hw05

February 15, 2019

1 Problem 1

1.1 Starting from $y_0 = 0$

1.1.1 Defining things

```
In [1]: %%file problem_1a1.m
        function [t,w] = rungekutta_a
        h = 0.1;
        t(1) = 0;
        w(1) = 0;
        fprintf(' Step 0: t = %12.8f, w = %12.8f\n', t, w);
        for i=1:10
            k1 = h*f(t(i),w(i));
            k2 = h*f(t(i)+h/2, w(i)+k1/2);
            k3 = h*f(t(i)+h/2, w(i)+k2/2);
            k4 = h*f(t(i)+h, w(i)+k3);
            w(i+1) = w(i) + (k1+2*k2+2*k3+k4)/6;
            t(i+1) = t(i) + h;
            fprintf('Step %d: t = %6.4f, w = %18.15f\n', i, t(i+1), w(i+1));
        end
        plot(t,w,'b--o')
        %%%%%%%%%%%%%%%%%
        function v = f(t,y)
        v = y^2+1;
```

Created file '/home/xren/Dropbox/numan/hw05/problem_1a1.m'.

```
In [2]: %%file problem_1a2.m
        function [t,w] = rungekutta_a
        h = 0.05;
        t(1) = 0;
        w(1) = 0;
        fprintf(' Step 0: t = %12.8f, w = %12.8f\n', t, w);
        for i=1:20
            k1 = h*f(t(i),w(i));
            k2 = h*f(t(i)+h/2, w(i)+k1/2);
```

```matlab
        k3 = h*f(t(i)+h/2, w(i)+k2/2);
        k4 = h*f(t(i)+h, w(i)+k3);
        w(i+1) = w(i) + (k1+2*k2+2*k3+k4)/6;
        t(i+1) = t(i) + h;
        fprintf('Step %d: t = %6.4f, w = %18.15f\n', i, t(i+1), w(i+1));
    end
    plot(t,w,'b--o')
    %%%%%%%%%%%%%%%%%%
    function v = f(t,y)
    v = y^2+1;
```

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1.1.2 Calculating timeseries

In [3]: [t1, w1] = problem_1a1

        [t2, w2] = problem_1a2

```
 Step 0: t =   0.00000000, w =   0.00000000
Step 1: t = 0.1000, w =  0.100334589078164
Step 2: t = 0.2000, w =  0.202709878231719
Step 3: t = 0.3000, w =  0.309336039344978
Step 4: t = 0.4000, w =  0.422792992854121
Step 5: t = 0.5000, w =  0.546302307583634
Step 6: t = 0.6000, w =  0.684136756653080
Step 7: t = 0.7000, w =  0.842288569394973
Step 8: t = 0.8000, w =  1.029639061115587
Step 9: t = 0.9000, w =  1.260158782791583
Step 10: t = 1.0000, w =  1.557406442844996
```

