$ or $v=\dfrac{5}{1-2\mathrm{i}}$, or equivalent	A1	
		Either: Multiply a numerator and denominator by conjugate of denominator, or equivalent		
		Or: Set u or v equal to $x + \mathrm{i}y$, obtain two equations by equating real and imaginary parts and solve for x or for y	M1	
		OR: Using $a + \mathrm{i}b$ and $c + \mathrm{i}d$ for u and v, equate real and imaginary parts and obtain four equations in a, b, c and d	M1	
		Obtain $b + 2d = 2$, $a + 2c = 0$, $a + d = 0$ and $-b + c = 3$, or equivalent	A1	
		Solve for one unknown	M1	
		Obtain final answer $u = -2 - 2\mathrm{i}$, or equivalent	A1	
		Obtain final answer $v = 1 + 2\mathrm{i}$, or equivalent	A1	**[5]**
	(b)	Show a circle with centre $-\mathrm{i}$	B1	
		Show a circle with radius 1	B1	
		Show correct half line from 2 at an angle of $\dfrac{3}{4}\pi$ to the real axis	B1	
		Use a correct method for finding the least value of the modulus	M1	
		Obtain final answer $\dfrac{3}{\sqrt{2}}-1$, or equivalent, e.g. 1.12 (allow 1.1)	A1	**[5]**

9	**(i)** *EITHER:*	Obtain a vector parallel to the plane, e.g. $\overrightarrow{AB} = -2\mathbf{i} + 4\mathbf{j} - \mathbf{k}$	B1	
		Use scalar product to obtain an equation in *a, b, c,* e.g. $-2a + 4b - c = 0$, $3a - 3b + 3c = 0$, or $a + b + 2c = 0$	M1	
		Obtain two correct equations in *a, b, c*	A1	
		Solve to obtain ratio $a : b : c$	M1	
		Obtain $a : b : c = 3 : 1 : -2$, or equivalent	A1	
		Obtain equation $3x + y - 2z = 1$, or equivalent	A1	
	OR1:	Substitute for two points, e.g. *A* and *B*, and obtain $2a - b + 2c = d$ and $3b + c = d$	B1	
		Substitute for another point, e.g. *C*, to obtain a third equation and eliminate one unknown entirely from the three equations	M1	
		Obtain two correct equations in three unknowns, e.g. in *a, b, c*	A1	
		Solve to obtain their ratio, e.g. $a : b : c$	M1	
		Obtain $a : b : c = 3 : 1 : -2$, $a : c : d = 3 : -2 : 1$, $a : b : d = 3 : 1 : 1$ or $b : c : d = -1 : -2 : 1$	A1	
		Obtain equation $3x + y - 2z = 1$, or equivalent	A1	
	OR2:	Obtain a vector parallel to the plane, e.g. $\overrightarrow{BC} = 3\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$	B1	
		Obtain a second such vector and calculate their vector product e.g. $(-2\mathbf{i} + 4\mathbf{j} - \mathbf{k}) \times (3\mathbf{i} - 3\mathbf{j} + 3\mathbf{k})$	M1	
		Obtain two correct components of the product	A1	
		Obtain correct answer, e.g. $9\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$	A1	
		Substitute in $9x + 3y - 6z = d$ to find d	M1	
		Obtain equation $9x + 3y - 6z = 3$, or equivalent	A1	
	OR3:	Obtain a vector parallel to the plane, e.g. $\overrightarrow{AC} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$	B1	
		Obtain a second such vector and form correctly a 2-parameter equation for the plane	M1	
		Obtain a correct equation, e.g. $\mathbf{r} = 3\mathbf{i} + 4\mathbf{k} + \lambda(-2\mathbf{i} + 4\mathbf{j} - \mathbf{k}) + \mu(\mathbf{i} + \mathbf{j} + 2\mathbf{k})$	A1	
		State three correct equations in x, y, z, λ, μ	A1	
		Eliminate λ and μ	M1	
		Obtain equation $3x + y - 2z = 1$, or equivalent	A1	**[6]**
	(ii)	Obtain answer $\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$, or equivalent	B1	**[1]**

(iii) *EITHER:* Use $\dfrac{\overrightarrow{OA}.\overrightarrow{OD}}{\left|\overrightarrow{OD}\right|}$ to find projection *ON* of *OA* onto *OD* M1

Obtain $ON = \dfrac{4}{3}$ A1

Use Pythagoras in triangle *OAN* to find *AN* M1

Obtain the given answer A1

OR1: Calculate the vector product of $\overrightarrow{OA}$ and $\overrightarrow{OD}$ M1

Obtain answer $6\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$ A1

Divide the modulus of the vector product by the modulus of $\overrightarrow{OD}$ M1

Obtain the given answer A1

OR2: Taking general point *P* of *OD* to have position vector $\lambda(\mathbf{i}+2\mathbf{j}+2\mathbf{k})$, form an equation in λ by either equating the scalar product of $\overrightarrow{AP}$ and $\overrightarrow{OP}$ to zero, or using Pythagoras in triangle *OPA,* or setting the derivative of $\left|\overrightarrow{AP}\right|$ to zero M1

Solve and obtain $\lambda = \dfrac{4}{9}$ A1

Carry out method to calculate *AP* when $\lambda = \dfrac{4}{9}$ M1

Obtain the given answer A1

OR3: Use a relevant scalar product to find the cosine of *AOD* or *ADO* M1

Obtain $\cos AOD = \dfrac{4}{9}$ or $\cos ADO = \dfrac{5}{3\sqrt{10}}$, or equivalent A1

Use trig to find the length of the perpendicular M1

Obtain the given answer A1

OR4: Use cosine formula in triangle *AOD* to find cos *AOD* or cos *ADO* M1

Obtain $\cos AOD = \dfrac{8}{18}$ or $\cos ADO = \dfrac{10}{6\sqrt{10}}$, or equivalent A1

Use trig to find the length of the perpendicular M1

Obtain the given answer A1 **[4]**

10 **(i)** State or imply $V = \pi h^3$ B1

State or imply $\dfrac{\mathrm{d}V}{\mathrm{d}t} = -k\sqrt{h}$ B1

Use $\dfrac{\mathrm{d}V}{\mathrm{d}t} = \dfrac{\mathrm{d}V}{\mathrm{d}h}.\dfrac{\mathrm{d}h}{\mathrm{d}t}$, or equivalent M1

Obtain the given equation A1 **[4]**

[The M1 is only available if $\dfrac{\mathrm{d}V}{\mathrm{d}h}$ is in terms of *h* and has been obtained by a correct method.]

[Allow B1 for $\dfrac{\mathrm{d}V}{\mathrm{d}t} = k\sqrt{h}$ but withhold the final A1 until the polarity of the constant $\dfrac{k}{3\pi}$ has been justified.]

(ii)	Separate variables and integrate at least one side	M1	
	Obtain terms $\frac{2}{5}h^{\frac{5}{2}}$ and $-At$, or equivalent	A1	
	Use $t = 0, h = H$ in a solution containing terms of the form $ah^{\frac{5}{2}}$ and $bt + c$	M1	
	Use $t = 60$, $h = 0$ in a solution containing terms of the form $ah^{\frac{5}{2}}$ and $bt + c$	M1	
	Obtain a correct solution in any form, e.g. $\frac{2}{5}h^{\frac{5}{2}} = \frac{1}{150}H^{\frac{5}{2}}t + \frac{2}{5}H^{\frac{5}{2}}$	A1	
(ii)	Obtain final answer $t = 60\left(1-\left(\frac{h}{H}\right)^{\frac{5}{2}}\right)$, or equivalent	A1	**[6]**
(iii)	Substitute $h = \frac{1}{2}H$ and obtain answer $t = 49.4$	B1	**[1]**

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the October/November 2013 series

9709 MATHEMATICS

9709/32 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2013 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

CAMBRIDGE
International Examinations

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through $\sqrt{}$" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation.
The PA –1 penalty is usually discussed at the meeting.

1		Use correct quotient or product rule	M1	
		Obtain correct derivative in any form	A1	
		Justify the given statement	A1	[3]
2	*EITHER:*	State or imply non-modular equation $2^2\left(3^x-1\right)^2=\left(3^x\right)^2$, or pair of equations $2\left(3^x-1\right)=\pm 3^x$	M1	
		Obtain $3^x=2$ and $3^x=\frac{2}{3}$ (or $3^{x+1}=2$)	A1	
	OR:	Obtain $3^x=2$ by solving an equation or by inspection	B1	
		Obtain $3^x=\frac{2}{3}$ (or $3^{x+1}=2$) by solving an equation or by inspection	B1	
		Use correct method for solving an equation of the form $3^x=a$ (or $3^{x+1}=a$), where $a>0$	M1	
		Obtain final answers 0.631 and –0.369	A1	[4]
3	*EITHER:*	Integrate by parts and reach $kx^{\frac{1}{2}}\ln x-m\int x^{\frac{1}{2}}.\frac{1}{x}\,\mathrm{d}x$	M1*	
		Obtain $2x^{\frac{1}{2}}\ln x-2\int\frac{1}{x^{\frac{1}{2}}}\,\mathrm{d}x$, or equivalent	A1	
		Integrate again and obtain $2x^{\frac{1}{2}}\ln x-4x^{\frac{1}{2}}$, or equivalent	A1	
		Substitute limits $x=1$ and $x=4$, having integrated twice	M1(dep*)	
		Obtain answer $4(\ln 4-1)$, or exact equivalent	A1	
	OR1:	Using $u=\ln x$, or equivalent, integrate by parts and reach $kue^{\frac{1}{2}u}-m\int e^{\frac{1}{2}u}\,\mathrm{d}u$	M1*	
		Obtain $2ue^{\frac{1}{2}u}-2\int e^{\frac{1}{2}u}\,\mathrm{d}u$, or equivalent	A1	
		Integrate again and obtain $2ue^{\frac{1}{2}u}-4e^{\frac{1}{2}u}$, or equivalent	A1	
		Substitute limits $u=0$ and $u=\ln 4$, having integrated twice	M1(dep*)	
		Obtain answer $4\ln 4-4$, or exact equivalent	A1	
	OR2:	Using $u=\sqrt{x}$, or equivalent, integrate and obtain $ku\ln u-m\int u.\frac{1}{u}\mathrm{d}u$	M1*	
		Obtain $4u\ln u-4\int 1\mathrm{d}u$, or equivalent	A1	
		Integrate again and obtain $4u\ln u-4u$, or equivalent	A1	
		Substitute limits $u=1$ and $u=2$, having integrated twice or quoted $\int\ln u\,\mathrm{d}u$ as $u\ln u\pm u$	M1(dep*)	
		Obtain answer $8\ln 2-4$, or exact equivalent	A1	
	OR3:	Integrate by parts and reach $I=\frac{x\ln x\pm x}{\sqrt{x}}+k\int\frac{x\ln x\pm x}{x\sqrt{x}}\,\mathrm{d}x$	M1*	
		Obtain $I=\frac{x\ln x-x}{\sqrt{x}}+\frac{1}{2}I-\frac{1}{2}\int\frac{1}{\sqrt{x}}\,\mathrm{d}x$	A1	
		Integrate and obtain $I=2\sqrt{x}\ln x-4\sqrt{x}$, or equivalent	A1	
		Substitute limits $x=1$ and $x=4$, having integrated twice	M1(dep*)	
		Obtain answer $4\ln 4-4$, or exact equivalent	A1	[5]

4		Use correct product or quotient rule at least once	M1*	
		Obtain $\dfrac{dx}{dt} = e^{-t}\sin t - e^{-t}\cos t$ or $\dfrac{dy}{dt} = e^{-t}\cos t - e^{-t}\sin t$, or equivalent	A1	
		Use $\dfrac{dy}{dx} = \dfrac{dy}{dt} \div \dfrac{dx}{dt}$	M1	
		Obtain $\dfrac{dy}{dx} = \dfrac{\sin t - \cos t}{\sin t + \cos t}$, or equivalent	A1	
		EITHER: Express $\dfrac{dy}{dx}$ in terms of tan t only	M1(dep*)	
		Show expression is identical to $\tan\left(t - \dfrac{1}{4}\pi\right)$	A1	
		OR: Express $\tan\left(t - \dfrac{1}{4}\pi\right)$ in terms of tan t	M1	
		Show expression is identical to $\dfrac{dy}{dx}$	A1	**[6]**
5	**(i)**	Use Pythagoras	M1	
		Use the sin2A formula	M1	
		Obtain the given result	A1	**[3]**
	(ii)	Integrate and obtain a $k\ln\sin\theta$ or $m\ln\cos\theta$ term, or obtain integral of the form $p\ln\tan\theta$	M1*	
		Obtain indefinite integral $\dfrac{1}{2}\ln\sin\theta - \dfrac{1}{2}\ln\cos\theta$, or equivalent, or $\dfrac{1}{2}\ln\tan\theta$	A1	
		Substitute limits correctly	M1(dep)*	
		Obtain the given answer correctly having shown appropriate working	A1	**[4]**
6	**(i)**	State or imply $AB = 2r\cos\theta$ or $AB^2 = 2r^2 - 2r^2\cos(\pi - 2\theta)$	B1	
		Use correct formula to express the area of sector ABC in terms of r and θ	M1	
		Use correct area formulae to express the area of a segment in terms of r and θ	M1	
		State a correct equation in r and θ in any form	A1	
		Obtain the given answer	A1	**[5]**
		[SR: If the complete equation is approached by adding two sectors to the shaded area above BO and OC give the first M1 as on the scheme, and the second M1 for using correct area formulae for a triangle AOB or AOC, and a sector AOB or AOC.]		
	(ii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 0.95	A1	
		Show sufficient iterations to 4 d.p. to justify 0.95 to 2 d.p., or show there is a sign change in the interval (0.945, 0.955)	A1	**[3]**

7	(i)	State or imply partial fractions are of the form $\frac{A}{x-2}+\frac{Bx+C}{x^2+3}$	B1	
		Use a relevant method to determine a constant	M1	
		Obtain one of the values $A=-1$, $B=3$, $C=-1$	A1	
		Obtain a second value	A1	
		Obtain the third value	A1	**[5]**
	(ii)	Use correct method to obtain the first two terms of the expansions of $(x-2)^{-1}$, $\left(1-\frac{1}{2}x\right)^{-1}$, $(x^2+3)^{-1}$ or $\left(1+\frac{1}{3}x^2\right)^{-1}$	M1	
		Substitute correct unsimplified expansions up to the term in x^2 into each partial fraction	A1√+A1√	
		Multiply out fully by $Bx+C$, where $BC \neq 0$	M1	
		Obtain final answer $\frac{1}{6}+\frac{5}{4}x+\frac{17}{72}x^2$, or equivalent	A1	**[5]**
		[Symbolic binomial coefficients, e.g. $\begin{pmatrix}-1\\1\end{pmatrix}$ are not sufficient for the M1. The f.t. is on *A, B, C.*]		
		[In the case of an attempt to expand $(2x^2-7x-1)(x-2)^{-1}(x^2+3)^{-1}$, give M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for the final answer.]		
		[If *B* or *C* omitted from the form of partial fractions, give B0M1A0A0A0 in **(i)**; M1A1√A1√ in **(ii)**]		

8	(a)	*EITHER:* Solve for u or for v	M1	
		Obtain $u=\frac{2i-6}{1-2i}$ or $v=\frac{5}{1-2i}$, or equivalent	A1	
		Either: Multiply a numerator and denominator by conjugate of denominator, or equivalent		
		Or: Set u or v equal to $x + iy$, obtain two equations by equating real and imaginary parts and solve for x or for y	M1	
		OR: Using $a + ib$ and $c + id$ for u and v, equate real and imaginary parts and obtain four equations in a, b, c and d	M1	
		Obtain $b+2d=2$, $a+2c=0$, $a+d=0$ and $-b+c=3$, or equivalent	A1	
		Solve for one unknown	M1	
		Obtain final answer $u=-2-2i$, or equivalent	A1	
		Obtain final answer $v=1+2i$, or equivalent	A1	**[5]**
	(b)	Show a circle with centre $-i$	B1	
		Show a circle with radius 1	B1	
		Show correct half line from 2 at an angle of $\frac{3}{4}\pi$ to the real axis	B1	
		Use a correct method for finding the least value of the modulus	M1	
		Obtain final answer $\frac{3}{\sqrt{2}}-1$, or equivalent, e.g. 1.12 (allow 1.1)	A1	**[5]**

9	(i)	*EITHER:*	Obtain a vector parallel to the plane, e.g. $\overrightarrow{AB} = -2\mathbf{i} + 4\mathbf{j} - \mathbf{k}$	B1	
			Use scalar product to obtain an equation in *a, b, c,* e.g. $-2a + 4b - c = 0$, $3a - 3b + 3c = 0$, or $a + b + 2c = 0$	M1	
			Obtain two correct equations in *a, b, c*	A1	
			Solve to obtain ratio $a : b : c$	M1	
			Obtain $a : b : c = 3 : 1 : -2$, or equivalent	A1	
			Obtain equation $3x + y - 2z = 1$, or equivalent	A1	
		OR1:	Substitute for two points, e.g. *A* and *B*, and obtain $2a - b + 2c = d$ and $3b + c = d$	B1	
			Substitute for another point, e.g. *C*, to obtain a third equation and eliminate one unknown entirely from the three equations	M1	
			Obtain two correct equations in three unknowns, e.g. in *a, b, c*	A1	
			Solve to obtain their ratio, e.g. $a : b : c$	M1	
			Obtain $a : b : c = 3 : 1 : -2$, $a : c : d = 3 : -2 : 1$, $a : b : d = 3 : 1 : 1$ or $b : c : d = -1 : -2 : 1$	A1	
			Obtain equation $3x + y - 2z = 1$, or equivalent	A1	
		OR2:	Obtain a vector parallel to the plane, e.g. $\overrightarrow{BC} = 3\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$	B1	
			Obtain a second such vector and calculate their vector product e.g. $(-2\mathbf{i} + 4\mathbf{j} - \mathbf{k}) \times (3\mathbf{i} - 3\mathbf{j} + 3\mathbf{k})$	M1	
			Obtain two correct components of the product	A1	
			Obtain correct answer, e.g. $9\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$	A1	
			Substitute in $9x + 3y - 6z = d$ to find d	M1	
			Obtain equation $9x + 3y - 6z = 3$, or equivalent	A1	
		OR3:	Obtain a vector parallel to the plane, e.g. $\overrightarrow{AC} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$	B1	
			Obtain a second such vector and form correctly a 2-parameter equation for the plane	M1	
			Obtain a correct equation, e.g. $\mathbf{r} = 3\mathbf{i} + 4\mathbf{k} + \lambda(-2\mathbf{i} + 4\mathbf{j} - \mathbf{k}) + \mu(\mathbf{i} + \mathbf{j} + 2\mathbf{k})$	A1	
			State three correct equations in x, y, z, λ, μ	A1	
			Eliminate λ and μ	M1	
			Obtain equation $3x + y - 2z = 1$, or equivalent	A1	[6]
	(ii)		Obtain answer $\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$, or equivalent	B1	[1]

(iii) *EITHER:* Use $\dfrac{\overrightarrow{OA}.\overrightarrow{OD}}{\left|\overrightarrow{OD}\right|}$ to find projection *ON* of *OA* onto *OD* — M1

Obtain $ON = \dfrac{4}{3}$ — A1

Use Pythagoras in triangle *OAN* to find *AN* — M1

Obtain the given answer — A1

OR1: Calculate the vector product of $\overrightarrow{OA}$ and $\overrightarrow{OD}$ — M1

Obtain answer $6\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$ — A1

Divide the modulus of the vector product by the modulus of $\overrightarrow{OD}$ — M1

Obtain the given answer — A1

OR2: Taking general point *P* of *OD* to have position vector $\lambda(\mathbf{i}+2\mathbf{j}+2\mathbf{k})$, form an equation in λ by either equating the scalar product of $\overrightarrow{AP}$ and $\overrightarrow{OP}$ to zero, or using Pythagoras in triangle *OPA,* or setting the derivative of $\left|\overrightarrow{AP}\right|$ to zero — M1

Solve and obtain $\lambda = \dfrac{4}{9}$ — A1

Carry out method to calculate *AP* when $\lambda = \dfrac{4}{9}$ — M1

Obtain the given answer — A1

OR3: Use a relevant scalar product to find the cosine of *AOD* or *ADO* — M1

Obtain $\cos AOD = \dfrac{4}{9}$ or $\cos ADO = \dfrac{5}{3\sqrt{10}}$, or equivalent — A1

Use trig to find the length of the perpendicular — M1

Obtain the given answer — A1

OR4: Use cosine formula in triangle *AOD* to find cos *AOD* or cos *ADO* — M1

Obtain $\cos AOD = \dfrac{8}{18}$ or $\cos ADO = \dfrac{10}{6\sqrt{10}}$, or equivalent — A1

Use trig to find the length of the perpendicular — M1

Obtain the given answer — A1 **[4]**

10 (i) State or imply $V = \pi h^3$ — B1

State or imply $\dfrac{dV}{dt} = -k\sqrt{h}$ — B1

Use $\dfrac{dV}{dt} = \dfrac{dV}{dh}.\dfrac{dh}{dt}$, or equivalent — M1

Obtain the given equation — A1 **[4]**

[The M1 is only available if $\dfrac{dV}{dh}$ is in terms of *h* and has been obtained by a correct method.]

[Allow B1 for $\dfrac{dV}{dt} = k\sqrt{h}$ but withhold the final A1 until the polarity of the constant $\dfrac{k}{3\pi}$ has been justified.]

(ii)	Separate variables and integrate at least one side	M1	
	Obtain terms $\frac{2}{5}h^{\frac{5}{2}}$ and $-At$, or equivalent	A1	
	Use $t = 0, h = H$ in a solution containing terms of the form $ah^{\frac{5}{2}}$ and $bt + c$	M1	
	Use $t = 60$, $h = 0$ in a solution containing terms of the form $ah^{\frac{5}{2}}$ and $bt + c$	M1	
	Obtain a correct solution in any form, e.g. $\frac{2}{5}h^{\frac{5}{2}} = \frac{1}{150}H^{\frac{5}{2}}t + \frac{2}{5}H^{\frac{5}{2}}$	A1	
(ii)	Obtain final answer $t = 60\left(1 - \left(\frac{h}{H}\right)^{\frac{5}{2}}\right)$, or equivalent	A1	**[6]**
(iii)	Substitute $h = \frac{1}{2}H$ and obtain answer $t = 49.4$	B1	**[1]**

1	Apply at least one logarithm property correctly	*M1	
	Obtain $\frac{(x+4)^2}{x} = x + a$ or equivalent **without logarithm** involved	A1	
	Rearrange to express x in terms of a	M1	d*M
	Obtain $\frac{16}{a-8}$ or equivalent	A1	[4]
2	Carry out complete substitution including the use of $\frac{\mathrm{d}u}{\mathrm{d}x} = 3$	M1	
	Obtain $\int \left(\frac{1}{3} - \frac{1}{3u} \right) \mathrm{d}u$	A1	
	Integrate to obtain form $k_1 u + k_2 \ln u$ or $k_1 u + k_2 \ln 3u$ where $k_1 k_2 \neq 0$	M1	
	Obtain $\frac{1}{3}(3x+1) - \frac{1}{3}\ln(3x+1)$ or equivalent, condoning absence of modulus signs and + c	A1	[4]
3	**(i)** Substitute –2 and equate to zero or divide by $x + 2$ and equate remainder to zero or use –2 in synthetic division	M1	
	Obtain $a = -1$	A1	[2]
	(ii) Attempt to find quadratic factor by division reaching $x^2 + kx$, or inspection as far as $(x+2)(x^2 + Bx + c)$ and equations for one or both of B and C, or $(x+2)(Ax^2 + Bx + 7)$ and equations for one or both of A and B.	M1	
	Obtain $x^2 - 3x + 7$	A1	
	Use discriminant to obtain –19, or equivalent, and **confirm one root** cwo	A1	[3]
4	Differentiate y^3 to obtain $3y^2 \frac{\mathrm{d}y}{\mathrm{d}x}$	B1	
	Use correct product rule at least once	*M1	
	Obtain $6\mathrm{e}^{2x} y + 3\mathrm{e}^{2x} \frac{\mathrm{d}y}{\mathrm{d}x} + \mathrm{e}^x y^3 + 3\mathrm{e}^x y^2 \frac{\mathrm{d}y}{\mathrm{d}x}$ as derivative of LHS	A1	
	Equate derivative of LHS to zero, substitute $x = 0$ and $y = 2$ and find value of $\frac{\mathrm{d}y}{\mathrm{d}x}$	M1(d*M)	
	Obtain $-\frac{4}{3}$ or equivalent as **final answer**	A1	[5]
5	**(i)** Use integration by parts to obtain $ax\mathrm{e}^{-\frac{1}{2}x} + \int b\mathrm{e}^{-\frac{1}{2}x} \mathrm{d}x$	M1*	
	Obtain $-8x\mathrm{e}^{-\frac{1}{2}x} + \int 8\mathrm{e}^{-\frac{1}{2}x} \mathrm{d}x$ or unsimplified equivalent	A1	
	Obtain $-8x\mathrm{e}^{-\frac{1}{2}x} - 16\mathrm{e}^{-\frac{1}{2}x}$	A1	
	Use limits correctly and equate to 9	M1(d*M)	
	Obtain given answer $p = 2\ln\left(\frac{8p+16}{7}\right)$ correctly	A1	[5]

	(ii)	Use correct iteration formula correctly at least once		M1	
		Obtain final answer 3.77		A1	
		Show sufficient iterations to 5sf or better to justify accuracy 3.77 or show sign change in interval (3.765, 3.775)		A1	[3]
		$[\,3.5 \to 3.6766 \to 3.7398 \to 3.7619 \to 3.7696 \to 3.7723\,]$			
6	(i)	Find scalar product of the normals to the planes		M1	
		Using the correct process for the moduli, divide the scalar product by the product of the moduli and find $\cos^{-1}$ of the result.		M1	
		Obtain 67.8° (or 1.18 radians)		A1	[3]
	(ii)	EITHER	Carry out complete method for finding point on line	M1	
			Obtain one such point, e.g. $(2,-3,0)$ or $\left(\frac{17}{7},0,\frac{6}{7}\right)$ or $(0,-17,-4)$ or …	A1…	
			Either State $3a-b+2c=0$ and $a+b-4c=0$ or equivalent	B1	
			Attempt to solve for one ratio, e.g. $a:b$	M1	
			Obtain $a:b:c=1:7:2$ or equivalent	A1	
			State a correct final answer, e.g. r = [2, –3, 0] + λ[1, 7, 2]	A1√	
			Or 1 Obtain a second point on the line	A1	
			Subtract position vectors to obtain direction vector	M1	
			Obtain [1, 7, 2] or equivalent	A1	
			State a correct final answer, e.g. r = [2, –3, 0] + λ[1, 7, 2]	A1√	
			Or 2 Use correct method to calculate vector product of two normals	M1	
			Obtain two correct components	A1	
			Obtain [2, 14, 4] or equivalent	A1	
			State a correct final answer, e.g. r = [2, –3, 0] + λ[1, 7, 2]	A1√	
			[√ is dependent on both M marks in all three cases]		
		OR 3	Express one variable in terms of a second variable	M1	
			Obtain a correct simplified expression, e.g. $x=\frac{1}{2}(4+z)$	A1	
			Express the first variable in terms of third variable	M1	
			Obtain a correct simplified expression, e.g. $x=\frac{1}{7}(17+y)$	A1	
			Form a vector equation for the line	M1	
			State a correct final answer, e.g. r = [0, –17, –4] + λ[1, 7, 2]	A1	
		OR 4	Express one variable in terms of a second variable	M1	
			Obtain a correct simplified expression, e.g. $z=2x-4$	A1	
			Express third variable in terms of the second variable	M1	
			Obtain a correct simplified expression, e.g. $y=7x-17$	A1	
			Form a vector equation for the line	M1	
			State a correct final answer, e.g. r = [0, –17, –4] + λ[1, 7, 2]	A1	[6]

7	**(i)**	Use $\sec\theta = \dfrac{1}{\cos\theta}$ and $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$		B1	
		Use $\sin 2\theta = 2\sin\theta\cos\theta$ and to form a horizontal equation in $\sin\theta$ and $\cos\theta$ or fractions with common denominators		M1	
		Obtain given equation $2\sin\theta + 4\cos\theta = 3$ correctly		A1	[3]
	(ii)	State or imply $R = \sqrt{20}$ or 4.47 or equivalent		B1	
		Use correct trigonometry to find α		M1	
		Obtain 63.43 or 63.44 with no errors seen		A1	[3]
	(iii)	Carry out a correct method to find one value in given range		M1	
		Obtain 74.4° (or 338.7°)		A1	
		Carry out a correct method to find second value in given range		M1	
		Obtain 338.7° (or 74.4°) and no others between 0° and 360°		A1	[4]
8	**(i)**	<u>Either</u>	State or imply form $\dfrac{A}{1+x} + \dfrac{B}{(1+x)^2} + \dfrac{C}{2-3x}$	B1	
			Use any relevant method to find at least one constant	M1	
			Obtain $A = -1$	A1	
			Obtain $B = 3$	A1	
			Obtain $C = 4$	A1	
		<u>Or</u>	State or imply form $\dfrac{A}{1+x} + \dfrac{Bx}{(1+x)^2} + \dfrac{C}{2-3x}$	B1	
			Use any relevant method to find at least one constant	M1	
			Obtain $A = 2$	A1	
			Obtain $B = -3$	A1	
			Obtain $C = 4$	A1	
		<u>Or</u>	State or imply form $\dfrac{Dx+E}{(1+x)^2} + \dfrac{F}{2-3x}$	B1	
			Use any relevant method to find at least one constant	M1	
			Obtain $D = -1$	A1	
			Obtain $E = 2$	A1	
			Obtain $F = 4$	A1	[5]

(ii) Either Use correct method to find first two terms of expansion of $(1+x)^{-1}$ or $(1+x)^{-2}$ or $(2-3x)^{-1}$ or $\left(1-\frac{3}{2}x\right)^{-1}$ M1

Obtain correct unsimplified expansion of first partial fraction up to x^2 term A1$\sqrt{}$

Obtain correct unsimplified expansion of second partial fraction up to x^2 term A1$\sqrt{}$

Obtain correct unsimplified expansion of third partial fraction up to x^2 term A1$\sqrt{}$

Obtain final answer $4-2x+\frac{25}{2}x^2$ A1

Or 1 Use correct method to find first two terms of expansion of $(1+x)^{-2}$ or $(2-3x)^{-1}$ or $\left(1-\frac{3}{2}x\right)^{-1}$ M1

Obtain correct unsimplified expansion of first partial fraction up to x^2 term A1$\sqrt{}$

Obtain correct unsimplified expansion of second partial fraction up to x^2 term A1$\sqrt{}$

Expand and obtain sufficient terms to obtain three terms M1

Obtain final answer $4-2x+\frac{25}{2}x^2$ A1

Or 2 (expanding original expression)

Use correct method to find first two terms of expansion of $(1+x)^{-2}$ or $(2-3x)^{-1}$ or $\left(1-\frac{3}{2}x\right)^{-1}$ M1

Obtain correct expansion $1-2x+3x^2$ or unsimplified equivalent A1

Obtain correct expansion $\frac{1}{2}\left(1+\frac{3}{2}x+\frac{9}{4}x^2\right)$ or unsimplified equivalent A1

Expand and obtain sufficient terms to obtain three terms M1

Obtain final answer $4-2x+\frac{25}{2}x^2$ A1

Or 3 (McLaurin expansion)

Obtain first derivative $f'(x)=(1+x)^{-2}-6(1+x)^{-3}+12(2-3x)^{-2}$ M1

Obtain $f'(0)=1-6+3$ or equivalent A1

Obtain $f''(0)=-2+18+9$ or equivalent A1

Use correct form for McLaurin expansion M1

Obtain final answer $4-2x+\frac{25}{2}x^2$ A1 [5]

9 **(a)** Solve using formula, including simplification under square root sign M1*

Obtain $\frac{-2\pm 4\mathrm{i}}{2(2-\mathrm{i})}$ or similarly simplified equivalents A1

Multiply by $\frac{2+\mathrm{i}}{2+\mathrm{i}}$ or equivalent in at least one case M1(d*M)

Obtain final answer $-\frac{4}{5}+\frac{3}{5}\mathrm{i}$ A1

Obtain final answer –i A1 [5]

(b)	Show w in first quadrant with modulus and argument relatively correct	B1	
	Show w^3 in second quadrant with modulus and argument relatively correct	B1	
	Show w^* in fourth quadrant with modulus and argument relatively correct	B1	
	Use correct method for area of triangle	M1	
	Obtain 10 by calculation	A1	[5]

10	Use $2\cos^2 x = 1 + \cos 2x$ or equivalent	B1	
	Separate variables and integrate at least one side	M1	
	Obtain $\ln\left(y^3+1\right) = \ldots$ or equivalent	A1	
	Obtain $\ldots = 2x + \sin 2x$ or equivalent	A1	
	Use $x = 0$, $y = 2$ to find constant of integration (or as limits) in an expression containing at least two terms of the form $a\ln\left(y^3+1\right)$, bx or $c\sin 2x$	M1*	
	Obtain $\ln\left(y^3+1\right) = 2x + \sin 2x + \ln 9$ or equivalent e.g. implied by correct constant	A1	
	Identify at least one of $\frac{1}{2}\pi$ and $\frac{3}{2}\pi$ as x-coordinate at stationary point	B1	
	Use correct process to find y-coordinate for at least one x-coordinate	M1(d*M)	
	Obtain 5.9	A1	
	Obtain 48.1	A1	[10]

