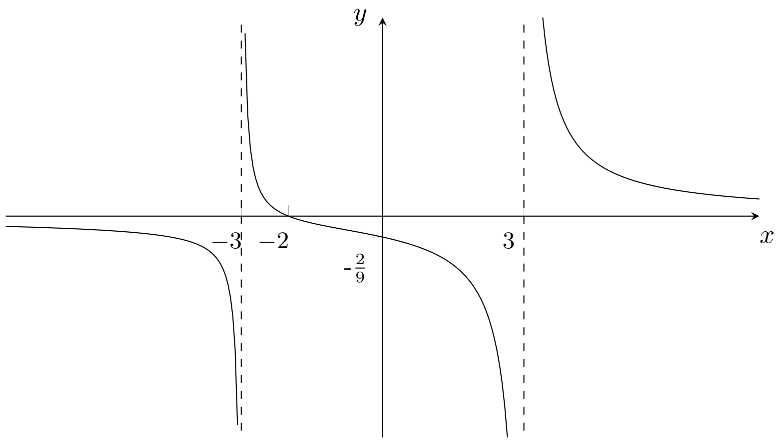
Rational function graph

1

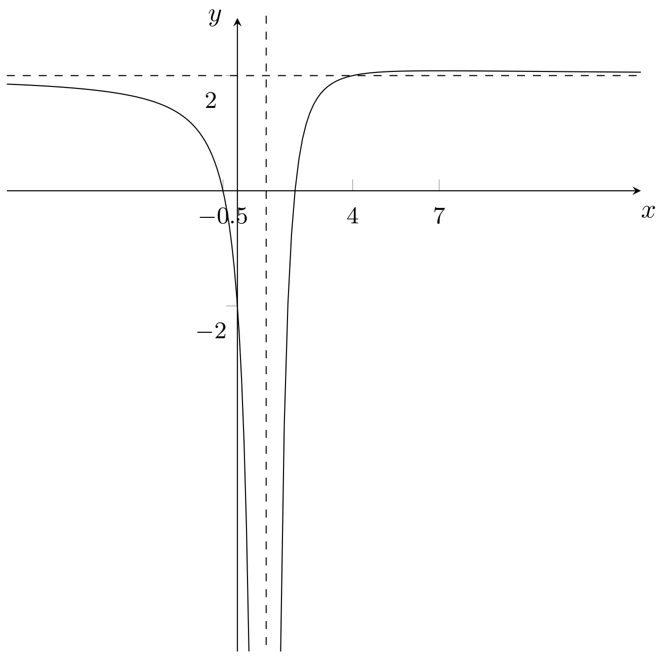
$\frac{dy}{dx} = \frac{(x^2-9) - 2x(x+2)}{(x^2-9)^2}$	**B1**
$= \frac{-x^2-4x-9}{(x^2-9)^2} = \frac{-(x+2)^2-5}{(x^2-9)^2} \Rightarrow \frac{dy}{dx} < 0$	**M1A1** [3]
Asymptotes: $x = \pm 3$; $y = 0$	**B1B1** [2]
Sketch: Axes and asymptotes ; Outside branches Middle branch, showing $\left(0, -\frac{2}{9}\right)$ and $(-2, 0)$. (Deduct at most 1 mark for poor forms at infinity.)	**B1B1** **B1** [3]

$y = \frac{x+2}{x^2-9}$:



2	States asymptotes.	Vertical: $x=1$ and Horizontal: $y=2$	B1B1	2	
	Obtains quadratic form in *x*.	$yx^2-2yx+y=2x^2-3x-2$ $\Rightarrow(y-2)x^2-(2y-3)x+(y+2)=0$	M1A1		
	Uses $B^2-4AC\ge 0$ for real roots.	For real x $(2y-3)^2-4(y-2)(y+2)\ge 0$ $\Rightarrow 12y\le 25\Rightarrow y\le\frac{25}{12}$.	M1 A1	 4	
	Finds condition for $y'=0$.	$y'=0\Rightarrow$ $(x^2-2x+1)(4x-3)-(2x^2-3x-2)(2x-2)=0$	M1		
	Solves	$\Rightarrow x^2-8x+7=0\Rightarrow(x-7)(x-1)=0$ $\Rightarrow x=7$, (since $x=1$ is vertical asymptote).	A1		
	Obtains stationary point.	Stationary point is $\left(7,\frac{25}{12}\right)$	A1	3	
	Sketch showing:	Axes and asymptotes (-0.5,0) , (2,0) , (0,–2) and (4,2) Left hand branch. Right hand branch.	B1 B1 B1 B1	4	 **[13]**

