# P3 Exercise Solution

updated: December 16, 2021

**Exercise** 1

$y = |2x + 1|$

$y = 2|x + 3| - 1$
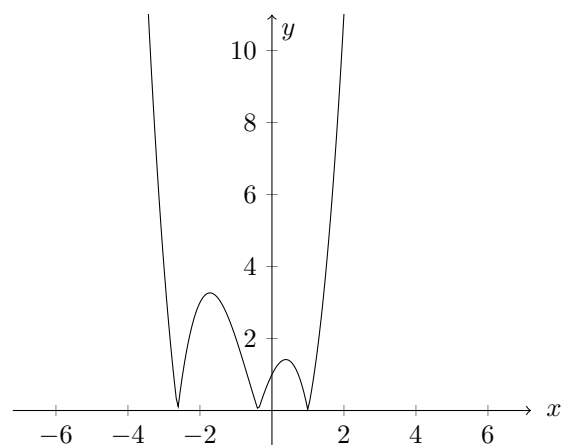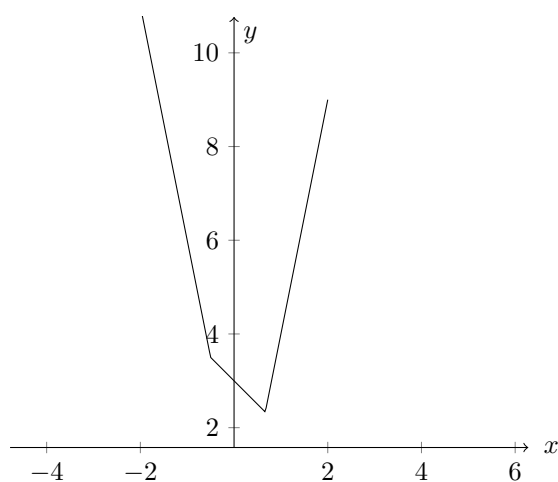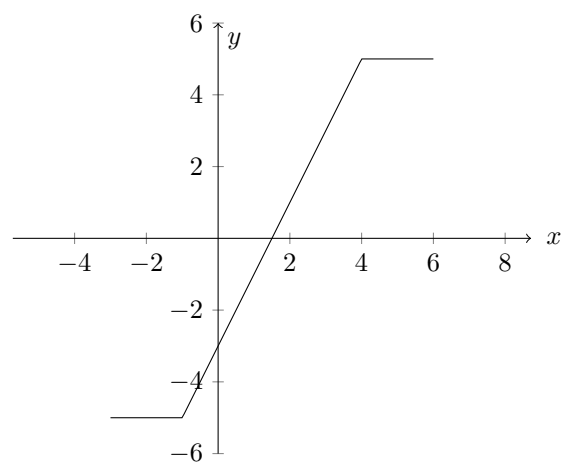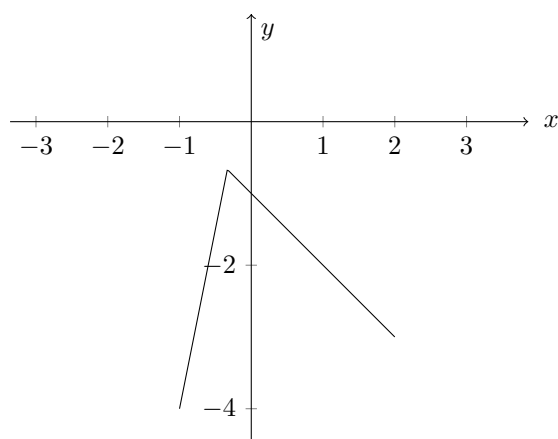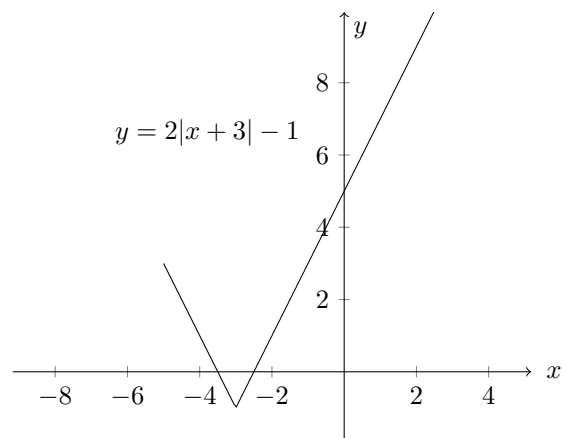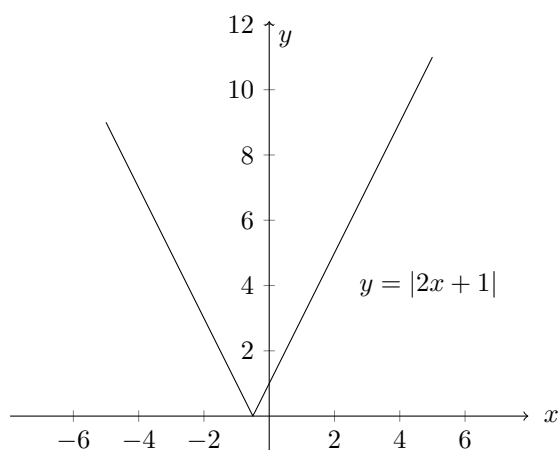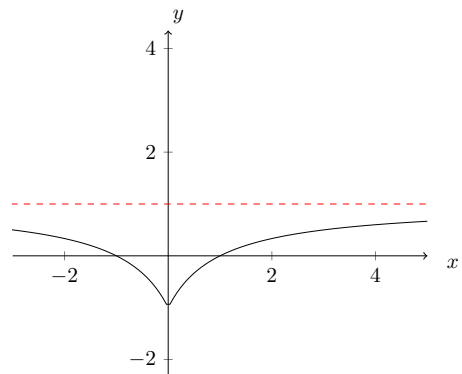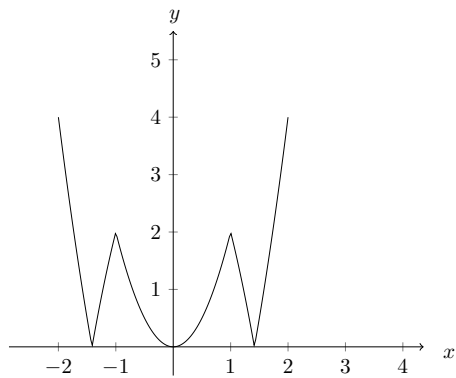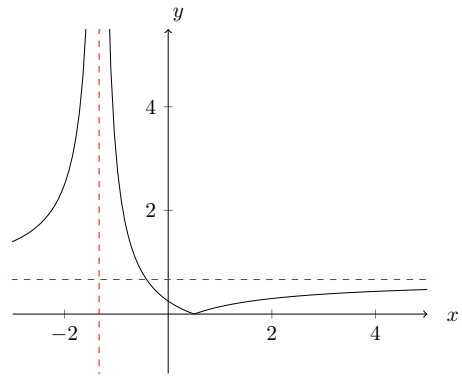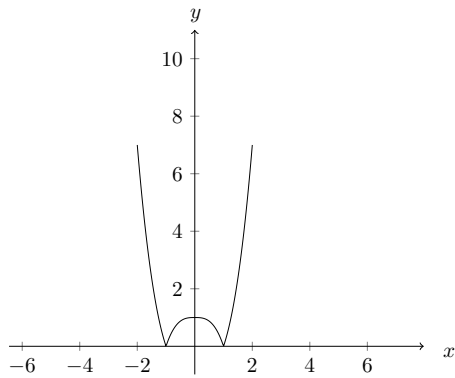
**Exercise** 2

