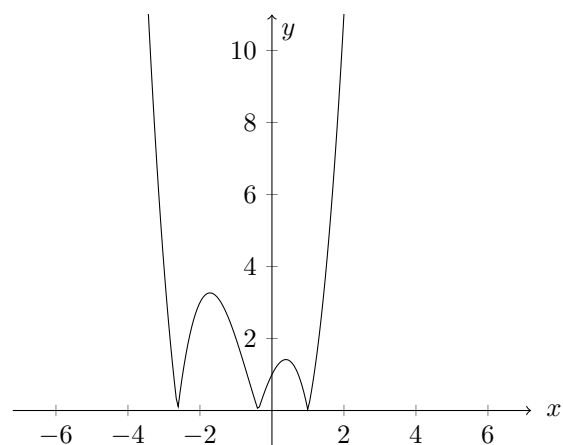
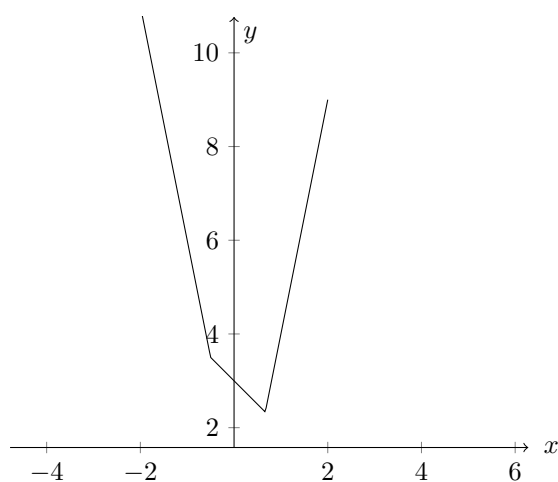
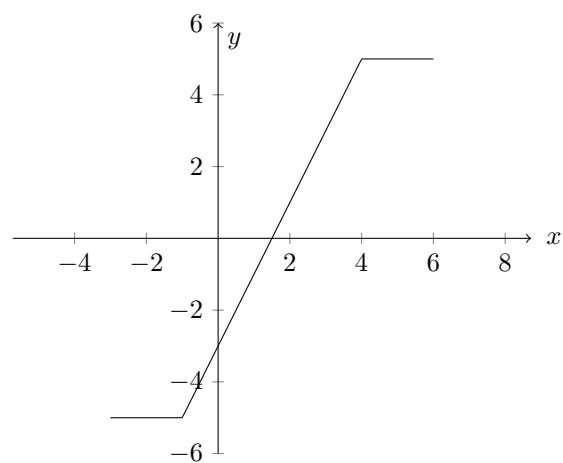
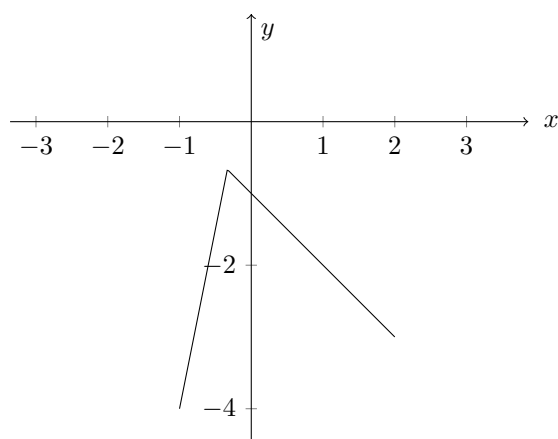
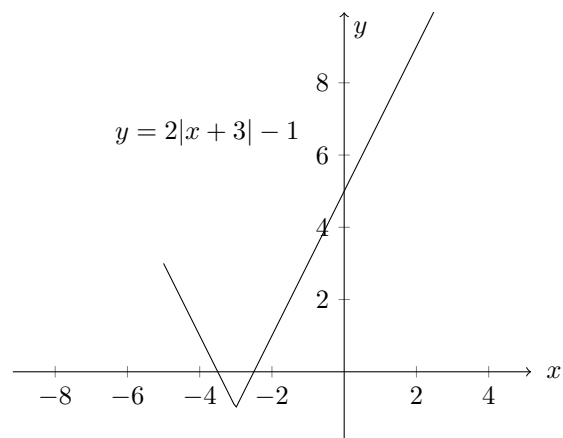
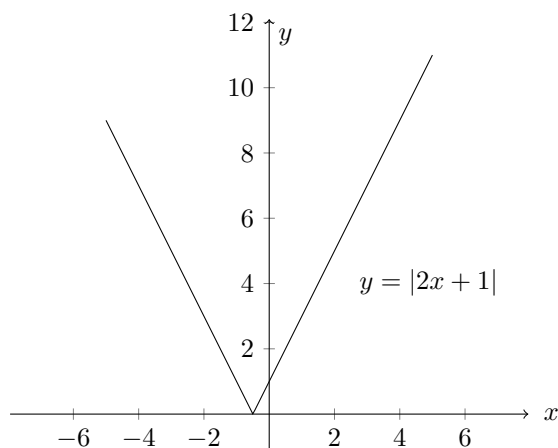
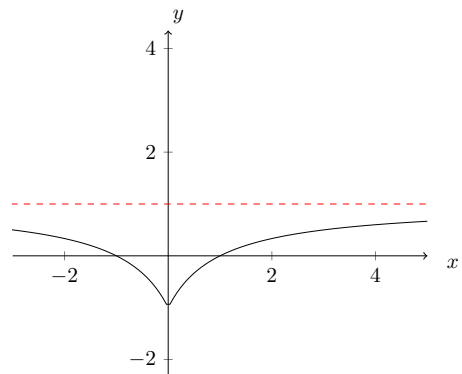
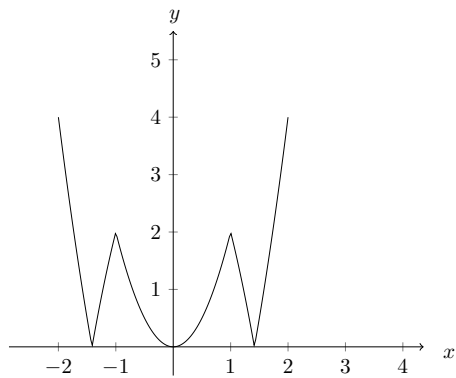
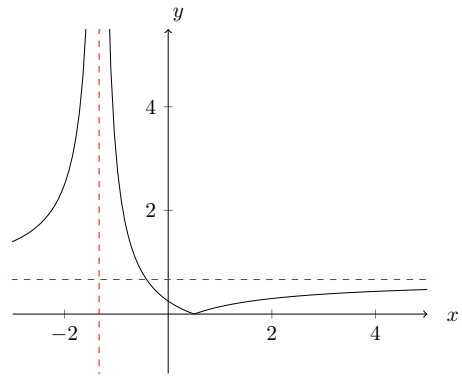
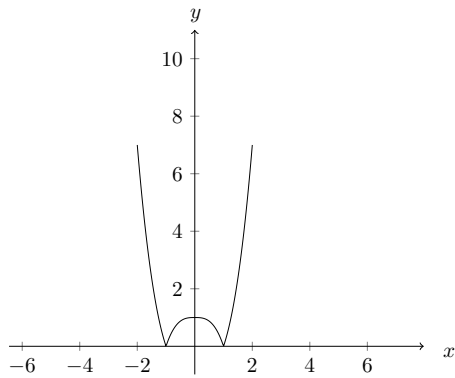