1		Use law of the logarithm of a power	M1	
		Obtain a correct linear equation in any form, e.g. $x = (x-2)\ln 3$	A1	
		Obtain answer $x = 22.281$	A1	**[3]**
2	**(i)**	State or imply ordinates 2, 1.1547…, 1, 1.1547…	B1	
		Use correct formula, or equivalent, with $h = \frac{1}{6}\pi$ and four ordinates	M1	
		Obtain answer 1.95	A1	**[3]**
	(ii)	Make recognisable sketch of $y = \operatorname{cosec} x$ for the given interval	B1	
		Justify a statement that the estimate will be an overestimate	B1	**[2]**
3		Substitute $x = -\frac{1}{3}$, equate result to zero or divide by $3x + 1$ and equate the remainder to zero and obtain a correct equation, e.g. $-\frac{1}{27}a + \frac{1}{9}b - \frac{1}{3} + 3 = 0$	B1	
		Substitute $x = 2$ and equate result to 21 or divide by $x - 2$ and equate constant remainder to 21	M1	
		Obtain a correct equation, e.g. $8a + 4b + 5 = 21$	A1	
		Solve for a or for b	M1	
		Obtain $a = 12$ and $b = -20$	A1	**[5]**
4	**(i)**	Use chain rule correctly at least once	M1	
		Obtain either $\frac{dx}{dt} = \frac{3\sin t}{\cos^4 t}$ or $\frac{dy}{dt} = 3\tan^2 t\sec^2 t$, or equivalent	A1	
		Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
		Obtain the given answer	A1	**[4]**
	(ii)	State a correct equation for the tangent in any form	B1	
		Use Pythagoras	M1	
		Obtain the given answer	A1	**[3]**
5	**(i)**	Substitute $z = 1 + i$ and obtain $w = \frac{1+2i}{1+i}$	B1	
		EITHER: Multiply numerator and denominator by the conjugate of the denominator, or equivalent	M1	
		Simplify numerator to $3 + i$ or denominator to 2	A1	
		Obtain final answer $\frac{3}{2} + \frac{1}{2}i$, or equivalent	A1	
		OR: Obtain two equations in x and y, and solve for x or for y	M1	
		Obtain $x = \frac{3}{2}$ or $y = \frac{1}{2}$, or equivalent	A1	
		Obtain final answer $\frac{3}{2} + \frac{1}{2}i$, or equivalent	A1	**[4]**

	(ii)	*EITHER*: Substitute $w = z$ and obtain a 3-term quadratic equation in z, e.g. $\mathrm{i}z^2 + z - \mathrm{i} = 0$	B1	
		Solve a 3-term quadratic for z or substitute $z = x + \mathrm{i}y$ and use a correct method to solve for x and y	M1	
		OR: Substitute $w = x + \mathrm{i}y$ and obtain two correct equations in x and y by equating real and imaginary parts	B1	
		Solve for x and y	M1	
		Obtain a correct solution in any form, e.g. $z = \dfrac{-1 \pm \sqrt{3}\,\mathrm{i}}{2\mathrm{i}}$	A1	
		Obtain final answer $-\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}\mathrm{i}$	A1	[4]
6	(i)	Integrate and reach $bx\ln 2x - c\displaystyle\int x.\frac{1}{x}\,\mathrm{d}x$, or equivalent	M1*	
		Obtain $x\ln 2x - \displaystyle\int x.\frac{1}{x}\,\mathrm{d}x$, or equivalent	A1	
		Obtain integral $x\ln 2x - x$, or equivalent	A1	
		Substitute limits correctly and equate to 1, having integrated twice	M1(dep*)	
		Obtain a correct equation in any form, e.g. $a\ln 2a - a + 1 - \ln 2 = 1$	A1	
		Obtain the given answer	A1	[6]
	(ii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.94	A1	
		Show sufficient iterations to 4 d.p. to justify 1.94 to 2d.p. or show that there is a sign change in the interval (1.935, 1.945).	A1	[3]
7	(i)	Separate variables correctly and attempt to integrate at least one side	B1	
		Obtain term $\ln R$	B1	
		Obtain $\ln x - 0.57x$	B1	
		Evaluate a constant or use limits $x = 0.5$, $R = 16.8$, in a solution containing terms of the form $a\ln R$ and $b\ln x$	M1	
		Obtain correct solution in any form	A1	
		Obtain a correct expression for R, e.g. $R = xe^{(3.80 - 0.57x)}$, $R = 44.7xe^{-0.57x}$ or $R = 33.6xe^{(0.285 - 0.57x)}$	A1	[6]
	(ii)	Equate $\dfrac{\mathrm{d}R}{\mathrm{d}x}$ to zero and solve for x	M1	
		State or imply $x = 0.57^{-1}$, or equivalent, e.g. 1.75	A1	
		Obtain $R = 28.8$ (allow 28.9)	A1	[3]
8	(i)	Use $\sin(A + B)$ formula to express $\sin 3\theta$ in terms of trig. functions of 2θ and θ	M1	
		Use correct double angle formulae and Pythagoras to express $\sin 3\theta$ in terms of $\sin\theta$	M1	
		Obtain a correct expression in terms of $\sin\theta$ in any form	A1	
		Obtain the given identity	A1	[4]
		[SR: Give M1 for using correct formulae to express RHS in terms of $\sin\theta$ and $\cos 2\theta$, then M1A1 for expressing in terms of $\sin\theta$ and $\sin 3\theta$ only, or in terms of $\cos\theta$, $\sin\theta$, $\cos 2\theta$ and $\sin 2\theta$, then A1 for obtaining the given identity.]		

	(ii)	Substitute for x and obtain the given answer	B1	**[1]**
	(iii)	Carry out a correct method to find a value of x	M1	
		Obtain answers 0.322, 0.799, –1.12	A1 + A1 + A1	**[4]**
		[Solutions with more than 3 answers can only earn a maximum of A1 + A1.]		
9	**(i)**	State or imply the form $\dfrac{A}{1-x}+\dfrac{B}{2-x}+\dfrac{C}{(2-x)^2}$	B1	
		Use a correct method to determine a constant	M1	
		Obtain one of $A = 2$, $B = -1$, $C = 3$	A1	
		Obtain a second value	A1	
		Obtain a third value	A1	**[5]**
		[The alternative form $\dfrac{A}{1-x}+\dfrac{Dx+E}{(2-x)^2}$, where $A = 2$, $D = 1$, $E = 1$ is marked B1M1A1A1A1 as above.]		
	(ii)	Use correct method to find the first two terms of the expansion of $(1-x)^{-1}$, $(2-x)^{-1}$, $(2-x)^{-2}$, $(1-\frac{1}{2}x)^{-1}$ or $(1-\frac{1}{2}x)^{-2}$	M1	
		Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	A1✓ + A1✓ + A1✓	
		Obtain final answer $\frac{9}{4}+\frac{5}{2}x+\frac{39}{16}x^2$, or equivalent	A1	**[5]**
		[Symbolic binomial coefficients, e.g. $\begin{pmatrix}-1\\1\end{pmatrix}$ are not sufficient for M1. The ✓ is on A,B,C.]		
		[For the A,D,E form of partial fractions, give M1 A1✓ A1✓ for the expansions then, if $D \neq 0$, M1 for multiplying out fully and A1 for the final answer.]		
		[In the case of an attempt to expand $(x^2-8x+9)(1-x)^{-1}(2-x)^{-2}$, give M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for the final answer.]		

10	**(i)**	*EITHER*:	Find $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) for a point P on l with parameter λ, e.g. $\mathbf{i}-17\mathbf{j}+4\mathbf{k}+\lambda(-2\mathbf{i}+\mathbf{j}-2\mathbf{k})$	B1
			Calculate scalar product of $\overrightarrow{AP}$ and a direction vector for l and equate to zero	M1
			Solve and obtain $\lambda = 3$	A1
			Carry out a complete method for finding the length of AP	M1
			Obtain the given answer 15 correctly	A1
		*OR*1:	Calling (4, –9, 9) B, state $\overrightarrow{BA}$ (or $\overrightarrow{AB}$) in component form, e.g. $-\mathbf{i}+17\mathbf{j}-4\mathbf{k}$	B1
			Calculate vector product of $\overrightarrow{BA}$ and a direction vector for l, e.g. $(-\mathbf{i}+17\mathbf{j}-4\mathbf{k})\times(-2\mathbf{i}+\mathbf{j}-2\mathbf{k})$	M1
			Obtain correct answer, e.g. $-30\mathbf{i}+6\mathbf{j}+33\mathbf{k}$	A1
			Divide the modulus of the product by that of the direction vector	M1
			Obtain the given answer correctly	A1
		*OR*2:	State $\overrightarrow{BA}$ (or $\overrightarrow{AB}$) in component form	B1
			Use a scalar product to find the projection of BA (or AB) on l	M1
			Obtain correct answer in any form, e.g. $\dfrac{27}{\sqrt{9}}$	A1
			Use Pythagoras to find the perpendicular	M1

	Obtain the given answer correctly	A1	
*OR*3:	State $\overrightarrow{BA}$ (or $\overrightarrow{AB}$) in component form	B1	
	Use a scalar product to find the cosine of *ABP*	M1	
	Obtain correct answer in any form, e.g. $\dfrac{27}{\sqrt{9}.\sqrt{306}}$	A1	
	Use trig. to find the perpendicular	M1	
	Obtain the given answer correctly	A1	
*OR*4:	State $\overrightarrow{BA}$ (or $\overrightarrow{AB}$) in component form	B1	
	Find a second point *C* on *l* and use the cosine rule in triangle *ABC* to find the cosine of angle *A, B*, or *C*, or use a vector product to find the area of *ABC*	M1	
	Obtain correct answer in any form	A1	
	Use trig. or area formula to find the perpendicular	M1	
	Obtain the given answer correctly	A1	
*OR*5:	State correct $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) for a point *P* on *l* with parameter λ in any form	B1	
	Use correct method to express AP^2 (or *AP*) in terms of λ	M1	
	Obtain a correct expression in any form, e.g. $(1-2\lambda)^2+(-17+\lambda)^2+(4-2\lambda)^2$	A1	
	Carry out a method for finding its minimum (using calculus, algebra or Pythagoras)	M1	
	Obtain the given answer correctly	A1	**[5]**
(**ii**) *EITHER*:	Substitute coordinates of a general point of l in equation of plane and either equate constant terms or equate the coefficient of λ to zero, obtaining an equation in *a* and *b*	M1*	
	Obtain a correct equation, e.g. $4a-9b-27+1=0$	A1	
	Obtain a second correct equation, e.g. $-2a+b+6=0$	A1	
	Solve for *a* or for *b*	M1(dep*)	
	Obtain $a = 2$ and $b = -2$	A1	
OR:	Substitute coordinates of a point of *l* and obtain a correct equation, e.g. $4a-9b=26$	B1	
	EITHER: Find a second point on *l* and obtain an equation in *a* and *b*	M1*	
	Obtain a correct equation	A1	
	OR: Calculate scalar product of a direction vector for *l* and a vector normal to the plane and equate to zero	M1*	
	Obtain a correct equation, e.g. $-2a+b+6=0$	A1	
	Solve for *a* or for *b*	M1(dep*)	
	Obtain $a = 2$ and $b = -2$	A1	**[5]**

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Level

MARK SCHEME for the October/November 2014 series

9709 MATHEMATICS

9709/32 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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CAMBRIDGE
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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

<u>Penalties</u>

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through $\sqrt{}$" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1		Use law of the logarithm of a power	M1	
		Obtain a correct linear equation in any form, e.g. $x = (x-2)\ln 3$	A1	
		Obtain answer $x = 22.281$	A1	**[3]**
2	(**i**)	State or imply ordinates 2, 1.1547…, 1, 1.1547…	B1	
		Use correct formula, or equivalent, with $h = \frac{1}{6}\pi$ and four ordinates	M1	
		Obtain answer 1.95	A1	**[3]**
	(**ii**)	Make recognisable sketch of $y = \operatorname{cosec} x$ for the given interval	B1	
		Justify a statement that the estimate will be an overestimate	B1	**[2]**
3		Substitute $x = -\frac{1}{3}$, equate result to zero or divide by $3x + 1$ and equate the remainder to zero and obtain a correct equation, e.g. $-\frac{1}{27}a + \frac{1}{9}b - \frac{1}{3} + 3 = 0$	B1	
		Substitute $x = 2$ and equate result to 21 or divide by $x - 2$ and equate constant remainder to 21	M1	
		Obtain a correct equation, e.g. $8a + 4b + 5 = 21$	A1	
		Solve for a or for b	M1	
		Obtain $a = 12$ and $b = -20$	A1	**[5]**
4	(**i**)	Use chain rule correctly at least once	M1	
		Obtain either $\frac{dx}{dt} = \frac{3\sin t}{\cos^4 t}$ or $\frac{dy}{dt} = 3\tan^2 t \sec^2 t$, or equivalent	A1	
		Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
		Obtain the given answer	A1	**[4]**
	(**ii**)	State a correct equation for the tangent in any form	B1	
		Use Pythagoras	M1	
		Obtain the given answer	A1	**[3]**
5	(**i**)	Substitute $z = 1 + \mathrm{i}$ and obtain $w = \frac{1+2\mathrm{i}}{1+\mathrm{i}}$	B1	
		EITHER: Multiply numerator and denominator by the conjugate of the denominator, or equivalent	M1	
		Simplify numerator to $3 + \mathrm{i}$ or denominator to 2	A1	
		Obtain final answer $\frac{3}{2} + \frac{1}{2}\mathrm{i}$, or equivalent	A1	
		OR: Obtain two equations in x and y, and solve for x or for y	M1	
		Obtain $x = \frac{3}{2}$ or $y = \frac{1}{2}$, or equivalent	A1	
		Obtain final answer $\frac{3}{2} + \frac{1}{2}\mathrm{i}$, or equivalent	A1	**[4]**

	(ii)	*EITHER*: Substitute $w = z$ and obtain a 3-term quadratic equation in z, e.g. $\mathrm{i}z^2 + z - \mathrm{i} = 0$	B1	
		Solve a 3-term quadratic for z or substitute $z = x + \mathrm{i}y$ and use a correct method to solve for x and y	M1	
		OR: Substitute $w = x + \mathrm{i}y$ and obtain two correct equations in x and y by equating real and imaginary parts	B1	
		Solve for x and y	M1	
		Obtain a correct solution in any form, e.g. $z = \dfrac{-1 \pm \sqrt{3}\,\mathrm{i}}{2\mathrm{i}}$	A1	
		Obtain final answer $-\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}\mathrm{i}$	A1	**[4]**
6	**(i)**	Integrate and reach $bx\ln 2x - c\int x.\dfrac{1}{x}\,\mathrm{d}x$, or equivalent	M1*	
		Obtain $x\ln 2x - \int x.\dfrac{1}{x}\,\mathrm{d}x$, or equivalent	A1	
		Obtain integral $x\ln 2x - x$, or equivalent	A1	
		Substitute limits correctly and equate to 1, having integrated twice	M1(dep*)	
		Obtain a correct equation in any form, e.g. $a\ln 2a - a + 1 - \ln 2 = 1$	A1	
		Obtain the given answer	A1	**[6]**
	(ii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.94	A1	
		Show sufficient iterations to 4 d.p. to justify 1.94 to 2d.p. or show that there is a sign change in the interval (1.935, 1.945).	A1	**[3]**
7	**(i)**	Separate variables correctly and attempt to integrate at least one side	B1	
		Obtain term $\ln R$	B1	
		Obtain $\ln x - 0.57x$	B1	
		Evaluate a constant or use limits $x = 0.5$, $R = 16.8$, in a solution containing terms of the form $a\ln R$ and $b\ln x$	M1	
		Obtain correct solution in any form	A1	
		Obtain a correct expression for R, e.g. $R = xe^{(3.80 - 0.57x)}$, $R = 44.7xe^{-0.57x}$ or $R = 33.6xe^{(0.285 - 0.57x)}$	A1	**[6]**
	(ii)	Equate $\dfrac{\mathrm{d}R}{\mathrm{d}x}$ to zero and solve for x	M1	
		State or imply $x = 0.57^{-1}$, or equivalent, e.g. 1.75	A1	
		Obtain $R = 28.8$ (allow 28.9)	A1	**[3]**
8	**(i)**	Use $\sin(A + B)$ formula to express $\sin 3\theta$ in terms of trig. functions of 2θ and θ	M1	
		Use correct double angle formulae and Pythagoras to express $\sin 3\theta$ in terms of $\sin\theta$	M1	
		Obtain a correct expression in terms of $\sin\theta$ in any form	A1	
		Obtain the given identity	A1	**[4]**
		[SR: Give M1 for using correct formulae to express RHS in terms of $\sin\theta$ and $\cos 2\theta$, then M1A1 for expressing in terms of $\sin\theta$ and $\sin 3\theta$ only, or in terms of $\cos\theta$, $\sin\theta$, $\cos 2\theta$ and $\sin 2\theta$, then A1 for obtaining the given identity.]		

(ii) Substitute for x and obtain the given answer — B1 **[1]**

(iii) Carry out a correct method to find a value of x — M1

Obtain answers 0.322, 0.799, –1.12 — A1 + A1 + A1 **[4]**

[Solutions with more than 3 answers can only earn a maximum of A1 + A1.]

9 (i) State or imply the form $\frac{A}{1-x}+\frac{B}{2-x}+\frac{C}{(2-x)^2}$ — B1

Use a correct method to determine a constant — M1

Obtain one of $A = 2$, $B = -1$, $C = 3$ — A1

Obtain a second value — A1

Obtain a third value — A1 **[5]**

[The alternative form $\frac{A}{1-x}+\frac{Dx+E}{(2-x)^2}$, where $A = 2$, $D = 1$, $E = 1$ is marked B1M1A1A1A1 as above.]

(ii) Use correct method to find the first two terms of the expansion of $(1-x)^{-1}$, $(2-x)^{-1}$, $(2-x)^{-2}$, $(1-\frac{1}{2}x)^{-1}$ or $(1-\frac{1}{2}x)^{-2}$ — M1

Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction — A1✓ + A1✓ + A1✓

Obtain final answer $\frac{9}{4}+\frac{5}{2}x+\frac{39}{16}x^2$, or equivalent — A1 **[5]**

[Symbolic binomial coefficients, e.g. $\begin{pmatrix}-1\\1\end{pmatrix}$ are not sufficient for M1. The ✓ is on A,B,C.]

[For the A,D,E form of partial fractions, give M1 A1✓ A1✓ for the expansions then, if $D \neq 0$, M1 for multiplying out fully and A1 for the final answer.]

[In the case of an attempt to expand $(x^2-8x+9)(1-x)^{-1}(2-x)^{-2}$, give M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for the final answer.]

10 (i) *EITHER*: Find $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) for a point P on l with parameter λ, e.g. $\mathbf{i}-17\mathbf{j}+4\mathbf{k}+\lambda(-2\mathbf{i}+\mathbf{j}-2\mathbf{k})$ — B1

Calculate scalar product of $\overrightarrow{AP}$ and a direction vector for l and equate to zero — M1

Solve and obtain $\lambda = 3$ — A1

Carry out a complete method for finding the length of AP — M1

Obtain the given answer 15 correctly — A1

*OR*1: Calling (4, –9, 9) B, state $\overrightarrow{BA}$ (or $\overrightarrow{AB}$) in component form, e.g. $-\mathbf{i}+17\mathbf{j}-4\mathbf{k}$ — B1

Calculate vector product of $\overrightarrow{BA}$ and a direction vector for l, e.g. $(-\mathbf{i}+17\mathbf{j}-4\mathbf{k})\times(-2\mathbf{i}+\mathbf{j}-2\mathbf{k})$ — M1

Obtain correct answer, e.g. $-30\mathbf{i}+6\mathbf{j}+33\mathbf{k}$ — A1

Divide the modulus of the product by that of the direction vector — M1

Obtain the given answer correctly — A1

*OR*2: State $\overrightarrow{BA}$ (or $\overrightarrow{AB}$) in component form — B1

Use a scalar product to find the projection of BA (or AB) on l — M1

Obtain correct answer in any form, e.g. $\frac{27}{\sqrt{9}}$ — A1

Use Pythagoras to find the perpendicular — M1

	Obtain the given answer correctly	A1	
*OR*3:	State $\overrightarrow{BA}$ (or $\overrightarrow{AB}$) in component form	B1	
	Use a scalar product to find the cosine of *ABP*	M1	
	Obtain correct answer in any form, e.g. $\dfrac{27}{\sqrt{9}.\sqrt{306}}$	A1	
	Use trig. to find the perpendicular	M1	
	Obtain the given answer correctly	A1	
*OR*4:	State $\overrightarrow{BA}$ (or $\overrightarrow{AB}$) in component form	B1	
	Find a second point *C* on *l* and use the cosine rule in triangle *ABC* to find the cosine of angle *A, B*, or *C*, or use a vector product to find the area of *ABC*	M1	
	Obtain correct answer in any form	A1	
	Use trig. or area formula to find the perpendicular	M1	
	Obtain the given answer correctly	A1	
*OR*5:	State correct $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) for a point *P* on *l* with parameter λ in any form	B1	
	Use correct method to express AP^2 (or *AP*) in terms of λ	M1	
	Obtain a correct expression in any form, e.g. $(1-2\lambda)^2+(-17+\lambda)^2+(4-2\lambda)^2$	A1	
	Carry out a method for finding its minimum (using calculus, algebra or Pythagoras)	M1	
	Obtain the given answer correctly	A1	**[5]**
(ii) *EITHER*:	Substitute coordinates of a general point of l in equation of plane and either equate constant terms or equate the coefficient of λ to zero, obtaining an equation in *a* and *b*	M1*	
	Obtain a correct equation, e.g. $4a - 9b - 27 + 1 = 0$	A1	
	Obtain a second correct equation, e.g. $-2a + b + 6 = 0$	A1	
	Solve for *a* or for *b*	M1(dep*)	
	Obtain $a = 2$ and $b = -2$	A1	
OR:	Substitute coordinates of a point of *l* and obtain a correct equation, e.g. $4a - 9b = 26$	B1	
	EITHER: Find a second point on *l* and obtain an equation in *a* and *b*	M1*	
	Obtain a correct equation	A1	
	OR: Calculate scalar product of a direction vector for *l* and a vector normal to the plane and equate to zero	M1*	
	Obtain a correct equation, e.g. $-2a + b + 6 = 0$	A1	
	Solve for *a* or for *b*	M1(dep*)	
	Obtain $a = 2$ and $b = -2$	A1	**[5]**

1	Either	State or imply non-modular inequality $(3x-1)^2 < (2x+5)^2$ or corresponding quadratic equation or pair of linear equations $3x-1 = \pm(2x+5)$	B1	
		Solve a three-term quadratic or two linear equations $5x^2 - 26x - 24 < 0$	M1	
		Obtain $-\frac{4}{5}$ and 6	A1	
		State $-\frac{4}{5} < x < 6$	A1	
	Or	Obtain value 6 from graph, inspection or solving linear equation	B1	
		Obtain value $-\frac{4}{5}$ similarly	B2	
		State $-\frac{4}{5} < x < 6$	B1	[4]

2	Use correct product rule or correct chain rule to differentiate y	M1	
	Use $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$	M*1	
	Obtain $\frac{-4\cos\theta\sin^2\theta + 2\cos^3\theta}{\sec^2\theta}$ or equivalent	A1	
	Express $\frac{dy}{dx}$ in terms of $\cos\theta$	DM*1	
	Confirm given answer $6\cos^5\theta - 4\cos^3\theta$ legitimately	A1	[5]

3	**(i)**	Either	Equate p(−1) or p(−2) to zero or divide by $(x+1)$ or $(x+2)$ and equate constant remainder to zero.	M*1	
			Obtain two equations $a - b = 6$ and $4a - 2b = 34$ or equivalents	A1	
			Solve pair of equations for a or b	DM*1	
			Obtain $a = 11$ and $b = 5$	A1	
		Or	State or imply third factor is $4x - 1$	B1	
			Carry out complete expansion of $(x+1)(x+2)(4x-1)$ or $(x+1)(x+2)(Cx+D)$	M1	
			Obtain $a = 11$	A1	
			Obtain $b = 5$	A1	[4]
	(ii)		Use division or equivalent and obtaining linear remainder	M1	
			Obtain quotient $4x + a$, following their value of a	A1√	
			Indicate remainder $x - 13$	A1	[3]

4	(i)	Either	Use $\cos(A \pm B)$ correctly at least once	M1	
			State correct complete expansion	A1	
			Confirm given answer $\cos\theta$ with explicit use of $\cos 60^o = \frac{1}{2}$	A1	
			SR: "correct" answer from sign errors in both expansions is B1 only		
		Or	Use correct $\cos A + \cos B$ formula	M1	
			State correct result e.g. $2\cos\left(\frac{2\theta}{2}\right)\cos\left(\frac{-120}{2}\right)$	A1	
			Confirm given answer $\cos\theta$ with explicit use of $\cos(\pm 60^o) = \frac{1}{2}$	A1	[3]
	(ii)		State or imply $\frac{\cos 2x}{\cos x} = 3$	B1	
			Obtain equation $2\cos^2 x - 3\cos x - 1 = 0$	B1	
			Solve a three-term quadratic equation for $\cos x$	M1	
			Obtain $\frac{1}{4}\left(3 - \sqrt{17}\right)$ or exact equivalent and, finally, no other	A1	[4]
5	(i)		State or imply $\mathrm{i}w = -3 + 5\mathrm{i}$	B1	
			Carry out multiplication by $\frac{4 - \mathrm{i}}{4 - \mathrm{i}}$	M1	
			Obtain final answer $-\frac{7}{17} + \frac{23}{17}\mathrm{i}$ or equivalent	A1	[3]
	(ii)		Multiply w by z to obtain $17 + 17\mathrm{i}$	B1	
			State $\arg w = \tan^{-1}\frac{3}{5}$ or $\arg z = \tan^{-1}\frac{1}{4}$	B1	
			State $\arg wz = \arg w + \arg z$	M1	
			Confirm given result $\tan^{-1}\frac{3}{5} + \tan^{-1}\frac{1}{4} = \frac{1}{4}\pi$ legitimately	A1	[4]
6	(i)		State or imply correct ordinates 1, 0.94259…, 0.79719…, 0.62000…	B1	
			Use correct formula or equivalent with $h = 0.1$ and four y values	M1	
			Obtain 0.255 with no errors seen	A1	[3]
	(ii)		Obtain or imply $a = -6$	B1	
			Obtain x^4 term including correct attempt at coefficient	M1	
			Obtain or imply $b = 27$	A1	
		Either	Integrate to obtain $x - 2x^3 + \frac{27}{5}x^5$, following their values of a and b	B1$\sqrt{}$	
			Obtain 0.259	B1	
		Or	Use correct trapezium rule with at least 3 ordinates	M1	
			Obtain 0.259 (from 4)	A1	[5]

7	**(i)**	State at least two of the equations $1+\lambda = a+\mu$, $4 = 2+2\mu$, $-2+3\lambda = -2+3a\mu$	B1	
		Solve for λ or for μ	M1	
		Obtain $\lambda = a$ (or $\lambda = a+\mu-1$) and $\mu = 1$	A1	
		Confirm values satisfy third equation	A1	[4]
	(ii)	State or imply point of intersection is $(a+1, 4, 3a-2)$	B1	
		Use correct method for the modulus of the position vector and equate to 9, following their point of intersection	M*1	
		Solve a three-term quadratic equation in a $\quad(a^2 - a - 6 = 0)$	DM*1	
		Obtain –2 and 3	A1	[4]
8	**(i)**	Sensibly separate variables and attempt integration of at least one side	M1	
		Obtain $2y^{\frac{1}{2}} = \ldots$ or equivalent	A1	
		Correct integration by parts of $x\sin\frac{1}{3}x$ as far as $ax\cos\frac{1}{3}x \pm \int b\cos\frac{1}{3}x\mathrm{d}x$	M1	
		Obtain $-3x\cos\frac{1}{3}x + \int 3\cos\frac{1}{3}x\mathrm{d}x$ or equivalent	A1	
		Obtain $-3x\cos\frac{1}{3}x + 9\sin\frac{1}{3}x$ or equivalent	A1	
		Obtain $y = \left(-\frac{3}{10}x\cos\frac{1}{3}x + \frac{9}{10}\sin\frac{1}{3}x + c\right)^2$ or equivalent	A1	[6]
	(ii)	Use $x = 0$ and $y = 100$ to find constant	M*1	
		Substitute 25 and calculate value of y	DM*1	
		Obtain 203	A1	[3]
9	**(i)**	Sketch increasing curve with correct curvature passing through origin, for $x \geq 0$	B1	
		Recognisable sketch of $y = 40 - x^3$, with equation stated, for $x > 0$	B1	
		Indicate in some way the one intersection, dependent on both curves being roughly correct and both existing for some $x < 0$	B1	[3]
	(ii)	Consider signs of $x^3 + \ln(x+1) - 40$ at 3 and 4 or equivalent or compare values of relevant expressions for $x = 3$ and $x = 4$	M1	
		Complete argument correctly with correct calculations (–11.6 and 25.6)	A1	[2]
	(iii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 3.377	A1	
		Show sufficient iterations to justify accuracy to 3 d.p. or show sign change in interval (3.3765, 3.3775)	A1	[3]
	(iv)	Attempt value of $\ln(x+1)$	M1	
		Obtain 1.48	A1	[2]

10	State or imply $\dfrac{\mathrm{d}u}{\mathrm{d}x} = \mathrm{e}^x$	B1	
	Substitute throughout for x and dx	M1	
	Obtain $\displaystyle\int \frac{u}{u^2+3u+2}\,\mathrm{d}u$ or equivalent (ignoring limits so far)	A1	
	State or imply partial fractions of form $\dfrac{A}{u+2}+\dfrac{B}{u+1}$, following their integrand	B1	
	Carry out a correct process to find at least one constant for their integrand	M1	
	Obtain correct $\dfrac{2}{u+2}-\dfrac{1}{u+1}$	A1	
	Integrate to obtain $a\ln(u+2)+b\ln(u+1)$	M1	
	Obtain $2\ln(u+2)-\ln(u+1)$ or equivalent, follow their A and B	A1$\sqrt{}$	
	Apply appropriate limits and use at least one logarithm property correctly	M1	
	Obtain given answer $\ln\dfrac{8}{5}$ legitimately	A1	[10]
	SR for integrand $\dfrac{u^2}{u(u+1)(u+2)}$		
	State or imply partial fractions of form $\dfrac{A}{u}+\dfrac{B}{u+1}+\dfrac{C}{u+2}$	(B1)	
	Carry out a correct process to find at least one constant	(M1)	
	Obtain correct $\dfrac{2}{u+2}-\dfrac{1}{u+1}$	(A1)	
	…complete as above.		

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Level

MARK SCHEME for the October/November 2015 series

9709 MATHEMATICS

9709/31 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)

CWO Correct Working Only – often written by a ‘fortuitous’ answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through √” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	*EITHER*: State or imply non-modular inequality $(2x-5)^2 > (3(2x+1))^2$, or corresponding quadratic equation, or pair of linear equations $(2x-5) = \pm 3(2x+1)$	**B1**	
	Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations for x	**M1**	
	Obtain critical values -2 and $\frac{1}{4}$	**A1**	
	State final answer $-2 < x < \frac{1}{4}$	**A1**	
	OR: Obtain critical value $x = -2$ from a graphical method, or by inspection, or by solving a linear equation or inequality	**B1**	
	Obtain critical value $x = \frac{1}{4}$ similarly	**B2**	
	State final answer $-2 < x < \frac{1}{4}$	**B1**	[4]
	[Do not condone $\leqslant$ for $<$]		
2	State or imply $1 + u = u^2$	**B1**	
	Solve for u	**M1**	
	Obtain root $\frac{1}{2}(1+\sqrt{5})$, or decimal in [1.61, 1.62]	**A1**	
	Use correct method for finding x from a positive root	**M1**	
	Obtain $x = 0.438$ and no other answer	**A1**	[5]
3	Use $\tan(A \pm B)$ and obtain an equation in $\tan\theta$ and $\tan\phi$	**M1***	
	Substitute throughout for $\tan\theta$ or for $\tan\phi$	**dep M1***	
	Obtain $3\tan^2\theta - \tan\theta - 4 = 0$ or $3\tan^2\phi - 5\tan\phi - 2 = 0$, or 3-term equivalent	**A1**	
	Solve a 3-term quadratic and find an angle	**M1**	
	Obtain answer $\theta = 135°$, $\phi = 63.4°$	**A1**	
	Obtain answer $\theta = 53.1°$, $\phi = 161.6°$	**A1**	[6]
	[Treat answers in radians as a misread. Ignore answers outside the given interval.]		
	[SR: Two correct values of θ (or ϕ) score A1; then A1 for both correct θ, ϕ pairs.]		
4	**(i)** Evaluate, or consider the sign of, $x^3 - x^2 - 6$ for two integer values of x, or equivalent	**M1**	
	Obtain the pair $x = 2$ and $x = 3$, with no errors seen	**A1**	[2]
	(ii) State a suitable equation, e.g. $x = \sqrt{(x + (6/x))}$	**B1**	
	Rearrange this as $x^3 - x^2 - 6 = 0$, or work *vice versa*	**B1**	[2]
	(iii) Use the iterative formula correctly at least once	**M1**	
	Obtain final answer 2.219	**A1**	
	Show sufficient iterates to 5 d.p. to justify 2.219 to 3 d.p., or show there is a sign change in the interval (2.2185, 2.2195)	**A1**	[3]

5	(i)	State or imply that the derivative of e^{-2x} is $-2e^{-2x}$	B1	
		Use product or quotient rule	M1	
		Obtain correct derivative in any form	A1	
		Use Pythagoras	M1	
		Justify the given form	A1	[5]
	(ii)	Fully justify the given statement	B1	[1]
	(iii)	State answer $x = \frac{1}{4}\pi$	B1	[1]
6	(i)	Substitute $x = -1$, equate to zero and simplify at least as far as $-8 + a - b - 1 = 0$	B1	
		Substitute $x = -\frac{1}{2}$ and equate the result to 1	M1	
		Obtain a correct equation in any form, e.g. $-1 + \frac{1}{4}a - \frac{1}{2}b - 1 = 1$	A1	
		Solve for a or for b	M1	
		Obtain $a = 6$ and $b = -3$	A1	[5]
	(ii)	Commence division by $(x + 1)$ reaching a partial quotient $8x^2 + kx$	M1	
		Obtain quadratic factor $8x^2 - 2x - 1$	A1	
		Obtain factorisation $(x+1)(4x+1)(2x-1)$	A1	[3]
		[The M1 is earned if inspection reaches an unknown factor $8x^2 + Bx + C$ and an equation in B and/or C, or an unknown factor $Ax^2 + Bx - 1$ and an equation in A and/or B.] [If linear factors are found by the factor theorem, give B1B1 for $(2x - 1)$ and $(4x + 1)$, and B1 for the complete factorisation.]		
7	(i)	Use correct method to form a vector equation for AB	M1	
		Obtain a correct equation, e.g. $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \lambda(2\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ or $\mathbf{r} = 3\mathbf{i} + \mathbf{k} + \mu(2\mathbf{i} - 2\mathbf{j} + \mathbf{k})$	A1	[2]
	(ii)	Using a direction vector for AB and a relevant point, obtain an equation for m in any form	M1	
		Obtain answer $2x - 2y + z = 4$, or equivalent	A1	[2]
	(iii)	Express general point of AB in component form, e.g. $(1 + 2\lambda, 2 - 2\lambda, \lambda)$ or $(3 + 2\mu, -2\mu, 1 + \mu)$	B1√	
		Substitute in equation of m and solve for λ or for μ	M1	
		Obtain final answer $\frac{7}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$ for the position vector of N, from $\lambda = \frac{2}{3}$ or $\mu = -\frac{1}{3}$	A1	
		Carry out a correct method for finding CN	M1	
		Obtain the given answer $\sqrt{13}$	A1	[5]
		[The f.t. is on the direction vector for AB.]		