1. 3 or $-2$

2. $\frac{2}{3}$ or $-5$

3. $\frac{3}{2}$

4. 2 or $-\frac{4}{3}$

5. 2

6. $-\frac{8}{7}$ or $\frac{12}{5}$

7. $-2$, $\frac{7}{3}$, 1, $-\frac{4}{3}$

8. $-3$, $-1$, 3,1

9. 1 or $-1$

10. 2 or $-2$

**Exercise** 3

1. $\left(-\infty, -\frac{1}{2}\right) \cup (1, \infty)$

2. $[-3, 1]$

3. $\left(-\frac{3}{2}, 4\right)$

4. $[1, \infty]$

5. $(-\infty, -2) \cup \left(\frac{1-\sqrt{17}}{2}, 1\right) \cup \left(\frac{1+\sqrt{17}}{2}, \infty\right)$

6. $\left(-\infty, \frac{5}{3}\right) \cup (5, \infty)$

7. $(-3, -1)$

8. $\left(-\infty, -\frac{3}{4}\right] \cup \left[\frac{7}{4}, \infty\right)$

9. $\left(-\infty, \frac{5-\sqrt{17}}{2}\right) \cup (2, 3) \cup \left(\frac{5+\sqrt{17}}{2}, \infty\right)$

10. $\left(-\infty, -\frac{7}{2}\right) \cup (-2, 0) \cup (0, 2) \cup \left(\frac{7}{2}, \infty\right)$

**Exercise** 4

1. Polynomial: $\frac{(x+1)(x+2)\cdot(x+n)}{n!}$, degree: $n$, leading term: $\frac{1}{n!}$, constant term 1.

2. $6x^4 + 3x^3 - 18x^2 - 9x - 2$

3. $A = 2$, $B = 3$, $C = -5$.

4. (a) $m$ or $n$

   (b) $m$ or $n$

   (c) $m + n$.

**Exercise** 5

1. quotient: $2x^2 - 3x + 4$, remainder: $-13$

2. quotient: $3x^2 + 4x + 1$, remainder: $9x - 7$

3. quotient: $x^4 - x^3 + x - 1$, remainder: $2$

4. quotient: $x^{n-1} + kx^{n-2} + k^2 x^{n-3} + \cdot + k^{n-1} x + k^n$, remainder: $k^{n+1}$

**Exercise** 6

1. (a) $302$

   (b) $-\frac{1}{3}$

   (c) $2\frac{3}{4}$

2. $a = -\frac{5}{3}$, $b = \frac{1}{3}$

3. $k = -9$, $(x - 2)(2x + 1)(3x + 1)$

4. $x > 1$

5. (a) $f(1) \neq 0$, and $f(-1) \neq 0$,

   (b) quotient: $x^3 + 3x^2 + 3x + 5$, remainder: $6x + 4$

   (c) omit

(d) $-1$, $-1+\sqrt{2}$, $-1-\sqrt{2}$.

## Exercise 7

1.   (a) $1 + 6x + 27x^2 + 108x^3$
   (b) $1 + \frac{2}{3}x - \frac{4}{9}x^2 + \frac{40}{81}x^3$
   (c) $2 - \frac{1}{4}x - \frac{1}{64} - \frac{1}{512}x^3$
   (d) $\frac{1}{27} - \frac{2}{27}x + \frac{8}{81}x^2 - \frac{80}{729}x^3$.

2.   (a) $4 + x - \frac{1}{16}x^2$
   (b) $2 - \frac{1}{32}x - \frac{131}{4096}x^2$
   (c) $\frac{1}{2} + \frac{1}{4}x^2$
   (d) $\frac{1}{2} - \frac{1}{16}x - \frac{117}{256}x^2$.

3. $a = 4$, $b = 12$, $c = -192$

4.   (a) $(-1)^n 2^n$
   (b) $\dfrac{(2n-1)!!}{2^n n!}$
   (c) $\dbinom{|k| + n - 1}{n}$.

5. By part $4(c)$, $x^r : \frac{1}{2}(r+1)(r+2)$ ; $x^{r-1} : -\frac{1}{2}r(r+1)$; $x^{r-2} : r(r-1)$.
   Add them together to get $r^2 + 1$.

   (a) 8
   (b) 12.

## Exercise 8

1. $-\frac{1}{x} - \frac{\frac{1}{2}}{x+1} + \frac{\frac{3}{2}}{x-1}$.

