REVIEW PROBLEMS

$\mathbb{F}$: $\mathbb{R}$ or $\mathbb{C}$

V: v.s. of all sequences $\{a_n\}$ of elements in $\mathbb{F}$ such that $a_n \neq 0$ for only finitely many $n \in \mathbb{N}$

(so: if $\{a_n\} \in V$, then there exists $m \in \mathbb{N}$ such that $a_k = 0$ for all $k \geq m$)

operations : componentwise

$$\{a_n\} + \{b_n\} := \{a_n + b_n\}$$

$$c\{a_n\} := \{c a_n\}$$

For $\{a_n\}, \{b_n\} \in V$,

$$\langle \{a_n\}, \{b_n\} \rangle := \sum_{i=1}^{\infty} a_i \overline{b_i}$$

#11 : Show $\langle \, , \rangle$ is an inner product

Define $e_k := \{\delta_{k,n}\}$ δ: Kronecker δ

$$e_1 = (1, 0, 0, \ldots)$$

$$e_2 = (0, 1, 0, 0 \ldots)$$

$$e_6 = (0, 0, 0, 0, 0, 1, 0, \ldots)$$ etc.

#12: $(e_1, e_2, \dots)$ is an orthonormal basis

Define $w_n := e_1 + e_n, \quad n \geq 2$

so: $w_2 = (1, 1, 0, 0, \dots)$

$w_3 = (1, 0, 1, 0, 0, \dots)$

$w_5 = (1, 0, 0, 0, 1, 0, \dots)$

$W := \text{span}(\{w_n : n \geq 2\})$

#13: (a) $e_1 \notin W$, so $W \neq V$

(b) $W^\perp = \{0\}$, so $W \neq (W^\perp)^\perp$

$T : V \to V$ defined by

$$\boxed{T(\{a_n\})_k := \sum_{i=k}^{\infty} a_i}$$

$$T(\{a_n\}) = \left\{ \sum_{i=n}^{\infty} a_i \right\}_n$$

k^{TH} term in <u>sequence</u> $T(\{a_n\})$ is the sum <u>after</u> the first $k-1$ terms (starting w/ k^{TH} term)

Ex $T(e_5)$

$$e_5 = (0, 0, 0, 0, 1, 0, 0, \ldots)$$

$$T(e_5) = (1, 1, 1, 1, 1, 0, 0, \ldots)$$

(14) (a): Prove $T(e_n) = \sum_{i=1}^{n} e_i$

(b): Prove that T has no adjoint.

HINT: consider $\langle T(e_k), e_n \rangle$, where $k > n$.

Q: Can we assume $\det(AB) = \det(A)\det(B)$ on #5? YES.