8		Separate variables and integrate one side	B1	
		Obtain term $\ln(x+2)$	B1	
		Use cos 2A formula to express $\sin^2 2\theta$ in the form $a + b\cos 4\theta$	M1	
		Obtain correct form $(1-\cos 4\theta)/2$, or equivalent	A1	
		Integrate and obtain term $\frac{1}{2}\theta - \frac{1}{8}\sin 4\theta$, or equivalent	A1√	
		Evaluate a constant, or use $\theta = 0$, $x = 0$ as limits in a solution containing terms $c\ln(x+2), d\sin(4\theta), e\theta$	M1	
		Obtain correct solution in any form, e.g. $\ln(x+2) = \frac{1}{2}\theta - \frac{1}{8}\sin 4\theta + \ln 2$	A1	
		Use correct method for solving an equation of the form $\ln(x+2) = f$	M1	
		Obtain answer $x = 0.962$	A1	[9]
9	(i)	Show u in a relatively correct position	B1	
		Show u^* in a relatively correct position	B1	
		Show $u^* - u$ in a relatively correct position	B1	
		State or imply that $OABC$ is a parallelogram	B1	[4]
	(ii)	*EITHER*: Substitute for u and multiply numerator and denominator by 3 + i, or equivalent	M1	
		Simplify the numerator to 8 + 6i or the denominator to 10	A1	
		Obtain final answer $\frac{4}{5} + \frac{3}{5}\mathrm{i}$, or equivalent	A1	
		OR: Substitute for u, obtain two equations in x and y and solve for x or for y	M1	
		Obtain $x = \frac{4}{5}$ or $y = \frac{3}{5}$, or equivalent	A1	
		Obtain final answer $\frac{4}{5} + \frac{3}{5}\mathrm{i}$, or equivalent	A1	[3]
	(iii)	State or imply $\arg(u^*/u) = \tan^{-1}(\frac{3}{4})$	B1	
		Substitute exact arguments in $\arg(u^*/u) = \arg u^* - \arg u$	M1	
		Fully justify the given statement using exact values	A1	[3]
10	(i)	Use the quotient rule	M1	
		Obtain correct derivative in any form	A1	
		Equate derivative to zero and solve for x	M1	
		Obtain answer $x = \sqrt[3]{2}$, or exact equivalent	A1	[4]
	(ii)	State or imply indefinite integral is of the form $k\ln(1+x^3)$	M1	
		State indefinite integral $\frac{1}{3}\ln(1+x^3)$	A1	
		Substitute limits correctly in an integral of the form $k\ln(1+x^3)$	M1	
		State or imply that the area of R is equal to $\frac{1}{3}\ln(1+p^3) - \frac{1}{3}\ln 2$, or equivalent	A1	
		Use a correct method for finding p from an equation of the form $\ln(1+p^3) = a$ or $\ln((1+p^3)/2) = b$	M1	
		Obtain answer $p = 3.40$	A1	[2]

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Level

MARK SCHEME for the October/November 2015 series

9709 MATHEMATICS

9709/32 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2015 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol $\sqrt{}$ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)

CWO Correct Working Only – often written by a ‘fortuitous’ answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through √” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1 *EITHER*: State or imply non-modular inequality $(2x-5)^2 > (3(2x+1))^2$, or corresponding quadratic
equation, or pair of linear equations $(2x-5) = \pm 3(2x+1)$ **B1**
Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations for x **M1**
Obtain critical values -2 and $\frac{1}{4}$ **A1**
State final answer $-2 < x < \frac{1}{4}$ **A1**
OR: Obtain critical value $x = -2$ from a graphical method, or by inspection, or by solving a linear
equation or inequality **B1**
Obtain critical value $x = \frac{1}{4}$ similarly **B2**
State final answer $-2 < x < \frac{1}{4}$ **B1** [4]
[Do not condone $\leqslant$ for $<$]

2 State or imply $1 + u = u^2$ **B1**
Solve for u **M1**
Obtain root $\frac{1}{2}(1+\sqrt{5})$, or decimal in [1.61, 1.62] **A1**
Use correct method for finding x from a positive root **M1**
Obtain $x = 0.438$ and no other answer **A1** [5]

3 Use $\tan(A \pm B)$ and obtain an equation in $\tan\theta$ and $\tan\phi$ **M1***
Substitute throughout for $\tan\theta$ or for $\tan\phi$ **dep M1***
Obtain $3\tan^2\theta - \tan\theta - 4 = 0$ or $3\tan^2\phi - 5\tan\phi - 2 = 0$, or 3-term equivalent **A1**
Solve a 3-term quadratic and find an angle **M1**
Obtain answer $\theta = 135°$, $\phi = 63.4°$ **A1**
Obtain answer $\theta = 53.1°$, $\phi = 161.6°$ **A1** [6]
[Treat answers in radians as a misread. Ignore answers outside the given interval.]
[SR: Two correct values of θ (or ϕ) score A1; then A1 for both correct θ, ϕ pairs.]

4 **(i)** Evaluate, or consider the sign of, $x^3 - x^2 - 6$ for two integer values of x, or equivalent **M1**
Obtain the pair $x = 2$ and $x = 3$, with no errors seen **A1** [2]

(ii) State a suitable equation, e.g. $x = \sqrt{(x + (6/x))}$ **B1**
Rearrange this as $x^3 - x^2 - 6 = 0$, or work *vice versa* **B1** [2]

(iii) Use the iterative formula correctly at least once **M1**
Obtain final answer 2.219 **A1**
Show sufficient iterates to 5 d.p. to justify 2.219 to 3 d.p., or show there is a sign change in the interval (2.2185, 2.2195) **A1** [3]

5	(i)	State or imply that the derivative of e^{-2x} is $-2e^{-2x}$	B1	
		Use product or quotient rule	M1	
		Obtain correct derivative in any form	A1	
		Use Pythagoras	M1	
		Justify the given form	A1	[5]
	(ii)	Fully justify the given statement	B1	[1]
	(iii)	State answer $x = \frac{1}{4}\pi$	B1	[1]
6	(i)	Substitute $x = -1$, equate to zero and simplify at least as far as $-8 + a - b - 1 = 0$	B1	
		Substitute $x = -\frac{1}{2}$ and equate the result to 1	M1	
		Obtain a correct equation in any form, e.g. $-1 + \frac{1}{4}a - \frac{1}{2}b - 1 = 1$	A1	
		Solve for a or for b	M1	
		Obtain $a = 6$ and $b = -3$	A1	[5]
	(ii)	Commence division by $(x + 1)$ reaching a partial quotient $8x^2 + kx$	M1	
		Obtain quadratic factor $8x^2 - 2x - 1$	A1	
		Obtain factorisation $(x+1)(4x+1)(2x-1)$	A1	[3]
		[The M1 is earned if inspection reaches an unknown factor $8x^2 + Bx + C$ and an equation in B and/or C, or an unknown factor $Ax^2 + Bx - 1$ and an equation in A and/or B.] [If linear factors are found by the factor theorem, give B1B1 for $(2x - 1)$ and $(4x + 1)$, and B1 for the complete factorisation.]		
7	(i)	Use correct method to form a vector equation for AB	M1	
		Obtain a correct equation, e.g. $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \lambda(2\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ or $\mathbf{r} = 3\mathbf{i} + \mathbf{k} + \mu(2\mathbf{i} - 2\mathbf{j} + \mathbf{k})$	A1	[2]
	(ii)	Using a direction vector for AB and a relevant point, obtain an equation for m in any form	M1	
		Obtain answer $2x - 2y + z = 4$, or equivalent	A1	[2]
	(iii)	Express general point of AB in component form, e.g. $(1 + 2\lambda, 2 - 2\lambda, \lambda)$ or $(3 + 2\mu, -2\mu, 1 + \mu)$	B1√	
		Substitute in equation of m and solve for λ or for μ	M1	
		Obtain final answer $\frac{7}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$ for the position vector of N, from $\lambda = \frac{2}{3}$ or $\mu = -\frac{1}{3}$	A1	
		Carry out a correct method for finding CN	M1	
		Obtain the given answer $\sqrt{13}$	A1	[5]
		[The f.t. is on the direction vector for AB.]		

8 Separate variables and integrate one side **B1**
Obtain term $\ln(x+2)$ **B1**

Use cos $2A$ formula to express $\sin^2 2\theta$ in the form $a + b\cos 4\theta$ **M1**
Obtain correct form $(1 - \cos 4\theta)/2$, or equivalent **A1**

Integrate and obtain term $\frac{1}{2}\theta - \frac{1}{8}\sin 4\theta$, or equivalent **A1**√

Evaluate a constant, or use $\theta = 0$, $x = 0$ as limits in a solution containing terms $c\ln(x+2), d\sin(4\theta), e\theta$ **M1**

Obtain correct solution in any form, e.g. $\ln(x+2) = \frac{1}{2}\theta - \frac{1}{8}\sin 4\theta + \ln 2$ **A1**

Use correct method for solving an equation of the form $\ln(x+2) = f$ **M1**
Obtain answer $x = 0.962$ **A1** [9]

9 **(i)** Show u in a relatively correct position **B1**
Show u^* in a relatively correct position **B1**
Show $u^* - u$ in a relatively correct position **B1**
State or imply that $OABC$ is a parallelogram **B1** [4]

(ii) *EITHER*: Substitute for u and multiply numerator and denominator by 3 + i, or equivalent **M1**
Simplify the numerator to 8 + 6i or the denominator to 10 **A1**
Obtain final answer $\frac{4}{5} + \frac{3}{5}\mathrm{i}$, or equivalent **A1**

OR: Substitute for u, obtain two equations in x and y and solve for x or for y **M1**
Obtain $x = \frac{4}{5}$ or $y = \frac{3}{5}$, or equivalent **A1**

Obtain final answer $\frac{4}{5} + \frac{3}{5}\mathrm{i}$, or equivalent **A1** [3]

(iii) State or imply $\arg(u^*/u) = \tan^{-1}(\frac{3}{4})$ **B1**

Substitute exact arguments in $\arg(u^*/u) = \arg u^* - \arg u$ **M1**
Fully justify the given statement using exact values **A1** [3]

10 **(i)** Use the quotient rule **M1**
Obtain correct derivative in any form **A1**
Equate derivative to zero and solve for x **M1**
Obtain answer $x = \sqrt[3]{2}$, or exact equivalent **A1** [4]

(ii) State or imply indefinite integral is of the form $k\ln(1+x^3)$ **M1**

State indefinite integral $\frac{1}{3}\ln(1+x^3)$ **A1**

Substitute limits correctly in an integral of the form $k\ln(1+x^3)$ **M1**

State or imply that the area of R is equal to $\frac{1}{3}\ln(1+p^3) - \frac{1}{3}\ln 2$, or equivalent **A1**

Use a correct method for finding p from an equation of the form $\ln(1+p^3) = a$ or $\ln((1+p^3)/2) = b$ **M1**
Obtain answer $p = 3.40$ **A1** [2]

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Level

MARK SCHEME for the October/November 2015 series

9709 MATHEMATICS

9709/33 Paper 3, maximum raw mark 75

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PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	Draw curve with increasing gradient existing for negative and positive values of x	**M1**	
	Draw correct curve passing through the origin	**A1**	[2]
2	Either State correct unsimplified x^2 or x^3 term	**M1**	
	Obtain $a = -9$	**A1**	
	Obtain $b = 45$	**A1**	
	Or Use chain rule to differentiate twice to obtain form $k(1+9x)^{-\frac{5}{3}}$	**M1**	
	Obtain $f''(x) = -18(1+9x)^{-\frac{5}{3}}$ and hence $a = -9$	**A1**	
	Obtain $f'''(x) = 270(1+9x)^{-\frac{8}{3}}$ and hence $b = 45$	**A1**	[3]
3	Use correct quotient rule or equivalent to find first derivative	**M1***	
	Obtain $\dfrac{-(1+\tan x)\sec^2 x - \sec^2 x(2-\tan x)}{(1+\tan x)^2}$ or equivalent	**A1**	
	Substitute $x = \frac{1}{4}\pi$ to find gradient	**dep M1***	
	Obtain $-\frac{3}{2}$	**A1**	
	Form equation of tangent at $x = \frac{1}{4}\pi$	**M1**	
	Obtain $y = -\frac{3}{2}x + 1.68$ or equivalent	**A1**	[6]
4	**(i)** Use $\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}}$ and equate $\frac{dy}{dx}$ to 4	**M1**	
	Obtain $\dfrac{4p^3}{2p+3} = 4$ or equivalent	**A1**	
	Confirm given result $p = \sqrt[3]{2p+3}$ correctly	**A1**	[3]
	(ii) Evaluate $p - \sqrt[3]{2p+3}$ or $p^3 - 2p - 3$ or equivalent at 1.8 and 2.0	**M1**	
	Justify result with correct calculations and argument (–0.076 and 0.087 or –0.77 and 1 respectively)	**A1**	[2]
	(iii) Use the iterative process correctly at least once with $1.8 \leqslant p_n \leqslant 2.0$	**M1**	
	Obtain final answer 1.89	**A1**	
	Show sufficient iterations to at least 4 d.p. to justify 1.89 or show sign change in interval (1.885, 1.895)	**A1**	[3]

5	State $\mathrm{d}u = 3\sin x\,\mathrm{d}x$ or equivalent	**B1**	
	Use identity $\sin 2x = 2\sin x\cos x$	**B1**	
	Carry out complete substitution, for x and $\mathrm{d}x$	**M1**	
	Obtain $\int \frac{8-2u}{\sqrt{u}}\,\mathrm{d}u$, or equivalent	**A1**	
	Integrate to obtain expression of form $au^{\frac{1}{2}} + bu^{\frac{3}{2}}$, $ab \neq 0$	**M1***	
	Obtain correct $16u^{\frac{1}{2}} - \frac{4}{3}u^{\frac{3}{2}}$	**A1**	
	Apply correct limits correctly	**dep M1***	
	Obtain $\frac{20}{3}$ or exact equivalent	**A1**	[8]

6	State or imply $\sin A \times \cos 45 + \cos A \times \sin 45 = 2\sqrt{2}\cos A$	**B1**	
	Divide by $\cos A$ to find value of $\tan A$	**M1**	
	Obtain $\tan A = 3$	**A1**	
	Use identity $\sec^2 B = 1 + \tan^2 B$	**B1**	
	Solve three-term quadratic equation and find $\tan B$	**M1**	
	Obtain $\tan B = \frac{3}{2}$ only	**A1**	
	Substitute **numerical values** in $\frac{\tan A - \tan B}{1 + \tan A \tan B}$	**M1**	
	Obtain $\frac{3}{11}$	**A1**	[8]

7	**(i)**	Either	Substitute $x = -1$ and evaluate	**M1**	
			Obtain 0 and conclude $x+1$ is a factor	**A1**	
		Or	Divide by $x+1$ and obtain a constant remainder	**M1**	
			Obtain remainder = 0 and conclude $x+1$ is a factor	**A1**	[2]
	(ii)		Attempt division, or equivalent, at least as far as quotient $4x^2 + kx$	**M1**	
			Obtain complete quotient $4x^2 - 5x - 6$	**A1**	
			State form $\frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{4x+3}$	**A1**	
			Use relevant method for finding at least one constant	**M1**	
			Obtain one of $A = -2, B = 1, C = 8$	**A1**	
			Obtain all three values	**A1**	
			Integrate to obtain three terms each involving natural logarithm of linear form	**M1**	
			Obtain $-2\ln(x+1) + \ln(x-2) + 2\ln(4x+3)$, condoning no use of modulus signs and absence of … + c	**A1**	[8]

8	**(i)**	Express a general point on the line in single component form, e.g. $(\lambda, 2-3\lambda, -8+4\lambda)$, substitute in equation of plane and solve for λ		**M1**	
		Obtain $\lambda = 3$		**A1**	
		Obtain $(3, -7, 4)$		**A1**	[3]
	(ii)	State or imply normal vector to plane is 4**i** – **j** + 5**k**		**B1**	
		Carry out process for evaluating scalar product of two relevant vectors		**M1**	
		Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate $\sin^{-1}$ or $\cos^{-1}$ of the result.		**M1**	
		Obtain $54.8°$ or 0.956 radians		**A1**	[4]
	(iii)	Either	Find at least one position of C by translating by appropriate multiple of direction vector **i** – 3**j** + 4**k** from A or B	**M1**	
			Obtain $(-3, 11, -20)$	**A1**	
			Obtain $(9, -25, 28)$	**A1**	
		Or	Form quadratic equation in λ by considering $BC^2 = 4AB^2$	**M1**	
			Obtain $26\lambda^2 - 156\lambda - 702 = 0$ or equivalent and hence $\lambda = -3, \lambda = 9$	**A1**	
			Obtain $(-3, 11, -20)$ and $(9, -25, 28)$	**A1**	[3]
9	**(a)**	Either	Find w using conjugate of $1+3\mathrm{i}$	**M1**	
			Obtain $\frac{7-\mathrm{i}}{5}$ or equivalent	**A1**	
			Square $x + \mathrm{i}y$ form to find w^2	**M1**	
			Obtain $w^2 = \frac{48-14\mathrm{i}}{25}$ and confirm modulus is 2	**A1**	
			Use correct process for finding argument of w^2	**M1**	
			Obtain –0.284 radians or $-16.3°$	**A1**	
		Or 1	Find w using conjugate of $1+3\mathrm{i}$	**M1**	
			Obtain $\frac{7-\mathrm{i}}{5}$ or equivalent	**A1**	
			Find modulus of w and hence of w^2	**M1**	
			Confirm modulus is 2	**A1**	
			Find argument of w and hence of w^2	**M1**	
			Obtain –0.284 radians or $-16.3°$	**A1**	
		Or 2	Square both sides to obtain $(-8+6\mathrm{i})w^2 = -12+16\mathrm{i}$	**B1**	
			Find w^2 using relevant conjugate	**M1**	
			Use correct process for finding modulus of w^2	**M1**	
			Confirm modulus is 2	**A1**	
			Use correct process for finding argument of w^2	**M1**	
			Obtain –0.284 radians or $-16.3°$	**A1**	

Or 3	Find modulus of LHS and RHS	**M1**	
	Find argument of LHS and RHS	**M1**	
	Obtain $\sqrt{10}\,e^{1.249i}\;w = \sqrt{20}\,e^{1.107i}$ or equivalent	**A1**	
	Obtain $w = \sqrt{2}\,e^{-0.1419i}$ or equivalent	**A1**	
	Use correct process for finding w^2	**M1**	
	Obtain 2 and –0.284 radians or $-16.3°$	**A1**	
Or 4	Find moduli of $2+4i$ and $1+3i$	**M1**	
	Obtain $\sqrt{20}$ and $\sqrt{10}$	**A1**	
	Obtain $\left\|w^2\right\| = 2$ correctly	**A1**	
	Find $\arg(2+4i)$ and $\arg(1+3i)$	**M1**	
	Use correct process for $\arg(w^2)$	**A1**	
	Obtain –0.284 radians or $-16.3°$	**A1**	
Or 5	Let $w = a + ib$, form and solve simultaneous equations in *a* and *b*	**M1**	
	$a = \frac{7}{5}$ and $b = -\frac{1}{5}$	**A1**	
	Find modulus of *w* and hence of w^2	**M1**	
	Confirm modulus is 2	**A1**	
	Find argument of *w* and hence of w^2	**M1**	
	Obtain –0.284 radians or $-16.3°$	**A1**	
Or 6	Find *w* using conjugate of $1+3i$	**M1**	
	Obtain $\frac{7-i}{5}$ or equivalent	**A1**	
	Use $\left\|w^2\right\| = w\overline{w}$	**M1**	
	Confirm modulus is 2	**A1**	
	Find argument of *w* and hence of w^2	**M1**	
	Obtain –0.284 radians or $-16.3°$	**A1**	[6]

(b)	Draw circle with centre the origin and radius 5	**B1**	
	Draw straight line parallel to imaginary axis in correct position	**B1**	
	Use relevant trigonometry on a correct diagram to find argument(s)	**M1**	
	Obtain $5e^{\pm\frac{1}{3}\pi i}$ or equivalents in required form	**A1**	[4]

10	**(i)**	State $\frac{dN}{dt} = k(N - 150)$	**B1**	[1]
	(ii)	Substitute $\frac{dN}{dt} = 60$ and $N = 900$ to find value of k	**M1**	
		Obtain $k = 0.08$	**A1**	
		Separate variables and obtain general solution involving $\ln(N - 150)$	**M1***	
		Obtain $\ln(N - 150) = 0.08t + c$ (following their k) or $\ln(N - 150) = kt + c$	**A1√**	
		Substitute $t = 0$ and $N = 650$ to find c	**dep M1***	
		Obtain $\ln(N - 150) = 0.08t + \ln 500$ or equivalent	**A1**	
		Obtain $N = 500e^{0.08t} + 150$	**A1**	[7]
	(iii)	Either Substitute $t = 15$ to find N or solve for t with $N = 2000$	**M1**	
		Obtain Either $N = 1810$ or $t = 16.4$ and conclude target not met	**A1**	[2]

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/31**

Paper 3 **October/November 2016**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2016 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol $\sqrt{}$ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

<u>Penalties</u>

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through $\sqrt{}$ " marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	Solve for 3^x and obtain $3^x = \frac{18}{7}$	**B1**	
	Use correct method for solving an equation of the form $3^x = a$, where $a > 0$	**M1**	
	Obtain answer $x = 0.860$ 3 d.p. only	**A1**	[3]
2	State correct unsimplified first two terms of the expansion of $(1+2x)^{-\frac{3}{2}}$, e.g. $1+(-\frac{3}{2})(2x)$	**B1**	
	State correct unsimplified term in x^2, e.g. $(-\frac{3}{2})(-\frac{3}{2}-1)(2x)^2\ /\ 2!$	**B1**	
	Obtain sufficient terms of the product of $(2-x)$ and the expansion up to the term in x^2	**M1**	
	Obtain **final** answer $2-7x+18x^2$ Do not ISW	**A1**	[4]
3	*EITHER*: Correctly restate the equation in terms of $\sin\theta$ and $\cos\theta$	**B1**	
	Correct method to obtain a horizontal equation in $\sin\theta$	**M1**	
	Reduce the equation to a correct quadratic in any form, e.g. $3\sin^2\theta - \sin\theta - 2 = 0$	**A1**	
	Solve a three-term quadratic for $\sin\theta$	**M1**	
	Obtain final answer $\theta = -41.8°$ only	**A1**	
	[Ignore answers outside the given interval.]		
	OR 1: Square both sides of the equation and use $1+\tan^2\theta = \sec^2\theta$	**B1**	
	Correct method to obtain a horizontal equation in $\sin\theta$	**M1**	
	Reduce the equation to a correct quadratic in any form, e.g. $9\sin^2\theta - 6\sin\theta - 8 = 0$	**A1**	
	Solve a three-term quadratic for $\sin\theta$	**M1**	
	Obtain final answer $\theta = -41.8°$ only	**A1**	
	OR 2: Multiply through by $(\sec\theta + \tan\theta)$	**M1**	
	Use $\sec^2\theta - \tan^2\theta = 1$	**B1**	
	Obtain $1 = 3 + 3\sin\theta$	**A1**	
	Solve for $\sin\theta$	**M1**	
	Obtain final answer $\theta = -41.8°$ only	**A1**	[5]

4	*EITHER*: *EITHER*: State $2xy + x^2\dfrac{dy}{dx}$, or equivalent, as derivative of x^2y	B1	
	State $6y^2 + 12xy\dfrac{dy}{dx}$, or equivalent, as derivative of $6xy^2$	B1	
	OR: Differentiating LHS using correct product rule, state term $xy(1-6\dfrac{dy}{dx})$, or equivalent	B1	
	State term $(y + x\dfrac{dy}{dx})(x-6y)$, or equivalent	B1	
	Equate attempted derivative of LHS to zero and set $\dfrac{dy}{dx}$ equal to zero	M1*	
	Obtain a horizontal equation, e.g. $6y^2 - 2xy = 0$ (from correct work only)	A1	
	Explicitly reject $y = 0$ as a possibility $py^2 - qxy = 0$	A1	
	Obtain an equation in x or y	DM1	
	Obtain answer $(-3a, -a)$	A1	
	OR: Rearrange to $y = \dfrac{9a^3}{x(x-6y)}$ and use correct quotient rule to obtain $-\dfrac{9a^3}{x^2(x-6y)^2}\times.....$	B1	
	State term $(x-6y) + x(1-6y')$, or equivalent	B1	
	Justify division by $x(x-6y)$	B1	
	Set $\dfrac{dy}{dx}$ equal to zero	M1*	
	Obtain a horizontal equation, e.g. $6y^2 - 2xy = 0$ (from correct work only)	A1	
	Obtain an equation in x or y	DM1	
	Obtain answer $(-3a, -a)$	A1	[7]
5 (i)	*EITHER*: Use tan 2A formula to express LHS in terms of $\tan\theta$	M1	
	Express as a single fraction in any correct form	A1	
	Use Pythagoras or cos 2A formula	M1	
	Obtain the given result correctly	A1	
	OR: Express LHS in terms of sin 2θ, cos 2θ, sin θ and cos θ	M1	
	Express as a single fraction in any correct form	A1	
	Use Pythagoras or cos 2A formula or sin($A - B$) formula	M1	
	Obtain the given result correctly	A1	[4]
(ii)	Integrate and obtain a term of the form $a\ln(\cos 2\theta)$ or $b\ln(\cos\theta)$ (or secant equivalents)	M1*	
	Obtain integral $-\frac{1}{2}\ln(\cos 2\theta) + \ln(\cos\theta)$, or equivalent	A1	
	Substitute limits correctly (expect to see use of <u>both</u> limits)	DM1	
	Obtain the given answer following full and correct working	A1	[4]

6	**(i)**	Make recognizable sketch of a relevant graph Sketch the other relevant graph and justify the given statement	**B1** **B1**	 [2]
	(ii)	Use calculations to consider the value of a relevant expression at x = 1.4 and x = 1.6, or the values of relevant expressions at x = 1.4 and x = 1.6 Complete the argument correctly with correct calculated values	**M1** **A1**	 [2]
	(iii)	State $x = 2\sin^{-1}\left(\frac{3}{x+3}\right)$ Rearrange this in the form $\operatorname{cosec}\frac{1}{2}x = \frac{1}{3}x + 1$ If working in reverse, need $\sin\frac{x}{2} = \left(\frac{3}{x+3}\right)$ for first B1	**B1** **B1**	 [2]
	(iv)	Use the iterative formula correctly at least once Obtain final answer 1.471 Show sufficient iterations to 5 d.p. to justify 1.471 to 3 d.p., or show there is a sign change in the interval (1.4705, 1.4715)	**M1** **A1** **A1**	 [3]
7	**(i)**	Use the correct product rule Obtain correct derivative in any form, e.g. $(2-2x)e^{\frac{1}{2}x} + \frac{1}{2}(2x-x^2)e^{\frac{1}{2}x}$ Equate derivative to zero and solve for x Obtain $x = \sqrt{5} - 1$ only	**M1** **A1** **M1** **A1**	 [4]
	(ii)	Integrate by parts and reach $a(2x-x^2)e^{\frac{1}{2}x} + b\int(2-2x)e^{\frac{1}{2}x}\,dx$ Obtain $2e^{\frac{1}{2}x}(2x-x^2) - 2\int(2-2x)e^{\frac{1}{2}x}dx$, or equivalent Complete the integration correctly, obtaining $(12x - 2x^2 - 24)e^{\frac{1}{2}x}$, or equivalent Use limits x = 0, x = 2 correctly having integrated by parts twice Obtain answer 24 – 8e, or <u>exact</u> simplified equivalent	**M1*** **A1** **A1** **DM1** **A1**	 [5]

8	(i)	State or imply a correct normal vector to either plane, e.g. $3\mathbf{i}+\mathbf{j}-\mathbf{k}$ or $\mathbf{i}-\mathbf{j}+2\mathbf{k}$ Use correct method to calculate their scalar product Show value is zero and planes are perpendicular	B1 M1 A1	[3]
	(ii)	*EITHER*: Carry out a complete strategy for finding a point on *l* the line of intersection Obtain such a point, e.g. (0, 7, 5) , (1, 0, 1), (5/4, –7/4, 0) *EITHER*: State two equations for a direction vector $a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$ for *l*, e.g. $3a+b-c=0$ and $a-b+2c=0$ Solve for one ratio, e.g. $a:b$ Obtain $a:b:c = 1:-7:-4$, or equivalent State a correct answer, e.g. $\mathbf{r}=7\mathbf{j}+5\mathbf{k}+\lambda(\mathbf{i}-7\mathbf{j}-4\mathbf{k})$	M1 A1 B1 M1 A1 A1√	
		*OR*1: Obtain a second point on *l*, e.g. (1, 0, 1) Subtract vectors and obtain a direction vector for *l* Obtain $-\mathbf{i}+7\mathbf{j}+4\mathbf{k}$, or equivalent State a correct answer, e.g. $\mathbf{r}=\mathbf{i}+\mathbf{k}+\lambda(-\mathbf{i}+7\mathbf{j}+4\mathbf{k})$	B1 M1 A1 A1√	
		*OR*2: Attempt to find the vector product of the two normal vectors Obtain two correct components of the product Obtain $\mathbf{i}-7\mathbf{j}-4\mathbf{k}$, or equivalent State a correct answer, e.g. $\mathbf{r}=7\mathbf{j}+5\mathbf{k}+\lambda(\mathbf{i}-7\mathbf{j}-4\mathbf{k})$	M1 A1 A1 A1√	
		*OR*1: Express one variable in terms of a second variable Obtain a correct simplified expression, e.g. $y = 7 - 7x$ Express the third variable in terms of the second Obtain a correct simplified expression, e.g. $z = 5 - 4x$ Form a vector equation for the line Obtain a correct equation, e.g. $\mathbf{r}=7\mathbf{j}+5\mathbf{k}+\lambda(\mathbf{i}-7\mathbf{j}-4\mathbf{k})$	M1 A1 M1 A1 M1 A1√	
		*OR*2: Express one variable in terms of a second variable Obtain a correct simplified expression, e.g. $z=5-4x$ Express the same variable in terms of the third Obtain a correct simplified expression e.g. $z=(7+4y)/7$ Form a vector equation for the line Obtain a correct equation, e.g. $\mathbf{r}=\frac{5}{4}\mathbf{i}-\frac{7}{4}\mathbf{j}+\lambda(-\frac{1}{4}\mathbf{i}+\frac{7}{4}\mathbf{j}+\mathbf{k})$	M1 A1 M1 A1 M1 A1√	[6]

9	(a)	EITHER: Use quadratic formula to solve for w	M1	
		Use $i^2 = -1$	M1	
		Obtain one of the answers $w = \dfrac{1}{2i+1}$ and $w = -\dfrac{5}{2i+1}$	A1	
		Multiply numerator and denominator of an answer by –2i + 1, or equivalent	M1	
		Obtain final answers $\frac{1}{5} - \frac{2}{5}i$ and $-1 + 2i$	A1	
		OR1: Multiply the equation by $1 - 2i$	M1	
		Use $i^2 = -1$	M1	
		Obtain $5w^2 + 4w(1-2i) - (1-2i)^2 = 0$, or equivalent	A1	
		Use quadratic formula or factorise to solve for w	M1	
		Obtain final answers $\frac{1}{5} - \frac{2}{5}i$ and $-1 + 2i$	A1	
		OR2: Substitute $w = x + iy$ and form equations for real and imaginary parts	M1	
		Use $i^2 = -1$	M1	
		Obtain $(x^2 - y^2) - 4xy + 4x - 1 = 0$ and $2(x^2 - y^2) + 2xy + 4y + 2 = 0$ o.e.	A1	
		Form equation in x only or y only and solve	M1	
		Obtain final answers $\frac{1}{5} - \frac{2}{5}i$ and $-1 + 2i$	A1	[5]
	(b)	Show a circle with centre $1 + i$	B1	
		Show a circle with radius 2	B1	
		Show half-line $\arg z = \frac{1}{4}\pi$	B1	
		Show half-line $\arg z = -\frac{1}{4}\pi$	B1	
		Shade the correct region	B1	[5]