2. $x^2 - x + 3 - \frac{\frac{16}{3}}{x+2} + \frac{\frac{1}{3}}{x-1}$.

3. $\frac{\frac{1}{2}}{x+1} - \frac{4}{x+2} + \frac{\frac{9}{2}}{x+3}$.

## Exercise 9

1. $\frac{1}{x} - \frac{1}{x-1} + \frac{2}{(x-1)^2}$.

2. $\frac{-\frac{7}{4}}{x} + \frac{\frac{11}{4}}{x-1} + \frac{\frac{3}{2}}{(x-1)^2}$.

3. $x + \frac{\frac{16}{9}}{x-2} + \frac{\frac{11}{9}}{x+2} + \frac{-\frac{1}{3}}{(x+1)^2}$.

4. $\frac{\frac{3}{4}}{(x-1)^2} + \frac{\frac{1}{4}}{(x+1)^2}$.
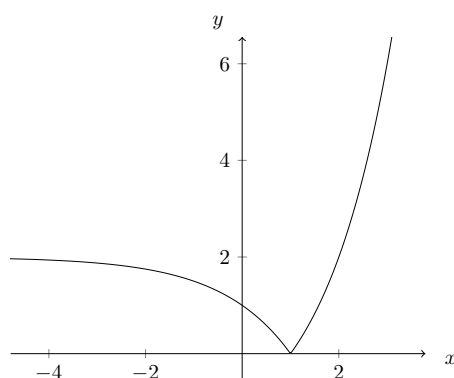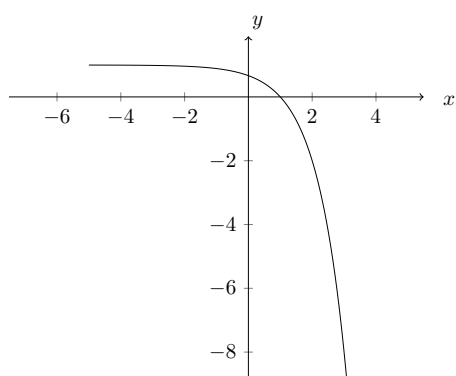
## Exercise 10

1. $\frac{1}{x} + \frac{-x+1}{x^2+1}$

2. $\frac{-1}{x+1} + \frac{1}{(x+1)^2} + \frac{x}{x^2+x+1}$

3. $x + 1 + \frac{2}{x-2} + \frac{-3x+1}{2x^2+2x+1}$.

4. $\frac{1}{x-1} + \frac{2}{x+2} + \frac{-2x-3}{x^2-2x+4}$.

5. $\frac{2x-3}{x^2-2x+2} + \frac{2x+3}{x^2+2x+2}$.

## Exercise 11

1. (a) $\frac{1}{x+1} - \frac{6}{x+2} + \frac{8}{x+4}$

   (b) $-\frac{1}{x+1} + \frac{1}{x+2} + \frac{2}{(x+2)^2}$.

   (c) $2 - \frac{2}{x} - \frac{3}{x^2} + \frac{4}{x-1}$

   (d) $\frac{1}{x} - \frac{2}{x-1} + \frac{3x-1}{x^2+1}$

   (e) $1 + \frac{\frac{1}{3}}{x-3} - \frac{\frac{1}{3}}{x+3} - \frac{2}{(x+3)^2}$.

2. (a) $\frac{1}{2} - \frac{3}{4}x + \frac{13}{24}x^2 - \frac{47}{144}x^3$.

   (b) $-1 - \frac{4}{3}x - x^2 - \frac{7}{9}x^3$

   (c) $\frac{5}{2} - \frac{3}{2}x + \frac{5}{4}x^2 - \frac{11}{4}x^3$.

3. $\frac{1}{x-5} - \frac{3}{x+5}$, $f'(x) = -(x-5)^{-2} + 3(x+5)^{-2}$.

4. $1 - \frac{1}{x-1} - \frac{2}{(x-1)^2} + \frac{2}{x-4}$, $f'(x) = (x-1)^{-2} + 4(x-1)^{-3} - 2(x-4)^{-2}$.

## Exercise 12

1. $x \in (-\infty, 0) \cup (0, \infty)$, $y \in (0, \infty)$

2. The graphs are as following:



3. $>, <, <, <, <$

4. $m < n$

5. $a > 1$, then $x < -6$

   $a < 1$, then $x > -6$

6. (a) $\sqrt{5}$

    (b) 7

    (c) $8\sqrt{5}$ or $-8\sqrt{5}$

7. no

8. 3 or $-\frac{1}{3}$

9. $f(0) = 1$, $f(\frac{1}{p} + \cdots + \frac{1}{p}) = \left[f(\frac{1}{p})\right]^p = f(1) = a$, then $f(\frac{1}{p}) = a^{\frac{1}{p}}$, which is $f(x) = a^x$.

**Exercise** 13

1. $3, -3, -\frac{7}{4}, \frac{37}{2}, \frac{5}{4}$

2. $\frac{121}{3}$

3. $\frac{5\ln 2 - \ln 3}{2\ln 3 - 3\ln 2}$.

4. $\frac{\ln 2}{\ln 3 + 2\ln 2} = 0.279$

5. $x = \frac{\ln 5}{\ln 15}$, $y = \frac{\ln 3}{\ln 15}$.