# P3 Exercise Solution

updated: November 30, 2021

## Exercise 1





## Exercise 2

1. 3 or  $-2$
2.  $\frac{2}{3}$  or  $-5$
3.  $\frac{3}{2}$
4. 2 or  $-\frac{4}{3}$
5. 2
6.  $-\frac{8}{7}$  or  $\frac{12}{5}$
7.  $-2, \frac{7}{3}, 1, -\frac{4}{3}$
8.  $-3, -1, 3, 1$
9. 1 or  $-1$
10. 2 or  $-2$

## Exercise 3

1.  $(-\infty, -\frac{1}{2}) \cup (1, \infty)$
2.  $[-3, 1]$

3.  $(-\frac{3}{2}, 4)$
4.  $[1, \infty]$
5.  $(-\infty, -2) \cup (\frac{1-\sqrt{17}}{2}, 1) \cup (\frac{1+\sqrt{17}}{2}, \infty)$
6.  $(-\infty, \frac{5}{3}) \cup (5, \infty)$
7.  $(-3, -1)$
8.  $(-\infty, -\frac{3}{4}] \cup [\frac{7}{4}, \infty)$
9.  $(-\infty, \frac{5-\sqrt{17}}{2}) \cup (2, 3) \cup (\frac{5+\sqrt{17}}{2}, \infty)$
10.  $(-\infty, -\frac{7}{2}) \cup (-2, 0) \cup (0, 2) \cup (\frac{7}{2}, \infty)$

#### Exercise 4

1. Polynomial:  $\frac{(x+1)(x+2) \cdot (x+n)}{n!}$ , degree:  $n$ , leading term:  $\frac{1}{n!}$ , constant term 1.
2.  $6x^4 + 3x^3 - 18x^2 - 9x - 2$
3.  $A = 2, B = 3, C = -5$ .
4. (a)  $m$  or  $n$   
 (b)  $m$  or  $n$   
 (c)  $m + n$ .

