1. **Proof:**

$n = 1$; LHS $= \dfrac{1+2}{1 \times 2 \times 2} = \dfrac{3}{4}$, RHS $= 1 - \dfrac{1}{2 \times 2} = \dfrac{3}{4}$

As LHS = RHS , the summation formula is true for $n = 1$.

Assume that the summation formula is true for $n = k$.

i.e. $\displaystyle\sum_{n=1}^{k} \frac{n+2}{n(n+1)2^n} = 1 - \frac{1}{(k+1)2^k}$.

With $n = k + 1$ terms the summation formula becomes:

$$\begin{aligned}
\sum_{r=1}^{k+1} \frac{n+2}{n(n+1)2^n} &= \sum_{r=1}^{k} \frac{n+2}{n(n+1)2^n} + \frac{k+3}{(k+1)(k+2)2^{k+1}} \\
&= 1 - \frac{1}{(k+1)2^k} + \frac{k+3}{(k+1)(k+2)2^{k+1}} \\
&= 1 + \frac{-2(k+2)+k+3}{(k+1)(k+2)2^{k+1}} \\
&= 1 + \frac{-k-1}{(k+1)(k+2)2^{k+1}} \\
&= 1 - \frac{1}{(k+1+1)2^{k+1}}
\end{aligned}$$

Therefore, the summation formula is true when $n = k + 1$.

If the summation formula is true for $n = k$ then it is shown to be true for $n = k + 1$. As the result is true for $n = 1$, it is now also true for all $n \geqslant 1$ and $n \in \mathbb{Z}^+$ by mathematical induction.

2

For $n=1$ 10 +192 + 5 = 207= $9\times 23 \Rightarrow \mathrm{H}_1$ is true.	**B1**
Assume H_k is true for some positive integer $k \Rightarrow 10^n + 3.4^{n+2} + 5 = 9\alpha$	**B1**
Let f(n) = $10^n + 3.4^{n+2} + 5$	
Hence $\mathrm{f}(n+1) - \mathrm{f}(n) = 10^n(10-1) + 3.4^{n+2}(4-1)$	**M1**
$= 9\left(10^n + 4^{n+2}\right)$	
$= 9\beta$	**A1**
Hence $\mathrm{f}(n+1)\left(=9(\beta+\alpha)\right) \Rightarrow \mathrm{H}_{k+1}$ is true	**A1**
H_1 is true and $\mathrm{H}_k \Rightarrow \mathrm{H}_{k+1}$, hence by PMI H_n is true for all positive integers n.	**A1**
N.B. Or can show $\mathrm{f}(n+1) = 9\left(10\alpha - 2.4^{n+2} - 5\right)$ for **M1A1A1**. (3rd ,4th&5th marks)	[6]

3

(States proposition.)	$(\mathrm{P}_n : u_n = 4\left(\frac{3}{4}\right)^n - 2)$	
Proves base case.	Let $n = 1$ $4 \times \frac{3}{4} - 2 = 3 - 2 = 1 \Rightarrow \mathrm{P}_1$ true.	B1
States Inductive hypothesis.	Assume P_k is true for some k.	B1
Proves inductive step.	$u_{k+1} = \dfrac{3\left\{4\left(\frac{3}{4}\right)^k - 2\right\} - 2}{4} = 4.\frac{3}{4}.\left(\frac{3}{4}\right)^k - \frac{6+2}{4}$	M1
	$= 4.\left(\frac{3}{4}\right)^{k+1} - 2 \quad \therefore \mathrm{P}_k \Rightarrow \mathrm{P}_{k+1}$	A1
States conclusion.	$\therefore$ By PMI P_n is true $\forall$ positive integers.	A1

4

$2! - S_1 = 1$, $3! - S_2 = 1$, $4! - S_3 = 1$, $5! - S_4 = 1$ (Two correct B1, all four correct B2)	B2,1,0 (2)
$S_n = (n+1)! - 1$	B1 (1)
$2! - 1 = 2 - 1 = 1 \Rightarrow H_1$ is true.	B1
H_k: $S_k = (k+1)! - 1$	B1
$(k+1)! - 1 + (k+1) \times (k+1)!$ $= (k+1)!(1 + k + 1) - 1$	M1
$= ([k+1]+1)! - 1$ Hence $H_k \Rightarrow H_{k+1}$ So result holds for all positive integers (by PMI).	A1 (4) **[7]**