6. $\frac{5}{6} + \log_{10} \frac{3}{2}$

7. (a) $a^{\ln b} = e^{\ln a \ln b} = b^{\ln a}$, then $\left(a^{\ln b}\right)^{\frac{1}{\ln c}} = \left(b^{\ln a}\right)^{\frac{1}{\ln c}}$

    (b) omit

    (c) omit

8. 2

9. 25

**Exercise** 14

1. $<, >, >, >$

2. The graph are as following:
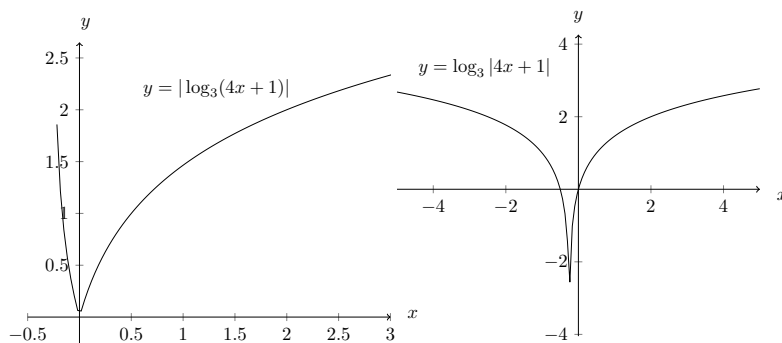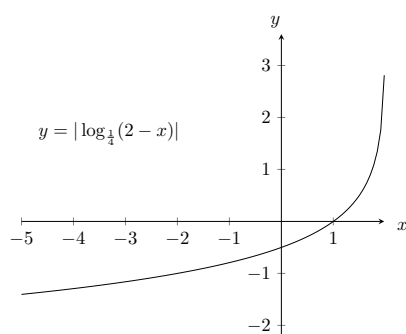


3. $m > n$

4. (a) $(-1, 1)$

    (b) even.

5. $y = a^x - 1$

6. $0 < a < 1$, $0 < x < \log_a(\sqrt{3} - 1)$, $a > 1$, $\log_a(\sqrt{3} - 1) < x < 0$

7. $k = 1$, $\mathbb{R}$

8. $m = n = 5$


## Exercise 15

1. (a) $0.323$

   (b) $130$

2. $a = \frac{3}{5 \lg 2}$, $b = \frac{5}{2} - \frac{3}{5 \lg 2}$

3. $A = 0.4$, $c = 1.01$, $x = 504$

4. (a) $5$

   (b) $1,1,9$

5. Consider $f(1) = 0$


## Exercise 16

1. The graph of $y = \sin x$ and $y = \cos x$:
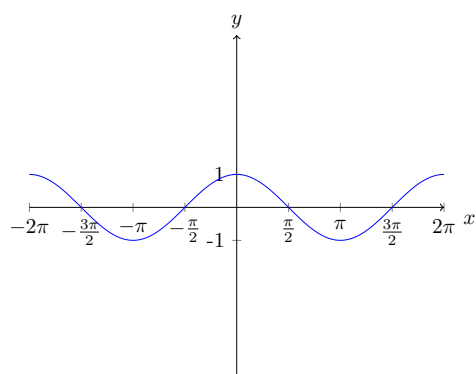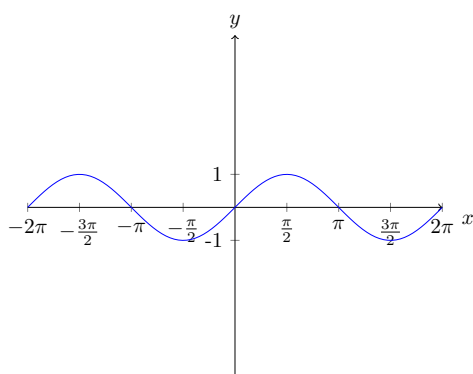


2. The graph of $y = \tan x$ and $y = \cot x$:
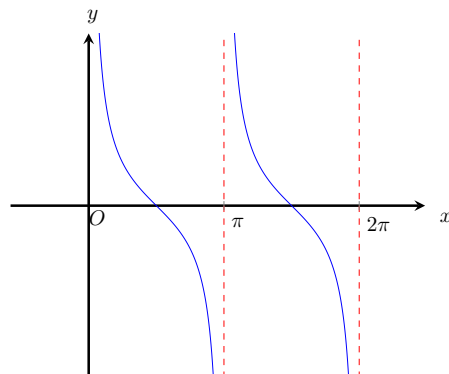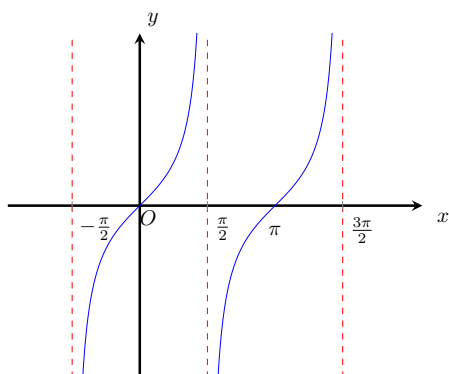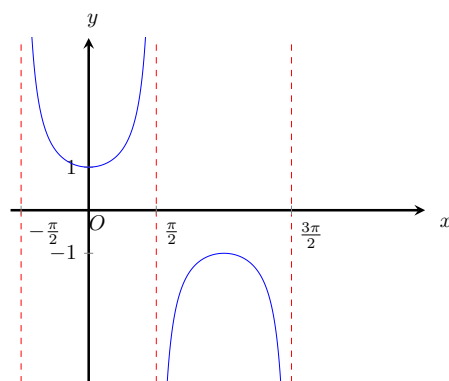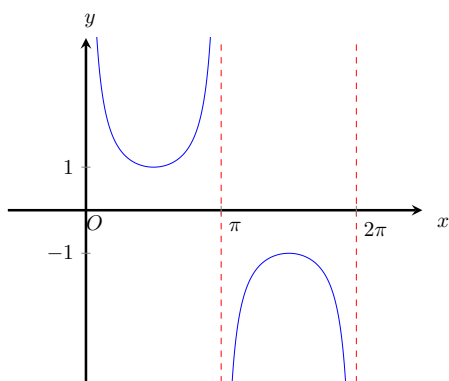
3. The graph of $y = \sec x$ and $y = \csc x$:

**Exercise** 17

1. (a) $0.5$
   (b) $\frac{131}{65}$
   (c) $\frac{457}{289}$
   (d) $\frac{3}{5}$

2. omit

3. (a) $73.9°, 286.1°$
   (b) $37.2°, 217.2°, 327.8°, 142.8°$
   (c) $30°, 210°, 150°, 330°$
   (d) $60°, 240°, 120°, 300°$

**Exercise** 18

1. $\frac{\sqrt{2}}{4}(\sqrt{3} - 1)$

2. $\frac{\sqrt{3}\tan x + 1}{\tan x - \sqrt{3}}$

3. $-\frac{56}{65}, \frac{63}{65}, -\frac{16}{63}$

4. $\pm\frac{\pi}{4}, \pm\frac{3\pi}{4}, \pm\frac{5\pi}{4}, \pm\frac{7\pi}{4}$

5. $0.12, 3.26, -3.02, -6.16$

6. $-\frac{59}{72}$

7. $-\frac{4}{3}, -\frac{3}{4}$

8. $0$

9. Use $\sin 2\xi = 2\sin\xi\cos\xi$, $\cos\xi = 1 - 2\sin^2\xi$.

**Exercise** 19

1. $\pm\frac{11}{6}, \pm\frac{5}{6}$

2. $-1 \pm \sqrt{2}, -1 + \sqrt{2}$

3. $\frac{\pi}{2}$, 5.44, 2.29.