#### Exercise 5

1. quotient:  $2x^2 - 3x + 4$ , remainder:  $-13$
2. quotient:  $3x^2 + 4x + 1$ , remainder:  $9x - 7$
3. quotient:  $x^4 - x^3 + x - 1$ , remainder:  $2$
4. quotient:  $x^{n-1} + kx^{n-2} + k^2x^{n-3} + \dots + k^{n-1}x + k^n$ , remainder:  $k^{n+1}$

#### Exercise 6

1. (a) 302  
 (b)  $-\frac{1}{3}$   
 (c)  $2\frac{3}{4}$
2.  $a = -\frac{5}{3}, b = \frac{1}{3}$
3.  $k = -9, (x-2)(2x+1)(3x+1)$
4.  $x > 1$
5. (a)  $f(1) \neq 0$ , and  $f(-1) \neq 0$ ,  
 (b) quotient:  $x^3 + 3x^2 + 3x + 5$ , remainder:  $6x + 4$   
 (c) omit

(d)  $-1, -1 + \sqrt{2}, -1 - \sqrt{2}.$

### Exercise 7

1. (a)  $1 + 6x + 27x^2 + 108x^3$   
 (b)  $1 + \frac{2}{3}x - \frac{4}{9}x^2 + \frac{40}{81}x^3$   
 (c)  $2 - \frac{1}{4}x - \frac{1}{64} - \frac{1}{512}x^3$   
 (d)  $\frac{1}{27} - \frac{2}{27}x + \frac{8}{81}x^2 - \frac{80}{729}x^3.$
2. (a)  $4 + x - \frac{1}{16}x^2$   
 (b)  $2 - \frac{1}{32}x - \frac{131}{4096}x^2$   
 (c)  $\frac{1}{2} + \frac{1}{4}x^2$   
 (d)  $\frac{1}{2} - \frac{1}{16}x - \frac{117}{256}x^2.$
3.  $a = 4, b = 12, c = -192$
4. (a)  $(-1)^n 2^n$   
 (b)  $\frac{(2n-1)!!}{2^n n!}$   
 (c)  $\binom{|k| + n - 1}{n}.$
5. By part 4(c),  $x^r : \frac{1}{2}(r+1)(r+2) ; x^{r-1} : -\frac{1}{2}r(r+1); x^{r-2} : r(r-1).$   
 Add them together to get  $r^2 + 1.$   
 (a) 8  
 (b) 12.

### Exercise 8

1.  $-\frac{1}{x} - \frac{\frac{1}{2}}{x+1} + \frac{\frac{3}{2}}{x-1}.$
2.  $x^2 - x + 3 - \frac{\frac{16}{3}}{x+2} + \frac{\frac{1}{3}}{x-1}.$
3.  $\frac{\frac{1}{2}}{x+1} - \frac{4}{x+2} + \frac{\frac{9}{2}}{x+3}.$

### Exercise 9

1.  $\frac{1}{x} - \frac{1}{x-1} + \frac{2}{(x-1)^2}.$
2.  $\frac{-\frac{7}{4}}{x} + \frac{\frac{11}{4}}{x-1} + \frac{\frac{3}{2}}{(x-1)^2}.$
3.  $x + \frac{\frac{16}{9}}{x-2} + \frac{\frac{11}{9}}{x+2} + \frac{-\frac{1}{3}}{(x+1)^2}.$
4.  $\frac{\frac{3}{4}}{(x-1)^2} + \frac{\frac{1}{4}}{(x+1)^2}.$

### Exercise 10

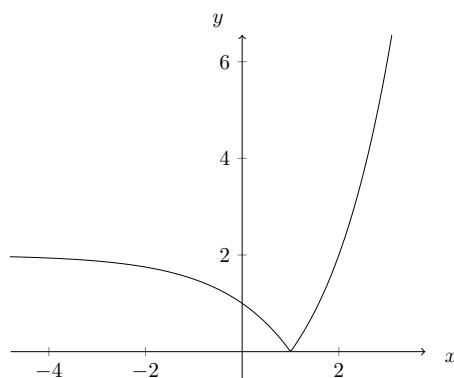
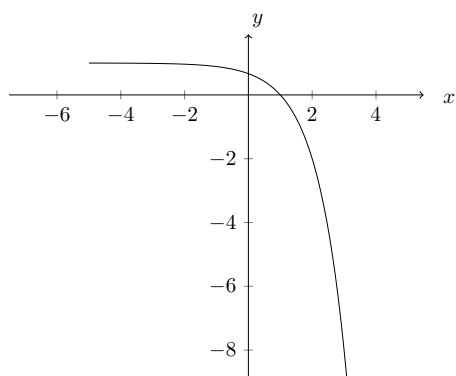
1.  $\frac{1}{x} + \frac{-x+1}{x^2+1}$
2.  $\frac{-1}{x+1} + \frac{1}{(x+1)^2} + \frac{x}{x^2+x+1}$
3.  $x+1 + \frac{2}{x-2} + \frac{-3x+1}{2x^2+2x+1}$ .
4.  $\frac{1}{x-1} + \frac{2}{x+2} + \frac{-2x-3}{x^2-2x+4}$ .
5.  $\frac{2x-3}{x^2-2x+2} + \frac{2x+3}{x^2+2x+2}$ .