$$y=\frac{2x^2-3x-2}{x^2-2x+1}:$$



3 There was much variation in the quality of responses to this question. About half of all candidates produced correct or nearly correct solutions to all parts, while a minority of candidates made almost no progress beyond part **(i)**.

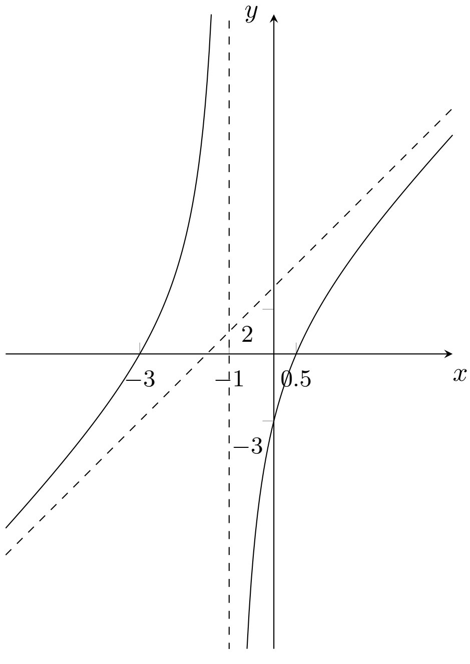
(i) Almost without exception the vertical asymptote was identified as $x = -1$. In contrast, a significant minority of candidates failed to obtain $y = 2x + 3$ as the diagonal asymptote and instead wrote down $y = 2x$. This particular inaccuracy almost totally undermined any prospect of success with the rest of the question.

(ii) Those candidates who wrote the equation of C in the form $y = 2x + 3 - \frac{6}{x+1}$ had little difficulty in determining the required set of x. In contrast, most of those who did not proceed in this way got involved in complicated and/or erroneous inequality arguments. Nevertheless, in one way or another, the majority of candidates did produce correct answers to this part even though, in many cases, there was no valid supporting argument.

(iii) Most sketches were satisfactory in that the asymptotes were placed appropriately relative to the axes and that two branches appeared whose location and intersections with the axes were correct. However, the majority of sketches had some deficiency with regard to the form at infinity. In this case there are four such forms and some care is needed to ensure that each of them relates to its associated asymptote in the right way.

Answers: **(i)** $x = -1$, $y = 2x + 3$; **(ii)** $x < -1$, $x > -1$.

$y = 2x + \frac{3(x-1)}{x+1}$:



4 This generated more responses than the alternative for **Question 12**. Generally, most of the ideas involved were understood, but elementary errors did some damage and much of the graphical work was of poor quality.

Most responses showed correct values for a, p and q, even when the underlying reasoning was invalid.

(i) Not a few responses began with a statement such as:

'$\dfrac{dy}{dx} = (4x+b)(x^2-5x+4)-(2x-5)(2x^2+bx+c)$' which as an application of the quotient rule is manifestly incorrect. Setting the right hand side of the above to zero together with $x = 2$, will, of course, lead to the correct value of c. Nevertheless, it must be emphasised here, as previously, that correct answers obtained from fundamentally erroneous working cannot gain full credit or, in some situations, any credit at all.

On the other hand the statement '$\dfrac{dy}{dx} = 0 \Rightarrow (4xb)(x^2-5x+4)-(2x-5)(2x^2+bx+c)=0$' (*) is correct and some candidates began part **(i)** in this way.

