Qu No	Commentary	Solution	Marks	Part Marks	Total
1	Finds normal to plane.	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & -1 \\ 1 & -3 & 4 \end{vmatrix} = \mathbf{i} - 9\mathbf{j} - 7\mathbf{k}$	M1A1		
(i)	Deduces equation.	Π:: $x - 9y - 7z = \text{constant}$			
		Sub e.g. $(1, -1, 2) \Rightarrow$ constant $= -4$			
		Π:: $x - 9y - 7z = -4$	M1A1	4	
	General point on line inserted in plane equation to find λ.	l: $x = 6 + 2\lambda \quad y = -2 + \lambda \quad z = 1 - 4\lambda$			
		Sub in $\Pi \Rightarrow 6 + 2\lambda + 18 - 9\lambda - 7 + 28\lambda = -4$	M1		
		$\Rightarrow \lambda = -1$	A1		
		Position vector of intersection is 4**i** – 3**j** + 5**k.**	A1	3	
(ii)	Distance of point from plane formula or triple scalar product method.	Either $\left\lvert \dfrac{6 + 18 - 7 + 4}{\sqrt{1 + 81 + 49}} \right\rvert$ Or $\dfrac{(2i + j - 4k).(i - 9j - 7k)}{\sqrt{1 + 81 + 49}}$	M1A1		
		$= \dfrac{21}{\sqrt{131}}$ (=1.83)	A1	3	
(iii)	Scalar product to find complement of angle.	$(2i + j - 4k).(i - 9j - 7k) = 21$	M1		
		$= \sqrt{4 + 1 + 16}\sqrt{1 + 81 + 49}\sin\theta$	A1		
		$\Rightarrow \sin\theta = \sqrt{\dfrac{21}{131}} \Rightarrow \theta = 23.6^0$ or 0.412 rad.	A1	3	**[13]**

Qu 2	Solution	Marks
(i)	Direction perpendicular to AB and CD: $\mathbf{n}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & 1 & 1\\ 4 & 3-\lambda & 1\end{vmatrix}=\begin{pmatrix}\lambda-2\\ 4\\ -4\end{pmatrix}$	**M1A1**
	$\overrightarrow{DB}=\begin{pmatrix}5\\ -4\\ -6\end{pmatrix}$. Hence $\left\lvert\dfrac{\begin{pmatrix}5\\ -4\\ -6\end{pmatrix}.\begin{pmatrix}\lambda-2\\ 4\\ -4\end{pmatrix}}{\sqrt{(\lambda-2)^2+16+16}}\right\rvert=3$ or equivalent	**M1A1**
(ii)	$\Rightarrow (5\lambda-2)^2=9\left(\lambda^2-4\lambda+36\right)$ $\Rightarrow \ldots \Rightarrow \lambda^2+\lambda-20=0$ (**AG**)	**M1M1** **A1** [7]
	$(\lambda+5)(\lambda-4)=0 \Rightarrow \lambda=-5,4$	**B1**
	$\lambda=4 \Rightarrow \mathbf{a}=\begin{pmatrix}2\\ 4\\ -3\end{pmatrix} \Rightarrow$ Normal to $ABD=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 4 & -1 & 1\\ -1 & 3 & 7\end{vmatrix}=\begin{pmatrix}-10\\ -29\\ 11\end{pmatrix}$	**M1A1**
	$\lambda=-5 \Rightarrow \mathbf{a}=\begin{pmatrix}2\\ -5\\ -3\end{pmatrix} \Rightarrow$ Normal to $ABD=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 4 & 8 & 1\\ -1 & 12 & 7\end{vmatrix}=\begin{pmatrix}44\\ -29\\ 56\end{pmatrix}$	**A1**
	$\cos\theta=\dfrac{-440+841+616}{\sqrt{10^2+29^2+11^2}\sqrt{44^2+29^2+56^2}}=\dfrac{1017}{\sqrt{1062}\sqrt{5913}}$	**M1A1**
	$\Rightarrow \theta=66.1°$	**A1** [7]