### Exercise 11

1. (a)  $\frac{1}{x+1} - \frac{6}{x+2} + \frac{8}{x+4}$   
 (b)  $-\frac{1}{x+1} + \frac{1}{x+2} + \frac{2}{(x+2)^2}$ .  
 (c)  $2 - \frac{2}{x} - \frac{3}{x^2} + \frac{4}{x-1}$   
 (d)  $\frac{1}{x} - \frac{2}{x-1} + \frac{3x-1}{x^2+1}$   
 (e)  $1 + \frac{\frac{1}{3}}{x-3} - \frac{\frac{1}{3}}{x+3} - \frac{2}{(x+3)^2}$ .
2. (a)  $\frac{1}{2} - \frac{3}{4}x + \frac{13}{24}x^2 - \frac{47}{144}x^3$ .  
 (b)  $-1 - \frac{4}{3}x - x^2 - \frac{7}{9}x^3$   
 (c)  $\frac{5}{2} - \frac{3}{2}x + \frac{5}{4}x^2 - \frac{11}{4}x^3$ .
3.  $\frac{1}{x-5} - \frac{3}{x+5}$ ,  $f'(x) = -(x-5)^{-2} + 3(x+5)^{-2}$ .
4.  $1 - \frac{1}{x-1} - \frac{2}{(x-1)^2} + \frac{2}{x-4}$ ,  $f'(x) = (x-1)^{-2} + 4(x-1)^{-3} - 2(x-4)^{-2}$ .

### Exercise 12

1.  $x \in (-\infty, 0) \cup (0, \infty)$ ,  $y \in (0, \infty)$
2. The graphs are as following:



3.  $>, <, <, <, <$
4.  $m < n$
5.  $a > 1$ , then  $x < -6$   
 $a < 1$ , then  $x > -6$

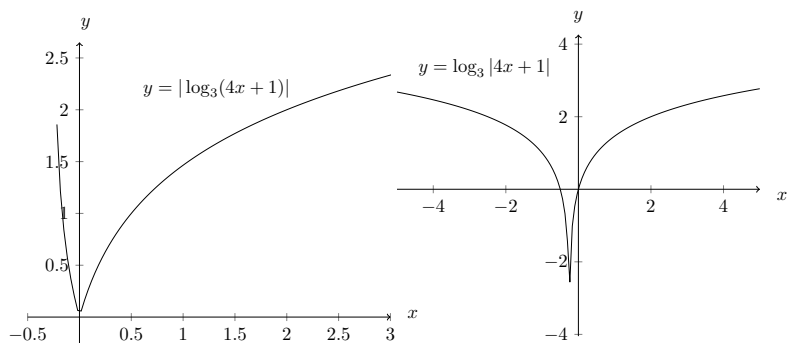
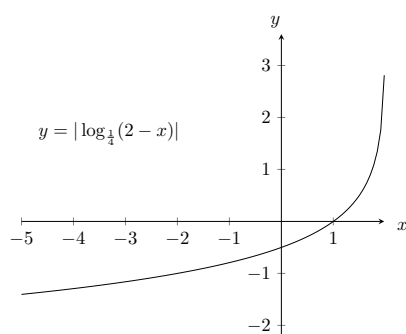
6. (a)  $\sqrt{5}$   
(b) 7  
(c)  $8\sqrt{5}$  or  $-8\sqrt{5}$
7. no
8. 3 or  $-\frac{1}{3}$
9.  $f(0) = 1$ ,  $f(\frac{1}{p} + \dots + \frac{1}{p}) = \left[f(\frac{1}{p})\right]^p = f(1) = a$ , then  $f(\frac{1}{p}) = a^{\frac{1}{p}}$ , which is  $f(x) = a^x$ .

### Exercise 13

1. 3, -3,  $-\frac{7}{4}$ ,  $\frac{37}{2}$ ,  $\frac{5}{4}$
2.  $\frac{121}{3}$
3.  $\frac{5 \ln 2 - \ln 3}{2 \ln 3 - 3 \ln 2}$ .
4.  $\frac{\ln 2}{\ln 3 + 2 \ln 2} = 0.279$
5.  $x = \frac{\ln 5}{\ln 15}$ ,  $y = \frac{\ln 3}{\ln 15}$ .
6.  $\frac{5}{6} + \log_{10} \frac{3}{2}$
7. (a)  $a^{\ln b} = e^{\ln a \ln b} = b^{\ln a}$ , then  $(a^{\ln b})^{\frac{1}{\ln c}} = (b^{\ln a})^{\frac{1}{\ln c}}$   
(b) omit  
(c) omit
8. 2
9. 25

### Exercise 14

1.  $<, >, >, >$
2. The graph are as following:



3.  $m > n$
4. (a)  $(-1, 1)$   
(b) even.