10	**(i)**	Separate variables correctly and integrate at least one side Integrate and obtain term kt, or equivalent	**M1** **A1**	
		Carry out a relevant method to obtain A and B such that $\dfrac{1}{x(4-x)} \equiv \dfrac{A}{x} + \dfrac{B}{4-x}$, or equivalent	**M1***	
		Obtain $A = B = \dfrac{1}{4}$, or equivalent	**A1**	
		Integrate and obtain terms $\dfrac{1}{4}\ln x - \dfrac{1}{4}\ln(4-x)$, or equivalent	**A1**$\sqrt{}$	
		EITHER: Use a pair of limits in an expression containing $p\ln x$, $q\ln(4-x)$ and rt and evaluate a constant	**DM1**	
		Obtain correct answer in any form, e.g. $\ln x - \ln(4-x) = 4kt - \ln 9$, or $\ln\left(\dfrac{x}{4-x}\right) = 4kt - 8k$	**A1**	
		Use a second pair of limits and determine k Obtain the given exact answer correctly	**DM1** **A1**	
		OR: Use both pairs of limits in a definite integral Obtain the given exact answer correctly	**M1*** **A1**	
		Substitute k and either pair of limits in an expression containing $p\ln x$, $q\ln(4-x)$ and rt and evaluate a constant	**DM1**	
		Obtain $\ln\dfrac{x}{4-x} = t\ln 3 - \ln 9$ or equivalent	**A1**	[9]
	(ii)	Substitute $x = 3.6$ and solve for t Obtain answer $t = 4$	**M1** **A1**	[2]

Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/32**

Paper 3 **October/November 2016**

MARK SCHEME

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1	Solve for 3^x and obtain $3^x = \frac{18}{7}$	B1	
	Use correct method for solving an equation of the form $3^x = a$, where $a > 0$ Obtain answer $x = 0.860$ 3 d.p. only	M1 A1	[3]
2	State correct unsimplified first two terms of the expansion of $(1+2x)^{-\frac{3}{2}}$, e.g. $1+(-\frac{3}{2})(2x)$	B1	
	State correct unsimplified term in x^2, e.g. $(-\frac{3}{2})(-\frac{3}{2}-1)(2x)^2 / 2!$	B1	
	Obtain sufficient terms of the product of $(2 - x)$ and the expansion up to the term in x^2 Obtain **final** answer $2-7x+18x^2$ Do not ISW	M1 A1	[4]
3	*EITHER*: Correctly restate the equation in terms of $\sin\theta$ and $\cos\theta$ Correct method to obtain a horizontal equation in $\sin\theta$ Reduce the equation to a correct quadratic in any form, e.g. $3\sin^2\theta - \sin\theta - 2 = 0$ Solve a three-term quadratic for $\sin\theta$ Obtain final answer $\theta = -41.8°$ only [Ignore answers outside the given interval.]	B1 M1 A1 M1 A1	
	OR 1: Square both sides of the equation and use $1+\tan^2\theta = \sec^2\theta$ Correct method to obtain a horizontal equation in $\sin\theta$ Reduce the equation to a correct quadratic in any form, e.g. $9\sin^2\theta - 6\sin\theta - 8 = 0$ Solve a three-term quadratic for $\sin\theta$ Obtain final answer $\theta = -41.8°$ only	B1 M1 A1 M1 A1	
	OR 2: Multiply through by $(\sec\theta + \tan\theta)$ Use $\sec^2\theta - \tan^2\theta = 1$ Obtain $1 = 3 + 3\sin\theta$ Solve for $\sin\theta$ Obtain final answer $\theta = -41.8°$ only	M1 B1 A1 M1 A1	[5]

4	*EITHER*:	*EITHER*: State $2xy + x^2\dfrac{dy}{dx}$, or equivalent, as derivative of x^2y	B1	
		State $6y^2 + 12xy\dfrac{dy}{dx}$, or equivalent, as derivative of $6xy^2$	B1	
		OR: Differentiating LHS using correct product rule, state term $xy(1-6\dfrac{dy}{dx})$, or equivalent	B1	
		State term $(y + x\dfrac{dy}{dx})(x-6y)$, or equivalent	B1	
		Equate attempted derivative of LHS to zero and set $\dfrac{dy}{dx}$ equal to zero	M1*	
		Obtain a horizontal equation, e.g. $6y^2 - 2xy = 0$ (from correct work only)	A1	
		Explicitly reject $y = 0$ as a possibility $py^2 - qxy = 0$	A1	
		Obtain an equation in x or y	DM1	
		Obtain answer $(-3a, -a)$	A1	
	OR:	Rearrange to $y = \dfrac{9a^3}{x(x-6y)}$ and use correct quotient rule to obtain $-\dfrac{9a^3}{x^2(x-6y)^2}\times.....$	B1	
		State term $(x-6y) + x(1-6y')$, or equivalent	B1	
		Justify division by $x(x-6y)$	B1	
		Set $\dfrac{dy}{dx}$ equal to zero	M1*	
		Obtain a horizontal equation, e.g. $6y^2 - 2xy = 0$ (from correct work only)	A1	
		Obtain an equation in x or y	DM1	
		Obtain answer $(-3a, -a)$	A1	[7]
5 (i)	*EITHER*:	Use tan $2A$ formula to express LHS in terms of $\tan\theta$	M1	
		Express as a single fraction in any correct form	A1	
		Use Pythagoras or cos $2A$ formula	M1	
		Obtain the given result correctly	A1	
	OR:	Express LHS in terms of sin 2θ, cos 2θ, sin θ and cos θ	M1	
		Express as a single fraction in any correct form	A1	
		Use Pythagoras or cos $2A$ formula or sin$(A-B)$ formula	M1	
		Obtain the given result correctly	A1	[4]
(ii)		Integrate and obtain a term of the form $a\ln(\cos 2\theta)$ or $b\ln(\cos\theta)$ (or secant equivalents)	M1*	
		Obtain integral $-\frac{1}{2}\ln(\cos 2\theta) + \ln(\cos\theta)$, or equivalent	A1	
		Substitute limits correctly (expect to see use of <u>both</u> limits)	DM1	
		Obtain the given answer following full and correct working	A1	[4]

6 **(i)**	Make recognizable sketch of a relevant graph Sketch the other relevant graph and justify the given statement	**B1** **B1**	 [2]
(ii)	Use calculations to consider the value of a relevant expression at $x = 1.4$ and $x = 1.6$, or the values of relevant expressions at $x = 1.4$ and $x = 1.6$ Complete the argument correctly with correct calculated values	**M1** **A1**	 [2]
(iii)	State $x = 2\sin^{-1}\left(\frac{3}{x+3}\right)$ Rearrange this in the form $\operatorname{cosec}\frac{1}{2}x = \frac{1}{3}x + 1$ If working in reverse, need $\sin\frac{x}{2} = \left(\frac{3}{x+3}\right)$ for first B1	**B1** **B1**	 [2]
(iv)	Use the iterative formula correctly at least once Obtain final answer 1.471 Show sufficient iterations to 5 d.p. to justify 1.471 to 3 d.p., or show there is a sign change in the interval (1.4705, 1.4715)	**M1** **A1** **A1**	 [3]
7 **(i)**	Use the correct product rule Obtain correct derivative in any form, e.g. $(2-2x)e^{\frac{1}{2}x} + \frac{1}{2}(2x - x^2)e^{\frac{1}{2}x}$ Equate derivative to zero and solve for x Obtain $x = \sqrt{5} - 1$ only	**M1** **A1** **M1** **A1**	 [4]
(ii)	Integrate by parts and reach $a(2x - x^2)e^{\frac{1}{2}x} + b\int(2-2x)e^{\frac{1}{2}x}\,dx$ Obtain $2e^{\frac{1}{2}x}(2x - x^2) - 2\int(2-2x)e^{\frac{1}{2}x}dx$, or equivalent Complete the integration correctly, obtaining $(12x - 2x^2 - 24)e^{\frac{1}{2}x}$, or equivalent Use limits $x = 0$, $x = 2$ correctly having integrated by parts twice Obtain answer 24 – 8e, or <u>exact</u> simplified equivalent	**M1*** **A1** **A1** **DM1** **A1**	 [5]

8	(i)	State or imply a correct normal vector to either plane, e.g. $3\mathbf{i}+\mathbf{j}-\mathbf{k}$ or $\mathbf{i}-\mathbf{j}+2\mathbf{k}$ Use correct method to calculate their scalar product Show value is zero and planes are perpendicular	**B1** **M1** **A1**	[3]
	(ii)	*EITHER*: Carry out a complete strategy for finding a point on l the line of intersection Obtain such a point, e.g. (0, 7, 5) , (1, 0, 1), (5/4, –7/4, 0) *EITHER*: State two equations for a direction vector $a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$ for l, e.g. $3a+b-c=0$ and $a-b+2c=0$ Solve for one ratio, e.g. $a:b$ Obtain $a:b:c = 1:-7:-4$, or equivalent State a correct answer, e.g. $\mathbf{r}=7\mathbf{j}+5\mathbf{k}+\lambda(\mathbf{i}-7\mathbf{j}-4\mathbf{k})$ *OR*1: Obtain a second point on l, e.g. (1, 0, 1) Subtract vectors and obtain a direction vector for l Obtain $-\mathbf{i}+7\mathbf{j}+4\mathbf{k}$, or equivalent State a correct answer, e.g. $\mathbf{r}=\mathbf{i}+\mathbf{k}+\lambda(-\mathbf{i}+7\mathbf{j}+4\mathbf{k})$ *OR*2: Attempt to find the vector product of the two normal vectors Obtain two correct components of the product Obtain $\mathbf{i}-7\mathbf{j}-4\mathbf{k}$, or equivalent State a correct answer, e.g. $\mathbf{r}=7\mathbf{j}+5\mathbf{k}+\lambda(\mathbf{i}-7\mathbf{j}-4\mathbf{k})$ *OR*1: Express one variable in terms of a second variable Obtain a correct simplified expression, e.g. $y = 7 - 7x$ Express the third variable in terms of the second Obtain a correct simplified expression, e.g. $z = 5 - 4x$ Form a vector equation for the line Obtain a correct equation, e.g. $\mathbf{r}=7\mathbf{j}+5\mathbf{k}+\lambda(\mathbf{i}-7\mathbf{j}-4\mathbf{k})$ *OR*2: Express one variable in terms of a second variable Obtain a correct simplified expression, e.g. $z=5-4x$ Express the same variable in terms of the third Obtain a correct simplified expression e.g. $z=(7+4y)/7$ Form a vector equation for the line Obtain a correct equation, e.g. $\mathbf{r}=\frac{5}{4}\mathbf{i}-\frac{7}{4}\mathbf{j}+\lambda(-\frac{1}{4}\mathbf{i}+\frac{7}{4}\mathbf{j}+\mathbf{k})$	**M1** **A1** **B1** **M1** **A1** **A1**$\sqrt{}$ **B1** **M1** **A1** **A1**$\sqrt{}$ **M1** **A1** **A1** **A1**$\sqrt{}$ **M1** **A1** **M1** **A1** **M1** **A1**$\sqrt{}$ **M1** **A1** **M1** **A1** **M1** **A1**$\sqrt{}$	[6]

9	(a)	EITHER:	Use quadratic formula to solve for w	M1	
			Use $i^2 = -1$	M1	
			Obtain one of the answers $w = \dfrac{1}{2i+1}$ and $w = -\dfrac{5}{2i+1}$	A1	
			Multiply numerator and denominator of an answer by –2i + 1, or equivalent	M1	
			Obtain final answers $\frac{1}{5} - \frac{2}{5}i$ and $-1 + 2i$	A1	
		OR1:	Multiply the equation by $1 - 2i$	M1	
			Use $i^2 = -1$	M1	
			Obtain $5w^2 + 4w(1-2i) - (1-2i)^2 = 0$, or equivalent	A1	
			Use quadratic formula or factorise to solve for w	M1	
			Obtain final answers $\frac{1}{5} - \frac{2}{5}i$ and $-1 + 2i$	A1	
		OR2:	Substitute $w = x + iy$ and form equations for real and imaginary parts	M1	
			Use $i^2 = -1$	M1	
			Obtain $(x^2 - y^2) - 4xy + 4x - 1 = 0$ and $2(x^2 - y^2) + 2xy + 4y + 2 = 0$ o.e.	A1	
			Form equation in x only or y only and solve	M1	
			Obtain final answers $\frac{1}{5} - \frac{2}{5}i$ and $-1 + 2i$	A1	[5]
	(b)	Show a circle with centre $1 + i$		B1	
		Show a circle with radius 2		B1	
		Show half-line $\arg z = \frac{1}{4}\pi$		B1	
		Show half-line $\arg z = -\frac{1}{4}\pi$		B1	
		Shade the correct region		B1	[5]

10	(i)	Separate variables correctly and integrate at least one side Integrate and obtain term kt, or equivalent	M1 A1	
		Carry out a relevant method to obtain A and B such that $\frac{1}{x(4-x)} \equiv \frac{A}{x} + \frac{B}{4-x}$, or equivalent	M1*	
		Obtain $A = B = \frac{1}{4}$, or equivalent	A1	
		Integrate and obtain terms $\frac{1}{4}\ln x - \frac{1}{4}\ln(4-x)$, or equivalent	A1√	
		EITHER: Use a pair of limits in an expression containing $p\ln x$, $q\ln(4-x)$ and rt and evaluate a constant	DM1	
		Obtain correct answer in any form, e.g. $\ln x - \ln(4-x) = 4kt - \ln 9$, or $\ln\left(\frac{x}{4-x}\right) = 4kt - 8k$	A1	
		Use a second pair of limits and determine k Obtain the given exact answer correctly	DM1 A1	
		OR: Use both pairs of limits in a definite integral Obtain the given exact answer correctly	M1* A1	
		Substitute k and either pair of limits in an expression containing $p\ln x$, $q\ln(4-x)$ and rt and evaluate a constant	DM1	
		Obtain $\ln\frac{x}{4-x} = t\ln 3 - \ln 9$ or equivalent	A1	[9]
	(ii)	Substitute $x = 3.6$ and solve for t Obtain answer $t = 4$	M1 A1	[2]

Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/33**

Paper 3 **October/November 2016**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2016 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.

CAMBRIDGE International Examinations

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol $\sqrt{}$ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √ " marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1		Use law of the logarithm of a quotient	**M1**	
		Remove logarithms and obtain a correct equation, e.g. $e^z = \frac{y+2}{y+1}$	**A1**	
		Obtain answer $y = \frac{2-e^z}{e^z-1}$, or equivalent	**A1**	[3]
2		Use correct quotient or product rule	**M1**	
		Obtain correct derivative in any form	**A1**	
		Use Pythagoras to simplify the derivative to $\frac{1}{1+\cos x}$, or equivalent	**A1**	
		Justify the given statement, $-1 < \cos x < 1$ statement, or equivalent	**A1**	[4]
3		Use the tan $2A$ formula to obtain an equation in tan θ only	**M1**	
		Obtain a correct horizontal equation	**A1**	
		Rearrange equation as a quadratic in tan θ, e.g. $3\tan^2\theta + 2\tan\theta - 1 = 0$	**A1**	
		Solve for θ (usual requirements for solution of quadratic)	**M1**	
		Obtain answer, e.g. 18.4°	**A1**	
		Obtain second answer, e.g. 135° , and no others in the given interval	**A1**	[6]
4	**(i)**	Commence division by $x^2 - x + 2$ and reach a partial quotient $4x^2 + kx$	**M1**	
		Obtain quotient $4x^2 + 4x + a - 4$ or $4x^2 + 4x + b/2$	**A1**	
		Equate x or constant term to zero and solve for a or b	**M1**	
		Obtain $a = 1$	**A1**	
		Obtain $b = -6$	**A1**	[5]
	(ii)	Show that $x^2 - x + 2 = 0$ has no real roots	**B1**	
		Obtain roots $\frac{1}{2}$ and $-\frac{3}{2}$ from $4x^2 + 4x - 3 = 0$	**B1**	[2]
5	**(i)**	State equation $\frac{dy}{dx} = \frac{1}{2}xy$	**B1**	[1]
	(ii)	Separate variables correctly and attempts to integrate one side of equation	**M1**	
		Obtain terms of the form $a\ln y$ and bx^2	**A1**	
		Use $x = 0$ and $y = 2$ to evaluate a constant, or as limits, in expression containing a ln y or bx^2	**M1**	
		Obtain correct solution in any form, e.g. $\ln y = \frac{1}{4}x^2 + \ln 2$	**A1**	
		Obtain correct expression for y, e.g. $y = 2e^{\frac{1}{4}x^2}$	**A1**	[5]
	(iii)	Show correct sketch for $x \geqslant 0$. Needs through (0, 2) and rapidly increasing positive gradient.	**B1**	[1]

6 (i)	State or imply $\mathrm{d}u = \dfrac{1}{2\sqrt{x}}\mathrm{d}x$	B1	
	Substitute for x and dx throughout	M1	
	Justify the change in limits and obtain the given answer	A1	[3]
(ii)	Convert integrand into the form $A + \dfrac{B}{u+1}$	M1*	
	Obtain integrand $A = 1$, $B = -2$	A1	
	Integrate and obtain $u - 2\ln(u+1)$	A1√ + A1√	
	Substitute limits correctly in an integral containing terms au and $b\ln(u + 1)$, where $ab \neq 0$	DM1	
	Obtain the given answer following full and correct working	A1	[6]
	[The f.t. is on A and B.]		
7 (i)	State modulus $2\sqrt{2}$, or equivalent	B1	
	State argument $-\frac{1}{3}\pi$ (or $-60°$)	B1	[2]
(ii) (a)	State answer $3\sqrt{2} + \sqrt{6}\,\mathrm{i}$	B1	
(b)	*EITHER:* Substitute for z and multiply numerator and denominator by conjugate of iz	M1	
	Simplify the numerator to $4\sqrt{3} + 4\,\mathrm{i}$ or the denominator to 8	A1	
	Obtain final answer $\frac{1}{2}\sqrt{3} + \frac{1}{2}\mathrm{i}$	A1	
	OR: Substitute for z, obtain two equations in x and y and solve for x or for y	M1	
	Obtain $x = \frac{1}{2}\sqrt{3}$ or $y = \frac{1}{2}$	A1	
	Obtain final answer $\frac{1}{2}\sqrt{3} + \frac{1}{2}\mathrm{i}$	A1	[4]
(iii)	Show points A and B in relatively correct positions	B1	
	Carry out a complete method for finding angle AOB, e.g. calculate the argument of $\dfrac{z^*}{\mathrm{i}z}$	M1	
	Obtain the given answer	A1	[3]
8 (i)	State or imply the form $\dfrac{A}{x+2} + \dfrac{Bx+C}{x^2+4}$	B1	
	Use a correct method to determine a constant	M1	
	Obtain one of $A = 2$, $B = 1$, $C = -1$	A1	
	Obtain a second value	A1	
	Obtain a third value	A1	[5]

(ii)	Use correct method to find the first two terms of the expansion of $(x+2)^{-1}$, $(1+\frac{1}{2}x)^{-1}$, $(4+x^2)^{-1}$ or $(1+\frac{1}{4}x^2)^{-1}$	M1	
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	A1√ + A1√	
	Multiply out fully by $Bx + C$, where $BC \neq 0$	M1	
	Obtain final answer $\frac{3}{4}-\frac{1}{4}x+\frac{5}{16}x^2$, or equivalent	A1	[5]
	[Symbolic binomial coefficients, e.g. $\begin{pmatrix}-1\\1\end{pmatrix}$ are not sufficient for the M1. The f.t. is on A, B, C.]		
	[In the case of an attempt to expand $(3x^2+x+6)(x+2)^{-1}(x^2+4)^{-1}$, give M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for the final answer.]		
9 (i)	Differentiate both equations and equate derivatives	M1*	
	Obtain equation $\cos a - a\sin a = -\dfrac{k}{a^2}$	A1 + A1	
	State $a\cos a = \dfrac{k}{a}$ and eliminate k	DM1	
	Obtain the given answer showing sufficient working	A1	[5]
(ii)	Show clearly correct use of the iterative formula at least once	M1	
	Obtain answer 1.077	A1	
	Show sufficient iterations to 5 d.p. to justify 1.077 to 3 d.p., or show there is a sign change in the interval (1.0765, 1.0775)	A1	[3]
(iii)	Use a correct method to determine k	M1	
	Obtain answer $k = 0.55$	A1	[2]
10 (i)	Express general point of l in component form e.g. $(1+2\lambda,\ 2-\lambda,\ 1+\lambda)$	B1	
	Using the correct process for the modulus form an equation in λ	M1*	
	Reduce the equation to a quadratic, e.g. $6\lambda^2+2\lambda-4=0$	A1	
	Solve for λ (usual requirements for solution of a quadratic)	DM1	
	Obtain final answers $-\mathbf{i}+3\mathbf{j}$ and $\frac{7}{3}\mathbf{i}+\frac{4}{3}\mathbf{j}+\frac{5}{3}\mathbf{k}$	A1	[5]
(ii)	Using the correct process, find the scalar product of a direction vector for l and a normal for p	M1	
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and equate the result to $\frac{2}{3}$	M1	
	State a correct equation in any form, e.g. $\dfrac{2a-1+1}{\sqrt{(a^2+1+1)}.\sqrt{(2^2+(-1)^2+1)}}=\pm\dfrac{2}{3}$	A1	
	Solve for a^2	M1	
	Obtain answer $a = \pm 2$	A1	[5]

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/31**

Paper 3 **October/November 2017**

MARK SCHEME

Maximum Mark: 75

Published

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Mark Scheme Notes

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A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous’ answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks
1	Commence division and reach a partial quotient $x^2 + kx$	**M1**
	Obtain quotient $x^2 - 2x + 5$	**A1**
	Obtain remainder $-12x + 5$	**A1**
		3

Question	Answer	Marks
2	Plot the four points and draw straight line	**B1**
	State or imply that $\ln y = \ln C + x \ln a$	**B1**
	Carry out a completely correct method for finding $\ln C$ or $\ln a$	**M1**
	Obtain answer $C = 3.7$	**A1**
	Obtain answer $a = 1.5$	**A1**
		5

Question	Answer	Marks
3(i)	Calculate value of a relevant expression or expressions at $x = 2$ and $x = 3$	**M1**
	Complete the argument correctly with correct calculated values	**A1**
		2
3(ii)	Use an iterative formula correctly at least once	**M1**
	Show that (B) fails to converge	**A1**
	Using (A), obtain final answer 2.43	**A1**
	Show sufficient iterations to justify 2.43 to 2 d.p., or show there is a sign change in (2.425, 2.435)	**A1**
		4

Question	Answer	Marks
4(i)	Use correct $\tan(A \pm B)$ formula and express the LHS in terms of $\tan x$	M1
	Using $\tan 45° = 1$ express LHS as a single fraction	A1
	Use Pythagoras or correct double angle formula	M1
	Obtain given answer	A1
		4
4(ii)	Show correct sketch for one branch	B1
	Both branches correct and nothing else seen in the interval	B1
	Show asymptote at $x = 45°$	B1
		3

Question	Answer	Marks
5(i)	State or imply $y^3 + 3xy^2 \frac{\mathrm{d}y}{\mathrm{d}x}$ as derivative of xy^3	B1
	State or imply $4y^3 \frac{\mathrm{d}y}{\mathrm{d}x}$ as derivative of y^4	B1
	Equate derivative of the LHS to zero and solve for $\frac{\mathrm{d}y}{\mathrm{d}x}$	M1
	Obtain the given answer	A1
		4
5(ii)	Equate numerator to zero	*M1
	Obtain $y = -2x$, or equivalent	A1
	Obtain an equation in x or y	DM1
	Obtain final answer $x = -1$, $y = 2$ and $x = 1$, $y = -2$	A1
		4

Question	Answer	Marks
6	Separate variables correctly and attempt integration of one side	**B1**
	Obtain term $\tan y$, or equivalent	**B1**
	Obtain term of the form $k \ln \cos x$, or equivalent	**M1**
	Obtain term $-4 \ln \cos x$, or equivalent	**A1**
	Use $x = 0$ and $y = \frac{1}{4}\pi$ in solution containing $a \tan y$ and $b \ln \cos x$ to evaluate a constant, or as limits	**M1**
	Obtain correct solution in any form, e.g. $\tan y = 4 \ln \sec x + 1$	**A1**
	Substitute $y = \frac{1}{3}\pi$ in solution containing terms $a \tan y$ and $b \ln \cos x$, and use correct method to find x	**M1**
	Obtain answer $x = 0.587$	**A1**
		8

Question	Answer	Marks
7(a)	Square $x + iy$ and equate real and imaginary parts to 8 and -15	**M1**
	Obtain $x^2 - y^2 = 8$ and $2xy = -15$	**A1**
	Eliminate one unknown and find a horizontal equation in the other	**M1**
	Obtain $4x^4 - 32x^2 - 225 = 0$ or $4y^4 + 32y^2 - 225 = 0$, or three term equivalent	**A1**
	Obtain answers $\pm\frac{1}{\sqrt{2}}(5 - 3\mathrm{i})$ or equivalent	**A1**
		5
7(b)	Show a circle with centre $2 + \mathrm{i}$ in a relatively correct position	**B1**
	Show a circle with radius 2 and centre not at the origin	**B1**
	Show line through i at an angle of $\frac{1}{4}\pi$ to the real axis	**B1**
	Shade the correct region	**B1**
		4

Question	Answer	Marks
8(i)	Use a relevant method to determine a constant	**M1**
	Obtain one of the values $A = 2$, $B = 2$, $C = -1$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
		4
8(ii)	Integrate and obtain terms $2x + 2\ln(x+2) - \frac{1}{2}\ln(2x-1)$ (deduct **B1** for each error or omission) [The FT is on A, B and C]	**B2 FT**
	Substitute limits correctly in an integral containing terms $a\ln(x+2)$ and $b\ln(2x-1)$, where $ab \neq 0$	***M1**
	Use at least one law of logarithms correctly	**DM1**
	Obtain the given answer after full and correct working	**A1**
		5

Question	Answer	Marks
9(i)	Use correct product or quotient rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and obtain a 3 term quadratic equation in x	**M1**
	Obtain answers $x = 2 \pm \sqrt{3}$	**A1**
		4
9(ii)	Integrate by parts and reach $k(1+x^2)e^{-\frac{1}{2}x} + l\int xe^{-\frac{1}{2}x}\,dx$	***M1**
	Obtain $-2(1+x^2)e^{-\frac{1}{2}x} + 4\int xe^{-\frac{1}{2}x}\,dx$, or equivalent	**A1**
	Complete the integration and obtain $(-18-8x-2x^2)e^{-\frac{1}{2}x}$, or equivalent	**A1**
	Use limits $x = 0$ and $x = 2$ correctly, having fully integrated twice by parts	**DM1**
	Obtain the given answer	**A1**
		5

Question	Answer	Marks
10(i)	Equate at least two pairs of components of general points on l and m and solve for λ or for μ	**M1**
	Obtain correct answer for λ or μ, e.g. $\lambda = 3$ or $\mu = -2$; $\lambda = 0$ or $\mu = -\frac{1}{2}$; or $\lambda = \frac{3}{2}$ or $\mu = -\frac{7}{2}$	**A1**
	Verify that not all three pairs of equations are satisfied and that the lines fail to intersect	**A1**
		3
10(ii)	Carry out correct process for evaluating scalar product of direction vectors for l and m	***M1**
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	**DM1**
	Obtain answer 45° or $\frac{1}{4}\pi$ (0.785) radians	**A1**
		3
10(iii)	*EITHER:* Use scalar product to obtain a relevant equation in a, b and c, e.g. $-a + b + 4c = 0$	**B1**
	Obtain a second equation, e.g. $2a + b - 2c = 0$ and solve for one ratio, e.g. $a : b$	**M1**
	Obtain $a : b : c = 2 : -2 : 1$, or equivalent	**A1**
	Substitute (3, –2, –1) and values of a, b and c in general equation and find d	**M1**
	Obtain answer $2x - 2y + z = 9$, or equivalent	**A1**
	*OR*1: Attempt to calculate vector product of relevant vectors, e.g $(-\mathbf{i} + \mathbf{j} + 4\mathbf{k}) \times (2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$	**(M1**
	Obtain two correct components	**A1**
	Obtain correct answer, e.g. $-6\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}$	**A1**
	Substitute (3, –2, –1) in $-6x + 6y - 3z = d$, or equivalent, and find d	**M1**
	Obtain answer $-2x + 2y - z = -9$, or equivalent	**A1)**
	*OR*2: Using the relevant point and relevant vectors, form a 2-parameter equation for the plane	**(M1**
	State a correct equation, e.g. $\mathbf{r} = 3\mathbf{i} - 2\mathbf{j} - \mathbf{k} + \lambda(-\mathbf{i} + \mathbf{j} + 4\mathbf{k}) + \mu(2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$	**A1**
	State three correct equations in x, y, z, λ and μ	**A1**
	Eliminate λ and μ	**M1**

Question		Answer	Marks
		Obtain answer $2x - 2y + z = 9$, or equivalent	**A1)**
	*OR*3:	Using the relevant point and relevant vectors, form a determinant equation for the plane	**(M1**
		State a correct equation, e.g. $\begin{vmatrix} x-3 & y+2 & z+1 \\ -1 & 1 & 4 \\ 2 & 1 & -2 \end{vmatrix} = 0$	**A1**
		Attempt to expand the determinant	**M1**
		Obtain two correct cofactors	**A1**
		Obtain answer $-2x + 2y - z = -9$, or equivalent	**A1)**
			5

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/32**

Paper 3 **October/November 2017**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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Mark Scheme Notes

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B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously 'correct' answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
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PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks
1(i)	State or imply ordinates 0.915929…, 1, 1.112485…	**B1**
	Use correct formula, or equivalent, with h = 1.2 and three ordinates	**M1**
	Obtain answer 2.42 only	**A1**
		3
1(ii)	Justify the given statement	**B1**
		1

Question	Answer	Marks
2	Use law for the logarithm of a power or a quotient on the given equation	**M1**
	Use $\log_2 8 = 3$ or $2^3 = 8$	**M1**
	Obtain $x^2 - 8x - 8 = 0$, or horizontal equivalent	**A1**
	Solve a 3-term quadratic equation	**M1**
	Obtain final answer x = 8.90 only	**A1**
		5

Question	Answer	Marks
3	Use correct $\tan(A \pm B)$ formula and express LHS in terms of tan θ	**M1**
	Using tan 60° = $\sqrt{3}$ and $\cot\theta = 1/\tan\theta$, obtain a correct equation in $\tan\theta$ in any form	**A1**
	Reduce the equation to one in $\tan^2\theta$ only	**M1**
	Obtain $11\tan^2\theta = 1$, or equivalent	**A1**
	Obtain answer 16.8°	**A1**
		5

Question	Answer	Marks
4(i)	Use correct product or quotient rule or rewrite as $2\sec x - \tan x$ and differentiate	M1
	Obtain correct derivative in any form	A1
	Equate the derivative to zero and solve for x	M1
	Obtain $x = \frac{1}{6}\pi$	A1
	Obtain $y = \sqrt{3}$	A1
		5
4(ii)	Carry out an appropriate method for determining the nature of a stationary point	M1
	Show the point is a minimum point with no errors seen	A1
		2

Question	Answer	Marks
5	Separate variables and obtain $\int \frac{1}{y}\mathrm{d}y = \int \frac{x+2}{x+1}\mathrm{d}x$	B1
	Obtain term $\ln y$	B1
	Use an appropriate method to integrate $(x+2)/(x+1)$	*M1
	Obtain integral $x + \ln(x+1)$, or equivalent, e.g. $\ln(x+1) + x + 1$	A1
	Use $x = 1$ and $y = 2$ to evaluate a constant, or as limits	DM1
	Obtain correct solution in x and y in any form e.g. $\ln y = x + \ln(x+1) - 1$	A1
	Obtain answer $y = (x+1)\mathrm{e}^{x-1}$	A1
		7

Question	Answer	Marks
6(i)	State or imply $3x^2y + x^3\dfrac{dy}{dx}$ as derivative of x^3y	**B1**
	State or imply $9xy^2\dfrac{dy}{dx} + 3y^3$ as derivative of $3xy^3$	**B1**
	Equate derivative of the LHS to zero and solve for $\dfrac{dy}{dx}$	**M1**
	Obtain the given answer AG	**A1**
		4
6(ii)	Equate numerator to zero and use $x = -y$ to obtain an equation in x or in y	**M1**
	Obtain answer $x = a$ and $y = -a$	**A1**
	Obtain answer $x = -a$ and $y = a$	**A1**
	Consider and reject $y = 0$ and $x = y$ as possibilities	**B1**
		4

Question	Answer	Marks
7(i)	State modulus 2	**B1**
	State argument $-\frac{1}{3}\pi$ or $-60°$ ($\frac{5}{3}\pi$ or $300°$)	**B1**
		2
7(ii)	*EITHER:* Expand $(1-(\sqrt{3})\mathrm{i})^3$ completely and process i^2 and i^3	**(M1**
	Verify that the given relation is satisfied	**A1)**
	OR: $u^3 = 2^3\left(\cos(-\pi) + i\sin(-\pi)\right)$ or equivalent: follow their answers to **(i)**	**(M1**
	Verify that the given relation is satisfied	**A1)**
		2