(i)	$\mathbf{p} = \begin{pmatrix} 8+\lambda \\ 2-2\lambda \\ 3 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} 5 \\ 3+2\mu \\ -14-3\mu \end{pmatrix} \Rightarrow \overrightarrow{QP} = \begin{pmatrix} 3+\lambda \\ -1-2\lambda-2\mu \\ 17+3\mu \end{pmatrix}$	M1A1
	$\begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 3+\lambda \\ -1-2\lambda-2\mu \\ 17+3\mu \end{pmatrix} = 0 \Rightarrow 5\lambda + 4\mu = -5$	M1A1
	$\begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 3+\lambda \\ -1-2\lambda-2\mu \\ 17+3\mu \end{pmatrix} = 0 \Rightarrow -4\lambda - 13\mu = 53$	A1
	Solving : $\lambda = 3, \mu = -5$	M1A1
	Whence: $\mathbf{p} = \begin{pmatrix} 11 \\ -4 \\ 3 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} 5 \\ -7 \\ 1 \end{pmatrix}$.	A1 (8)
(ii) (a)	$\overrightarrow{AP} = \begin{pmatrix} 3 \\ -6 \\ 0 \end{pmatrix}$, $\overrightarrow{AQ} = \begin{pmatrix} -3 \\ -9 \\ -2 \end{pmatrix}$	B1
	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -6 & 0 \\ -3 & -9 & -2 \end{vmatrix} = \begin{pmatrix} 12 \\ 6 \\ -45 \end{pmatrix}$ (CAO) ; Area = $\frac{1}{2}\sqrt{2205}$ (=23.5)	M1A1 A1
(b)	$\overrightarrow{AB} = \begin{pmatrix} -3 \\ 1 \\ -17 \end{pmatrix} \Rightarrow$ Volume $= \frac{1}{3} \times \frac{1}{2}\sqrt{2205} \left\| \frac{\begin{pmatrix} -3 \\ 1 \\ -17 \end{pmatrix} \cdot \begin{pmatrix} 12 \\ 6 \\ -45 \end{pmatrix}}{\sqrt{2205}} \right\| = \frac{735}{6}$ (= 122.5)	M1A1 (6) **Total: 14**

Qu No	Commentary	Solution	Marks	Part Mark	Total
4	Finds vector normal to *Π*.	$\mathbf{n}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 1 & -2 & 2\\ 3 & 1 & -2\end{vmatrix}=2\mathbf{i}+8\mathbf{j}+7\mathbf{k}$	M1A1		
	Dot product of this with general point on l_1.	$\begin{pmatrix}3+8t\\6+5t\\12-8t\end{pmatrix}\cdot\begin{pmatrix}2\\8\\7\end{pmatrix}=138$ or $\begin{pmatrix}8\\5\\-8\end{pmatrix}\cdot\begin{pmatrix}2\\8\\7\end{pmatrix}=0$	A1√		
	Deduces result.	Independent of $t \Rightarrow$ parallel, or $\Rightarrow$ parallel.	A1	4	
	Cartesian equation of *Π*.	*Π*: $2x+8y+7z=21$	B1		
	Substitutes general point of l_2.	Sub. $x=5+2s$, $y=-4-s$, $z=7+s$	M1		
	Finds value of parameter.	$\Rightarrow s=-2$	A1		
	Finds p.v. of intersection.	and line meets *Π* at point with p.v. **i** – 2**j** + 5**k**	A1	4	
		Take (9,11,2) as *A*, (3,6,12) as *B* and let *C* be foot of perpendicular from A to *l*.			
	Finds distance from point to known point on *l*.	$\text{AB}=\sqrt{6^2}+5^2+10^2=\sqrt{161}$	B1		
	Finds distance along *l* from known point to foot of perpendicular from given point to *l*.	$\text{BC}=\frac{1}{\sqrt{(6^2\ 5^2+8^2)(6\text{i}+5\text{j}-10\text{k})(8\text{i}+5\text{j}-8\text{k})}}=\frac{153}{\sqrt{153}}=\sqrt{153}$	M1		
			A1	4	[12]
	F.t. on non-hypotenuse side (must be real).	$\text{AC}=\sqrt{161-153=\sqrt{8\,\text{or}\,2\sqrt{2}}}$ (= 2.83)	A1√		
		Alternatively:			
	Writes a set of three equations in three unknowns for the intersection of *l* with *Π*.	$5+2s=2+\lambda+3\mu$ $-4-s=3-2\lambda+\mu$ $7+s=-1+2\lambda-2\mu$	(B1)		
	Solves the set of equations.	$\Rightarrow s=-2$, $\Rightarrow \lambda=2$, $\mu=-1$	(M1A1)		
	Finds p.v. of intersection.	and line meets *Π* at point with p.v. **i** – 2**j** + 5**k**	(A1)		