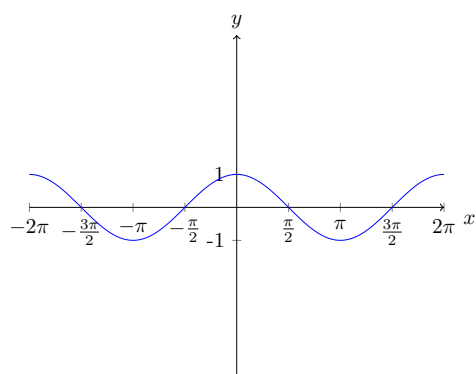
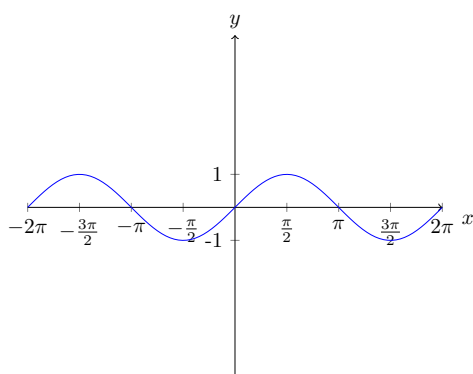
5.  $y = a^x - 1$
6.  $0 < a < 1, 0 < x < \log_a(\sqrt{3} - 1), a > 1, \log_a(\sqrt{3} - 1) < x < 0$
7.  $k = 1, \mathbb{R}$
8.  $m = n = 5$

### Exercise 15

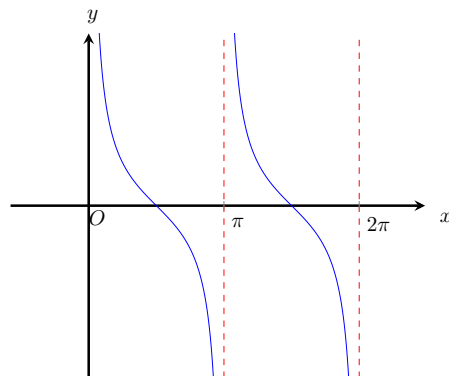
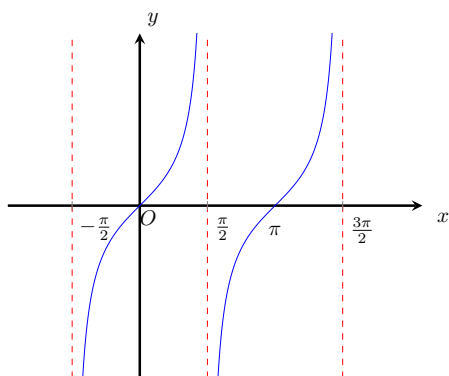
1. (a) 0.323  
(b) 130
2.  $a = \frac{3}{5 \lg 2}, b = \frac{5}{2} - \frac{3}{5 \lg 2}$
3.  $A = 0.4, c = 1.01, x = 504$
4. (a) 5  
(b) 1,1,9
5. Consider  $f(1) = 0$

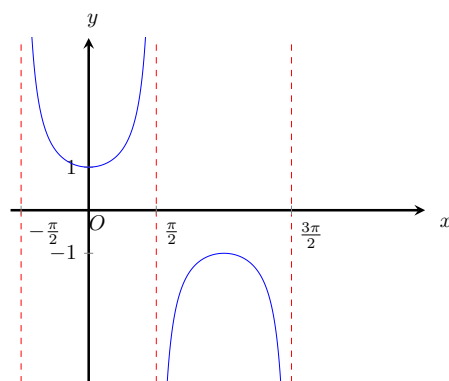
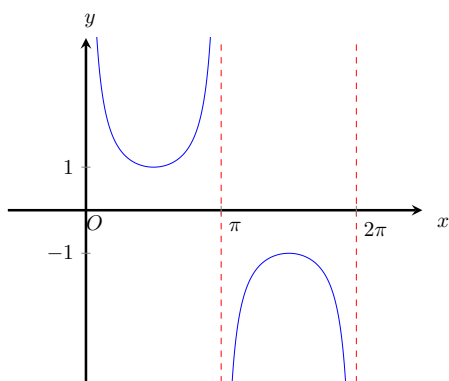
### Exercise 16