Question	Answer	Marks
7(iii)	Show a circle with centre $1-(\sqrt{3})\mathrm{i}$ in a relatively correct position	**B1**
	Show a circle with radius 2 passing through the origin	**B1**
	Show the line Re $z = 2$	**B1**
	Shade the correct region	**B1**
		4

Question	Answer	Marks
8(i)	State or imply the form $\dfrac{A}{1-x}+\dfrac{B}{2x+3}+\dfrac{C}{(2x+3)^2}$	**B1**
	Use a relevant method to determine a constant	**M1**
	Obtain one of the values $A = 1$, $B = -2$, $C = 5$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
		5
	[Mark the form $\dfrac{A}{1-x}+\dfrac{Dx+E}{(2x+3)^2}$, where $A = 1$, $D = -4$, $E = -1$, **B1M1A1A1A1** as above.]	
8(ii)	Use a correct method to find the first two terms of the expansion of $(1-x)^{-1}$, $(1+\frac{2}{3}x)^{-1}$, $(2x+3)^{-1}$, $(1+\frac{2}{3}x)^{-2}$ or $(2x+3)^{-2}$	**M1**
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A3 FT**
	Obtain final answer $\frac{8}{9}+\frac{19}{27}x+\frac{13}{9}x^2$, or equivalent	**A1**
		5

Question	Answer	Marks
9(i)	Integrate by parts and reach $ax^{\frac{3}{2}}\ln x + b\int x^{\frac{3}{2}}.\frac{1}{x}\,dx$	*M1
	Obtain $\frac{2}{3}x^{\frac{3}{2}}\ln x - \frac{2}{3}\int x^{\frac{1}{2}}\,dx$	A1
	Obtain integral $\frac{2}{3}x^{\frac{3}{2}}\ln x - \frac{4}{9}x^{\frac{3}{2}}$, or equivalent	A1
	Substitute limits correctly and equate to 2	DM1
	Obtain the given answer correctly AG	A1
		5
9(ii)	Evaluate a relevant expression or pair of expressions at $x = 2$ and $x = 4$	M1
	Complete the argument correctly with correct calculated values	A1
		2
9(iii)	Use the iterative formula correctly at least once	M1
	Obtain final answer 3.031	A1
	Show sufficient iterations to 5 d.p. to justify 3.031 to 3 d.p., or show there is a sign change in the interval (3.0305, 3.0315)	A1
		3

Question	Answer	Marks
10(i)	State or imply a correct normal vector to either plane, e.g. $\mathbf{i}+\mathbf{j}+3\mathbf{k}$ or $2\mathbf{i}-2\mathbf{j}+\mathbf{k}$	B1
	Carry out correct process for evaluating the scalar product of two normal vectors	M1
	Using the correct process for the moduli, divide the scalar product of the two normals by the product of their moduli and evaluate the inverse cosine of the result	M1
	Obtain final answer 72.5° or 1.26 radians	A1
		4
10(ii)	*EITHER*: Substitute $y = 2$ in both plane equations and solve for x or for z	(M1
	Obtain $x = 3$ and $z = 1$	A1)
	OR: Find the equation of the line of intersection of the planes	
	Substitute $y = 2$ in line equation and solve for x or for z	(M1
	Obtain $x = 3$ and $z = 1$	A1)

Question	Answer	Marks
	EITHER: Use scalar product to obtain an equation in a, b and c, e.g. $a + b + 3c = 0$	**(B1**
	Form a second relevant equation, e.g. $2a - 2b + c = 0$, and solve for one ratio, e.g. $a : b$	***M1**
	Obtain final answer $a : b : c = 7 : 5 : -4$	**A1**
	Use coordinates of A and values of a, b and c in general equation and find d	**DM1**
	Obtain answer $7x + 5y - 4z = 27$, or equivalent	**A1 FT)**
	*OR*1: Calculate the vector product of relevant vectors, e.g. $(\mathbf{i} + \mathbf{j} + 3\mathbf{k}) \times (2\mathbf{i} - 2\mathbf{j} + \mathbf{k})$	**(*M1**
	Obtain two correct components	**A1**
	Obtain correct answer, e.g. $7\mathbf{i} + 5\mathbf{j} - 4\mathbf{k}$	**A1**
	Substitute coordinates of A in plane equation with their normal and find d	**DM1**
	Obtain answer $7x + 5y - 4z = 27$, or equivalent	**A1 FT)**
	*OR*2: Using relevant vectors, form a two-parameter equation for the plane	**(*M1**
	State a correct equation, e.g. $\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} + \mathbf{j} + 3\mathbf{k}) + \mu(2\mathbf{i} - 2\mathbf{j} + \mathbf{k})$	**A1 FT**
	State 3 correct equations in x, y, z, λ and μ	**A1 FT**
	Eliminate λ and μ	**DM1**
	Obtain answer $7x + 5y - 4z = 27$, or equivalent	**A1 FT)**
	*OR*3: Use the direction vector of the line of intersection of the two planes as normal vector to the plane	**(*M1**
	Two correct components	**A1**
	Three correct components	**A1**
	Substitute coordinates of A in plane equation with their normal and find d	**DM1**
	Obtain answer $7x + 5y - 4z = 27$, or equivalent	**A1 FT)**
		7

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/33**

Paper 3 **October/November 2017**

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	Obtain quotient $x^2 - 2x + 5$	**A1**
	Obtain remainder $-12x + 5$	**A1**
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	State or imply that $\ln y = \ln C + x \ln a$	**B1**
	Carry out a completely correct method for finding $\ln C$ or $\ln a$	**M1**
	Obtain answer $C = 3.7$	**A1**
	Obtain answer $a = 1.5$	**A1**
		5

Question	Answer	Marks
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	Complete the argument correctly with correct calculated values	**A1**
		2
3(ii)	Use an iterative formula correctly at least once	**M1**
	Show that (B) fails to converge	**A1**
	Using (A), obtain final answer 2.43	**A1**
	Show sufficient iterations to justify 2.43 to 2 d.p., or show there is a sign change in (2.425, 2.435)	**A1**
		4

Question	Answer	Marks
4(i)	Use correct $\tan(A \pm B)$ formula and express the LHS in terms of $\tan x$	**M1**
	Using $\tan 45° = 1$ express LHS as a single fraction	**A1**
	Use Pythagoras or correct double angle formula	**M1**
	Obtain given answer	**A1**
		4
4(ii)	Show correct sketch for one branch	**B1**
	Both branches correct and nothing else seen in the interval	**B1**
	Show asymptote at $x = 45°$	**B1**
		3

Question	Answer	Marks
5(i)	State or imply $y^3 + 3xy^2 \dfrac{dy}{dx}$ as derivative of xy^3	**B1**
	State or imply $4y^3 \dfrac{dy}{dx}$ as derivative of y^4	**B1**
	Equate derivative of the LHS to zero and solve for $\dfrac{dy}{dx}$	**M1**
	Obtain the given answer	**A1**
		4
5(ii)	Equate numerator to zero	***M1**
	Obtain $y = -2x$, or equivalent	**A1**
	Obtain an equation in x or y	**DM1**
	Obtain final answer $x = -1$, $y = 2$ and $x = 1$, $y = -2$	**A1**
		4

Question	Answer	Marks
6	Separate variables correctly and attempt integration of one side	**B1**
	Obtain term $\tan y$, or equivalent	**B1**
	Obtain term of the form $k\ln\cos x$, or equivalent	**M1**
	Obtain term $-4\ln\cos x$, or equivalent	**A1**
	Use $x = 0$ and $y = \frac{1}{4}\pi$ in solution containing $a\tan y$ and $b\ln\cos x$ to evaluate a constant, or as limits	**M1**
	Obtain correct solution in any form, e.g. $\tan y = 4\ln\sec x + 1$	**A1**
	Substitute $y = \frac{1}{3}\pi$ in solution containing terms $a\tan y$ and $b\ln\cos x$, and use correct method to find x	**M1**
	Obtain answer $x = 0.587$	**A1**
		8

Question	Answer	Marks
7(a)	Square $x + iy$ and equate real and imaginary parts to 8 and -15	**M1**
	Obtain $x^2 - y^2 = 8$ and $2xy = -15$	**A1**
	Eliminate one unknown and find a horizontal equation in the other	**M1**
	Obtain $4x^4 - 32x^2 - 225 = 0$ or $4y^4 + 32y^2 - 225 = 0$, or three term equivalent	**A1**
	Obtain answers $\pm\dfrac{1}{\sqrt{2}}(5 - 3i)$ or equivalent	**A1**
		5
7(b)	Show a circle with centre $2 + i$ in a relatively correct position	**B1**
	Show a circle with radius 2 and centre not at the origin	**B1**
	Show line through i at an angle of $\frac{1}{4}\pi$ to the real axis	**B1**
	Shade the correct region	**B1**
		4

Question	Answer	Marks
8(i)	Use a relevant method to determine a constant	**M1**
	Obtain one of the values $A = 2$, $B = 2$, $C = -1$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
		4
8(ii)	Integrate and obtain terms $2x + 2\ln(x+2) - \frac{1}{2}\ln(2x-1)$ (deduct **B1** for each error or omission) [The FT is on A, B and C]	**B2 FT**
	Substitute limits correctly in an integral containing terms $a\ln(x+2)$ and $b\ln(2x-1)$, where $ab \neq 0$	***M1**
	Use at least one law of logarithms correctly	**DM1**
	Obtain the given answer after full and correct working	**A1**
		5

Question	Answer	Marks
9(i)	Use correct product or quotient rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and obtain a 3 term quadratic equation in x	**M1**
	Obtain answers $x = 2 \pm \sqrt{3}$	**A1**
		4
9(ii)	Integrate by parts and reach $k(1+x^2)e^{-\frac{1}{2}x} + l\int xe^{-\frac{1}{2}x}\,dx$	***M1**
	Obtain $-2(1+x^2)e^{-\frac{1}{2}x} + 4\int xe^{-\frac{1}{2}x}\,dx$, or equivalent	**A1**
	Complete the integration and obtain $(-18-8x-2x^2)e^{-\frac{1}{2}x}$, or equivalent	**A1**
	Use limits $x = 0$ and $x = 2$ correctly, having fully integrated twice by parts	**DM1**
	Obtain the given answer	**A1**
		5

Question	Answer	Marks
10(i)	Equate at least two pairs of components of general points on l and m and solve for λ or for μ	M1
	Obtain correct answer for λ or μ, e.g. $\lambda = 3$ or $\mu = -2$; $\lambda = 0$ or $\mu = -\frac{1}{2}$; or $\lambda = \frac{3}{2}$ or $\mu = -\frac{7}{2}$	A1
	Verify that not all three pairs of equations are satisfied and that the lines fail to intersect	A1
		3
10(ii)	Carry out correct process for evaluating scalar product of direction vectors for l and m	*M1
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	DM1
	Obtain answer 45° or $\frac{1}{4}\pi$ (0.785) radians	A1
		3
10(iii)	*EITHER:* Use scalar product to obtain a relevant equation in a, b and c, e.g. $-a + b + 4c = 0$	B1
	Obtain a second equation, e.g. $2a + b - 2c = 0$ and solve for one ratio, e.g. $a : b$	M1
	Obtain $a : b : c = 2 : -2 : 1$, or equivalent	A1
	Substitute (3, –2, –1) and values of a, b and c in general equation and find d	M1
	Obtain answer $2x - 2y + z = 9$, or equivalent	A1
	*OR*1: Attempt to calculate vector product of relevant vectors, e.g $(-\mathbf{i} + \mathbf{j} + 4\mathbf{k}) \times (2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$	(M1
	Obtain two correct components	A1
	Obtain correct answer, e.g. $-6\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}$	A1
	Substitute (3, –2, –1) in $-6x + 6y - 3z = d$, or equivalent, and find d	M1
	Obtain answer $-2x + 2y - z = -9$, or equivalent	A1)
	*OR*2: Using the relevant point and relevant vectors, form a 2-parameter equation for the plane	(M1
	State a correct equation, e.g. $\mathbf{r} = 3\mathbf{i} - 2\mathbf{j} - \mathbf{k} + \lambda(-\mathbf{i} + \mathbf{j} + 4\mathbf{k}) + \mu(2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$	A1
	State three correct equations in x, y, z, λ and μ	A1
	Eliminate λ and μ	M1

Question	Answer		Marks
		Obtain answer $2x-2y+z=9$, or equivalent	**A1)**
	*OR*3:	Using the relevant point and relevant vectors, form a determinant equation for the plane	**(M1**
		State a correct equation, e.g. $\begin{vmatrix} x-3 & y+2 & z+1 \\ -1 & 1 & 4 \\ 2 & 1 & -2 \end{vmatrix} = 0$	**A1**
		Attempt to expand the determinant	**M1**
		Obtain two correct cofactors	**A1**
		Obtain answer $-2x+2y-z=-9$, or equivalent	**A1)**
			5

Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/31**

Paper 3 **October/November 2018**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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Cambridge Assessment International Education

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)

CWO Correct Working Only – often written by a ‘fortuitous’ answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	*EITHER*: State or imply non-modular inequality $2^2\left(2x-a\right)^2<\left(x+3a\right)^2$, or corresponding quadratic equation, or pair of linear equations $2(2x-a)=\pm(x+3a)$	**B1**	
	Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	**M1**	
	Obtain critical values $x=\frac{5}{3}a$ and $x=-\frac{1}{5}a$	**A1**	
	State final answer $-\frac{1}{5}a<x<\frac{5}{3}a$	**A1**	
	OR: Obtain critical value $x=\frac{5}{3}a$ from a graphical method, or by inspection, or by solving a linear equation or an inequality	**B1**	
	Obtain critical value $x=-\frac{1}{5}a$ similarly	**B2**	
	State final answer $-\frac{1}{5}a<x<\frac{5}{3}a$ [Do not condone $\leqslant$ for $<$ in the final answer.]	**B1**	
		4	

Question	Answer	Marks	Guidance
2	Rearrange the equation in the form $ae^{2x} = b$ or $ae^{x} = be^{-x}$	**M1**	
	Obtain correct equation in either form with $a = 2$ and $b = 5$	**A1**	
	Use correct method to solve for x	**M1**	
	Obtain answer $x = 0.46$	**A1**	
		4	

Question	Answer	Marks	Guidance
3 (i)	Sketch a relevant graph, e.g. $y = x^3$	**B1**	
	Sketch a second relevant graph, e.g. $y = 3 - x$, and justify the given statement	**B1**	Consideration of behaviour for $x < 0$ is needed for the second B1
		2	
3(ii)	State or imply the equation $x = \left(2x^3 + 3\right) / \left(3x^2 + 1\right)$	**B1**	
	Rearrange this in the form $x^3 = 3 - x$, or commence work *vice versa*	**B1**	
		2	

Question	Answer	Marks	Guidance
3(iii)	Use the iterative formula correctly at least once	**M1**	
	Obtain final answer 1.213	**A1**	
	Show sufficient iterations to 5 d.p. or more to justify 1.213 to 3 d.p., or show there is a sign change in the interval (1.2125, 1.2135)	**A1**	
		3	

Question	Answer	Marks	Guidance
4(i)	Obtain $\dfrac{dx}{d\theta} = 2\cos\theta + 2\cos 2\theta$ or $\dfrac{dy}{d\theta} = -2\sin\theta - 2\sin 2\theta$	**B1**	
	Use $dy/dx = dy/d\theta \div dx/d\theta$	**M1**	
	Obtain correct $\dfrac{dy}{dx}$ in any form, e.g. $-\dfrac{2\sin\theta + 2\sin 2\theta}{2\cos\theta + 2\cos 2\theta}$	**A1**	
		3	
4(ii)	Equate denominator to zero and use any correct double angle formula	**M1***	
	Obtain correct 3-term quadratic in $\cos\theta$ in any form	**A1**	
	Solve for θ	**depM1***	
	Obtain $x = 3\sqrt{3}/2$ and $y = \dfrac{1}{2}$, or exact equivalents	**A1**	
		4	

Question	Answer	Marks	Guidance
5	Separate variables correctly and integrate at least one side	**B1**	
	Obtain term $\ln y$	**B1**	
	Obtain terms $2 \ln x - \frac{1}{2}x^2$	**B1+B1**	
	Use $x = 1$, $y = 1$ to evaluate a constant, or as limits	**M1**	
	Obtain correct solution in any form, e.g. $\ln y = 2\ln x - \frac{1}{2}x^2 + \frac{1}{2}$	**A1**	
	Rearrange as $y = x^2 \exp\left(\frac{1}{2} - \frac{1}{2}x^2\right)$, or equivalent	**A1**	
		7	

Question	Answer	Marks	Guidance
6(i)	Rearrange in the form $\sqrt{3}\sin x - \cos x = \sqrt{2}$	**B1**	
	State $R = 2$	**B1**	
	Use trig formulae to obtain α	**M1**	
	Obtain $\alpha = 30°$ with no errors seen	**A1**	
		4	

Question	Answer	Marks	Guidance
6(ii)	Evaluate $\sin^{-1}\left(\frac{\sqrt{2}}{R}\right)$	**B1ft**	
	Carry out a correct method to find a value of x in the given interval	**M1**	
	Obtain answer $x = 75°$	**A1**	
	Obtain a second answer e.g. $x = 165°$ and no others [Treat answers in radians as a misread. Ignore answers outside the given interval.]	**A1ft**	
		4	

Question	Answer	Marks	Guidance
7(i)	Use product rule	**M1***	
	Obtain correct derivative in any form	**A1**	
	Equate derivative to zero and obtain an equation in a single trig function	**depM1***	
	Obtain a correct equation, e.g. $3\tan^2 x = 2$	**A1**	
	Obtain answer $x = 0.685$	**A1**	
		5	

Question	Answer	Marks	Guidance
7(ii)	Use the given substitution and reach $a\int\left(u^2-u^4\right)\mathrm{d}u$	**M1**	
	Obtain correct integral with $a = 5$ and limits 0 and 1	**A1**	
	Use correct limits in an integral of the form $a\left(\frac{1}{3}u^3-\frac{1}{5}u^5\right)$	**M1**	
	Obtain answer $\frac{2}{3}$	**A1**	
		4	

Question	Answer	Marks	Guidance
8(i)	*EITHER*: Multiply numerator and denominator by $1 + 2i$, or equivalent, or equate to $x + iy$, obtain two equations in x and y and solve for x or for y	M1	
	Obtain quotient $-\frac{4}{5}+\frac{7}{5}i$, or equivalent	A1	
	Use correct method to find either r or θ	M1	
	Obtain $r = 1.61$	A1	
	Obtain $\theta = 2.09$	A1	
	OR: Find modulus or argument of $2 + 3i$ or of $1 - 2i$	B1	
	Use correct method to find r	M1	
	Obtain $r = 1.61$	A1	
	Use correct method to find θ	M1	
	Obtain $\theta = 2.09$	A1	
		5	
8(ii)	Show a circle with centre $3 - 2i$	B1	
	Show a circle with radius 1	B1ft	Centre not at the origin
	Carry out a correct method for finding the least value of $\lvert z \rvert$	M1	
	Obtain answer $\sqrt{13} - 1$	A1	
		4	

Question	Answer	Marks	Guidance
9(i)	State or imply the form $\dfrac{A}{2-x}+\dfrac{B}{3+2x}+\dfrac{C}{(3+2x)^2}$	**B1**	
	Use a correct method to find a constant	**M1**	
	Obtain one of $A = 1$, $B = -1$, $C = 3$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value [Mark the form $\dfrac{A}{2-x}+\dfrac{Dx+E}{(3+2x)^2}$, where $A = 1$, $D = -2$ and $E = 0$, B1M1A1A1A1 as above.]	**A1**	
		5	
9(ii)	Integrate and obtain terms $-\ln(2-x)-\dfrac{1}{2}\ln(3+2x)-\dfrac{3}{2(3+2x)}$	**B3ft**	The f.t is on A, B, C; or on A, D, E.
	Substitute correctly in an integral with terms $a\ln(2-x)$, $b\ln(3+2x)$ and $c/(3+2x)$ where $abc \neq 0$	**M1**	
	Obtain the given answer after full and correct working [Correct integration of the A, D, E form gives an extra constant term if integration by parts is used for the second partial fraction.]	**A1**	
		5	

Question	Answer	Marks	Guidance
10(i)	*EITHER*: Expand scalar product of a normal to m and a direction vector of l	**M1**	
	Verify scalar product is zero	**A1**	
	Verify that one point of l does not lie in the plane	**A1**	
	OR: Substitute coordinates of a general point of l in the equation of the plane m	**M1**	
	Obtain correct equation in λ in any form	**A1**	
	Verify that the equation is not satisfied for any value of λ	**A1**	
		3	
10(ii)	Use correct method to evaluate a scalar product of normal vectors to m and n	**M1**	
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	**M1**	
	Obtain answer 74.5° or 1.30 radians	**A1**	
		3	
10(iii)	*EITHER*: Using the components of a general point P of l form an equation in λ by equating the perpendicular distance from n to 2	**M1**	
	OR: Take a point Q on l, e.g. (5, 3, 3) and form an equation in λ by equating the length of the projection of QP onto a normal to plane n to 2	**M1**	
	Obtain a correct modular or non-modular equation in any form	**A1**	
	Solve for λ and obtain a position vector for P, e.g. $7\mathbf{i} + 5\mathbf{j} + 7\mathbf{j}$ from $\lambda = 3$	**A1**	
	Obtain position vector of the second point, e.g. $3\mathbf{i} + \mathbf{j} - \mathbf{k}$ from $\lambda = -1$	**A1**	
		4	

Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/32**

Paper 3 **October/November 2018**

MARK SCHEME

Maximum Mark: 75

Published

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

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B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

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- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
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Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
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Question	Answer	Marks	Guidance
1	State or imply non-modular inequality $3^2(2x-1)^2 > (x+4)^2$, or corresponding quadratic equation, or pair of linear equations/inequalities $3(2x-1) = \pm(x+4)$	**B1**	$35x^2 - 44x - 7 = 0$
	Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	**M1**	Allow for reasonable attempt at factorising e.g. $(5x-7)(7x+1)$
	Obtain critical values $x = \frac{7}{5}$ and $x = -\frac{1}{7}$	**A1**	Accept 1.4 and -0.143 or better for penultimate A mark
	State final answer $x > \frac{7}{5}$, $x < -\frac{1}{7}$	**A1**	'and' is A0, $\frac{7}{5} < x < -\frac{1}{7}$ is A0. Must be exact values. Must be strict inequalities in final answer
	Alternative		
	Obtain critical value $x = \frac{7}{5}$ from a graphical method	**B1**	or by inspection, or by solving a linear equation or an inequality
	Obtain critical value $x = -\frac{1}{7}$ similarly	**B2**	
	State final answer $x > \frac{7}{5}$ or $x < -\frac{1}{7}$ or equivalent	**B1**	[Do not condone $\geqslant$ for $>$, or $\leqslant$ for $<$.]
		4	

Question	Answer	Marks	Guidance
2	Use trig formula and obtain an equation in $\sin\theta$ and $\cos\theta$	M1*	Condone sign error in expansion and/or omission of "$+\cos\theta$" $\sin\theta\cos 30° - \cos\theta\sin 30° + \cos\theta = 2\sin\theta$
	Obtain an equation in $\tan\theta$	M1(dep*)	e.g. $\tan\theta = \frac{1-\sin 30°}{2-\cos 30°}$ Can be implied by correct answer following correct expansion. Otherwise need to see working
	Obtain $\tan\theta = 1/\left(4-\sqrt{3}\right)$, or equivalent	A1	$\frac{4+\sqrt{3}}{13}$, $0.4409....$ (2 s.f or better)
	Obtain final answer $\theta = 23.8°$ and no others in range	A1	At least 3 sf $(23.7939....)$ ignore extra values outside range
		4	

Question	Answer	Marks	Guidance
3(i)	Integrate by parts and reach $a\dfrac{\ln x}{x^2} + b\int \dfrac{1}{x}.\dfrac{1}{x^2}\,\mathrm{d}x$	M1*	
	Obtain $\pm\dfrac{1}{2}\dfrac{\ln x}{x^2} \pm \int \dfrac{1}{x}.\dfrac{1}{2x^2}\,\mathrm{d}x$, or equivalent	A1	
	Complete integration correctly and obtain $-\dfrac{\ln x}{2x^2} - \dfrac{1}{4x^2}$, or equivalent	A1	Condone without '$+ C$' ISW
		3	

Question	Answer	Marks	Guidance
3(ii)	Substitute limits correctly in an expression of the form $a\dfrac{\ln x}{x^2}+\dfrac{b}{x^2}$ or equivalent	M1(dep*)	$-\frac{1}{8}\ln 2-\frac{1}{16}+\frac{1}{4}$
	Obtain the **given answer** following full and **exact** working	A1	The step $\ln 2=\frac{1}{2}\ln 4$ or 2ln2 = ln4 needs to be clear.
		2	

Question	Answer	Marks	Guidance
4	Substitute and obtain 3-term quadratic $3u^2+4u-1=0$, or equivalent	B1	e.g. $3\left(e^x\right)^2+4e^x-1=0$
	Solve a 3 term quadratic for u	M1	Must be an equation with real roots
	Obtain root $\left(\sqrt{7}-2\right)/3$, or decimal in [0.21, 0.22]	A1	Or equivalent. Ignore second root (even if incorrect)
	Use correct method for finding x **from a positive value of** e^x	M1	Must see some indication of method: use of $x=\ln u$
	Obtain answer $x = -1.536$ only	A1	CAO. Must be 3 dp
		5	

Question	Answer	Marks	Guidance
5(i)	Use product rule on a correct expression	**M1**	Condone with $+\frac{x}{8-x}$ unless there is clear evidence of incorrect product rule.
	Obtain correct derivative in any form	**A1**	$\frac{dy}{dx}=\ln(8-x)-\frac{x}{8-x}$
	Equate derivative to 1 and obtain $x=8-\frac{8}{\ln(8-x)}$	**A1**	**Given answer:** check carefully that it follows from correct working
			Condone the use of *a* for *x* throughout
		3	
5(ii)	Calculate values of a relevant expression or pair of relevant expressions at x = 2.9 and x = 3.1	**M1**	$8-\frac{8}{\ln 5.1}=3.09>2.9$, $8-\frac{8}{\ln 4.9}=2.97<3.1$ Clear linking of pairs needed for M1 by this method (0.19 and -0.13)
	Complete the argument correctly with correct calculated values	**A1**	Note: valid to consider gradient at 2.9 (1.06..) and 3.1 (0.95..) and comment on comparison with 1
		2	

Question	Answer	Marks	Guidance
5(iii)	Use the iterative process $x_{n+1} = 8 - \dfrac{8}{\ln(8 - x_n)}$ correctly to find at least two successive values. SR: Clear successive use of 0, 1, 2, 3 etc., or equivalent, scores M0.	**M1**	3, 3.0293, 3.0111, 3.0225, 3.0154, (3.0198) 2.9, 3.0897, 2.9728, 3.0460, 3.0006, 3.290, 3.0113, 3.0223, 3.0155 3.1, 2.9661, 3.0501, 2.9980, 3.0305, 3.0103, 3.0229, 3.0151 Allow M1 if values given to fewer than 4 dp
	Obtain final answer 3.02	**A1**	
	Show sufficient iterations to at least 4 d.p. to justify 3.02 to 2 d.p., or show there is a sign change in the interval (3.015, 3.025)	**A1**	Must have two consecutive values rounding correctly to 3.02
		3	

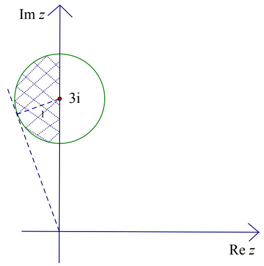
Question	Answer	Marks	Guidance
6	State equation $\frac{dy}{dx} = k\frac{y^2}{x}$, or equivalent	**B1**	SC: If $k = 1$ seen or implied give B0 and then allow B1B1B0M1, max 3/8.
	Separate variables correctly and integrate at least one side	**B1**	$\int \frac{k}{x}dx = \int \frac{1}{y^2}dy$ Allow with incorrect value substituted for k
	Obtain terms $-\frac{1}{y}$ and $k\ln x$	**B1 + B1**	Incorrect k used scores max. B1B0
	Use given coordinates correctly to find k and/or a constant of integration C in an equation containing terms $\frac{a}{y}$, $b\ln x$ and C	**M1**	SC: If an incorrect method is used to find k, M1 is allowable for a correct method to find C
	Obtain $k = \frac{1}{2}$ and $c = -1$, or equivalent	**A1 + A1**	$\frac{1}{2}\ln x = 1 - \frac{1}{y}$ A0 for fortuitous answers.
	Obtain answer $y = \frac{2}{2-\ln x}$, or equivalent, and ISW	**A1**	$y = \frac{-1}{-1+\ln\sqrt{x}}$
			SC: MR of the fraction. $\frac{dy}{dx} = k\frac{y^2}{x^2}$ B1 Separate variables and integrate B1 $\frac{-1}{y} = \frac{-k}{x}(+C)$ B1+B1 Substitute to find k and/or c M1 $k = \frac{e}{2(e-1)}$, $c = \frac{2-e}{2(e-1)}$ A1+A1 Answer A0
		8	

Question	Answer	Marks	Guidance
7(i)	Use correct quotient or product rule	**M1**	
	Obtain correct derivative in any form	**A1**	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-3\sin x(2+\sin x)-3\cos x\cos x}{(2+\sin x)^2}$ Condone invisible brackets if recovery implied later.
	Equate numerator to zero	**M1**	
	Use $\cos^2 x+\sin^2 x=1$ and solve for $\sin x$	**M1**	$-6\sin x-3=0 \Rightarrow \sin x=\ldots$
	Obtain coordinates $x=-\pi/6$ and $y=\sqrt{3}$ ISW	**A1 + A1**	From correct working. No others in range
			SR: A candidate who only states the numerator of the derivative, but justifies this, can have full marks. Otherwise they score M0A0M1M1A0A0
		6	
7(ii)	State indefinite integral of the form k ln $(2 + \sin x)$	**M1***	
	Substitute limits correctly, equate result to 1 and obtain 3 ln $(2 + \sin a)$ – 3 ln 2 = 1	**A1**	or equivalent
	Use correct method to solve for a	**M1(dep*)**	Allow for a correct method to solve an incorrect equation, so long as that equation has a solution. $1+\frac{1}{2}\sin a=e^{1/3} \Rightarrow a=\sin^{-1}\left[2\left(e^{1/3}-1\right)\right]$ Can be implied by 52.3°
	Obtain answer a = 0.913 or better	**A1**	Ignore additional solutions. Must be in radians.
		4	

Question	Answer	Marks	Guidance
8(i)	State or imply the form $\dfrac{A}{1-2x}+\dfrac{B}{2-x}+\dfrac{C}{(2-x)^2}$	**B1**	
	Use a correct method for finding a constant M1 is available following a single slip in working from their form but no A marks (even if a constant is "correct")	**M1**	$7=A+2B$ $-15=-4A-5B-2C$ $8=4A+2B+C$
	Obtain one of $A=1$, $B=3$, $C=-2$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
	[Mark the form $\dfrac{A}{1-2x}+\dfrac{Dx+E}{(2-x)^2}$, where $A=1$, $D=-3$ and $E=4$, B1M1A1A1A1 as above.]		
		5	

Question	Answer	Marks	Guidance
8(ii)	Use a correct method to find the first two terms of the expansion of $(1-2x)^{-1}$, $(2-x)^{-1}$, $\left(1-\frac{1}{2}x\right)^{-1}$, $(2-x)^{-2}$ or $\left(1-\frac{1}{2}x\right)^{-2}$	**M1**	Symbolic coefficients are not sufficient for the M1
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A3ft**	$1+2x+4x^2$ The ft is on *A*, *B*, *C*. $\frac{3}{2}+\frac{3}{4}x+\frac{3}{8}x^2$ $-\frac{1}{2}-\frac{1}{2}x-\frac{3}{8}x^2$
	Obtain **final** answer $2+\frac{9}{4}x+4x^2$	**A1**	
	[For the *A*, *D*, *E* form of fractions give M1A2ft for the expanded partial fractions, then, if $D \neq 0$, M1 for multiplying out fully, and A1 for the final answer.]		[The ft is on *A*, *D*, *E*.]
		5	

Question	Answer	Marks	Guidance
9(a)(i)	Multiply numerator and denominator by 1 + 2i, or equivalent	**M1**	Requires at least one of $2+10\mathrm{i}+12\mathrm{i}^2$ and $1-4\mathrm{i}^2$ together with use of $\mathrm{i}^2=-1$. Can be implied by $\frac{-10+10\mathrm{i}}{5}$
	Obtain quotient – 2 + 2i	**A1**	
	Alternative		
	Equate to x + iy, obtain two equations in x and y and solve for x or for y	**M1**	$x+2y=2,\quad y-2x=6$
	Obtain quotient – 2 + 2i	**A1**	
		2	
9(a)(ii)	Use correct method to find either r or θ	**M1**	If only finding θ, need to be looking for θ in the correct quadrant
	Obtain $r=2\sqrt{2}$, or exact equivalent	**A1ft**	ft their x + iy
	Obtain $\theta=\frac{3}{4}\pi$ from exact work	**A1ft**	ft on k(-1 + i) for $k>0$ Do not ISW
		3	