5

$a_1 > 5$ (given) $\Rightarrow \mathrm{H}_1$ is true.	B1
Assume H_k is true for some positive integer k, i.e. $a_k = 5 + \delta$, where $\delta > 0$.	B1
$a_{k+1} - 5 = \dfrac{4a_k^{\ 2} + 25}{5a_k} - 5 = \dfrac{4a_k^{\ 2} + 25 - 25a_k}{5a_k} = \dfrac{(4a_k - 5)(a_k - 5)}{5a_k} > 0$ $, \Rightarrow a_{k+1} > 5$	M1A1
Or	
$a_{k+1} = \dfrac{4}{5}(5+\delta) + \dfrac{5}{5+\delta}$, $= 4 + \dfrac{4}{5}\delta + (1 - \dfrac{\delta}{5} + \dfrac{\delta^2}{25} - ...)$ for $0 < \delta < 5$ $= 5 + \dfrac{3}{5}\delta + 0(\delta^2) \geqslant a_{k+1} > 5$, ($\delta \geqslant 5$ is trivial).	(M1) (A1)
$\mathrm{H}_k \Rightarrow \mathrm{H}_{k+1}$ and H_1 is true, hence by mathematical induction, the result is true for all $n \in \mathbf{Z}^+$ (N.B. The minimum requirement is ‘true for all positive integers’.)	A1 (5)
$a_{k+1} - a_k = \dfrac{5}{a_k} - \dfrac{1}{5}a_k$	M1
$\dfrac{5}{a_k} < 1$ and $\dfrac{1}{5}a_k > 1 \Rightarrow a_{k+1} - a_k < 0 \Rightarrow a_{k+1} < a_k$	A1 (2)
	Total: 7

6

(i)	Differentiates once,	$y' = 2(1+x)\ln(1+x) + (1+x)$	B1	
	twice	$y'' = 2\ln(1+x) + 3$	B1	
	and three times.	$y''' = \dfrac{2}{1+x}$	B1	3
		(Allow B1√ if constant term in previous line incorrect.)		
(ii)	Proves base case.	$\dfrac{d^3y}{dx^3} = \dfrac{(-1)^2.2.0!}{1+x} = \dfrac{2}{1+x} \Rightarrow H_3$ is true.	B1	
	States inductive hypothesis.	$H_k : \dfrac{d^k y}{dx^k} = \dfrac{(-1)^{k-1}.2.(k-3)!}{(1+x)^{k-2}}$ for some k.	B1	
	Differentiates	$\dfrac{d^{k+1}y}{dx^{k+1}} = (-1)^{k-1}.2(k-3)!(-1)(k-2)(1+x)^{-(k-1)}$	M1	
	Proves inductive step and	$= \dfrac{(-1)^k.2.(k-2)!}{(1+x)^{k-1}} \Rightarrow H_{k+1}$ is true	A1	
	states conclusion.	Hence by PMI H_n is true for all integers $\geqslant 3$	A1	5

7

States proposition.	Let P_n be the proposition: $\mathbf{A} = \begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix} \Rightarrow \mathbf{A}^n = \begin{pmatrix} 2^n & 3(2^n - 1) \\ 0 & 1 \end{pmatrix}$	
Shows base case is true.	$\mathbf{A}^1 = \begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2^1 & 3\times(2-1) \\ 0 & 1 \end{pmatrix} \Rightarrow P_1$ is true.	B1
	Assume P_k is true for some integer k.	B1
Proves inductive step.	$\mathbf{A}^{k+1} = \begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2^k & 3(2^k - 1) \\ 0 & 1 \end{pmatrix}$	M1
	$= \begin{pmatrix} 2^{k+1} & 3.2(2^k - 1) + 3 \\ 0 & 1 \end{pmatrix}$	
	$= \begin{pmatrix} 2^{k+1} & 3(2^{k+1} - 1) \\ 0 & 1 \end{pmatrix}$	A1
States conclusion.	Since P_1 is true and $P_k \Rightarrow P_{k+1}$, hence by PMI P_n is true $\forall$ positive integers n.	A1

Most candidates found this question beyond them and so there were very few complete and correct responses.

(i) At this level it was to be expected that the successive differentiation of $\frac{\ln x}{x}$ with respect to x would be a routine task for candidates. Nevertheless, a significant number of elementary errors appeared and this indicated a deficiency in basic mathematical technique in the candidature. Beyond that, some candidates did not even comprehend that three differentiations were required in order to establish the values of a_1, a_2, a_3, and this lack of perception inevitably led to an incorrect conjecture for the form of a_n, as required in the remainder of this question.

(ii) A minority of candidates wrote down a correct inductive hypothesis. Among those who did work from $H_k : a_k = (-1)^k k!$, few went on to prove convincingly that $H_k \Rightarrow a_{k+1} = (-1)^{k+1}(k + 1)!$ and hence to complete the inductive argument. Again, it was evident that lack of technique was the main cause of failure.

Answers: **(i)** $a_1 = -1$, $a_2 = 2$, $a_3 = -6$; **(ii)** $a_n = (1)^n n!$