1. The graph of  $y = \sin x$  and  $y = \cos x$ :



2. The graph of  $y = \tan x$  and  $y = \cot x$ :





3. The graph of  $y = \sec x$  and  $y = \csc x$ :

### Exercise 17

1. (a) 0.5  
(b)  $\frac{131}{65}$   
(c)  $\frac{457}{289}$   
(d)  $\frac{3}{5}$
2. omit
3. (a)  $73.9^\circ, 286.1^\circ$   
(b)  $37.2^\circ, 217.2^\circ, 327.8^\circ, 142.8^\circ$   
(c)  $30^\circ, 210^\circ, 150^\circ, 330^\circ$   
(d)  $60^\circ, 240^\circ, 120^\circ, 300^\circ$

### Exercise 18

1.  $\frac{\sqrt{2}}{4}(\sqrt{3} - 1)$
2.  $\frac{\sqrt{3} \tan x + 1}{\tan x - \sqrt{3}}$
3.  $-\frac{56}{65}, \frac{63}{65}, -\frac{16}{63}$
4.  $\pm \frac{\pi}{4}, \pm \frac{3\pi}{4}, \pm \frac{5\pi}{4}, \pm \frac{7\pi}{4}$
5. 0.12, 3.26, -3.02, -6.16
6.  $-\frac{59}{72}$
7.  $-\frac{4}{3}, -\frac{3}{4}$
8. 0
9. Use  $\sin 2\xi = 2 \sin \xi \cos \xi$ ,  $\cos \xi = 1 - 2 \sin^2 \xi$ .

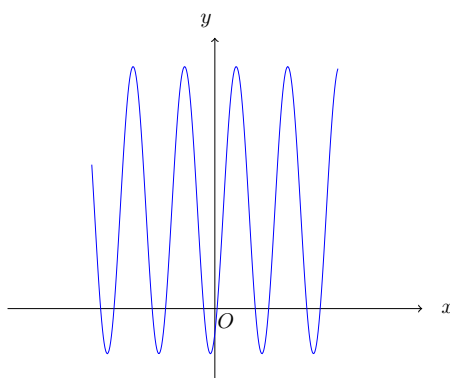
### Exercise 19



1.  $\pm \frac{11}{6}, \pm \frac{5}{6}$
2.  $-1 \pm \sqrt{2}, -1 + \sqrt{2}$
3.  $\frac{\pi}{2}, 5.44, 2.29$ .
4.  $x = k\pi, 0.869 + k\pi, -0.869 + k\pi$ .
5.  $\sin^2 \frac{1}{2}A = \pm \frac{\sqrt{2}}{2} \sqrt{1 - \cos A}, \cos^2 \frac{1}{2}A = \pm \frac{\sqrt{2}}{2} \sqrt{1 + \cos A}$ .
6.  $\sin 3A = 3 \sin A - 4 \sin^3 A, \cos 3A = 4 \cos^3 A - 3 \cos A, \tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$ .
7. Omit.
8. Omit.
9. (a)  $\tan \theta$   
(b)  $\frac{1}{2}$   
(c) 1
10.  $1, -1 + \sqrt{2}$
11.  $\frac{7\sqrt{65}}{65}$ .

#### Exercise 20

1. (a)  $\sqrt{53}, 15.9^\circ, 74.1^\circ$   
(b)  $\sqrt{41}, 51.3^\circ$   
(c)  $\sqrt{34}, 59.0^\circ$
2. 5.57
3.  $-2 \sin \alpha$
4.  $[\frac{2}{37}, \frac{2}{3}]$
5. 9, -17, 0.588, 0.983
6. the graph is as follows: period  $\frac{2\pi}{3}$



7.  $\pi$

#### Exercise 21

- 1
  - $-\frac{3}{4}$
- $k = -1, m = -1$
- $a_n = \cos \frac{1}{2}x \cos \frac{1}{4}x \cdots \cos^2 \frac{1}{2^{n-1}}x, b_n = \cos \frac{1}{2}x \cos \frac{1}{4}x \cdots \cos \frac{1}{2^{n-1}}x$
- omit
- prove  $\tan \frac{q+p}{2} = \tan \frac{r+s}{2}$

### Exercise 22

- $3e^{3x+1}$
  - $\frac{1}{2} \cdot 2^{\sqrt{x}}(\ln 2) \cdot (x^{-\frac{1}{2}})$
  - $3(2 + e^{1-x})^{-2}e^{1-x}$
  - $\frac{1}{3}e^{3x-x^2}(3-2x) - \frac{2}{3}e^{1-x-x^2}(-1-2x)$ .
  - omit
- Maximum,  $(\frac{\ln 2}{2}, 4\sqrt{2})$
- Minimum,  $(-1, e^{-3})$
- $x = 0$ .
- Omit.