(ii) The substitution $c = 8$ in (*) leads to $(-10 - b)x^2 + 40 + 4b = 0$, or some equivalent, and it was here that most candidates failed to see the relevance of the condition $b \neq -10$ to this result and so did not make any further progress.

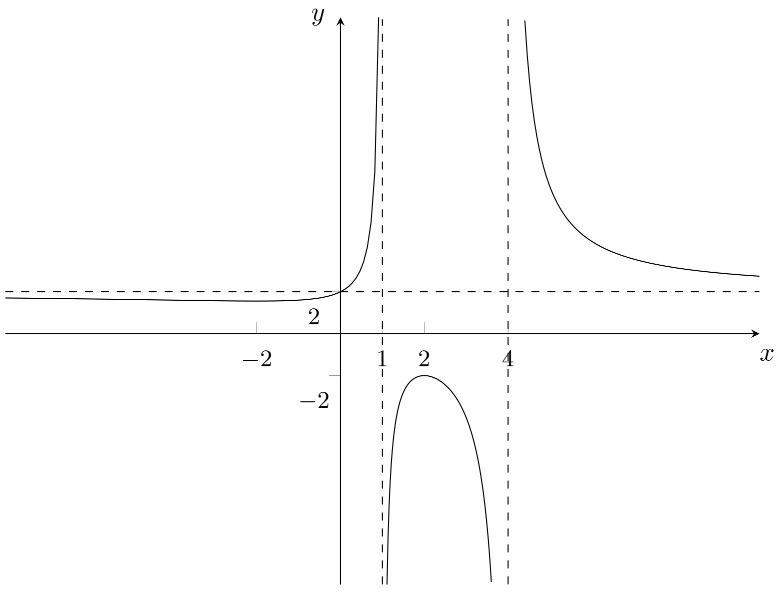
A few candidates resolved y into partial fractions, all of a, c, p, q now having numerical values, set the derivative of this form to zero, cancelled the factor $b + 10$, usually without giving a reason, and so obtained the equation $4(x - 1)^2 = (x - 4)^2$. From there it was easy to establish the required conclusion.

(iii) Most sketch graphs showed the 3 asymptotes correctly placed in relation to the axes. Also, they usually showed Γ to have 3 branches. However, apart from the inclusion of these basic aspects, most sketch graphs were deficient in some way. The main errors were as follows:

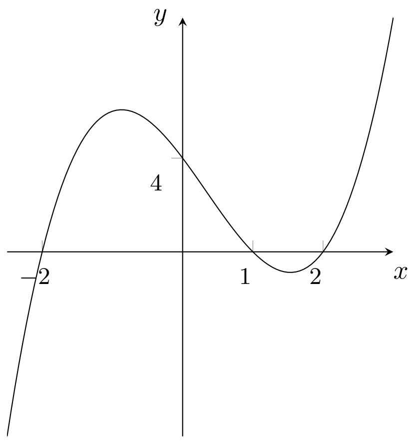
- the left hand branch did not have a minimum at a point where $x = -2$, and below the horizontal asymptote, and/or was not asymptotic to both $x = 1$ and $y = 2$
- the maximum point of the middle branch was not located below the x-axis at point where $x = 2$ and the asymptotic approach to at least one of $x = 1$ and $x = 4$ was carelessly drawn
- the asymptotic behaviour of the right hand branch was unclear particularly as $x \to +\infty$.

Answers: $a = 2$, $p = -5$, $q = 4$; **(i)** $c = 8$.