4. $x = k\pi, 0.869 + k\pi, -0.869 + k\pi$.

5. $\sin^2 \frac{1}{2}A = \pm\frac{\sqrt{2}}{2}\sqrt{1 - \cos A}$, $\cos^2 \frac{1}{2}A = \pm\frac{\sqrt{2}}{2}\sqrt{1 + \cos A}$.

6. $\sin 3A = 3\sin A - 4\sin^3 a$, $\cos 3A = 4\cos^3 A - 3\cos A$, $\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$.

7. Omit.

8. Omit.

9. (a) $\tan\theta$
   (b) $\frac{1}{2}$
   (c) 1

10. $1, -1 + \sqrt{2}$

11. $\frac{7\sqrt{65}}{65}$.

**Exercise** 20

1. (a) $\sqrt{53}, 15.9°, 74.1°$
   (b) $\sqrt{41}, 51.3°$
   (c) $\sqrt{34}, 59.0°$

2. 5.57

3. $-2\sin\alpha$

4. $[\frac{2}{37}, \frac{2}{3}]$

5. $9, -17, 0.588, 0.983$

6. the graph is as follows: period $\frac{2\pi}{3}$



7. $\pi$

**Exercise** 21

1. (a) 1

   (b) $-\frac{3}{4}$

2. $k = -1$, $m = -1$
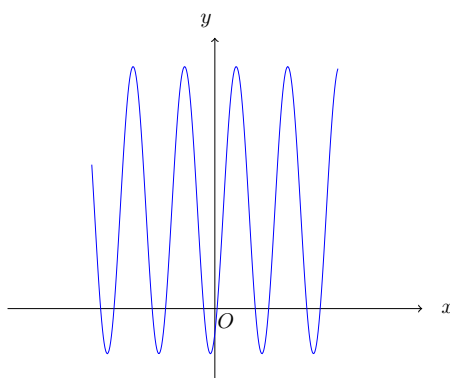
3. $a_n = \cos \frac{1}{2}x \cos \frac{1}{4}x \cdots \cos^2 \frac{1}{2^{n-1}}x$, $b_n = \cos \frac{1}{2}x \cos \frac{1}{4}x \cdots \cos \frac{1}{2^{n-1}}x$

4. omit

5. prove $\tan \frac{q+p}{2} = \tan \frac{r+s}{2}$

**Exercise 22**

1. (a) $3e^{3x+1}$

   (b) $\frac{1}{2} \cdot 2^{\sqrt{x}}(\ln 2) \cdot (x^{-\frac{1}{2}})$

   (c) $3(2 + e^{1-x})^{-2}e^{1-x}$

   (d) $\frac{1}{3}e^{3x-x^2}(3 - 2x) - \frac{2}{3}e^{1-x-x^2}(-1 - 2x)$.

   (e) omit

2. Maximum, $\left(\frac{\ln 2}{2}, 4\sqrt{2}\right)$

3. Minimum, $(-1, e^{-3})$

4. $x = 0$.

5. Omit.

**Exercise 23**

1. (a) $\frac{2x}{x^2+1}$

   (b) $\frac{2}{(2x-5)\ln 3}$

   (c) $\frac{3}{x} + \frac{8x}{x^2-1}$

   (d) $\frac{4}{4x+3} - \frac{3x^2}{x^3-1}$

   (e) $-\frac{1}{(x+1)(\ln(x+1))^2}$

   (f) $-\frac{2}{x(\ln x)^2}$.

2. $(1, \ln 2)$

3. $y = -x + 4$

4. $y - 2e = -2x$

5. $x = -3$. $x < -3$, then $\frac{dy}{dx} < 0$, $x > -3$, then $\frac{dy}{dx} > 0$.

6. (a) $(-\infty, \infty)$

   (b) $f^{-1}(x) = \frac{x+\sqrt{x^2+4}}{2}$.

   (c) $y - 1 = \frac{1}{2}(x - 0)$.

**Exercise 24**

1. (a) $2x\cos(x^2)$.
   (b) $-\frac{1}{2}\sin x(1+\cos x)^{-\frac{1}{2}}$.
   (c) $6\cos 2x e^{3\sin 2x}$.
   (d) $\tan x$.
   (e) $2e^{2x}\cos(e^{2x})$.
   (f) $-\sin 2x$.
   (g) $\sin 2x$.

2. $y - \ln\sqrt{2} = x - \frac{\pi}{4}$

3. $y + \frac{1}{3} = -\frac{2}{3\sqrt{3}}(x - \frac{\pi}{6})$

4. $x = \frac{\pi}{6}$, max; $x = \frac{5}{6}\pi$, min.

5. $\cos 2x(1 - \sin 2x)$.

6. omit, $-(\csc x - \sin x)^{-2}(-\csc x \cot x - \cos x)$.

7. Omit.

8. (a) $\frac{1}{2}\cos\frac{1}{2}x + \frac{1}{3}\sin\frac{1}{3}x$.
   (b) $-1$ and $\frac{1}{2}$
   (c) $12\pi$
   (d) $12\pi$

9. $\frac{1}{\sqrt{1-x^2}}$, $-\frac{1}{\sqrt{1-x^2}}$

10. (a) $-\frac{1}{2}x^{-\frac{1}{2}}\sin\sqrt{x}$
    (b) $-\frac{1}{2}\sin x(\cos x)^{-\frac{1}{2}}$
    (c) $2x\cos x^2$
    (d) $(-x^2)\cos\frac{1}{x}$.