Question	Answer	Marks	Guidance
9(b)	Show a circle with centre 3i	**B1**	
	Show a circle with radius 1	**B1ft**	Follow through their centre provided not at the origin For clearly unequal scales, should be an ellipse
	All correct with even scales and **shade** the correct region	**B1**	
	Carry out a correct method for calculating greatest value of arg z	**M1**	e.g. $\arg z = \frac{\pi}{2} + \sin^{-1}\frac{1}{3}$
	Obtain answer 1.91	**A1**	
		5	

Question	Answer	Marks	Guidance
10(i)	Substitute for **r** and expand the scalar product to obtain an equation in λ	**M1***	e.g. $3(5+\lambda)+(-3-2\lambda)+(-1+\lambda)=5\ \ (2\lambda=5-11)$ or $3(4+\lambda)+1(-5-2\lambda)+(-1+\lambda)=0$ Must attempt to deal with **i** + 2**j**
	Solve a linear equation for λ	**M1(dep*)**	
	Obtain $\lambda = -3$ and position vector $\mathbf{r}_A = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$ for A	**A1**	Accept coordinates
		3	
10(ii)	State or imply a normal vector of p is 3**i** + **j** + **k**, or equivalent	**B1**	
	Use correct method to evaluate a scalar product of relevant vectors e.g. (**i** – 2**j** + **k**).(3**i** + **j** + **k**)	**M1**	
	Using the correct process for calculating the moduli, divide the scalar product by the product of the moduli and evaluate the inverse sine or cosine of the result	**M1**	$\cos\theta = \dfrac{2}{\sqrt{6}\sqrt{11}}$ Second M1 available if working with the wrong vectors
	Obtain answer 14.3° or 0.249 radians	**A1**	Or better

Question	Answer	Marks	Guidance
10(ii)	**Alternative 1**		
	Use of a point on l and Cartesian equation $3x+y+z=5$ to find distance of point from plane e.g. $B(5,-3,-1)$ $d=\frac{3\times5-3-1-5}{\sqrt{9+1+1}}$	**M1**	
	$=\frac{6}{\sqrt{11}}\quad(=1.809...)$	**A1**	
	Complete method to find angle e.g. $\sin\theta=\dfrac{d}{AB}$	**M1**	
	$\theta=\sin^{-1}\left(\dfrac{6}{\sqrt{11}\sqrt{54}}\right)=0.249$	**A1**	Or better
	Alternative 2		
	State or imply a normal vector of p is $3\mathbf{i}+\mathbf{j}+\mathbf{k}$, or equivalent	**B1**	
	Use correct method to evaluate a vector product of relevant vectors e.g. $(\mathbf{i}-2\mathbf{j}+\mathbf{k})\mathbf{x}(3\mathbf{i}+\mathbf{j}+\mathbf{k})$	**M1**	$3\mathbf{i}-2\mathbf{j}+7\mathbf{k}$
	Using the correct process for calculating the moduli, divide the vector product by the product of the moduli and evaluate the inverse sine or cosine of the result	**M1**	$\sin\theta=\dfrac{\sqrt{3^2+2^2+7^2}}{\sqrt{11}\sqrt{6}}$. Second M1 available if working with the wrong vectors
	Obtain answer 14.3° or 0.249 radians	**A1**	Or better
		4	

Question	Answer	Marks	Guidance
10(iii)	Taking the direction vector of the line to be $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$, state a relevant equation in a, b, c, e.g. $3a + b + c = 0$	**B1**	
	State a second relevant equation, e.g. $a - 2b + c = 0$, and solve for one ratio, e.g. $a : b$	**M1**	
	Obtain $a : b : c = 3 : -2 : -7$, or equivalent	**A1**	
	State answer $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k} + \mu(3\mathbf{i} - 2\mathbf{j} - 7\mathbf{k})$	**A1ft**	Or equivalent. The f.t. is on $\mathbf{r}_A$ Requires '$\mathbf{r}$ = ….'
	Alternative		
	Attempt to calculate the vector product of relevant vectors, e.g. $(3\mathbf{i} + \mathbf{j} + \mathbf{k}) \times (\mathbf{i} - 2\mathbf{j} + \mathbf{k})$	**M1**	
	Obtain two correct components of the product	**A1**	
	Obtain correct product, e.g. $3\mathbf{i} - 2\mathbf{j} - 7\mathbf{k}$	**A1**	
	State answer $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k} + \mu(3\mathbf{i} - 2\mathbf{j} - 7\mathbf{k})$	**A1ft**	Or equivalent. The f.t. is on $\mathbf{r}_A$. Requires " $\mathbf{r}$ = …."
		4	

Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/33**

Paper 3 **October/November 2018**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2018 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

Cambridge Assessment International Education

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
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MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	*EITHER*: State or imply non-modular inequality $2^2(2x-a)^2 < (x+3a)^2$, or corresponding quadratic equation, or pair of linear equations $2(2x-a) = \pm(x+3a)$	B1	
	Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	M1	
	Obtain critical values $x = \frac{5}{3}a$ and $x = -\frac{1}{5}a$	A1	
	State final answer $-\frac{1}{5}a < x < \frac{5}{3}a$	A1	
	OR: Obtain critical value $x = \frac{5}{3}a$ from a graphical method, or by inspection, or by solving a linear equation or an inequality	B1	
	Obtain critical value $x = -\frac{1}{5}a$ similarly	B2	
	State final answer $-\frac{1}{5}a < x < \frac{5}{3}a$ [Do not condone $\leqslant$ for $<$ in the final answer.]	B1	
		4	

Question	Answer	Marks	Guidance
2	Rearrange the equation in the form $ae^{2x} = b$ or $ae^{x} = be^{-x}$	**M1**	
	Obtain correct equation in either form with $a = 2$ and $b = 5$	**A1**	
	Use correct method to solve for x	**M1**	
	Obtain answer $x = 0.46$	**A1**	
		4	

Question	Answer	Marks	Guidance
3 (i)	Sketch a relevant graph, e.g. $y = x^3$	**B1**	
	Sketch a second relevant graph, e.g. $y = 3 - x$, and justify the given statement	**B1**	Consideration of behaviour for $x < 0$ is needed for the second B1
		2	
3(ii)	State or imply the equation $x = \left(2x^3 + 3\right) / \left(3x^2 + 1\right)$	**B1**	
	Rearrange this in the form $x^3 = 3 - x$, or commence work *vice versa*	**B1**	
		2	

Question	Answer	Marks	Guidance
3(iii)	Use the iterative formula correctly at least once	**M1**	
	Obtain final answer 1.213	**A1**	
	Show sufficient iterations to 5 d.p. or more to justify 1.213 to 3 d.p., or show there is a sign change in the interval (1.2125, 1.2135)	**A1**	
		3	

Question	Answer	Marks	Guidance
4(i)	Obtain $\frac{\mathrm{d}x}{\mathrm{d}\theta} = 2\cos\theta + 2\cos 2\theta$ or $\frac{\mathrm{d}y}{\mathrm{d}\theta} = -2\sin\theta - 2\sin 2\theta$	**B1**	
	Use $\mathrm{d}y/\mathrm{d}x = \mathrm{d}y/\mathrm{d}\theta \div \mathrm{d}x/\mathrm{d}\theta$	**M1**	
	Obtain correct $\frac{\mathrm{d}y}{\mathrm{d}x}$ in any form, e.g. $-\frac{2\sin\theta + 2\sin 2\theta}{2\cos\theta + 2\cos 2\theta}$	**A1**	
		3	
4(ii)	Equate denominator to zero and use any correct double angle formula	**M1***	
	Obtain correct 3-term quadratic in $\cos\theta$ in any form	**A1**	
	Solve for θ	**depM1***	
	Obtain $x = 3\sqrt{3}/2$ and $y = \frac{1}{2}$, or exact equivalents	**A1**	
		4	

Question	Answer	Marks	Guidance
5	Separate variables correctly and integrate at least one side	**B1**	
	Obtain term ln y	**B1**	
	Obtain terms $2 \ln x - \frac{1}{2} x^2$	**B1+B1**	
	Use $x = 1$, $y = 1$ to evaluate a constant, or as limits	**M1**	
	Obtain correct solution in any form, e.g. $\ln y = 2\ln x - \frac{1}{2}x^2 + \frac{1}{2}$	**A1**	
	Rearrange as $y = x^2 \exp\left(\frac{1}{2} - \frac{1}{2}x^2\right)$, or equivalent	**A1**	
		7	

Question	Answer	Marks	Guidance
6(i)	Rearrange in the form $\sqrt{3}\sin x - \cos x = \sqrt{2}$	**B1**	
	State $R = 2$	**B1**	
	Use trig formulae to obtain α	**M1**	
	Obtain $\alpha = 30°$ with no errors seen	**A1**	
		4	

Question	Answer	Marks	Guidance
6(ii)	Evaluate $\sin^{-1}\left(\frac{\sqrt{2}}{R}\right)$	**B1ft**	
	Carry out a correct method to find a value of x in the given interval	**M1**	
	Obtain answer $x = 75°$	**A1**	
	Obtain a second answer e.g. $x = 165°$ and no others [Treat answers in radians as a misread. Ignore answers outside the given interval.]	**A1ft**	
		4	

Question	Answer	Marks	Guidance
7(i)	Use product rule	**M1***	
	Obtain correct derivative in any form	**A1**	
	Equate derivative to zero and obtain an equation in a single trig function	**depM1***	
	Obtain a correct equation, e.g. $3\tan^2 x = 2$	**A1**	
	Obtain answer $x = 0.685$	**A1**	
		5	

Question	Answer	Marks	Guidance
7(ii)	Use the given substitution and reach $a\int\left(u^2 - u^4\right)\mathrm{d}u$	**M1**	
	Obtain correct integral with a = 5 and limits 0 and 1	**A1**	
	Use correct limits in an integral of the form $a\left(\frac{1}{3}u^3 - \frac{1}{5}u^5\right)$	**M1**	
	Obtain answer $\frac{2}{3}$	**A1**	
		4	

Question	Answer		Marks	Guidance
8(i)	*EITHER*:	Multiply numerator and denominator by 1 + 2i, or equivalent, or equate to $x + iy$, obtain two equations in x and y and solve for x or for y	**M1**	
		Obtain quotient $-\frac{4}{5}+\frac{7}{5}$ i, or equivalent	**A1**	
		Use correct method to find either r or θ	**M1**	
		Obtain $r = 1.61$	**A1**	
		Obtain $\theta = 2.09$	**A1**	
	OR:	Find modulus or argument of 2 + 3i or of 1 – 2i	**B1**	
		Use correct method to find r	**M1**	
		Obtain $r = 1.61$	**A1**	
		Use correct method to find θ	**M1**	
		Obtain $\theta = 2.09$	**A1**	
			5	
8(ii)	Show a circle with centre 3 – 2i		**B1**	
	Show a circle with radius 1		**B1ft**	Centre not at the origin
	Carry out a correct method for finding the least value of $\lvert z \rvert$		**M1**	
	Obtain answer $\sqrt{13} - 1$		**A1**	
			4	

Question	Answer	Marks	Guidance
9(i)	State or imply the form $\dfrac{A}{2-x}+\dfrac{B}{3+2x}+\dfrac{C}{(3+2x)^2}$	B1	
	Use a correct method to find a constant	M1	
	Obtain one of $A = 1$, $B = -1$, $C = 3$	A1	
	Obtain a second value	A1	
	Obtain the third value [Mark the form $\dfrac{A}{2-x}+\dfrac{Dx+E}{(3+2x)^2}$, where $A = 1$, $D = -2$ and $E = 0$, B1M1A1A1A1 as above.]	A1	
		5	
9(ii)	Integrate and obtain terms $-\ln(2-x)-\dfrac{1}{2}\ln(3+2x)-\dfrac{3}{2(3+2x)}$	B3ft	The f.t is on A, B, C; or on A, D, E.
	Substitute correctly in an integral with terms $a \ln(2-x)$, $b \ln(3+2x)$ and $c/(3+2x)$ where $abc \neq 0$	M1	
	Obtain the given answer after full and correct working [Correct integration of the A, D, E form gives an extra constant term if integration by parts is used for the second partial fraction.]	A1	
		5	

Question	Answer	Marks	Guidance
10(i)	*EITHER*: Expand scalar product of a normal to *m* and a direction vector of *l*	**M1**	
	Verify scalar product is zero	**A1**	
	Verify that one point of *l* does not lie in the plane	**A1**	
	OR: Substitute coordinates of a general point of *l* in the equation of the plane *m*	**M1**	
	Obtain correct equation in λ in any form	**A1**	
	Verify that the equation is not satisfied for any value of λ	**A1**	
		3	
10(ii)	Use correct method to evaluate a scalar product of normal vectors to *m* and *n*	**M1**	
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	**M1**	
	Obtain answer 74.5° or 1.30 radians	**A1**	
		3	
10(iii)	*EITHER*: Using the components of a general point *P* of *l* form an equation in λ by equating the perpendicular distance from *n* to 2	**M1**	
	OR: Take a point *Q* on *l*, e.g. (5, 3, 3) and form an equation in λ by equating the length of the projection of *QP* onto a normal to plane *n* to 2	**M1**	
	Obtain a correct modular or non-modular equation in any form	**A1**	
	Solve for λ and obtain a position vector for *P*, e.g. $7\mathbf{i} + 5\mathbf{j} + 7\mathbf{j}$ from $\lambda = 3$	**A1**	
	Obtain position vector of the second point, e.g. $3\mathbf{i} + \mathbf{j} - \mathbf{k}$ from $\lambda = -1$	**A1**	
		4	

Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS	**9709/31**
Paper 3	**October/November 2019**
MARK SCHEME	
Maximum Mark: 75	

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2019 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

Cambridge Assessment International Education

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Mark Scheme Notes

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Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
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CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	State $1+e^{2y}=e^x$	**B1**	
	Make y the subject	**M1**	Rearrange to $e^{2y}=\ldots$ and use logs
	Obtain answer $y=\frac{1}{2}\ln\left(e^x-1\right)$	**A1**	OE
		3	

Question	Answer	Marks	Guidance
2	State or imply non-modular inequality $(2x-3)^2 > 4^2(x+1)^2$, or corresponding quadratic equation, or pair of linear equations $(2x-3)=\pm 4(x+1)$	**B1**	$12x^2+44x+7<0$
	Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	**M1**	Correct method seen, or implied by correct answers
	Obtain critical values $x=-\frac{7}{2}$ and $x=-\frac{1}{6}$	**A1**	
	State final answer $-\frac{7}{2}<x<-\frac{1}{6}$	**A1**	
	Alternative method for question 2		
	Obtain critical value $x=-\frac{7}{2}$ from a graphical method, or by inspection, or by solving a linear equation or an inequality	**B1**	
	Obtain critical value $x=-\frac{1}{6}$ similarly	**B2**	
	State final answer $-\frac{7}{2}<x<-\frac{1}{6}$	**B1**	
		4	

Question	Answer	Marks	Guidance
3	State $\dfrac{dx}{dt} = 2 + 2\cos 2t$	**B1**	
	Use the chain rule to find the derivative of y	**M1**	
	Obtain $\dfrac{dy}{dt} = \dfrac{2\sin 2t}{1 - \cos 2t}$	**A1**	OE
	Use $\dfrac{dy}{dx} = \dfrac{dy}{dt} \div \dfrac{dx}{dt}$	**M1**	
	Obtain $\dfrac{dy}{dx} = \operatorname{cosec} 2t$ correctly	**A1**	AG
		5	

Question	Answer	Marks	Guidance
4(i)	State $\frac{dN}{dt} = ke^{-0.02t}N$ and show $k = -0.01$	B1	OE $(-10 = k \times 1 \times 1000)$
		1	
4(ii)	Separate variables correctly and integrate at least one side	B1	$\int \frac{1}{N}dN = \int -0.01e^{-0.02t}dt$
	Obtain term $\ln N$	B1	OE
	Obtain term $0.5e^{-0.02t}$	B1	OE
	Use $N = 1000$, $t = 0$ to evaluate a constant, or as limits, in a solution with terms $a\ln N$ and $be^{-0.02t}$, where $ab \neq 0$	M1	
	Obtain correct solution in any form e.g. $\ln N - \ln 1000 = 0.5\left(e^{-0.02t} - 1\right)$	A1	$\ln 1000 - \frac{1}{2} = 6.41$
	Substitute $N = 800$ and obtain $t = 29.6$	A1	
		6	
4(iii)	State that N approaches $\frac{1000}{\sqrt{e}}$	B1	Accept 606 or 607 or 606.5
		1	

Question	Answer	Marks	Guidance
5(i)	Use correct product rule	**M1**	
	Obtain correct derivative in any form $\frac{\mathrm{d}y}{\mathrm{d}x}=-2\mathrm{e}^{-2x}\ln(x-1)+\frac{\mathrm{e}^{-2x}}{x-1}$	**A1**	
	Equate derivative to zero and derive $x=1+e^{\frac{1}{2(x-1)}}$ or $p=1+^{\frac{1}{2(p-1)}}$	**A1**	AG
		3	
5(ii)	Calculate values of a relevant expression or pair of relevant expressions at x = 2.2 and x = 2.6 $\mathrm{f}(x)=\ln(x-1)-\frac{1}{2(x-1)}\Rightarrow\mathrm{f}(2.2)=-0.234,\ \mathrm{f}(2.6)=0.317$ $\mathrm{f}(x)=2\mathrm{e}^{-2x}\ln(x-1)+\frac{\mathrm{e}^{-2x}}{x-1}\Rightarrow\mathrm{f}(2.2)=0.005\ldots,\ \mathrm{f}(2.6)=-0.0017...$	**M1**	
	Complete the argument correctly with correct calculated values	**A1**	
		2	

Question	Answer	Marks	Guidance
5(iii)	Use the iterative process $p_{n+1} = 1 + \exp\left(\frac{1}{2(p_n - 1)}\right)$ correctly at least once	**M1**	
	Obtain final answer 2.42	**A1**	
	Show sufficient iterations to 4 d.p. to justify 2.42 to 2 d.p., or show there is a sign change in the interval (2.415, 2.425)	**A1**	
		3	

Question	Answer	Marks	Guidance
6(i)	Use correct quotient rule	M1	
	Obtain $\frac{\mathrm{d}y}{\mathrm{d}x} = -\operatorname{cosec}^2 x$ correctly	A1	AG
		2	
6(ii)	Integrate by parts and reach $ax\cot x + b\int \cot x\,\mathrm{d}x$	*M1	
	Obtain $-x\cot x + \int \cot x\,\mathrm{d}x$	A1	OE
	State $\pm\ln\sin x$ as integral of $\cot x$	M1	
	Obtain complete integral $-x\cot x + \ln\sin x$	A1	OE
	Use correct limits correctly	DM1	$0+0+\frac{\pi}{4}-\ln\frac{1}{\sqrt{2}}$
	Obtain $\frac{1}{4}(\pi + \ln 4)$ following full and exact working	A1	AG
		6	

Question	Answer	Marks	Guidance
7(i)	Express general point of l or m in component form e.g. $(a + \lambda, 2 - 2\lambda, 3 + 3\lambda)$ or $(2 + 2\mu, 1 - \mu, 2 + \mu)$	**B1**	
	Equate at least two pairs of corresponding components and solve for λ or for μ	**M1**	
	Obtain **either** $\lambda = -2$ or $\mu = -5$ **or** $\lambda = \frac{1}{3}a$ or $\mu = \frac{2}{3}a - 1$ **or** $\lambda = \frac{1}{5}(a-4)$ or $\mu = \frac{1}{5}(3a-7)$	**A1**	
	Obtain $a = -6$	**A1**	
		4	
7(ii)	Use scalar product to obtain a relevant equation in a, b and c, e.g. $a - 2b + 3c = 0$	**B1**	
	Obtain a second equation, e.g. $2a - b + c = 0$ and solve for one ratio	**M1**	
	Obtain $a : b : c = 1 : 5 : 3$	**A1**	OE
	Substitute a relevant point and values of a, b, c in general equation and find d	**M1**	
	Obtain correct answer $x + 5y + 3z = 13$	**A1FT**	OE. The FT is on a from part (**i**), if used
	Alternative method for question 7(ii)		
	Attempt to calculate vector product of relevant vectors,	**M1**	e.g. $(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}).(2\mathbf{i} - \mathbf{j} + \mathbf{k})$
	Obtain two correct components	**A1**	
	Obtain correct answer, e.g. $\mathbf{i} + 5\mathbf{j} + 3\mathbf{k}$	**A1**	
	Substitute a relevant point and find d	**M1**	
	Obtain correct answer $x + 5y + 3z = 13$	**A1FT**	OE. The FT is on a from part (**i**), if used

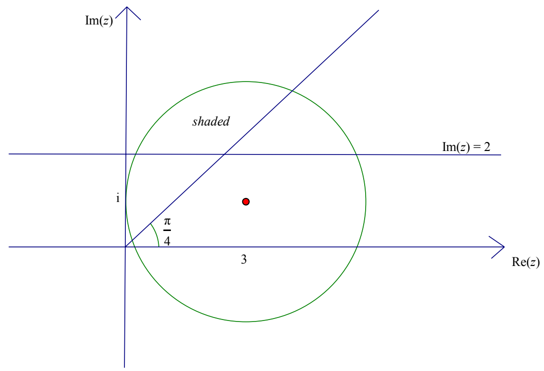
Question	Answer	Marks	Guidance
7(ii)	**Alternative method for question 7(ii)**		
	Using a relevant point and relevant vectors, form a 2-parameter equation for the plane	**M1**	
	State a correct equation, e.g. $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}) + \mu(2\mathbf{i} - \mathbf{j} + \mathbf{k})$	**A1FT**	
	State three correct equations in x, y, z, λ and μ	**A1FT**	
	Eliminate λ and μ	**M1**	
	Obtain correct answer $x + 5y + 3z = 13$	**A1FT**	OE. The FT is on a from part (**i**), if used
		5	

Question	Answer	Marks	Guidance
8(i)	State or imply the form $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+2}$	**B1**	
	Use a correct method for finding a constant	**M1**	
	Obtain one of $A = -1$, $B = 3$, $C = 2$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	Allow in the form $\frac{Ax+B}{x^2} + \frac{C}{x+2}$
		5	

Question	Answer	Marks	Guidance
8(ii)	Integrate and obtain terms $\ln x - \frac{3}{x} + 2\ln(x+2)$	**B1FT + B1FT + B1FT**	The FT is on A, B, C; or on A, D, E.
	Substitute limits correctly in an integral with terms $a\ln x$, $\frac{b}{x}$ and $c\ln(x+2)$, where $abc \neq 0$	**M1**	$-\ln 4 - \frac{3}{4} + 2\ln 6(+\ln 1) + 3 - 2\ln 3$
	Obtain $\frac{9}{4}$ following full and exact working	**A1**	AG – work to combine or simplify logs is required
		5	

Question	Answer	Marks	Guidance
9(i)	Use $\cos(A + B)$ formula to express $\cos 3x$ in terms of trig functions of $2x$ and x	**M1**	
	Use double angle formulae and Pythagoras to obtain an expression in terms of $\cos x$ only	**M1**	
	Obtain a correct expression in terms of $\cos x$ in any form	**A1**	
	Obtain $\cos 3x \equiv 4\cos^3 x - 3\cos x$	**A1**	AG
		4	
9(ii)	Use identity and solve cubic $4\cos^3 x = -1$ for x	**M1**	$\cos x = -0.6299...$
	Obtain answer 2.25 and no other in the interval	**A1**	Accept 0.717π M1A0 for 129.0°
		2	

Question	Answer	Marks	Guidance
9(iii)	Obtain indefinite integral $\frac{1}{12}\sin 3x + \frac{3}{4}\sin x$	**B1 + B1**	
	Substitute limits in an indefinite integral of the form $a\sin 3x + b\sin x$, where $ab \neq 0$	**M1**	$\frac{1}{4}\left[\frac{1}{3}\sin\pi + 3\sin\frac{\pi}{3} - \frac{1}{3}\sin\frac{\pi}{2} - 3\sin\frac{\pi}{6}\right]$
	Obtain answer $\frac{1}{24}\left(9\sqrt{3} - 11\right)$, or exact equivalent	**A1**	
	Alternative method for question 9(iii)		
	$\int \cos x\left(1 - \sin^2 x\right)\mathrm{d}x = \sin x - \frac{1}{3}\sin^3 x\,(+C)$	**B1 + B1**	
	Substitute limits in an indefinite integral of the form $a\sin x + b\sin^3 x$ where $ab \neq 0$	**M1**	$\left(\frac{\sqrt{3}}{2} - \frac{1}{2} - \frac{1}{4}\frac{\sqrt{3}}{2} + \frac{1}{24}\right)$
	Obtain answer $\frac{1}{24}\left(9\sqrt{3} - 11\right)$, or exact equivalent	**A1**	
		4	

Question	Answer	Marks	Guidance
10(a)	Square $a + ib$ and equate real and imaginary parts to -3 and $-2\sqrt{10}$ respectively	*M1	
	Obtain $a^2 - b^2 = -3$ and $2ab = -2\sqrt{10}$	A1	
	Eliminate one unknown and find an equation in the other	DM1	
	Obtain $a^4 + 3a^2 - 10 = 0$, or $b^4 - 3b^2 - 10 = 0$, or horizontal 3-term equivalent	A1	
	Obtain answers $\pm\left(\sqrt{2} - \sqrt{5}\mathrm{i}\right)$, or exact equivalent	A1	
		5	
10(b)	Show point representing 3 + i in relatively correct position	B1	
	Show a circle with radius 3 and centre not at the origin	B1	
	Show correct half line from the origin at $\frac{1}{4}\pi$ to the real axis	B1	
	Show horizontal line $y = 2$	B1	
	Shade the correct region	B1	
		5	

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MATHEMATICS **9709/32**

Paper 3 **October/November 2019**

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AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Remove logarithms and state $4-3^x = e^{1.2}$, or equivalent	**B1**	Accept $4-3^x = 3.32(01169.....)$ 3 s.f. or better
	Use correct method to solve an equation of the form $3^x = a$, where $a > 0$.	**M1**	$(3^x = 0.67988..)$ Complete method to $x = \ldots$ If using $\log_3$ the subscript can be implied
	Obtain answer $x = -0.351$ only	**A1**	CAO must be to 3 d.p.
		3	

Question	Answer	Marks	Guidance
2	Use correct quotient rule or correct product rule	**M1**	
	Obtain correct derivative in any form	**A1**	$\dfrac{dy}{dx} = \dfrac{-2e^{-2x}\left(1-x^2\right)+2xe^{-2x}}{\left(1-x^2\right)^2}$
	Equate derivative to zero and obtain a 3 term quadratic in x	**M1**	
	Obtain a correct 3-term equation e.g. $2x^2+2x-2=0$ or $x^2+x=1$	**A1**	From correct work only
	Solve and obtain $x = 0.618$ only	**A1**	From correct work only
		5	

Question	Answer	Marks	Guidance
3	Commence division and reach partial quotient $x^2 + kx$	**M1**	
	Obtain correct quotient $x^2 + 2x - 1$	**A1**	
	Set their linear remainder equal to $2x + 3$ and solve for a or for b	**M1**	Remainder = $(a+3)x + (b-1)$
	Obtain answer $a = -1$	**A1**	
	Obtain answer $b = 4$	**A1**	
	Alternative method for question 3		
	State $x^4 + 3x^3 + ax + b = (x^2 + x - 1)(x^2 + Ax + B) + 2x + 3$ and form and solve two equations in A and B	**M1**	e.g. $3 = 1 + A$ and $0 = -1 + A + B$
	Obtain $A = 2,\ B = -1$	**A1**	
	Form and solve equations for a or b	**M1**	e.g. $a = B - A + 2, \quad b = -B + 3$
	Obtain answer $a = -1$	**A1**	
	Obtain answer $b = 4$	**A1**	
		5	

Question	Answer	Marks	Guidance
3	**Alternative method for question 3**		
	Use remainder theorem with $x = \dfrac{-1 \pm \sqrt{5}}{2}$	**M1**	Allow for correct use of either root in exact or decimal form.
	Obtain $-\dfrac{a}{2} \pm \dfrac{a\sqrt{5}}{2} + b = \dfrac{9}{2} \mp \dfrac{\sqrt{5}}{2}$	**A1**	Expand brackets and obtain exact equation for either root. Accept exact equivalent.
	Solve simultaneous equations for a or b	**M1**	
	Obtain answer $a = -1$ from exact working	**A1**	
	Obtain answer $b = 4$ from exact working	**A1**	
		5	

Question	Answer	Marks	Guidance
4(i)	State $R = \sqrt{7}$	**B1**	
	Use correct trig formulae to find α	**M1**	e.g. $\tan\alpha = \dfrac{1}{\sqrt{6}}$, $\sin\alpha = \dfrac{1}{\sqrt{7}}$, or $\cos\alpha = \dfrac{\sqrt{6}}{\sqrt{7}}$
	Obtain $\alpha = 22.208°$	**A1**	ISW
		3	

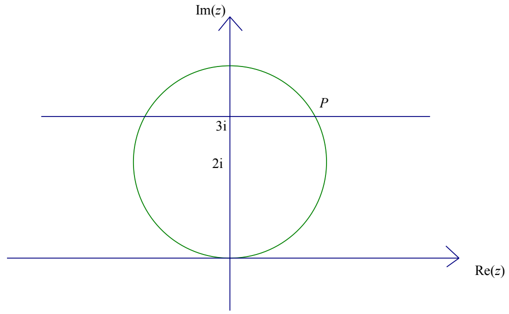
Question	Answer	Marks	Guidance
4(ii)	Evaluate $\sin^{-1}\left(\frac{2}{\sqrt{7}}\right)$ to at least 1 d.p.	**B1FT**	$49.107°$ to 3 d.p. B1 can be implied by correct answer(s) later. The FT is on *their R*
			SC: allow B1 for a correct alternative equation e.g. $3\tan^2\theta - 2\sqrt{6}\tan\theta + 1 = 0$
	Use correct method to find a value of θ in the interval	**M1**	Must get to θ
	Obtain answer, e.g. $13.4°$	**A1**	Accept correct over-specified answers. 13.449…, 54.3425…
	Obtain second answer, e.g. $54.3°$ and no extras in the given interval	**A1**	Ignore answers outside the given interval.
		4	

Question	Answer	Marks	Guidance
5	State $4xy + 2x^2 \dfrac{dy}{dx}$, or equivalent, as derivative of $2x^2y$	**B1**	
	State $y^2 + 2xy \dfrac{dy}{dx}$, or equivalent, as derivative of xy^2	**B1**	
	Equate attempted derivative of LHS to zero and set $\dfrac{dy}{dx}$ equal to zero (or set numerator equal to zero)	***M1**	$\dfrac{dy}{dx} = \dfrac{y^2 - 4xy}{2x^2 - 2xy}$
	Reject $y = 0$	**B1**	Allow from $y^2 - kxy = 0$
	Obtain $y = 4x$	**A1**	OE from correct numerator. ISW
	Obtain an equation in y (or in x) and solve for y (or for x) in terms of a	**DM1**	$8x^3 - 16x^3 = a^3$ or $\dfrac{y^3}{8} - \dfrac{y^3}{4} = a^3$
	Obtain $y = -2a$	**A1**	With no errors seen
		7	

Question	Answer	Marks	Guidance
5	**Alternative method for question 5**		
	Rewrite as $y = \dfrac{a^3}{2x^2 - xy}$ and differentiate	**M1**	Correct use of function of a function and implicit differentiation
	Obtain correct derivative (in any form)	**A1**	$\dfrac{dy}{dx} = \dfrac{-a^3\left(4x - y - x\dfrac{dy}{dx}\right)}{\left(2x^2 - xy\right)^2}$
	set $\dfrac{dy}{dx}$ equal to zero (or set numerator equal to zero)	***M1**	
	Obtain $4x - y = 0$	**A1**	
	Confirm $2x^2 - xy \neq 0$	**B1**	$x = 0$ and $2x = y$ both give $a = 0$
	Obtain an equation in y (or in x) and solve for y (or for x)	**DM1**	$8x^3 - 16x^3 = a^3$ or $\dfrac{y^3}{8} - \dfrac{y^3}{4} = a^3$
	Obtain $y = -2a$	**A1**	With no errors seen
		7	

Question	Answer	Marks	Guidance
6	Separate variables correctly to obtain $\int \frac{1}{x+2}\,dx = \int \cot\frac{1}{2}\theta\ d\theta$	B1	Or equivalent integrands. Integral signs SOI
	Obtain term $\ln(x+2)$	B1	Modulus signs not needed.
	Obtain term of the form $k\ln\sin\frac{1}{2}\theta$	M1	
	Obtain term $2\ln\sin\frac{1}{2}\theta$	A1	
	Use $x = 1$, $\theta = \frac{1}{3}\pi$ to evaluate a constant, or as limits, in an expression containing $p\ln(x+2)$ and $q\ln\left(\sin\frac{1}{2}\theta\right)$	M1	Reach C = an expression or a decimal value
	Obtain correct solution in any form e.g. $\ln(x+2) = 2\ln\sin\frac{1}{2}\theta + \ln 12$	A1	ln12 = 2.4849….. Accept constant to at least 3 s.f. Accept with $\ln 3 - 2\ln\frac{1}{2}$
	Remove logarithms and use correct double angle formula	M1	Need correct algebraic process. $\left(\frac{x+2}{12} = \frac{1-\cos\theta}{2}\right)$
	Obtain answer $x = 4 - 6\cos\theta$	A1	
		8	

Question	Answer	Marks	Guidance
7(a)	Substitute and obtain a correct horizontal equation in x and y in any form	**B1**	$zz^* + iz - 2z^* = 0 \Rightarrow$ $x^2 + y^2 + ix - y - 2x + 2iy = 0$ Allow if still includes brackets and/or i^2
	Use $i^2 = -1$ and equate real and imaginary parts to zero OE	***M1**	For their horizontal equation
	Obtain two correct equations e.g. $x^2 + y^2 - y - 2x = 0$ and $x + 2y = 0$	**A1**	Allow $ix + 2iy = 0$
	Solve for x or for y	**DM1**	
	Obtain answer $\frac{6}{5} - \frac{3}{5}i$ and no other	**A1**	OE, condone $\frac{1}{5}(6 - 3i)$
		5	