Qu No	Commentary	Solution	Marks	Part Mark	Total
4	Finds vector $\overrightarrow{BA}$.	$\overrightarrow{BA} = 6\mathbf{i} + 5\mathbf{j} - 10\mathbf{j}$	B1		
		$\left\lvert \dfrac{1}{\sqrt{8^2+5^2+8}} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & 5 & -10 \\ 8 & 5 & -8 \end{vmatrix} \right\rvert$	M1A1		
		$= \sqrt{\dfrac{1224}{153}} = \sqrt{8}$ or $2\sqrt{2}$ $\quad (= 2.83)$	A1	(4)	
		Alternatively: (A)			
	Finds distance from point to known point on l.	Take (9,11,2) as A, (3,6,12) as B and let C be foot of perpendicular from A to l. $AB = \sqrt{6^2+5^2+10^2} = \sqrt{161}$	(B1)		
	Finds distance along l from known point to foot of perpendicular from given point to l.	$BC = \dfrac{1}{\sqrt{8^2+5^2+8^2}}(6\mathbf{i}+5\mathbf{j}-10\mathbf{k})(8\mathbf{i}+5\mathbf{j}-8\mathbf{k})$	(M1)		
	F.t. on non-hypotenuse side (must be real).	$= \dfrac{153}{\sqrt{153}} = \sqrt{153}$	(A1)		
		$AC = \sqrt{161-153} = \sqrt{8}$ or $2\sqrt{2}$ $\quad (= 2.83)$	(A1√)	(4)	
		(B)			
	Finds vector . $\overrightarrow{AC}$	$\overrightarrow{AC} = \begin{pmatrix} 8t-6 \\ 5t-5 \\ 10-8t \end{pmatrix}$	(B1)		
	Uses $\overrightarrow{AC}$ perpendicular to l to find t.	$\begin{pmatrix} 8t-6 \\ 5t-5 \\ 10-8t \end{pmatrix} . \begin{pmatrix} 8 \\ 5 \\ -8 \end{pmatrix} = 0 \Rightarrow t = 1$	(M1A1)		
	Finds length AC.	$AC = \sqrt{2^2+0^2+2^2} = \sqrt{8}$	(A1)	(4)	**[12]**

5 (i) $(\mathbf{i} - (2\sin t)\mathbf{j}) \times (4\mathbf{j} - \mathbf{k}) = (2\sin t)\mathbf{i} + \mathbf{j} + 4\mathbf{k}$ M1A1

$PQ = (\mathbf{i} - \mathbf{j}).[(2\sin t)\mathbf{i} + \mathbf{j} + 4\mathbf{k}]/\sqrt{4\sin^2 t + 17}$ dM1A1

$PQ = |1 - 2\sin t|/\sqrt{4\sin^2 t + 17}$ A1

Condone disappearing modulus sign –

Deduct 1 mark if no modulus sign at all

(ii) $PQ = 0 \Rightarrow \sin t = 1/2 \Rightarrow t = \pi/6, 5\pi/6$ or 0.524, 2.62 M1A1 (both)