**Exercise** 25

1. (a) $\sec^2(x)$.
   (b) $-\csc^2 x$
   (c) $x + 2x\ln x$.
   (d) $2x\sec^2(x^2 + 1)$
   (e) $\dfrac{2xe^{2x} - (e^{2x} + 1)}{x^2}$.
   (f) $(2x)\sin x^2 + (x^2 - 2)(\cos x^2)(2x)$

2. Max $(-3, 6e^{-3})$, Min $(1, -2e)$

3. omit

4. $(e, \frac{1}{3})$

5. $(1, \frac{1}{2})$

6. (a) $x = \frac{\pi}{4}$
   (b) max

11

7. $u_1' u_2 u_3 \cdots u_k + u_1 u_2' \cdots u_k + \cdots + u_1 u_2 \cdots u_k'$

8. $\displaystyle\sum_{k=0}^{n} \binom{n}{k} u^{(n-k)} v^{(k)}$

## Exercise 26

Omit

## Exercise 27

1. $y - 4 = 3(x + 8)$

2. $x = a\cos t,\ y = a\sin t.\ \ AB = a.$

3. $\frac{dy}{dx} = \frac{e^{-t}}{1 - e^{-t}}.\ \ t = \ln 3 - \ln 2.$

4. $t = 3,\ \left(\frac{4}{3}, \frac{27}{e^3}\right).$

5. Omit.

6. (a) $\frac{5}{3}$

    (b) $-\frac{32}{27}.$

7. Omit.

## Exercise 28

1. $-\frac{1}{\pi + 2}$

2. $y - 2 = \frac{1}{3}(x - 1)$

3. Horizontal: $y = \pm 3\sqrt{3}$, vertical: $x = \pm 3$

4. $(-1, -2)$, max; $(0, 0)$, min.

5. $\left(\frac{8}{3}, -4\right)$

6. $-1$, 4.

7. Omit

8. $\left(2e, e^{\frac{1}{2}}\right).\ \ \ln y \neq 0.$

9. $(1, -4)$,min; $(-1, 4)$,max.

10. Omit.

## Exercise 29

1. (a) $\frac{2}{3}\ln|3x - 1| + C$

    (b) $\frac{1}{2}e^{2x+3} + C$

    (c) $\frac{\sqrt{3}}{3}\tan^{-1}\left(\frac{2}{\sqrt{3}}x\right) + C$

(d) $-\frac{1}{2}\cos 2x + C$

(e) $\frac{1}{2}x + \frac{1}{4}\sin 2x + C$

2. $x^4 - 3x^2 + x + 7$

## Exercise 30

Omit

## Exercise 31

1. $8\frac{5}{8} + 3\ln 2$

2. $\frac{1}{2} - \frac{1}{2e}$

3. $2$

4. $\frac{\pi}{8} - \frac{1}{12}$

5. $\frac{\pi}{8}$

6. $0$

## Exercise 32

1. $1 - \frac{\pi}{4}$

2. $2\frac{11}{12} - \ln 6$

3. Omit

4. (a) $4e^{\frac{x}{2}} - 2x - 1$

   (b) min.

5. Omit

6. Omit

7. Omit

8. Omit

9. Omit

## Exercise 33

1. If $p = 1$, infinite;

   if $p > 1$, $\int_0^1 \frac{1}{x^p}\,dx$ infinite, $\int_1^\infty \frac{1}{x^p}\,dx = \frac{1}{1-p}$;

   if $p < 1$, $\int_0^1 \frac{1}{x^p}\,dx = \frac{1}{1-p}$, $\int_1^\infty \frac{1}{x^p}\,dx$ infinite.

2. $\frac{1}{2}e$

3. Infinite

4. Omit

## Exercise 34

1. (a) $\frac{287}{192}\pi + \pi \ln 4$

   (b) $\frac{\pi}{4}(e^2 - e^{-2})$

   (c) $\frac{\pi^2}{2}$

   (d) $\pi(\sqrt{3} - 1 - \frac{\pi}{12})$.

2. $\frac{1}{2}$

3. $2\pi^2 r a^2$

4. (a) $\pi(\sqrt{3} - \frac{\pi}{3})$

   (b) Omit.

## Exercise 35

1. (a) $\int 1 + \dfrac{-2}{x-2} + \dfrac{-3}{x+3}\,\mathrm{d}x = x - 2\ln|x-2| - 3\ln|x+3| + C$.

   (b) $\int \dfrac{\frac{5}{3}}{x+4} + \dfrac{-\frac{3}{4}}{x+2} + \dfrac{\frac{1}{12}}{x-2}\,\mathrm{d}x = \dfrac{5}{3}\ln|x+4| - \dfrac{3}{4}\ln|x+2| + \dfrac{1}{12}\ln|x-2| + C$.

   (c) $\int \dfrac{1}{4}\dfrac{1}{1-x} + \dfrac{1}{4}\dfrac{1}{1+x} + \dfrac{1}{2}\dfrac{1}{1+x^2}\,\mathrm{d}x = -\dfrac{1}{4}\ln|1-x| + \dfrac{1}{4}\ln|1+x| + \dfrac{1}{2}\tan^{-1}x + C$.

   (d) $\int_{-\frac{1}{2}}^{1} \dfrac{1}{2}\dfrac{1}{x+1} - \dfrac{1}{x+2} + \dfrac{1}{2}\dfrac{1}{x+3}\,\mathrm{d}x = \dfrac{1}{2}(\ln 8 - \ln 5)$.

   (e) $\frac{3}{2}\ln 3 - \ln 4$.