### Exercise 23

- $\frac{2x}{x^2+1}$
  - $\frac{2}{(2x-5)\ln 3}$
  - $\frac{3}{x} + \frac{8x}{x^2-1}$
  - $\frac{4}{4x+3} - \frac{3x^2}{x^3-1}$
  - $-\frac{1}{(x+1)(\ln(x+1))^2}$
  - $-\frac{2}{x(\ln x)^2}$ .
- $(1, \ln 2)$
- $y = -x + 4$
- $y - 2e = -2x$
- $x = -3$ .  $x < -3$ , then  $\frac{dy}{dx} < 0$ ,  $x > -3$ , then  $\frac{dy}{dx} > 0$ .
- $(-\infty, \infty)$
  - $f^{-1}(x) = \frac{x + \sqrt{x^2 + 4}}{2}$ .
  - $y - 1 = \frac{1}{2}(x - 0)$ .

### Exercise 24

1. (a)  $2x \cos(x^2)$ .  
 (b)  $-\frac{1}{2} \sin x (1 + \cos x)^{-\frac{1}{2}}$ .  
 (c)  $6 \cos 2x e^{3 \sin 2x}$ .  
 (d)  $\tan x$ .  
 (e)  $2e^{2x} \cos(e^{2x})$ .  
 (f)  $-\sin 2x$ .  
 (g)  $\sin 2x$ .
2.  $y - \ln \sqrt{2} = x - \frac{\pi}{4}$
3.  $y + \frac{1}{3} = -\frac{2}{3\sqrt{3}}(x - \frac{\pi}{6})$
4.  $x = \frac{\pi}{6}$ , max;  $x = \frac{5}{6}\pi$ , min.
5.  $\cos 2x(1 - \sin 2x)$ .
6. omit,  $-(\csc x - \sin x)^{-2}(-\csc x \cot x - \cos x)$ .
7. Omit.
8. (a)  $\frac{1}{2} \cos \frac{1}{2}x + \frac{1}{3} \sin \frac{1}{3}x$ .  
 (b)  $-1$  and  $\frac{1}{2}$   
 (c)  $12\pi$   
 (d)  $12\pi$
9.  $\frac{1}{\sqrt{1-x^2}}, -\frac{1}{\sqrt{1-x^2}}$
10. (a)  $-\frac{1}{2}x^{-\frac{1}{2}} \sin \sqrt{x}$   
 (b)  $-\frac{1}{2} \sin x (\cos x)^{-\frac{1}{2}}$   
 (c)  $2x \cos x^2$   
 (d)  $(-x^2) \cos \frac{1}{x}$ .

### Exercise 25

1. (a)  $\sec^2(x)$ .  
 (b)  $-\csc^2 x$   
 (c)  $x + 2x \ln x$ .  
 (d)  $2x \sec^2(x^2 + 1)$   
 (e)  $\frac{2xe^{2x} - (e^{2x} + 1)}{x^2}$ .  
 (f)  $(2x) \sin x^2 + (x^2 - 2)(\cos x^2)(2x)$
2. Max  $(-3, 6e^{-3})$ , Min  $(1, -2e)$
3. omit
4.  $(e, \frac{1}{3})$
5.  $(1, \frac{1}{2})$
6. (a)  $x = \frac{\pi}{4}$   
 (b) max

7.  $u'_1 u_2 u_3 \cdots u_k + u_1 u'_2 \cdots u_k + \cdots + u_1 u_2 \cdots u'_k$
8.  $\sum_{k=0}^n \binom{n}{k} u^{(n-k)} v^{(k)}$

#### Exercise 26

Omit

#### Exercise 27

1.  $y - 4 = 3(x + 8)$
2.  $x = a \cos t, y = a \sin t. AB = a.$
3.  $\frac{dy}{dx} = \frac{e^{-t}}{1-e^{-t}}. t = \ln 3 - \ln 2.$
4.  $t = 3, (\frac{4}{3}, \frac{27}{e^3}).$
5. Omit.
6. (a)  $\frac{5}{3}$   
(b)  $-\frac{32}{27}.$
7. Omit.

#### Exercise 28

1.  $-\frac{1}{\pi+2}$
2.  $y - 2 = \frac{1}{3}(x - 1)$
3. Horizontal:  $y = \pm 3\sqrt{3}$ , vertical:  $x = \pm 3$
4.  $(-1, -2)$ , max;  $(0, 0)$ , min.
5.  $(\frac{8}{3}, -4)$
6.  $-1, 4.$
7. Omit
8.  $(2e, e^{\frac{1}{2}}). \ln y \neq 0.$
9.  $(1, -4)$ , min;  $(-1, 4)$ , max.
10. Omit.