(iii) Obtains some vector $\perp$ plane BPQ, e.g.,

$\left(\sqrt{2}\mathbf{i} + \mathbf{j} + 4\mathbf{k}\right) \times \left(\mathbf{i} - \sqrt{2}\mathbf{j}\right) = 4\sqrt{2}\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$ M1A1

$p(A : BPQ) = \left(4\sqrt{2}\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}\right).\left(\mathbf{i} - \mathbf{j}\right)/\sqrt{57}$ M1A1

$= 4\left(\sqrt{2} - 1\right)/\sqrt{57} = 0.219$ A1

or

For **(i)**

$$\ell_1 = r = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 4 \\ -1 \end{pmatrix}$$
$$\ell_2 = r = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -2\sin t \\ 0 \end{pmatrix} \qquad \Rightarrow \overrightarrow{PQ} = \begin{pmatrix} \mu - 1 \\ 1 - 4\lambda - 2\mu\sin t \\ \lambda \end{pmatrix}$$

$$\overrightarrow{PQ} \cdot \begin{pmatrix} 0 \\ 4 \\ -1 \end{pmatrix} = 0 \qquad \Rightarrow 17\lambda + 8\mu\sin t = 4$$

$$PQ \cdot \begin{pmatrix} 1 \\ -2\sin t \\ 0 \end{pmatrix} = 0 \qquad \Rightarrow 8\sin t\lambda + \left(1 + 4\sin^2 t\right)\mu = 1 + 2\sin t$$ M1

Whence $\lambda = \dfrac{4(1 - 2\sin t)}{\left(17 + 4\sin^2 t\right)}$ and $\mu = \dfrac{17 + 2\sin t}{\left(17 + 4\sin^2 t\right)}$ A1 (both)

6 Candidates generally found some difficulty with this question. Many failures occurred in the first part of the question.

Responses generally exhibited p = 24 and subsequently stated or implied that the vector 2**i** + 3**j** + 4**k** is perpendicular to the plane Π. However, about half of all candidates were unable to obtain possible values for q, r, s and t. In this context it was common to see complicated arguments based on the observation that (q **i** + r **j**) × (s **i** + t **k**) is parallel to the normal vector 2**i** + 3**j** + 4**k**, but in most cases these were not successful.

The next part of this question again divided candidates into two main categories. On the one hand, there were those who took the most direct route, that is they verified that, for all θ, the given scalar equation of Π is checked out when $x = 29 + 5\theta$, $y = -2 - 6\theta$, $z = -1 + 2\theta$. On the other, there were those who verified at least one of (a) that a particular point of l is in Π, (b) l is perpendicular to 2**i** + 3**j** + 4**k**. However, not all members of the second category of candidates verified both (a) and (b).

For the last part, strategies were soundly based and the working was generally accurate. There were many correct responses. Frequently these began with (2**i** + 3**j** + 4**k**) × (5**i** – 6**j** + 2**k**) = 30**i** + 16**j** – 27**k** so indicating that almost all candidates were familiar with the vector product and were able to apply it in a relevant way.

Answers: **r** = 24**i** + λ(3**i** – 2**j**) + μ(2**i** – **k**) or equivalent; $30x + 16y - 27z = 865$.