2. $10\frac{5}{12}\pi$; $20.8333\pi$

## Exercise 36

1. $\mathrm{d}t = 2x\,\mathrm{d}x$

2. $\mathrm{d}x = t\,\mathrm{d}t$

3. $\mathrm{d}t = \cos x\,\mathrm{d}x$

4. $\mathrm{d}x = e^t\,\mathrm{d}t$

5. $\mathrm{d}x = \frac{1}{t+1}\,\mathrm{d}t$

6. $\mathrm{d}t = \frac{1}{2}\sec^2 \frac{x}{2}\,\mathrm{d}x$

7. $\mathrm{d}t = \frac{2x}{1+x^4}\,\mathrm{d}x$

8. $\mathrm{d}t = -\tan x\,\mathrm{d}x$

**Exercise 37**

1. $e^{\sin x} + C$

2. $-\frac{1}{6}(3 - 2x^2)^{\frac{3}{2}} + C$

3. $\frac{1}{3}\ln|x^3 - 1| + C$

4. $\frac{\pi}{8}$

5. $\frac{7}{24}$

6. $\frac{3}{14}(x^2 - 1)^7 + \frac{1}{4}(x^2 - 1)^6 + C$

7. $x + 4\sqrt{x} + 4\ln|\sqrt{x} - 1| + C$

8. $\frac{27}{140}$

9. $\frac{1696}{2835}$

10. $\frac{3}{8}$

11. $\frac{2}{3}\pi$

12. $\pi$

13. $2x^{\frac{1}{2}} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\ln|1 + x^{\frac{1}{6}}| + C$

14. $\ln(\sec x + \tan x) + C$

15. $\frac{1}{3}\ln|1 + x| - \frac{1}{3}\left(\frac{1}{2}\ln|x^2 - x + 1| - \sqrt{3}\tan^{-1}\left(\frac{2}{\sqrt{3}}(x - \frac{1}{2})\right)\right) + C$


**Exercise 38**

1. $\frac{1}{3}xe^{3x} - \frac{1}{9}e^x + C$

2. $\frac{1}{5}x^5\ln 3x - \frac{1}{25}x^5 + C$

3. $\frac{-x\ln x - 1}{x} + C$

4. $\frac{\sqrt{2}}{2}(4 - \pi)$

5. $\frac{\pi^2}{4}$

6. $-\frac{1}{2}x^2\cos 2x + \frac{x}{2}\sin 2x + \frac{1}{4}\cos 2x + C$

7. $\frac{4}{25}e^{3x}\sin 4x + \frac{3}{25}e^{3x}\cos 4x + C$

8. $\frac{3}{10}(e^{-\pi} - e^{\pi})$

9. $\frac{\pi}{4} - \frac{1}{2}\ln 2$

10. $\frac{1}{3}x^3\ln^2 x - \frac{2}{9}x^3\ln x + \frac{2}{27}x^3 + C$

11. $\frac{1}{2}e^{x^2}(x^2 - 1) + C$

12. $2e^{\sin x}(\sin x - 1) + C$

13. $4\left(\sqrt{2} - 1\right)$


**Exercise 39**

1. $-\frac{\sqrt{1+x^2}}{x} + C$

2. $\frac{1}{11}\tan^{11} x + C$

3. $-2\cos\sqrt{x} + C$

4. $\frac{1}{2}\ln\left|\frac{e^x-1}{e^x+1}\right| + C$

5. $\frac{3}{2}(\sin x - \cos x)^{\frac{3}{2}} + C$

6. $-(\arcsin x)^{-1} + C$

7. $-\ln\left|\cos\sqrt{1+x^2}\right| + C$

8. $\frac{1}{2}\left(\ln\left|\sqrt{1+e^{2x}}-1\right| - \ln\left|\sqrt{1+e^{2x}}+1\right|\right) + C$

9. $-\frac{1}{3}\left(\frac{x}{\sqrt{1+x^2}}\right)^{-3} + \left(\frac{x}{\sqrt{1+x^2}}\right)^{-1} + C$

10. $-\ln\left|1+\sqrt{\frac{x-1}{x+1}}\right| + \ln\left|1-\sqrt{\frac{x-1}{x+1}}\right| - \left(1+\sqrt{\frac{x-1}{x+1}}\right)^{-1} + \left(1-\sqrt{\frac{x-1}{x+1}}\right)^{-1} + C$

11. $\frac{x}{\sqrt{1-x^2}} + C$

12. $\frac{1}{2}\arcsin x + \frac{1}{2}\ln\left|x+\sqrt{1-x^2}\right| + C$

13. $\sqrt{2x} - \ln\left|1+\sqrt{2x}\right| + C$

14. $\operatorname{arcsec} x + C$

15. $(\arctan\sqrt{x})^2 + C$

16. $\frac{1}{4}\ln\left|\frac{\sqrt[4]{1+x^4}-1}{\sqrt[4]{1+x^4}+1}\right| + \frac{1}{2}\arctan\sqrt[4]{1+x^4} + C$

17. $-(x\ln x)^{-1} + C$

18. $2\left(\sqrt{x+1}e^{\sqrt{x+1}} - e^{\sqrt{x+1}}\right) + C$

19. $x(-\cot x) + \ln|\sin x| + C$

20. $x\arcsin x + \sqrt{1-x^2} + C$

21. $\arctan\left(\frac{1}{3}x^3\right) - \frac{1}{3}\left(\frac{1}{2}x^2 - \frac{1}{2}\ln\left|1+x^2\right| + C\right)$