#### Exercise 29

1. (a)  $\frac{2}{3} \ln |3x - 1| + C$   
(b)  $\frac{1}{2} e^{2x+3} + C$   
(c)  $\frac{\sqrt{3}}{3} \tan^{-1}(\frac{2}{\sqrt{3}}x) + C$

- (d)  $-\frac{1}{2} \cos 2x + C$
  - (e)  $\frac{1}{2}x + \frac{1}{4} \sin 2x + C$
2.  $x^4 - 3x^2 + x + 7$

### Exercise 30

Omit

### Exercise 31

- 1.  $8\frac{5}{8} + 3 \ln 2$
- 2.  $\frac{1}{2} - \frac{1}{2e}$
- 3. 2
- 4.  $\frac{\pi}{8} - \frac{1}{12}$
- 5.  $\frac{\pi}{8}$
- 6. 0

### Exercise 32

- 1.  $1 - \frac{\pi}{4}$
- 2.  $2\frac{11}{12} - \ln 6$
- 3. Omit
- 4. (a)  $4e^{\frac{x}{2}} - 2x - 1$   
(b) min.
- 5. Omit
- 6. Omit
- 7. Omit
- 8. Omit
- 9. Omit

### Exercise 33

- 1. If  $p = 1$ , infinite;  
if  $p > 1$ ,  $\int_0^1 \frac{1}{x^p} dx$  infinite,  $\int_1^\infty \frac{1}{x^p} dx = \frac{1}{1-p}$ ;  
if  $p < 1$ ,  $\int_0^1 \frac{1}{x^p} dx = \frac{1}{1-p}$ ,  $\int_1^\infty \frac{1}{x^p} dx$  infinite.
- 2.  $\frac{1}{2}e$

3. Infinite
4. Omit

#### Exercise 34

1. (a)  $\frac{287}{192}\pi + \pi \ln 4$   
 (b)  $\frac{\pi}{4}(e^2 - e^{-2})$   
 (c)  $\frac{\pi^2}{2}$   
 (d)  $\pi(\sqrt{3} - 1 - \frac{\pi}{12})$ .
2.  $\frac{1}{2}$
3.  $2\pi^2 r a^2$
4. (a)  $\pi(\sqrt{3} - \frac{\pi}{3})$   
 (b) Omit.

#### Exercise 35

1. (a)  $\int 1 + \frac{-2}{x-2} + \frac{-3}{x+3} dx = x - 2 \ln |x-2| - 3 \ln |x+3| + C$ .  
 (b)  $\int \frac{\frac{5}{3}}{x+4} + \frac{\frac{-3}{4}}{x+2} + \frac{\frac{1}{12}}{x-2} dx = \frac{5}{3} \ln |x+4| - \frac{3}{4} \ln |x+2| + \frac{1}{12} \ln |x-2| + C$ .  
 (c)  $\int \frac{1}{4} \frac{1}{1-x} + \frac{1}{4} \frac{1}{1+x} + \frac{1}{2} \frac{1}{1+x^2} dx = -\frac{1}{4} \ln |1-x| + \frac{1}{4} \ln |1+x| + \frac{1}{2} \tan^{-1} x + C$ .  
 (d)  $\int_{-\frac{1}{2}}^1 \frac{1}{2} \frac{1}{x+1} - \frac{1}{x+2} + \frac{1}{2} \frac{1}{x+3} dx = \frac{1}{2}(\ln 8 - \ln 5)$ .  
 (e)  $\frac{3}{2} \ln 3 - \ln 4$ .
2.  $10\frac{5}{12}\pi$ ;  $20.8333\pi$

#### Exercise 36

1.  $dt = 2x dx$
2.  $dx = t dt$
3.  $dt = \cos x dx$
4.  $dx = e^t dt$
5.  $dx = \frac{1}{t+1} dt$
6.  $dt = \frac{1}{2} \sec^2 \frac{x}{2} dx$
7.  $dt = \frac{2x}{1+x^4} dx$
8.  $dt = -\tan x dx$

**Exercise 37**

1.  $e^{\sin x} + C$
2.  $-\frac{1}{6}(3 - 2x^2)^{\frac{3}{2}} + C$
3.  $\frac{1}{3} \ln |x^3 - 1| + C$
4.  $\frac{\pi}{8}$
5.  $\frac{7}{24}$
6.  $\frac{3}{14}(x^2 - 1)^7 + \frac{1}{4}(x^2 - 1)^6 + C$
7.  $x + 4\sqrt{x} + 4 \ln |\sqrt{x} - 1| + C$
8.  $\frac{27}{140}$
9.  $\frac{1696}{2835}$
10.  $\frac{3}{8}$
11.  $\frac{2}{3}\pi$
12.  $\pi$
13.  $2x^{\frac{1}{2}} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6 \ln |1 + x^{\frac{1}{6}}| + C$
14.  $\ln(\sec x + \tan x) + C$
15.  $\frac{1}{3} \ln |1 + x| - \frac{1}{3} \left( \frac{1}{2} \ln |x^2 - x + 1| - \sqrt{3} \tan^{-1} \left( \frac{2}{\sqrt{3}} \left( x - \frac{1}{2} \right) \right) \right) + C$