7

(i)	$\mathbf{r} = 4\mathbf{i} - 2\mathbf{j} + 2\mathbf{k} + \lambda(\mathbf{i} + 7\mathbf{j} + \mathbf{k}) + \mu(3\mathbf{i} + \mathbf{j} - \mathbf{k}) \Rightarrow A$ is in Π_1.	B1 [1]
(ii)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 7 & 1 \\ 3 & 1 & -1 \end{vmatrix} = \begin{pmatrix} -8 \\ 4 \\ -20 \end{pmatrix} \sim \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$	M1A1 [2]
(iii)	L is (12, –6, 6) $2x - y + 5z = 24 + 6 + 30 = 60$ $\mathbf{n} = t\,(2\mathbf{i} - \mathbf{j} + 5\mathbf{k}) \Rightarrow 4t + t + 25t = 60 \Rightarrow t = 2$ $\mathbf{n} = 4\mathbf{i} - 2\mathbf{j} + 10\mathbf{k}$	B1 B1 M1A1√ A1 [5]
(iv)	$\mathbf{m} = 4\mathbf{i} - 2\mathbf{j} + 2\mathbf{k} + \frac{3}{4}(8\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}) = 10\mathbf{i} - 5\mathbf{j} + 5\mathbf{k}$ (AG) M is (10, –5, 5) $\Rightarrow \overrightarrow{NM} = 6\mathbf{i} - 3\mathbf{j} - 5\mathbf{k}$ $(6\mathbf{i} - 3\mathbf{j} - 5\mathbf{k}) \times (2\mathbf{i} - \mathbf{j} + 5\mathbf{k}) = -\mathbf{i} + 2\mathbf{j}$ Perpendicular distance is $\frac{\lvert 20(-\mathbf{i} + 2\mathbf{j} \rvert}{\sqrt{30}} = \frac{20}{\sqrt{6}} = 8.16$ (Mark various alternative methods in a similar manner.)	B1 B1√ M1A1 M1A1 [6]

8	Normal to plane: (2, 3, 4) × (–1, 0, 1) = (3, –6, 3)	M1A1		
	$\mathbf{r}.(1, -2, 1) = d$ and point (2, 1, 4)	M1	substitute point into plane eqn	
	$d = 4 \quad x - 2y + z = 4$	A1		[4]
	<u>Alternative:</u>			
	$\left.\begin{array}{l} x = 2 + 2\lambda - \mu \\ y = 1 + 2\lambda \\ z = 4 + 4\lambda + \mu \end{array}\right\\} \quad x + z = 6 + 6\lambda$	M1A1		
	$\therefore x + z = 6 + 2(y - 1)$	M1		
	$\therefore x - 2y + z = 4$	A1		
	$x - 4y + 5z = 12$			
	$x - 2y + z = 4$ Solve by eliminating one variable	M1		
	Use parameter and express all 3 variables in terms of it	M1		
	e.g. $x = 3t - 4$, $y = 2t - 4$, $z = t$			
	$\mathbf{r} = (-4, -4, 0) + t\,(3, 2, 1)$	A1	or equivalent	[3]
	<u>Alternative:</u>			
	Direction of line = $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \times \begin{pmatrix} 1 \\ -4 \\ 5 \end{pmatrix} = t\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$	M1A1		
	Find any point on line e.g. $\begin{pmatrix} -4 \\ -4 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$ etc.			
	$\therefore \mathbf{r} = \begin{pmatrix} -4 \\ -4 \\ 0 \end{pmatrix} + t\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$	B1		
	Line l: $\mathbf{r} = (a, 2a + 1, -3) + \alpha(3c, -3, c)$			
	Plane: $x - 2y + z = 4$			
	Distance A to plane:			
	$\left\lvert \dfrac{a - 2(2a+1) - 3 - 4}{\sqrt{6}} \right\rvert = \dfrac{15}{\sqrt{6}}$	M1		
	$3a + 9 = 15$	M1	correct use of modulus sign	
	$a = 2$	A1		
	$\sin\theta = \dfrac{3c + 6 + c}{\sqrt{6}\sqrt{9c^2 + 9 + c^2}}$	M1A1		
	$\therefore \dfrac{4c + 6}{\sqrt{6}\sqrt{9 + 10c^2}} = \dfrac{2}{\sqrt{6}}$	M1	solve for c	
	$6c^2 - 12c = 0$: $c = 2$	A1		
	(Penalise only once for negative values.)			[7]