22. $x\ln^2 x - 2x\ln x + 2x + C$

23. $x\tan x + \ln|\cos x| - \frac{x^2}{2} + C$

24. $-x^{-1}\ln^3 x - 3x^{-1}\ln^2 x - 6x^{-1}\ln x - 6x^{-1} + C$

25. $\frac{1}{2}\left(\sin(\ln x) + \cos(\ln x)\right) + C$

26. $x(\arcsin x)^2 + 2(\arcsin x)(1-x^2)^{\frac{1}{2}} + 2x + C$

27. $e^{\sqrt{x}}(2x - 4\sqrt{x} + 4) + C$

28. $x\ln\left(x+\sqrt{1+x^2}\right) - (1+x^2)^{\frac{1}{2}} + C$

29. *Omit*

30. *Omit*

31. *Omit*

32. *Omit*

33. *Omit*

34. *Omit*

35. *Omit*

36. *Omit*

37. *Omit*

38. *Omit*


**Exercise** 40

1. *Omit*

2. *Omit*

3. *Omit*

4. *Omit*

5. *Omit*

6. *Omit*

7. *Omit*

8. *Omit*

9. *Omit*

10. *Omit*

11. *Omit*

12. *Omit*

13. *Omit*


**Exercise** 41

1. Omit

2. $-2, 1, 16$

3. 1.466

4. 7.725


**Exercise** 42

1. 1.17

2. 1.37

3. 2.38

4. 0.739

5. 0.79

**Exercise 43**

1. (a) Omit

   (b) $3x^2 > 1$ for $x \in (1, 2)$

   (c) $|\frac{1}{3}(x+1)^{-\frac{2}{3}}| < 1$ for $x \in (1, 2)$

2. (a) 3

   (b) 3.25

   (c) $x = -\sqrt{e^x - 5x + 1}$, $-5.19$; $x = \frac{e^x - x^2 + 1}{5}$, 0.48.

**Exercise 44**

1. 6.2417.

2. 1.95.

3. 0.7828, 0.7842, 0.7847. 0.7854

4. 7.500, 7.306.

**Exercise 45**

   Omit

**Exercise 46**

1. (a) $y = \frac{1}{2}x^2 + x + \ln|x - 1| + C$

   (b) $x = -\frac{\ln y}{y} - \frac{1}{y} + C$

   (c) $\frac{1}{2}y^2 = \frac{1}{3}x^3 + C$

   (d) $u = \frac{C}{\cos t}$

   (e) $\ln(1 + y^2) = xe^x - e^x + C$

   (f) $e^y = x \ln x + C$

   (g) $\ln|1 - w| - \frac{1}{w-1} = -t \cos t + \sin t + C$

2. $\ln y = \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + \frac{1}{4}$

3. $y = \frac{1}{2}(1 + x^2)$

4. $y = (\sqrt{x} + 1)^2$

5. (a) $\dfrac{\mathrm{d}x}{\mathrm{d}t} = 0.3\sqrt{x} - 0.02x$

(b) $x = \left( \dfrac{0.3 - e^{-100(t+C)}}{0.02} \right)^2$

(c) $x \longrightarrow 225$

6.  (a) $y = x - 1$

(b) $y = e^{-\int p(x)\,\mathrm{d}x} \left( \displaystyle\int e^{\int p(x)\,\mathrm{d}x} q(x)\,\mathrm{d}x \right)$

## Exercise 47
Omit

## Exercise 48
Omit

## Exercise 49

1. Opposite

2. $\overrightarrow{FO}, \overrightarrow{OC}, \overrightarrow{ED}$

## Exercise 50

1. $\begin{pmatrix} 1 \\ a \end{pmatrix}, \begin{pmatrix} -2 \\ -2 \end{pmatrix}.$

2. $E(1,1)$, $F(4,-3)$

3. $\vec{x} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

4. $\overrightarrow{GY} = \begin{pmatrix} -6 \\ -3 \end{pmatrix}$

## Exercise 51

1. $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$

2. $\begin{pmatrix} -4 \\ -4 \end{pmatrix}$

3.  (a) $\overrightarrow{DB}$
    (b) $\vec{e}$
    (c) $\overrightarrow{BA}$
    (d) $\overrightarrow{DA}$
    (e) $\vec{b}$

**Exercise** 52

 Omit

**Exercise** 53

1. Omit

2. $\vec{r} = \begin{pmatrix} \frac{5}{\sqrt{61}} \\ \frac{6}{\sqrt{61}} \end{pmatrix}$

3. $\overrightarrow{AB} = 2\mathbf{i} + \mathbf{j} + \mathbf{k}$, $\frac{1}{2}\overrightarrow{AC} = \mathbf{i} + \mathbf{j} - \mathbf{k}$, $-\overrightarrow{BC} = 6\mathbf{i} - \mathbf{j} - \mathbf{k}$

4. $E(10, 2, -3)$

5. $\overrightarrow{AB} = \overrightarrow{DC} = \begin{pmatrix} 1 \\ 1 \\ -6 \end{pmatrix}$, parallelogram

6. 0 or $-\frac{1}{3}$

7. Omit

8. $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$, distance between two points.

**Exercise** 54

 Omit

**Exercise** 55

1. 14, 1

2. Omit

3. NO.

4. Omit

5. NO.

**Exercise** 56

1. $\vec{a}$ and $\vec{d}$; $\vec{b}$ and $\vec{c}$

2. Omit

3. 98.1°, 151.9°, 166.0°, 122.5°, 27.2°, 161.9°, invalid, invalid.

4. 24.3°, 8.12

5. 45°

6. $\arccos\left(\frac{1}{3}\right)$

7. $\frac{3}{5}\mathbf{i} - \frac{4}{5}\mathbf{j}$ or $-\frac{3}{5}\mathbf{i} + \frac{4}{5}\mathbf{j}$

8. $x = -\frac{1}{2}$, $y = \pm 1$.

9. Omit

10. Omit

## Exercise 57
Omit

## Exercise 58
Omit

## Exercise 59

1. (a) Skew

   (b) intersect $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

   (c) parallel

   (d) the same line

   (e) Skew

2. (a) $\cos POQ = \dfrac{-3p - q}{\sqrt{14}\sqrt{p^2 + q^2 + 9}}$

   (b) $p = -\frac{4}{5}$, $q = \frac{12}{5}$

   (c) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 1 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

   (d) $\begin{pmatrix} -5 \\ -1 \\ -3 \end{pmatrix}$, $2\sqrt{2}$

3. (a) $79.9°$

   (b) $\frac{1}{\sqrt{29}} \begin{pmatrix} 2 \\ -4 \\ -3 \end{pmatrix}$

   (c) 7 or 1

   (d) Skew

4. (a) Omit

   (b) $\begin{pmatrix} -\frac{2}{3} \\ \frac{11}{3} \\ \frac{1}{3} \end{pmatrix}$

5. (a) 4.09

   (b) 1.90

   (c) 3.03