Question	Answer	Marks	Guidance
7(b)(i)	Show a circle with centre 2i and radius 2	**B1**	
	Show horizontal line $y = 3$ – in first and second quadrant	**B1**	
			SC: For clearly labelled axes not in the conventional directions, allow B1 for a fully 'correct' diagram.
		2	
7(b)(ii)	Carry out a complete method for finding the argument. (Not by measuring the sketch)	**M1**	$\left(z = \sqrt{3} + 3\text{i}\right)$ Must show working if using 1.7 in place of $\sqrt{3}$.
	Obtain answer $\frac{1}{3}\pi$ (or $60°$)	**A1**	**SC:** Allow B2 for 60° with no working
		2	

Question	Answer	Marks	Guidance
8(i)	State or imply the form $\frac{A}{2x-1}+\frac{Bx+C}{x^2+2}$	B1	
	Use a correct method for finding a constant	M1	
	Obtain one of $A = 4$, $B = -1$, $C = 0$	A1	
	Obtain a second value	A1	
	Obtain the third value	A1	
		5	
8(ii)	Integrate and obtain term $2\ln(2x-1)$	B1FT	The FT is on A. $\frac{1}{2}A\ln(2x-1)$
	Integrate and obtain term of the form $k\ln(x^2+2)$	*M1	From $\frac{nx}{x^2+2}$
	Obtain term $-\frac{1}{2}\ln(x^2+2)$	A1FT	The FT is on B
	Substitute limits correctly in an integral of the form $a\ln(2x-1)+b\ln(x^2+2)$, where $ab \neq 0$	DM1	$2\ln 9(-2\ln 1)-\frac{1}{2}\ln 27+\frac{1}{2}\ln 3$
	Obtain answer ln 27 after full and correct exact working	A1	ISW
		5	

Question	Answer	Marks	Guidance
9(i)	Commence integration by parts, reaching $ax\sin\frac{1}{3}x - b\int\sin\frac{1}{3}x\,\mathrm{d}x$	*M1	
	Obtain $3x\sin\frac{1}{3}x - 3\int\sin\frac{1}{3}x\,\mathrm{d}x$	A1	
	Complete integration and obtain $3x\sin\frac{1}{3}x + 9\cos\frac{1}{3}x$	A1	
	Substitute limits correctly and equate result to 3 in an integral of the form $px\sin\frac{1}{3}x + q\cos\frac{1}{3}x$	DM1	$3 = 3a\sin\frac{a}{3} + 9\cos\frac{a}{3}(-0) - 9$
	Obtain $a = \dfrac{4 - 3\cos\frac{a}{3}}{\sin\frac{a}{3}}$ correctly	A1	With sufficient evidence to show how they reach the given equation
		5	
9(ii)	Calculate values at $a = 2.5$ and $a = 3$ of a relevant expression or pair of expressions.	M1	$2.5 < 2.679$ and $3 > 2.827$ If using 2.679 and 2.827 must be linked explicitly to 2.5 and 3. Solving f(a) = 0, f(2.5) = 0.179. and f(3) = –0.173 or if $\mathrm{f}(a) = a\sin\frac{1}{3}a + 3\cos\frac{1}{3}a - 4 \Rightarrow \mathrm{f}(2.5) = -0.13.., \mathrm{f}(3) = 0.145...$
	Complete the argument correctly with correct calculated values	A1	Accept values to 1 sf. or better
		2	

Question	Answer	Marks	Guidance
9(iii)	Use the iterative process $a_{n+1} = a_{n+1}\dfrac{4-3\cos\frac{1}{3}a_n}{\sin\frac{1}{3}a_n}$ correctly at least once	**M1**	
	Show sufficient iterations to at least 5 d.p. to justify 2.736 to 3d.p., or show a sign change in the interval (2.7355, 2.7365)	**A1**	
	Obtain final answer 2.736	**A1**	
		3	

Question	Answer	Marks	Guidance
10(i)	Express general point of l in component form e.g. $(1 + \lambda, 3 - 2\lambda, -2 + 3\lambda)$	**B1**	
	Substitute in equation of p and solve for λ	**M1**	
	Obtain final answer $\frac{5}{3}\mathbf{i}+\frac{5}{3}\mathbf{j}$ from $\lambda = \frac{2}{3}$	**A1**	OE Accept $1.67\mathbf{i}+1.67\mathbf{j}$ or better
		3	

Question	Answer	Marks	Guidance
10(ii)	Use correct method to evaluate a scalar product of relevant vectors e.g. $(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}).(2\mathbf{i} + \mathbf{j} - 3\mathbf{k})$	**M1**	
	Using the correct process for calculating the moduli, divide the scalar product by the product of the moduli and evaluate the inverse sine or cosine of the result	**M1**	$\lvert\sin\theta\rvert = \frac{9}{14}$
	Obtain answer 40.0° or 0.698 radians	**A1**	AWRT
		3	
	Alternative method for question 10(ii)		
	Use correct method to evaluate a vector product of relevant vectors e.g. $(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})\text{x}(2\mathbf{i} + \mathbf{j} - 3\mathbf{k})$	**M1**	
	Using the correct process for calculating the moduli, divide the modulus of the vector product by the product of the moduli of the two vectors and evaluate the inverse sine or cosine of the result	**M1**	$\cos\theta = \frac{\sqrt{115}}{14}$
	Obtain answer 40.0° or 0.698 radians	**A1**	AWRT
		3	

Question	Answer	Marks	Guidance
10(iii)	State $a - 2b + 3c = 0$ or $2a + b - 3c = 0$	**B1**	
	Obtain two relevant equations and solve for one ratio, e.g. $a : b$	**M1**	Could use $2a + b - 3c = 0$ and $\begin{cases} a + 3b - 2c = d \\ \frac{5}{3}a + \frac{5}{3}b = d \end{cases}$ i.e. use two points on the line rather than the direction of the line. The second M1 is not scored until they solve for d.
	Obtain $a : b : c = 3 : 9 : 5$	**A1**	OE
	Substitute a, b, c and a relevant point in the plane equation and evaluate d	**M1**	Using their calculated normal and a relevant point
	Obtain answer $3x + 9y + 5z = 20$	**A1**	OE
	Alternative method for question 10(iii)		
	Attempt to calculate vector product of relevant vectors, e.g. $(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}) \times (2\mathbf{i} + \mathbf{j} - 3\mathbf{k})$	**M1**	
	Obtain two correct components	**A1**	
	Obtain correct answer, e.g. $3\mathbf{i} + 9\mathbf{j} + 5\mathbf{k}$	**A1**	
	Use the product and a relevant point to find d	**M1**	Using *their* calculated normal and a relevant point
	Obtain answer $3x + 9y + 5z = 20$, or equivalent	**A1**	OE

Question	Answer	Marks	Guidance
10(iii)	**Alternative method for question 10(iii)**		
	Attempt to form a 2-parameter equation with relevant vectors	**M1**	
	State a correct equation e.g. $\mathbf{r} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}) + \mu(2\mathbf{i} + \mathbf{j} - 3\mathbf{k})$	**A1**	
	State 3 equations in x, y, z, λ and μ	**A1**	
	Eliminate λ and μ	**M1**	
	Obtain answer $3x + 9y + 5z = 2$	**A1**	OE
		5	

Cambridge International A Level

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Cambridge International Advanced Level

MATHEMATICS **9709/33**

Paper 3 **October/November 2019**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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Cambridge International is publishing the mark schemes for the October/November 2019 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

Cambridge Assessment International Education

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	State or imply non-modular inequality $(x+2)^2 > (3x-1)^2$, or corresponding quadratic equation, or pair of linear equations $2(x + 2) = \pm (3x - 1)$	B1	
	Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	M1	
	Obtain critical values $x = -\frac{3}{5}$ and $x = 5$	A1	
	State final answer $-\frac{3}{5} < x < 5$	A1	
	Alternative method for question 1		
	Obtain critical value $x = 5$ from a graphical method, or by inspection, or by solving a linear equation or an inequality	B1	
	Obtain critical value $x = -\frac{3}{5}$ similarly	B2	
	State final answer $-\frac{3}{5} < x < 5$	B1	
		4	

Question	Answer	Marks	Guidance
2	Substitute $x = -\frac{1}{2}$, equate result to zero and obtain a correct equation, e.g. $-\frac{6}{8} + \frac{1}{4}a - \frac{1}{2}b - 2 = 0$	B1	
	Substitute $x = -2$ and equate result to -24	*M1	
	Obtain a correct equation, e.g. $-48 + 4a - 2b - 2 = -24$	A1	
	Solve for a or for b	DM1	
	Obtain $a = 5$ and $b = -3$	A1	
		5	

Question	Answer	Marks	Guidance
3	Reduce the equation to a horizontal equation in 3^{3x}, 3^{3x+1} or 27^x	M1	
	Simplify and reach $3(3^{3x}) = 5$, $3\left(27^x\right) = 5$, or equivalent	A1	
	Use correct method for finding x from a positive value of 3^{3x}, 3^{3x+1} or 27^x	M1	
	Obtain answer $x = 0.155$	A1	
		4	

Question	Answer	Marks	Guidance
4(i)	Use tan $(A + B)$ formula to express the LHS in terms of tan $2x$ and tan x	**M1**	
	Using the tan $2A$ formula, express the entire equation in terms of tan x	**M1**	
	Obtain a correct equation in tan x in any form	**A1**	
	Obtain the given form correctly	**A1**	AG
		4	
4(ii)	Use correct method to solve the given equation for x	**M1**	
	Obtain answer, e.g. $x = 26.8°$	**A1**	
	Obtain second answer, e.g. $x = 73.7°$ and no other	**A1**	Ignore answers outside the given interval
		3	

Question	Answer	Marks	Guidance
5(i)	Sketch a relevant graph, e.g. $y = \ln(x+2)$	**B1**	
	Sketch a second relevant graph, e.g. $y = 4e^{-x}$, and justify the given statement	**B1**	Consideration of behaviour for $x < 0$ is needed for the second B1
		2	
5(ii)	Calculate the values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.5$	**M1**	
	Complete the argument correctly with correct calculated values	**A1**	
		2	

Question	Answer	Marks	Guidance
5(iii)	Use the iterative formula correctly at least **twice** using output from a previous iteration	**M1**	
	Obtain final answer 1.23	**A1**	
	Show sufficient iterations to 4 d.p. to justify 1.23 to 2 d.p., or show there is a sign change in the interval (1.225, 1.235)	**A1**	
		3	

Question	Answer	Marks	Guidance
6(i)	Obtain answer $w = \frac{1}{2} + \frac{\sqrt{3}}{2}\mathrm{i}$	**B1**	
		1	
6(ii)	Show point representing u	**B1**	
	Show point representing v in relatively correct position	**B1**	
		2	
6(iii)	Explain why the moduli are equal	**B1**	
	Explain why the arguments are equal	**B1**	
	Use $\mathrm{i}^2 = -1$ and obtain $2uw$ in the given form	**M1**	
	Obtain answer $1 - 2\sqrt{3} + \left(2 + \sqrt{3}\right)\mathrm{i}$	**A1**	
		4	

Question	Answer	Marks	Guidance
7(i)	Substitute coordinates (5, 2, – 2) in $x + 4y - 8z = d$	**M1**	
	Obtain plane equation $x + 4y - 8z = 29$, or equivalent	**A1**	
		2	
7(ii)	Attempt to use perpendicular formula to find perpendicular from (5, 2, – 2) to m	**M1**	
	Obtain a correct unsimplified expression, e.g. $\dfrac{5+8+16-2}{\sqrt{(1+16+64)}}$	**A1**	
	Obtain answer 3	**A1**	
	Alternative method 1 for question 7(ii)		
	State or imply perpendicular from O to m is $\dfrac{2}{9}$ or from O to n is $\dfrac{29}{9}$	**B1**	
	Find difference in perpendiculars	**M1**	
	Obtain answer 3	**A1**	
	Alternative method 2 for question 7(ii)		
	Obtain correct parameter value, or position vector or coordinates of the foot of the perpendicular from (5, 2, – 2) to m, e.g. $\mu = \pm\dfrac{1}{3}$; $\left(\dfrac{14}{3}, \dfrac{2}{3}, \dfrac{2}{3}\right)$	**B1**	
	Calculate the length of the perpendicular	**M1**	
	Obtain answer 3	**B1**	
		3	

Question	Answer	Marks	Guidance
7(iii)	Calling the direction vector $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$, use a scalar product to form a relevant equation in a, b and c, e.g. $a + 4b - 8c = 0$ or $5a + 2b - 2z = 0$	**B1**	
	Solve two relevant equations for the ratio $a : b : c$	**M1**	
	Obtain $a : b : c = 4 : -19 : -9$	**A1**	OE
	State answer $\mathbf{r} = 5\mathbf{i} + 2\mathbf{j} - 2\mathbf{k} + \lambda(4\mathbf{i} - 19\mathbf{j} - 9\mathbf{k})$	**A1**	OE
	Alternative method for question 7(iii)		
	Attempt to calculate vector product of two relevant vectors, e.g. $(\mathbf{i} + 4\mathbf{j} - 8\mathbf{k})\times(5\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$	**M1**	
	Obtain two correct components	**A1**	
	Obtain $8\mathbf{i} - 38\mathbf{j} - 18\mathbf{k}$	**A1**	OE
	State answer $\mathbf{r} = 5\mathbf{i} + 2\mathbf{j} - 2\mathbf{k} + \lambda(4\mathbf{i} - 19\mathbf{j} - 9\mathbf{k})$	**A1**	OE
		4	

Question	Answer	Marks	Guidance
8(i)	State or imply ordinates 1, 1.2116…, 2.7597...	**B1**	
	Use correct formula, or equivalent, with $h = 0.6$	**M1**	
	Obtain answer 1.85	**A1**	
		3	
8(ii)	Explain why the rule gives an overestimate	**B1**	
		1	
8(iii)	Differentiate using quotient or chain rule	**M1**	
	Obtain correct derivative in terms of $\sin x$ and $\cos x$	**A1**	
	Equate derivative to 2, use Pythagoras and obtain an equation in $\sin x$	**M1**	
	Obtain $2\sin^2 x + \sin x - 2 = 0$	**A1**	OE
	Solve a 3-term quadratic for x	**M1**	
	Obtain answer $x = 0.896$ only	**A1**	
		6	

Question	Answer	Marks	Guidance
9(i)	Separate variables correctly and integrate one side	**B1**	
	Obtain term $0.2t$, or equivalent	**B1**	
	Carry out a relevant method to obtain A and B such that $\dfrac{1}{(20-x)(40-x)} \equiv \dfrac{A}{20-x} + \dfrac{B}{40-x}$	***M1**	OE
	Obtain $A = \dfrac{1}{20}$ and $B = -\dfrac{1}{20}$	**A1**	
	Integrate and obtain terms $-\dfrac{1}{20}\ln(20-x)+\dfrac{1}{20}\ln(40-x)$ OE	**A1FT** **+A1FT**	The FT is on A and B
	Use $x = 10$, $t = 0$ to evaluate a constant, or as limits	**DM1**	
	Obtain correct answer in any form	**A1**	
	Obtain final answer $x = \dfrac{60e^{4t}-40}{3e^{4t}-1}$	**A1**	OE
		9	
9(ii)	State that x approaches 20	**B1**	
		1	

Question	Answer	Marks	Guidance
10(i)	Use product rule and chain rule at least once	**M1**	
	Obtain correct derivative in any form	**A1**	
	Equate derivative to zero, use Pythagoras and obtain an equation in cos x	**M1**	
	Obtain $\cos^2 x + 3\cos x - 1 = 0$, or 3-term equivalent	**A1**	
	Obtain answer $x = 1.26$	**A1**	
		5	
10(ii)	Using du = $\pm$ sin x dx express integrand in terms of u and du	**M1**	
	Obtain integrand $e^u\left(u^2 - 1\right)$	**A1**	OE
	Commence integration by parts and reach $ae^u\left(u^2 - 1\right) + b\int ue^u \, du$	***M1**	
	Obtain $e^u\left(u^2 - 1\right) - 2\int ue^u \, du$	**A1**	OE
	Complete integration, obtaining $e^u\left(u^2 - 2u + 1\right)$	**A1**	OE
	Substitute limits $u = 1$ and $u = -1$ (or $x = 0$ and $x = \pi$), having integrated completely	**DM1**	
	Obtain answer $\dfrac{4}{e}$, or exact equivalent	**A1**	
		7	

Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **October/November 2020**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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This document consists of **21** printed pages.

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2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Make a recognisable sketch graph of $y = 2\|x-3\|$ and the line $y = 2 - 5x$	**B1**	Need to see correct V at $x = 3$, roughly symmetrical, $x = 3$ stated, domain at least $(-2, 5)$.
	Find x-coordinate of intersection with $y = 2 - 5x$	**M1**	Find point of intersection with $y = 2\|x - 3\|$ or solve $2 - 5x$ with $2(x - 3)$ or $-2(x - 3)$
	Obtain $x = -\frac{4}{3}$	**A1**	
	State final answer $x < -\frac{4}{3}$	**A1**	Do not accept $x < -1.33$ [Do not condone $\leqslant$ for $<$ in the final answer.]
	Alternative method for question 1		
	State or imply non-modular inequality/equality $(2-5x)^2 >, \geqslant, =, 2^2(x-3)^2$, or corresponding quadratic equation, or pair of linear equations $(2-5x) >, \geqslant, =, \pm 2(x-3)$	**B1**	Two correct linear equations only
	Make reasonable attempt at solving a 3-term quadratic, or solve one linear equation, or linear inequality for x	**M1**	$21x^2 + 4x - 32 = (3x+4)(7x-8) = 0$ $2 - 5x$ or $-(2-5x)$ with $2(x-3)$ or $-2(x-3)$
	Obtain critical value $x = -\frac{4}{3}$	**A1**	
	State final answer $x < -\frac{4}{3}$	**A1**	Do not accept $x < -1.33$ [Do not condone $\leqslant$ for $<$ in the final answer.]
		4	

Question	Answer	Marks	Guidance
2	Show a circle with centre the origin and radius 2	**B1**	
	Show the point representing 1 – i	**B1**	
	Show a circle with centre 1 – i and radius 1	**B1 FT**	The FT is on the position of 1 – i.
	Shade the appropriate region	**B1 FT**	The FT is on the position of 1 – i. Shaded region outside circle with centre the origin and radius 2 and inside circle with centre $\pm 1 \pm$ i and radius 1
		4	

Question	Answer	Marks	Guidance
3	State or imply $\dfrac{dx}{d\theta} = 2\sin 2\theta$ or $\dfrac{dy}{d\theta} = 2 + 2\cos 2\theta$	B1	
	Use $\dfrac{dy}{dx} = \dfrac{dy}{d\theta} \div \dfrac{dx}{d\theta}$	M1	
	Obtain correct answer $\dfrac{dy}{dx} = \dfrac{2+2\cos 2\theta}{2\sin 2\theta}$	A1	OE
	Use correct double angle formulae	M1	
	Obtain the given answer correctly $\dfrac{dy}{dx} = \cot\theta$	A1	AG. Must have simplified numerator in terms of cosθ.
	Alternative method for question 3		
	Start by using both correct double angle formulae e.g. $x = 3 - (2\cos^2\theta - 1)$, $y = 2\theta + 2\sin\theta\cos\theta$	M1	
	$\dfrac{dx}{d\theta}$ or $\dfrac{dy}{d\theta}$	B1	
	$\dfrac{dy}{dx} = \dfrac{\left(2+2\left(\cos^2\theta - \sin^2\theta\right)\right)}{4\cos\theta\sin\theta}$	M1 A1	
	Simplify to given answer correctly $\dfrac{dy}{dx} = \cot\theta$	A1	AG

Question	Answer	Marks	Guidance
3	**Alternative method for question 3**		
	Set $= 2\theta$. State $\frac{\mathrm{d}x}{\mathrm{d}t} = \sin t$ or $\frac{\mathrm{d}y}{\mathrm{d}t} = 1+\cos t$	**B1**	
	Use $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \div \frac{\mathrm{d}x}{\mathrm{d}t}$	**M1**	
	Obtain correct answer $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1+\cos t}{\sin t}$	**A1**	OE
	Use correct double angle formulae	**M1**	
	Obtain the given answer correctly $\frac{\mathrm{d}y}{\mathrm{d}x} = \cot\theta$	**A1**	
		5	
4	State or imply $\log_{10} 10 = 1$	**B1**	$\log_{10} 10^{-1} = -1$
	Use law of the logarithm of a power, product or quotient	**M1**	
	Obtain a correct equation in any form, free of logs	**A1**	e.g. $(2x + 1)/(x + 1)^2 = 10^{-1}$ or $10(2x + 1)/(x + 1)^2 = 10^0$ or 1 or $x^2 + 2x + 1 = 20x + 10$
	Reduce to $x^2 - 18x - 9 = 0$, or equivalent	**A1**	
	Solve a 3-term quadratic	**M1**	
	Obtain final answers $x = 18.487$ and $x = -0.487$	**A1**	Must be 3 d.p. Do not allow rejection.
		6	

Question	Answer	Marks	Guidance
5(a)	Sketch a relevant graph, e.g. $y = \operatorname{cosec} x$	**B1**	cosec x, U shaped, roughly symmetrical about $x = \frac{\pi}{2}$, $y\left(\frac{\pi}{2}\right) = 1$ and domain at least $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$.
	Sketch a second relevant graph, e.g. $y = 1 + \mathrm{e}^{-\frac{1}{2}x}$, and justify the given statement	**B1**	Exponential graph needs $y(0) = 2$, negative gradient, always increasing, and $y(\pi) > 1$ Needs to mark intersections with dots, crosses, or say roots at points of intersection, or equivalent
		2	
5(b)	Use the iterative formula correctly at least twice	**M1**	2, 2.3217, 2.2760, 2.2824… Need to see 2 iterations and following value inserted correctly
	Obtain final answer 2.28	**A1**	Must be supported by iterations
	Show sufficient iterations to at least 4 d.p. to justify 2.28 to 2 d.p., or show there is a sign change in the interval (2.275, 2.285)	**A1**	
		3	

Question	Answer	Marks	Guidance
6(a)	State $R = \sqrt{15}$	**B1**	
	Use trig formulae to find α	**M1**	$\frac{\sin\alpha}{\cos\alpha} = \frac{3}{\sqrt{6}}$ with no error seen or $\tan\alpha = \frac{3}{\sqrt{6}}$ quoted then allow M1
	Obtain $\alpha = 50.77$	**A1**	Must be 2 d.p. If radians 0.89 A0 MR
		3	
6(b)	Evaluate $\beta = \cos^{-1}\frac{2.5}{\sqrt{15}}$ (49.797° to 4 d.p.)	**B1 FT**	The FT is on incorrect R. $\frac{x}{3} = \beta - \alpha$ [–2.9° and –301.7°]
	Use correct method to find a value of $\frac{x}{3}$ in the interval	**M1**	Needs to use $\frac{x}{3}$
	Obtain answer rounding to $x = 301.6°$ to $301.8°$	**A1**	
	Obtain second answer rounding to $x = 2.9(0)°$ to $2.9(2)°$ and no others in the interval	**A1**	
		4	

Question	Answer	Marks	Guidance
7(a)	Substitute $-1+\sqrt{5}\,\mathrm{i}$ in the equation and attempt expansions of x^2 and x^3	**M1**	All working must be seen. Allow M1 if small errors in $1-2\sqrt{5}\mathrm{i}-5$ or $1-\sqrt{5}\mathrm{i}-\sqrt{5}\mathrm{i}-5$ and $4-2\sqrt{5}\mathrm{i}+10$ or $4-4\sqrt{5}\mathrm{i}+2\sqrt{5}\mathrm{i}+10$
	Use $\mathrm{i}^2=-1$ correctly at least once	**M1**	$1-5$ or $4+10$ seen
	Complete the verification correctly	**A1**	$2\left(14-2\sqrt{5}\mathrm{i}\right)+\left(-4-2\sqrt{5}\mathrm{i}\right)+6\left(-1+\sqrt{5}\mathrm{i}\right)-18=0$
		3	
7(b)	State second root $-1-\sqrt{5}\mathrm{i}$	**B1**	
	Carry out a complete method for finding a quadratic factor with zeros $-1+\sqrt{5}\mathrm{i}$ and $-1-\sqrt{5}\mathrm{i}$	**M1**	
	Obtain x^2+2x+6	**A1**	
	Obtain root $x=\dfrac{3}{2}$	**A1**	OE
	Alternative method for question 7(b)		
	State second root $-1-\sqrt{5}\mathrm{i}$	**B1**	
	$(x+1-\sqrt{5}\mathrm{i})(x+1+\sqrt{5}\mathrm{i})(2x+a)=2x^3+x^2+6x-18$	**M1**	
	$(1-\sqrt{5}\,\mathrm{i})(1+\sqrt{5}\,\mathrm{i})\,a=-18$	**A1**	
	$6a=-18$ $a=-3$ leading to $x=\dfrac{3}{2}$	**A1**	OE

Question	Answer	Marks	Guidance
7(b)	**Alternative method for question 7(b)**		
	State second root $-1-\sqrt{5}\text{i}$	**B1**	
	POR = 6 SOR = – 2	**M1**	
	Obtain $x^2 + 2x + 6$	**A1**	
	Obtain root $x=\dfrac{3}{2}$	**A1**	OE
	Alternative method for question 7(b)		
	State second root $-1-\sqrt{5}\text{i}$	**B1**	
	POR $\left(-1-\sqrt{5}\text{i}\right)\left(-1+\sqrt{5}\text{i}\right)a=9$	**M1 A1**	
	Obtain root $x=\dfrac{3}{2}$	**A1**	OE
	Alternative method for question 7(b)		
	State second root $-1-\sqrt{5}\text{i}$	**B1**	
	SOR $\left(-1-\sqrt{5}\text{i}\right)+\left(-1+\sqrt{5}\text{i}\right)+a=-\dfrac{1}{2}$	**M1 A1**	
	Obtain root $x=\dfrac{3}{2}$	**A1**	OE
		4	

Question	Answer	Marks	Guidance
8	Separate variables correctly and attempt integration of at least one side	**B1**	$\frac{1}{y}\mathrm{d}y = \frac{1-2x^2}{x}\mathrm{d}x$
	Obtain term $\ln y$	**B1**	
	Obtain terms $\ln x - x^2$	**B1**	
	Use $x = 1$, $y = 1$ to evaluate a constant, or as limits, in a solution containing at least 2 terms of the form $a\ln y$, $b\ln x$ and cx^2	**M1**	The 2 terms of required form must be from correct working e.g. $\ln y = \ln x - x^2 + 1$
	Obtain correct solution in any form	**A1**	
	Rearrange and obtain $y = xe^{1-x^2}$	**A1**	OE
		6	

Question	Answer	Marks	Guidance
9(a)	State or imply the form $\frac{A}{1-x}+\frac{B}{2+3x}+\frac{C}{(2+3x)^2}$	**B1**	
	Use a correct method for finding a coefficient	**M1**	
	Obtain one of $A = 1$, $B = -1$, $C = 6$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	In the form $\frac{A}{1-x}+\frac{Dx+E}{(2+3x)^2}$, where $A = 1$, $D = -3$ and $E = 4$ can score B1 M1 A1 A1 A1 as above.
		5	

Question	Answer	Marks	Guidance
9(b)	Use a correct method to find the first two terms of the expansion of $(1-x)^{-1}$, $(2+3x)^{-1}$, $\left(1+\frac{3}{2}x\right)^{-1}$, $(2+3x)^{-2}$ or $\left(1+\frac{3}{2}x\right)^{-2}$	M1	Symbolic coefficients are not sufficient for the M1 $A\left[\frac{1+(-1)(-x)+(-1)(-2)(-x)^2}{2\ldots}\right]$ $A=1$ $\frac{B}{2}\left[\frac{1+(-1)\left(\frac{3x}{2}\right)+(-1)(-2)\left(\frac{3x}{2}\right)^2}{2\ldots}\right]$ $B=1$ $\frac{C}{4}\left[\frac{1+(-2)\left(\frac{3x}{2}\right)+(-2)(-3)\left(\frac{3x}{2}\right)^2}{2\ldots}\right]$ $C=6$
	Obtain correct un-simplified expansions up to the term in of each partial fraction	A1 FT + A1 FT + A1 FT	$\left(1+x+x^2\right)+\left(-\frac{1}{2}+\left(\frac{3}{4}\right)x-\left(\frac{9}{8}\right)x^2\right)$ $+\left(\frac{6}{4}-\left(\frac{18}{4}\right)x+\left(\frac{81}{8}\right)x^2\right)$ [The FT is on A, B, C] $\left(1-\frac{1}{2}+\frac{6}{4}\right)+\left(1+\frac{3}{4}-\frac{18}{4}\right)x+\left(1-\frac{9}{8}+\frac{81}{8}\right)x^2$
	Obtain final answer $2-\frac{11}{4}x+10x^2$, or equivalent	A1	Allow unsimplified fractions $\frac{(Dx+E)}{4}\left[\frac{1+(-2)\left(\frac{3x}{2}\right)+(-2)(-3)\left(\frac{3x}{2}\right)^2}{2\ldots}\right]$ $D=-3$, $E=4$ The FT is on A, D, E.
		5	

Question	Answer	Marks	Guidance
10(a)	Use correct product or quotient rule	*M1	$\dfrac{dy}{dx} = \left(-\dfrac{1}{2}\right)(2-x)e^{-\frac{1}{2}x} - e^{-\frac{1}{2}x}$ M1 requires at least one of derivatives correct
	Obtain correct derivative in any form	A1	
	Equate derivative to zero and solve for x	DM1	
	Obtain $x = 4$	A1	ISW
	Obtain $y = -2e^{-2}$, or exact equivalent	A1	
		5	

Question	Answer	Marks	Guidance
10(b)	Commence integration and reach $a(2-x)e^{-\frac{1}{2}x}+b\int e^{-\frac{1}{2}x}dx$	*M1	Condone omission of dx $-2(2-x)e^{-\frac{1}{2}x}+4e^{-\frac{1}{2}x}$ or $2xe^{-\frac{1}{2}x}$
	Obtain $-2(2-x)e^{-\frac{1}{2}x}-2\int e^{-\frac{1}{2}x}dx$	A1	OE
	Complete integration and obtain $2xe^{-\frac{1}{2}x}$	A1	OE
	Use correct limits, $x = 0$ and $x = 2$, correctly, having integrated twice	DM1	Ignore omission of zeros and allow max of 1 error
	Obtain answer $4e^{-1}$, or exact equivalent	A1	ISW
	Alternative method for question 10(b)		
	$\dfrac{d\left(2xe^{-\frac{1}{2}x}\right)}{dx}=2e^{-\frac{1}{2}x}-xe^{-\frac{1}{2}x}$	*M1 A1	
	$\therefore 2xe^{-\frac{1}{2}x}$	A1	
	Use correct limits, $x = 0$ and $x = 2$, correctly, having integrated twice	DM1	Ignore omission of zeros and allow max of 1 error
	Obtain answer $4e^{-1}$, or exact equivalent	A1	ISW
		5	

Question	Answer	Marks	Guidance
11(a)	Express general point of at least one line correctly in component form, i.e. $(1 + a\lambda, 2 + 2\lambda, 1 - \lambda)$ or $(2 + 2\mu, 1 - \mu, -1 + \mu)$	**B1**	
	Equate at least two pairs of corresponding components and solve for λ or for μ	**M1**	May be implied $1 + a\lambda = 2 + 2\mu$ $2 + 2\lambda = 1 - \mu$ $1 - \lambda = -1 + \mu$
	Obtain $\lambda = -3$ or $\mu = 5$	**A1**	
	Obtain $a = -\dfrac{11}{3}$	**A1**	Allow $a = -3.667$
	State that the point of intersection has position vector $12\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$	**A1**	Allow coordinate form $(12, -4, 4)$
		5	

Question	Answer	Marks	Guidance
11(b)	Use correct process for finding the scalar product of direction vectors for the two lines	**M1**	$(a, 2, -1) \cdot (2, -1, 1) = 2a - 2 - 1$ or $2a - 3$
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and equate the result to $\pm\frac{1}{6}$	***M1**	
	State a correct equation in a in any form, e.g. $\frac{2a-2-1}{\sqrt{6}\sqrt{(a^2+5)}} = \pm\frac{1}{6}$	**A1**	
	Solve for a	**DM1**	Solve 3-term quadratic for a having expanded $(2a-3)^2$ to produce 3 terms e.g. $36(2a-3)^2 = 6(a^2+5)$ $138a^2 - 432a + 294 = 0$ $23a^2 - 72a + 49 = 0$ $(23a-49)(a-1) = 0$
	Obtain $a = 1$	**A1**	
	Obtain $a = \frac{49}{23}$	**A1**	Allow $a = 2.13$

Question	Answer	Marks	Guidance
11(b)	**Alternative method for question 11(b)**		
	$\cos(\theta) = [\lvert a^2 + 2^2 + (-1)^2\rvert^2 + \lvert 2^2 + (-1)^2 + 1^2\rvert^2$ $- \lvert(a-2)^2 + 3^2 + (-2)^2\rvert^2] / [2\lvert a^2 + 2^2 + (-1)^2\rvert.\lvert 2^2 + (-1)^2 + 1^2\rvert]$	**M1**	Use of cosine rule. Must be correct vectors.
	Equate the result to $\pm\frac{1}{6}$	***M1** **A1**	Allow M1* here for any two vectors
	Solve for a	**DM1**	Solve 3-term quadratic for a having expanded $(2a-3)^2$ to produce 3 terms e.g. $36(2a-3)^2 = 6(a^2+5)$ $138a^2 - 432a + 294 = 0$ $23a^2 - 72a + 49 = 0$ $(23a-49)(a-1) = 0$
	Obtain $a = 1$	**A1**	
	Obtain $a = \frac{49}{23}$	**A1**	Allow $a = 2.13$
		6	

Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **October/November 2020**

MARK SCHEME

Maximum Mark: 75

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Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

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Abbreviations

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AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	State that $1+e^{-3x}=e^2$	**B1**	With no errors seen to that point
	Use correct method to solve an equation of the form $e^{-3x}=a$, where $a>0$, for x or equivalent	**M1**	$\left(e^{-3x}=6.389...\right)$ Evidence of method must be seen.
	Obtain answer $x=-0.618$ only	**A1**	Must be 3 decimal places
	Alternative method for question 1		
	State that $1+e^{-3x}=e^2$	**B1**	
	Rearrange to obtain an expression for e^x and solve an equation of the form $e^x=a$, where $a>0$, or equivalent	**M1**	$e^x=\sqrt[3]{\dfrac{1}{e^2-1}}$
	Obtain answer $x=-0.618$ only	**A1**	Must be 3 decimal places
		3	

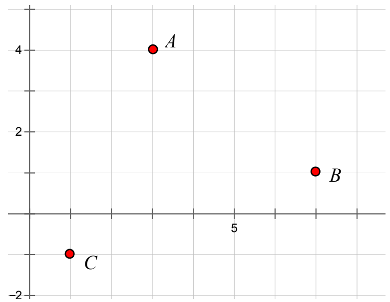
Question	Answer	Marks	Guidance
2(a)	State a correct unsimplified version of the x or x^2 or x^3 term	**M1**	For the given expression
	State correct first two terms $1+2x$	**A1**	
	Obtain the next two terms $-4x^2+\dfrac{40}{3}x^3$	**A1 + A1**	One mark for each correct term. ISW Accept $13\frac{1}{3}$ The question asks for simplified coefficients, so candidates should cancel fractions.
		4	
2(b)	State answer $\lvert x\rvert<\dfrac{1}{6}$	**B1**	OE. Strict inequality
		1	

Question	Answer	Marks	Guidance
3(a)	State or imply $y\log 2 = \log 3 - 2x\log 3$	**B1**	Accept $y\ln 2 = (1-2x)\ln 3$
	State that the graph of y against x has an equation which is linear in x and y, or is of the form $ay = bx + c$	**B1**	Correct equation. Need a clear statement/comparison with matching linear form.
	Clear indication that the gradient is $-\dfrac{2\ln 3}{\ln 2}$	**B1**	Must be exact. Any equivalent e.g. $-\dfrac{2\log_k 3}{\log_k 2}$, $\log_2 \dfrac{1}{9}$
		3	
3(b)	Substitute $y = 3x$ in an equation involving logarithms and solve for x	**M1**	
	Obtain answer $x = \dfrac{\ln 3}{\ln 72}$	**A1**	Allow M1A1 for the correct answer following decimals
		2	

Question	Answer	Marks	Guidance
4(a)	Use correct $\tan(A+B)$ formula and obtain an equation in $\tan\theta$	**M1**	e.g. $\dfrac{\tan\theta+\tan 60°}{1-\tan\theta\tan 60°}=\dfrac{2}{\tan\theta}$
	Use $\tan 60° = \sqrt{3}$ and obtain a correct horizontal equation in any form	**A1**	e.g. $\tan\theta\left(\tan\theta+\sqrt{3}\right)=2\left(1-\sqrt{3}\tan\theta\right)$
	Reduce to $\tan^2\theta+3\sqrt{3}\tan\theta-2=0$ correctly	**A1**	AG
		3	
4(b)	Solve the given quadratic to obtain a value for θ	**M1**	$\left(\tan\theta=\dfrac{-3\sqrt{3}\pm\sqrt{35}}{2}=0.3599,-5.556\right)$
	Obtain one correct answer e.g. $\theta = 19.8°$	**A1**	Accept 1d.p. or better. If over-specified must be correct. 19.797...., 100.2029...
	Obtain second correct answer $\theta = 100.2°$ and no others in the given interval	**A1**	Ignore answers outside the given interval.
		3	

Question	Answer	Marks	Guidance
5(a)	State $\dfrac{dx}{d\theta} = \sec^2\theta$ or $\dfrac{dy}{d\theta} = -2\sin\theta\cos\theta$	**B1**	CWO, AEF.
	Use $\dfrac{dy}{dx} = \dfrac{dy}{d\theta} \div \dfrac{dx}{d\theta}$	**M1**	
	Obtain $\dfrac{dy}{dx} = -2\sin\theta\cos^3\theta$ from correct working	**A1**	AG
	Alternative method for question 5(a)		
	Convert to Cartesian form and differentiate	**M1**	$y = \dfrac{1}{1+x^2}$
	$\dfrac{dy}{dx} = \dfrac{-2x}{\left(1+x^2\right)^2}$	**A1**	OE
	Obtain $\dfrac{dy}{dx} = -2\sin\theta\cos^3\theta$ from correct working	**A1**	AG
		3	

Question	Answer	Marks	Guidance
5(b)	Use correct product rule to obtain $\dfrac{d}{d\theta}\left(\pm 2\cos^3\theta\sin\theta\right)$	M1	Condone incorrect naming of the derivative For work done in correct context
	Obtain correct derivative in any form	A1	e.g. $\pm\left(-2\cos^4\theta + 6\sin^2\theta\cos^2\theta\right)$
	Equate derivative to zero and obtain an equation in one trig ratio	A1	e.g. $3\tan^2\theta = 1$, or $4\sin^2\theta = 1$ or $4\cos^2\theta = 3$
	Obtain answer $x = -\dfrac{1}{\sqrt{3}}$	A1	Or $-\dfrac{\sqrt{3}}{3}$
	Alternative method for question 5(b)		
	Use correct quotient rule to obtain $\dfrac{d^2y}{dx^2}$	M1	
	Obtain correct derivative in any form	A1	$\dfrac{-2\left(1+x^2\right)^2 + 2\times 2x\times 2x\left(1+x^2\right)}{\left(1+x^2\right)^4}$
	Equate derivative to zero and obtain an equation in x^2	A1	e.g. $6x^2 = 2$
	Obtain answer $x = -\dfrac{1}{\sqrt{3}}$	A1	
		4	

Question	Answer	Marks	Guidance
6(a)	Multiply numerator and denominator by 1 + i, or equivalent	**M1**	Must multiply out
	Obtain numerator 6 + 8i or denominator 2	**A1**	
	Obtain final answer $u = 3 + 4i$	**A1**	
	Alternative method for question 6(a)		
	Multiply out $(1-i)(x+iy)=7+i$ and compare real and imaginary parts	**M1**	
	Obtain $x+y=7$ or $y-x=1$	**A1**	
	Obtain final answer $u = 3 + 4i$	**A1**	
		3	
6(b)	Show the point A representing u in a relatively correct position	**B1 FT**	The FT is on $xy \neq 0$.
	Show the other two points B and C in relatively correct positions: approximately equal distance above / below real axis	**B1**	 Take the position of A as a guide to 'scale' if axes not marked
		2	

Question	Answer	Marks	Guidance
6(c)	State or imply arg(1 – i) = $-\frac{1}{4}\pi$	**B1**	Arg$C$
	Substitute exact arguments in arg(7 + i) – arg(1 – i) = arg u	**M1**	Must see a statement about the relationship between the Args e.g. Arg$A$ = Arg$B$ – Arg$C$ or equivalent exact method
	Obtain $\tan^{-1}\left(\frac{4}{3}\right) = \tan^{-1}\left(\frac{1}{7}\right) + \frac{1}{4}\pi$ correctly	**A1**	Obtain given answer correctly from *their* $u = k(3+4i)$
		3	

Question	Answer	Marks	Guidance
7(a)	Correct separation of variables	**B1**	$\int \sec^2 2x \, dx = \int e^{-3t} dt$ Needs correct structure
	Obtain term $-\frac{1}{3}e^{-3t}$	**B1**	
	Obtain term of the form $k \tan 2x$	**M1**	From correct working
	Obtain term $\frac{1}{2}\tan 2x$	**A1**	
	Use $x = 0$, $t = 0$ to evaluate a constant, or as limits in a solution containing terms of the form $a \tan 2x$ and be^{-3t}, where $ab \neq 0$	**M1**	
	Obtain correct solution in any form	**A1**	e.g. $\frac{1}{2}\tan 2x = -\frac{1}{3}e^{-3t} + \frac{1}{3}$
	Obtain final answer $x = \frac{1}{2}\tan^{-1}\left(\frac{2}{3}\left(1 - e^{-3t}\right)\right)$	**A1**	
		7	
7(b)	State that x approaches $\frac{1}{2}\tan^{-1}\left(\frac{2}{3}\right)$	**B1 FT**	Correct value. Accept $x \to 0.294$ The FT is dependent on letting $e^{-3t} \to 0$ in a solution containing e^{-3t}.
		1	

Question	Answer	Marks	Guidance
8(a)	Obtain $\overrightarrow{AB}=\begin{pmatrix}2\\-2\\-4\end{pmatrix}$ and $\overrightarrow{CD}=\begin{pmatrix}2\\1\\1\end{pmatrix}$	**B1**	Or equivalent seen or implied
	Use the correct process for calculating the modulus of both vectors to obtain AB and CD	**M1**	$AB=\sqrt{24}, CD=\sqrt{6}$
	Using exact values, verify that $AB=2CD$	**A1**	Obtain given statement from correct work Allow from $BA=2DC$, OE
		3	
8(b)	Use the correct process to calculate the scalar product of the relevant vectors (*their* $\overrightarrow{AB}$ and $\overrightarrow{CD}$)	**M1**	$\begin{pmatrix}2\\-2\\-4\end{pmatrix}$ and $\begin{pmatrix}2\\1\\1\end{pmatrix}$ or $\begin{pmatrix}2\\-2\\-4\end{pmatrix}$ and $\begin{pmatrix}4\\2\\2\end{pmatrix}$
	Divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	**M1**	
	Obtain answer $99.6°$ (or 1.74 radians) or better	**A1**	Do not ISW if go on to subtract from 180° (99.594…, 1.738...) Accept 260.4°
		3	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>8(c)</td><td>State correct vector equations for AB and CD in any form,
e.g. $(\mathbf{r}=)\begin{pmatrix}2\\1\\5\end{pmatrix}+\lambda\begin{pmatrix}2\\-2\\-4\end{pmatrix}$ and $(\mathbf{r}=)\begin{pmatrix}1\\1\\2\end{pmatrix}+\mu\begin{pmatrix}2\\1\\1\end{pmatrix}$</td><td>B1ft</td><td>Follow their $\overrightarrow{AB}$ and $\overrightarrow{CD}$
Alternative: $(\mathbf{r}=)\begin{pmatrix}4\\-1\\1\end{pmatrix}+\lambda\begin{pmatrix}2\\-2\\-4\end{pmatrix}$ and $(\mathbf{r}=)\begin{pmatrix}3\\2\\3\end{pmatrix}+\mu\begin{pmatrix}2\\1\\1\end{pmatrix}$</td></tr>
<tr><td></td><td>Equate at least two pairs of components of their lines and solve for λ or for μ</td><td>M1</td><td></td></tr>
<tr><td></td><td>Obtain correct pair of values from correct equations</td><td>A1</td><td>Alternatives when taking A or B as point on line
<table>
<tr><th>A</th><th>λ</th><th>μ</th><th></th><th>B</th><th>λ</th><th>μ</th><th></th></tr>
<tr><td>ij</td><td>$-\frac{1}{6}$</td><td>$\frac{1}{3}$</td><td>$\frac{17}{3}\neq\frac{7}{3}$</td><td>ij</td><td>$-\frac{7}{6}$</td><td>$-\frac{2}{3}$</td><td>$\frac{17}{3}\neq\frac{7}{3}$</td></tr>
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</table></td></tr>
<tr><td></td><td>Verify that all three equations are not satisfied and that the lines do not intersect</td><td>A1</td><td>CWO with conclusion
e.g. $\frac{17}{3}\neq\frac{7}{3}$ or $\frac{17}{3}=\frac{7}{3}$ is inconsistent or equivalent</td></tr>
<tr><td></td><td></td><td>4</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
9(a)	State or imply the form $\dfrac{A}{3x+2}+\dfrac{Bx+C}{x^2+4}$	**B1**	
	Use a correct method for finding a constant	**M1**	
	Obtain one of $A=3$, $B=-1$, $C=3$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
		5	
9(b)	Integrate and obtain $\ln(3x+2)\ldots$	**B1 FT**	The FT is on A
	State a term of the form $k\ln\left(x^2+4\right)$.	**M1**	From $\int\dfrac{\lambda x}{x^2+4}\mathrm{d}x$
	$\ldots-\dfrac{1}{2}\ln\left(x^2+4\right)\ldots$	**A1 FT**	The FT is on B
	$\ldots+\dfrac{3}{2}\tan^{-1}\dfrac{x}{2}$	**B1 FT**	The FT is on C
	Substitute limits correctly in an integral with at least two terms of the form $a\ln(3x+2)$, $b\ln\left(x^2+4\right)$ and $c\tan^{-1}\left(\dfrac{x}{2}\right)$, and subtract in correct order	**M1**	Using terms that have been obtained correctly from completed integrals
	Obtain answer $\dfrac{3}{2}\ln 2+\dfrac{3}{8}\pi$, or exact 2-term equivalent	**A1**	
		6	

Question	Answer	Marks	Guidance
10(a)	Use correct product rule	**M1**	
	Obtain correct derivative in any form	**A1**	e.g. $\frac{dy}{dx}=\frac{1}{2\sqrt{x}}\cos x-\sqrt{x}\sin x$. Accept in a or in x
	Equate derivative to zero and obtain $\tan a=\frac{1}{2a}$	**A1**	Obtain given answer from correct working**.** The question says 'show that ..' so there should be an intermediate step e.g. $\cos x=2x\sin x$. Allow $\tan x=\frac{1}{2x}$
		3	
10(b)	Use the iterative process correctly at least once (get one value and go on to use it in a second use of the formula)	**M1**	Must be working in radians Degrees gives 1, 12.6039, 5.4133, ... M0
	Obtain final answer 3.29	**A1**	Clear conclusion
	Show sufficient iterations to at least 4 d.p.to justify 3.29, or show there is a sign change in the interval (3.285, 3.295)	**A1**	3, 3.3067, 3.2917, 3.2923 Allow more than 4d.p. Condone truncation.
		3	

Question	Answer	Marks	Guidance
10(c)	State or imply the indefinite integral for the volume is $\pi\int\left(\sqrt{x}\cos x\right)^2 \mathrm{d}x$	B1	[If π omitted, or 2π or $\frac{1}{2}\pi$ used, give B0 and follow through. 4/6 available]
	Use correct cos $2A$ formula, commence integration by parts and reach $x(ax+b\sin 2x)\pm\int ax+b\sin 2x\mathrm{d}x$	*M1	Alternative: $\frac{x^2}{4}+\frac{x}{4}\sin 2x-\int\frac{1}{4}\sin 2x\mathrm{d}x$
	Obtain $x(\frac{1}{2}x+\frac{1}{4}\sin 2x)-\int\frac{1}{2}x+\frac{1}{4}\sin 2x\,\mathrm{d}x$, or equivalent	A1	
	Complete integration and obtain $\frac{1}{4}x^2+\frac{1}{4}x\sin 2x+\frac{1}{8}\cos 2x$	A1	OE
	Substitute limits $x = 0$ and $x = \frac{1}{2}\pi$, having integrated twice	DM1	$\frac{\pi}{2}\left[\frac{\pi^2}{8}+0-\frac{1}{4}-0-0-\frac{1}{4}\right]$
	Obtain answer $\frac{1}{16}\pi\left(\pi^2-4\right)$, or exact equivalent	A1	CAO
		6	

Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **October/November 2020**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2020 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
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Question	Answer	Marks	Guidance
1	Make a recognisable sketch graph of $y = 2\|x-3\|$ and the line $y = 2 - 5x$	**B1**	Need to see correct V at $x = 3$, roughly symmetrical, $x = 3$ stated, domain at least $(-2, 5)$.
	Find x-coordinate of intersection with $y = 2 - 5x$	**M1**	Find point of intersection with $y = 2\|x - 3\|$ or solve $2 - 5x$ with $2(x - 3)$ or $-2(x - 3)$
	Obtain $x = -\frac{4}{3}$	**A1**	
	State final answer $x < -\frac{4}{3}$	**A1**	Do not accept $x < -1.33$ [Do not condone $\leqslant$ for $<$ in the final answer.]
	Alternative method for question 1		
	State or imply non-modular inequality/equality $(2-5x)^2 >, \geqslant, =, 2^2(x-3)^2$, or corresponding quadratic equation, or pair of linear equations $(2-5x) >, \geqslant, =, \pm 2(x-3)$	**B1**	Two correct linear equations only
	Make reasonable attempt at solving a 3-term quadratic, or solve one linear equation, or linear inequality for x	**M1**	$21x^2 + 4x - 32 = (3x+4)(7x-8) = 0$ $2 - 5x$ or $-(2 - 5x)$ with $2(x - 3)$ or $-2(x - 3)$
	Obtain critical value $x = -\frac{4}{3}$	**A1**	
	State final answer $x < -\frac{4}{3}$	**A1**	Do not accept $x < -1.33$ [Do not condone $\leqslant$ for $<$ in the final answer.]
		4	

Question	Answer	Marks	Guidance
2	Show a circle with centre the origin and radius 2	**B1**	
	Show the point representing 1 – i	**B1**	
	Show a circle with centre 1 – i and radius 1	**B1 FT**	The FT is on the position of 1 – i.
	Shade the appropriate region	**B1 FT**	The FT is on the position of 1 – i. Shaded region outside circle with centre the origin and radius 2 and inside circle with centre $\pm 1 \pm$ i and radius 1
		4	

Question	Answer	Marks	Guidance
3	State or imply $\dfrac{dx}{d\theta} = 2\sin 2\theta$ or $\dfrac{dy}{d\theta} = 2 + 2\cos 2\theta$	B1	
	Use $\dfrac{dy}{dx} = \dfrac{dy}{d\theta} \div \dfrac{dx}{d\theta}$	M1	
	Obtain correct answer $\dfrac{dy}{dx} = \dfrac{2+2\cos 2\theta}{2\sin 2\theta}$	A1	OE
	Use correct double angle formulae	M1	
	Obtain the given answer correctly $\dfrac{dy}{dx} = \cot\theta$	A1	AG. Must have simplified numerator in terms of $\cos\theta$.
	Alternative method for question 3		
	Start by using both correct double angle formulae e.g. $x = 3 - (2\cos^2\theta - 1)$, $y = 2\theta + 2\sin\theta\cos\theta$	M1	
	$\dfrac{dx}{d\theta}$ or $\dfrac{dy}{d\theta}$	B1	
	$\dfrac{dy}{dx} = \dfrac{\left(2+2\left(\cos^2\theta - \sin^2\theta\right)\right)}{4\cos\theta\sin\theta}$	M1 A1	
	Simplify to given answer correctly $\dfrac{dy}{dx} = \cot\theta$	A1	AG

Question	Answer	Marks	Guidance
3	**Alternative method for question 3**		
	Set $= 2\theta$. State $\dfrac{dx}{dt} = \sin t$ or $\dfrac{dy}{dt} = 1 + \cos t$	**B1**	
	Use $\dfrac{dy}{dx} = \dfrac{dy}{dt} \div \dfrac{dx}{dt}$	**M1**	
	Obtain correct answer $\dfrac{dy}{dx} = \dfrac{1+\cos t}{\sin t}$	**A1**	OE
	Use correct double angle formulae	**M1**	
	Obtain the given answer correctly $\dfrac{dy}{dx} = \cot\theta$	**A1**	
		5	
4	State or imply $\log_{10} 10 = 1$	**B1**	$\log_{10} 10^{-1} = -1$
	Use law of the logarithm of a power, product or quotient	**M1**	
	Obtain a correct equation in any form, free of logs	**A1**	e.g. $(2x + 1)/(x + 1)^2 = 10^{-1}$ or $10(2x + 1)/(x + 1)^2 = 10^0$ or 1 or $x^2 + 2x + 1 = 20x + 10$
	Reduce to $x^2 - 18x - 9 = 0$, or equivalent	**A1**	
	Solve a 3-term quadratic	**M1**	
	Obtain final answers $x = 18.487$ and $x = -0.487$	**A1**	Must be 3 d.p. Do not allow rejection.
		6	

Question	Answer	Marks	Guidance
5(a)	Sketch a relevant graph, e.g. $y = \operatorname{cosec} x$	**B1**	cosec x, U shaped, roughly symmetrical about $x = \frac{\pi}{2}$, $y\left(\frac{\pi}{2}\right) = 1$ and domain at least $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$.
	Sketch a second relevant graph, e.g. $y = 1 + e^{-\frac{1}{2}x}$, and justify the given statement	**B1**	Exponential graph needs $y(0) = 2$, negative gradient, always increasing, and $y(\pi) > 1$ Needs to mark intersections with dots, crosses, or say roots at points of intersection, or equivalent
		2	
5(b)	Use the iterative formula correctly at least twice	**M1**	2, 2.3217, 2.2760, 2.2824… Need to see 2 iterations and following value inserted correctly
	Obtain final answer 2.28	**A1**	Must be supported by iterations
	Show sufficient iterations to at least 4 d.p. to justify 2.28 to 2 d.p., or show there is a sign change in the interval (2.275, 2.285)	**A1**	
		3	

Question	Answer	Marks	Guidance
6(a)	State $R = \sqrt{15}$	**B1**	
	Use trig formulae to find α	**M1**	$\frac{\sin\alpha}{\cos\alpha} = \frac{3}{\sqrt{6}}$ with no error seen or $\tan\alpha = \frac{3}{\sqrt{6}}$ quoted then allow M1
	Obtain $\alpha = 50.77$	**A1**	Must be 2 d.p. If radians 0.89 A0 MR
		3	
6(b)	Evaluate $\beta = \cos^{-1}\frac{2.5}{\sqrt{15}}$ (49.797° to 4 d.p.)	**B1 FT**	The FT is on incorrect R. $\frac{x}{3} = \beta - \alpha$ [–2.9° and –301.7°]
	Use correct method to find a value of $\frac{x}{3}$ in the interval	**M1**	Needs to use $\frac{x}{3}$
	Obtain answer rounding to $x = 301.6°$ to $301.8°$	**A1**	
	Obtain second answer rounding to $x = 2.9(0)°$ to $2.9(2)°$ and no others in the interval	**A1**	
		4	

Question	Answer	Marks	Guidance
7(a)	Substitute $-1+\sqrt{5}\,\mathrm{i}$ in the equation and attempt expansions of x^2 and x^3	**M1**	All working must be seen. Allow M1 if small errors in $1-2\sqrt{5}\mathrm{i}-5$ or $1-\sqrt{5}\mathrm{i}-\sqrt{5}\mathrm{i}-5$ and $4-2\sqrt{5}\mathrm{i}+10$ or $4-4\sqrt{5}\mathrm{i}+2\sqrt{5}\mathrm{i}+10$
	Use $\mathrm{i}^2=-1$ correctly at least once	**M1**	$1-5$ or $4+10$ seen
	Complete the verification correctly	**A1**	$2\left(14-2\sqrt{5}\mathrm{i}\right)+\left(-4-2\sqrt{5}\mathrm{i}\right)+6\left(-1+\sqrt{5}\mathrm{i}\right)-18=0$
		3	
7(b)	State second root $-1-\sqrt{5}\mathrm{i}$	**B1**	
	Carry out a complete method for finding a quadratic factor with zeros $-1+\sqrt{5}\mathrm{i}$ and $-1-\sqrt{5}\mathrm{i}$	**M1**	
	Obtain x^2+2x+6	**A1**	
	Obtain root $x=\dfrac{3}{2}$	**A1**	OE
	Alternative method for question 7(b)		
	State second root $-1-\sqrt{5}\mathrm{i}$	**B1**	
	$(x+1-\sqrt{5}\mathrm{i})(x+1+\sqrt{5}\mathrm{i})(2x+a)=2x^3+x^2+6x-18$	**M1**	
	$(1-\sqrt{5}\,\mathrm{i})(1+\sqrt{5}\,\mathrm{i})\,a=-18$	**A1**	
	$6a=-18$ $a=-3$ leading to $x=\dfrac{3}{2}$	**A1**	OE

Question	Answer	Marks	Guidance
7(b)	**Alternative method for question 7(b)**		
	State second root $-1-\sqrt{5}\mathrm{i}$	**B1**	
	POR = 6 SOR = – 2	**M1**	
	Obtain $x^2 + 2x + 6$	**A1**	
	Obtain root $x = \frac{3}{2}$	**A1**	OE
	Alternative method for question 7(b)		
	State second root $-1-\sqrt{5}\mathrm{i}$	**B1**	
	POR $\left(-1-\sqrt{5}\mathrm{i}\right)\left(-1+\sqrt{5}\mathrm{i}\right)a = 9$	**M1 A1**	
	Obtain root $x = \frac{3}{2}$	**A1**	OE
	Alternative method for question 7(b)		
	State second root $-1-\sqrt{5}\mathrm{i}$	**B1**	
	SOR $\left(-1-\sqrt{5}\mathrm{i}\right)+\left(-1+\sqrt{5}\mathrm{i}\right)+a = -\frac{1}{2}$	**M1 A1**	
	Obtain root $x = \frac{3}{2}$	**A1**	OE
		4	

Question	Answer	Marks	Guidance
8	Separate variables correctly and attempt integration of at least one side	**B1**	$\frac{1}{y}\mathrm{d}y = \frac{1-2x^2}{x}\mathrm{d}x$
	Obtain term $\ln y$	**B1**	
	Obtain terms $\ln x - x^2$	**B1**	
	Use $x = 1$, $y = 1$ to evaluate a constant, or as limits, in a solution containing at least 2 terms of the form $a\ln y$, $b\ln x$ and cx^2	**M1**	The 2 terms of required form must be from correct working e.g. $\ln y = \ln x - x^2 + 1$
	Obtain correct solution in any form	**A1**	
	Rearrange and obtain $y = xe^{1-x^2}$	**A1**	OE
		6	

Question	Answer	Marks	Guidance
9(a)	State or imply the form $\dfrac{A}{1-x}+\dfrac{B}{2+3x}+\dfrac{C}{(2+3x)^2}$	**B1**	
	Use a correct method for finding a coefficient	**M1**	
	Obtain one of $A = 1$, $B = -1$, $C = 6$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	In the form $\dfrac{A}{1-x}+\dfrac{Dx+E}{(2+3x)^2}$, where $A = 1$, $D = -3$ and $E = 4$ can score B1 M1 A1 A1 A1 as above.
		5	

Question	Answer	Marks	Guidance
9(b)	Use a correct method to find the first two terms of the expansion of $(1-x)^{-1}$, $(2+3x)^{-1}$, $\left(1+\frac{3}{2}x\right)^{-1}$, $(2+3x)^{-2}$ or $\left(1+\frac{3}{2}x\right)^{-2}$	**M1**	Symbolic coefficients are not sufficient for the M1 $A\left[\frac{1+(-1)(-x)+(-1)(-2)(-x)^2}{2\ldots}\right]$ $A=1$ $\frac{B}{2}\left[\frac{1+(-1)\left(\frac{3x}{2}\right)+(-1)(-2)\left(\frac{3x}{2}\right)^2}{2\ldots}\right]$ $B=1$ $\frac{C}{4}\left[\frac{1+(-2)\left(\frac{3x}{2}\right)+(-2)(-3)\left(\frac{3x}{2}\right)^2}{2\ldots}\right]$ $C=6$
	Obtain correct un-simplified expansions up to the term in of each partial fraction	**A1 FT** +	$\left(1+x+x^2\right)+\left(-\frac{1}{2}+\left(\frac{3}{4}\right)x-\left(\frac{9}{8}\right)x^2\right)$
		A1 FT +	$+\left(\frac{6}{4}-\left(\frac{18}{4}\right)x+\left(\frac{81}{8}\right)x^2\right)$ [The FT is on A, B, C]
		A1 FT	$\left(1-\frac{1}{2}+\frac{6}{4}\right)+\left(1+\frac{3}{4}-\frac{18}{4}\right)x+\left(1-\frac{9}{8}+\frac{81}{8}\right)x^2$
	Obtain final answer $2-\frac{11}{4}x+10x^2$, or equivalent	**A1**	Allow unsimplified fractions $\frac{(Dx+E)}{4}\left[\frac{1+(-2)\left(\frac{3x}{2}\right)+(-2)(-3)\left(\frac{3x}{2}\right)^2}{2\ldots}\right]$ $D=-3$, $E=4$ The FT is on A, D, E.
		5	

Question	Answer	Marks	Guidance
10(a)	Use correct product or quotient rule	*M1	$\frac{dy}{dx}=\left(-\frac{1}{2}\right)(2-x)e^{-\frac{1}{2}x}-e^{-\frac{1}{2}x}$ M1 requires at least one of derivatives correct
	Obtain correct derivative in any form	A1	
	Equate derivative to zero and solve for x	DM1	
	Obtain $x = 4$	A1	ISW
	Obtain $y = -2e^{-2}$, or exact equivalent	A1	
		5	

Question	Answer	Marks	Guidance
10(b)	Commence integration and reach $a(2-x)e^{-\frac{1}{2}x}+b\int e^{-\frac{1}{2}x}\,dx$	*M1	Condone omission of dx $-2(2-x)e^{-\frac{1}{2}x}+4e^{-\frac{1}{2}x}$ or $2xe^{-\frac{1}{2}x}$
	Obtain $-2(2-x)e^{-\frac{1}{2}x}-2\int e^{-\frac{1}{2}x}\,dx$	A1	OE
	Complete integration and obtain $2xe^{-\frac{1}{2}x}$	A1	OE
	Use correct limits, $x = 0$ and $x = 2$, correctly, having integrated twice	DM1	Ignore omission of zeros and allow max of 1 error
	Obtain answer $4e^{-1}$, or exact equivalent	A1	ISW
	Alternative method for question 10(b)		
	$\frac{d\left(2xe^{-\frac{1}{2}x}\right)}{dx}=2e^{-\frac{1}{2}x}-xe^{-\frac{1}{2}x}$	*M1 A1	
	$\therefore 2xe^{-\frac{1}{2}x}$	A1	
	Use correct limits, $x = 0$ and $x = 2$, correctly, having integrated twice	DM1	Ignore omission of zeros and allow max of 1 error
	Obtain answer $4e^{-1}$, or exact equivalent	A1	ISW
		5	

Question	Answer	Marks	Guidance
11(a)	Express general point of at least one line correctly in component form, i.e. $(1 + a\lambda, 2 + 2\lambda, 1 - \lambda)$ or $(2 + 2\mu, 1 - \mu, -1 + \mu)$	**B1**	
	Equate at least two pairs of corresponding components and solve for λ or for μ	**M1**	May be implied $1 + a\lambda = 2 + 2\mu$ $2 + 2\lambda = 1 - \mu$ $1 - \lambda = -1 + \mu$
	Obtain $\lambda = -3$ or $\mu = 5$	**A1**	
	Obtain $a = -\frac{11}{3}$	**A1**	Allow $a = -3.667$
	State that the point of intersection has position vector $12\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$	**A1**	Allow coordinate form (12, – 4, 4)
		5	

Question	Answer	Marks	Guidance
11(b)	Use correct process for finding the scalar product of direction vectors for the two lines	**M1**	$(a, 2, -1) \, . \, (2, -1, 1) = 2a - 2 - 1$ or $2a - 3$
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and equate the result to $\pm\frac{1}{6}$	***M1**	
	State a correct equation in a in any form, e.g. $\dfrac{2a-2-1}{\sqrt{6}\sqrt{(a^2+5)}} = \pm\dfrac{1}{6}$	**A1**	
	Solve for a	**DM1**	Solve 3-term quadratic for a having expanded $(2a - 3)^2$ to produce 3 terms e.g. $36(2a-3)^2 = 6(a^2+5)$ $138a^2 - 432a + 294 = 0$ $23a^2 - 72a + 49 = 0$ $(23a-49)(a-1) = 0$
	Obtain $a = 1$	**A1**	
	Obtain $a = \dfrac{49}{23}$	**A1**	Allow $a = 2.13$

Question	Answer	Marks	Guidance
11(b)	**Alternative method for question 11(b)**		
	$\cos(\theta) = [\lvert a^2 + 2^2 + (-1)^2\rvert^2 + \lvert 2^2 + (-1)^2 + 1^2\rvert^2$ $- \lvert(a-2)^2 + 3^2 + (-2)^2\rvert^2] / [2\lvert a^2 + 2^2 + (-1)^2\rvert.\lvert 2^2 + (-1)^2 + 1^2\rvert]$	**M1**	Use of cosine rule. Must be correct vectors.
	Equate the result to $\pm\frac{1}{6}$	***M1** **A1**	Allow M1* here for any two vectors
	Solve for a	**DM1**	Solve 3-term quadratic for a having expanded $(2a-3)^2$ to produce 3 terms e.g. $36(2a-3)^2 = 6(a^2+5)$ $138a^2 - 432a + 294 = 0$ $23a^2 - 72a + 49 = 0$ $(23a-49)(a-1) = 0$
	Obtain $a = 1$	**A1**	
	Obtain $a = \frac{49}{23}$	**A1**	Allow $a = 2.13$
		6	