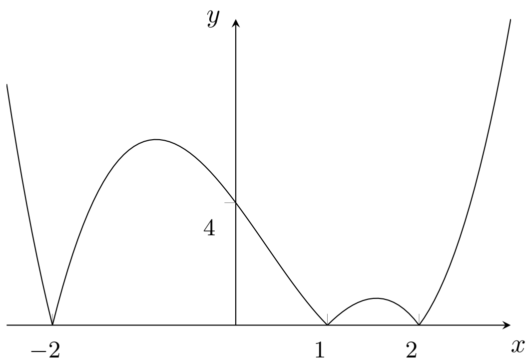
$$y = \frac{2x^2 - 6x + 8}{x^2 - 5x + 4}:$$



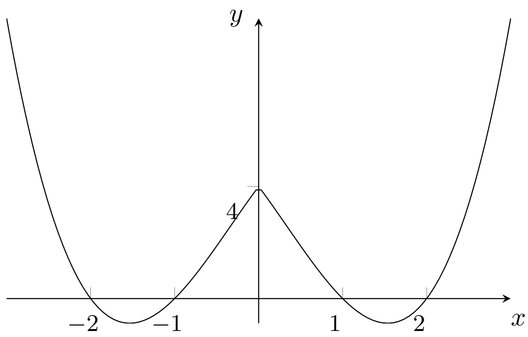
5 (a) $y = \mathrm{f}(x)$:



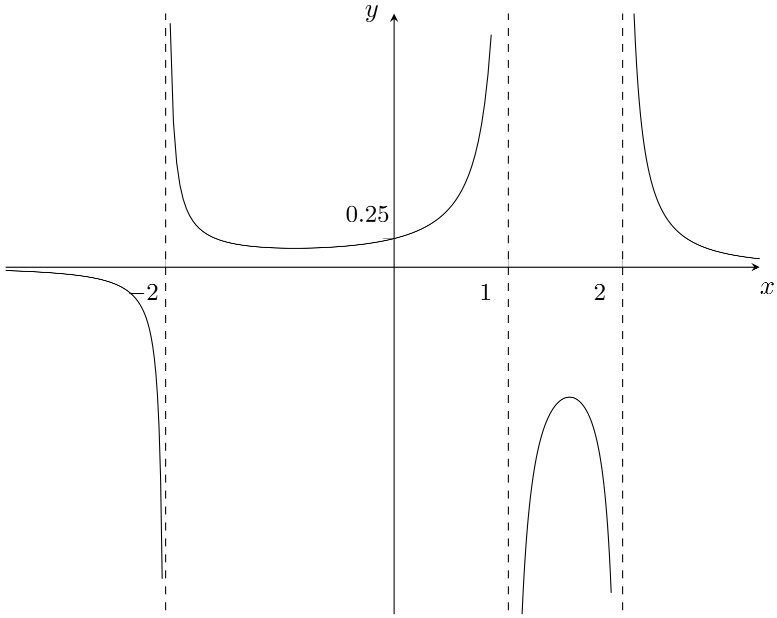
(b) $y = |\mathrm{f}(x)|$:



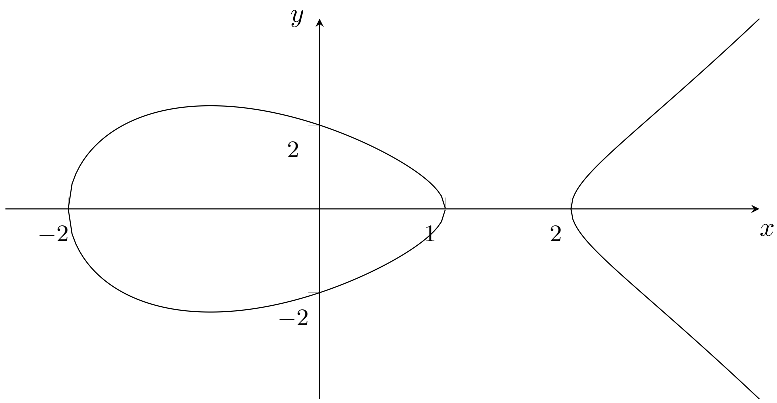
(c) $y = \mathrm{f}(|x|)$:

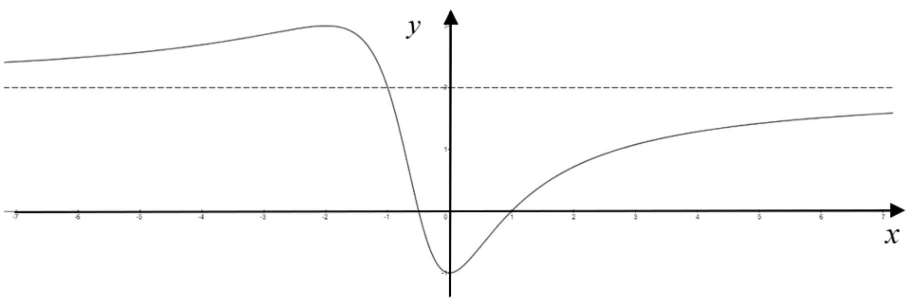
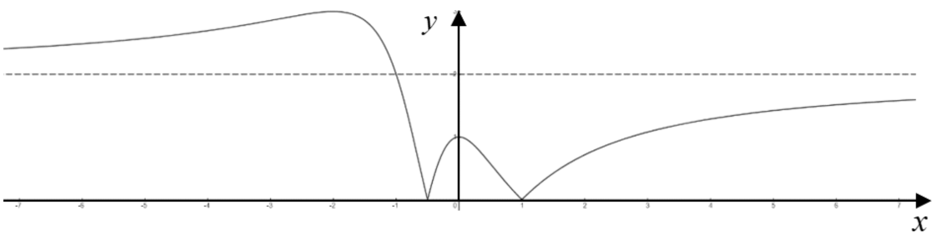


(d) $y = \dfrac{1}{\mathrm{f}(x)}$:

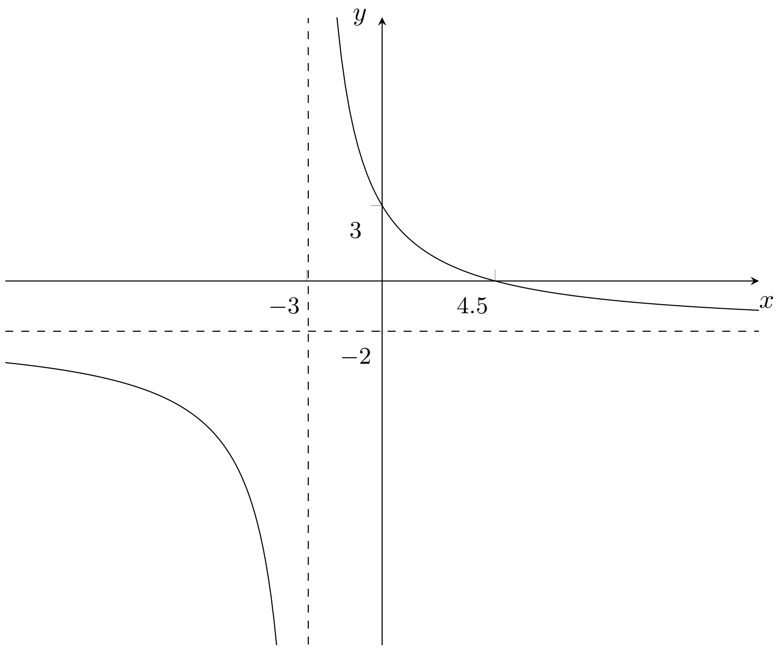


(e) $y^2 = \mathrm{f}(x)$:

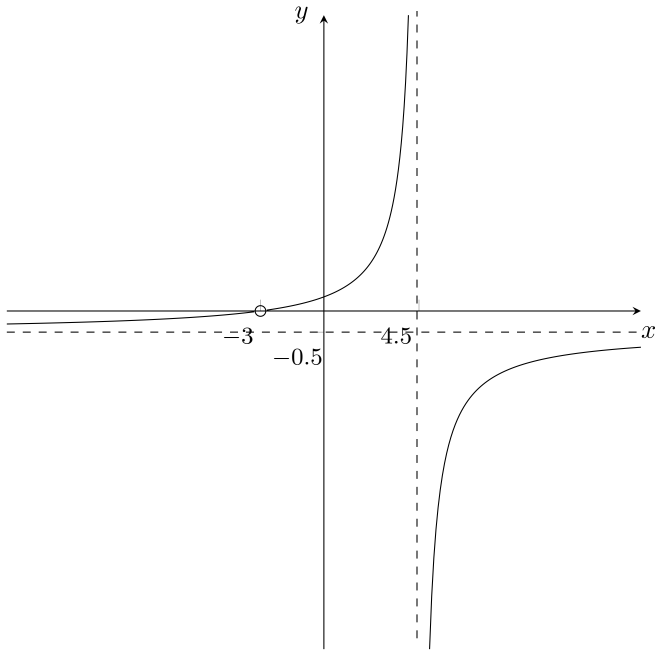


6(a)	$-3<0$	**M1 A1**
	$y=2$	**B1**
		3
6(b)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(x^2+x+1)(4x-1)-(2x^2-x-1)(2x+1)}{\left(x^2+x+1\right)^2}$	**M1**
	$3x^2+6x=0$	**M1**
	$\left(0,-1\right)$ $\quad\left(-2,3\right)$	**A1 A1**
	Alternative method for question 5(b)	
	$2x^2-x-1-y(x^2+x+1)=0$	**M1**
	Finds discriminant $(y+1)^2-4(y+1)(y-2)$ AND states y exists if discriminant $\geq$ 0 OR does not exist if discriminant < 0	**M1**
	Finds (0, -1) and (-2, 3)	**A1**
	Explains why they are stationary values.	**A1**
		4
6(c)		**B1** **B1**
	$\left(1,0\right)$, $\quad\left(-\frac{1}{2},0\right)$, $\quad(0,-1)$	**B1**
		3
6(d)		**B1 FT**
	$0<k<1$	**B1**
		2

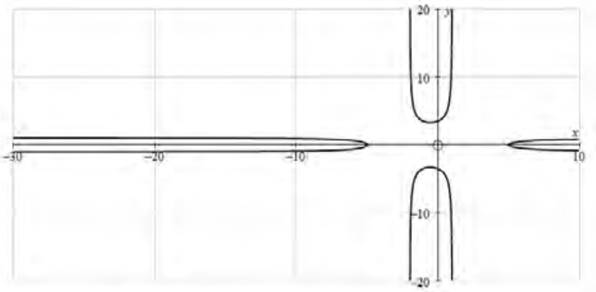
7 (a) The graph of $y = \mathrm{f}(x)$ is



(b) When $x = 4.5$ and $x = -3$, $y = \dfrac{1}{\mathrm{f}(x)}$ is undefined.



Cut the axis at $\left(0, \dfrac{1}{3}\right)$, asymptote: $x = 4.5$.

Question			Answer	Marks	
8	(i)		$\dfrac{x^2-25}{(x-1)(x+2)} = A + \dfrac{B}{(x-1)} + \dfrac{C}{(x+2)}$ $x^2-25 = A(x-1)(x+2)+B(x+2)+C(x-1)$ 3 processes of equating coefficients or substituting: e.g. $x=1 \;\; \Rightarrow -24 = 3B \Rightarrow B=-8$ $x=-2 \Rightarrow -21=-3C \Rightarrow C=7$ coeff of x^2 : $A=1$ $\dfrac{x^2-25}{(x-1)(x+2)} = 1 - \dfrac{8}{(x-1)} + \dfrac{7}{(x+2)}$	**M1** **B1** **A1** **A1**	Splitting in correct way to give partial fractions (may be seen anywhere) For A For B For C
				4	
	(ii)		$x=1,\, x=-2$ $y=1$	**B1** **B1**	
				2	
	(iii)		$y=1 \Rightarrow (x-1)(x+2)=x^2-25$ $x^2+x-2=x^2-25 \Rightarrow x=-23$	**M1** **A1**	
				2	
	(iv)			**B1** **B1**	4 bits as shown, roughly symmetric about axes, approaching asymptotes Lh side crosses asymptotes and upper section approaches from above and lower section approaches from below
				2	