t1 =

  Columns 1 through 7

         0   0.1000   0.2000   0.3000   0.4000   0.5000   0.6000

  Columns 8 through 11

    0.7000   0.8000   0.9000   1.0000

w1 =

  Columns 1 through 7

         0   0.1003   0.2027   0.3093   0.4228   0.5463   0.6841

```
   Columns 8 through 11

    0.8423     1.0296     1.2602     1.5574

 Step 0: t =   0.00000000, w =   0.00000000
Step 1: t = 0.0500, w =  0.050041705773956
Step 2: t = 0.1000, w =  0.100334666951460
Step 3: t = 0.1500, w =  0.151135210554240
Step 4: t = 0.2000, w =  0.202710025897922
Step 5: t = 0.2500, w =  0.255341909881852
Step 6: t = 0.3000, w =  0.309336237055508
Step 7: t = 0.3500, w =  0.365028481737598
Step 8: t = 0.4000, w =  0.422793205980792
Step 9: t = 0.4500, w =  0.483055054310007
Step 10: t = 0.5000, w =  0.546302481394858
Step 11: t = 0.5500, w =  0.613105209448994
Step 12: t = 0.6000, w =  0.684136811257641
Step 13: t = 0.6500, w =  0.760204411363160
Step 14: t = 0.7000, w =  0.842288404922268
Step 15: t = 0.7500, w =  0.931596499674640
Step 16: t = 0.8000, w =  1.029638614595370
Step 17: t = 0.8500, w =  1.138332789155729
Step 18: t = 0.9000, w =  1.260158307118358
Step 19: t = 0.9500, w =  1.398382674997960
Step 20: t = 1.0000, w =  1.557407759438250

t2 =

  Columns 1 through 7

        0    0.0500    0.1000    0.1500    0.2000    0.2500    0.3000

  Columns 8 through 14

   0.3500    0.4000    0.4500    0.5000    0.5500    0.6000    0.6500

  Columns 15 through 21

   0.7000    0.7500    0.8000    0.8500    0.9000    0.9500    1.0000


w2 =

  Columns 1 through 7

        0    0.0500    0.1003    0.1511    0.2027    0.2553    0.3093
```

```
Columns 8 through 14

   0.3650    0.4228    0.4831    0.5463    0.6131    0.6841    0.7602

Columns 15 through 21

   0.8423    0.9316    1.0296    1.1383    1.2602    1.3984    1.5574
```
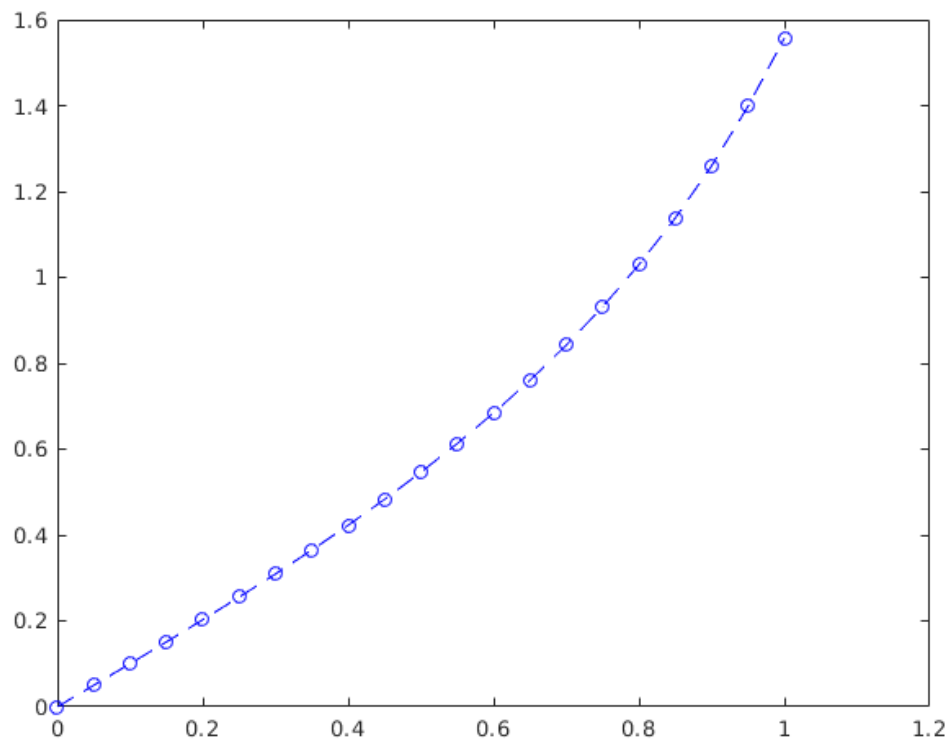

1.2 Starting from $y_0 = 1$

```
In [4]: %%file problem_2a1.m
        function [t,w] = rungekutta_a
        h = 0.1;
        t(1) = 0;
        w(1) = 1;
        fprintf(' Step 0: t = %12.8f, w = %12.8f\n', t, w);
        for i=1:10
            k1 = h*f(t(i),w(i));
            k2 = h*f(t(i)+h/2, w(i)+k1/2);
```

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```matlab
        k3 = h*f(t(i)+h/2, w(i)+k2/2);
        k4 = h*f(t(i)+h, w(i)+k3);
        w(i+1) = w(i) + (k1+2*k2+2*k3+k4)/6;
        t(i+1) = t(i) + h;
        fprintf('Step %d: t = %6.4f, w = %18.15f\n', i, t(i+1), w(i+1));
    end
    plot(t,w,'b--o')
    %%%%%%%%%%%%%%%%%%
    function v = f(t,y)
    v = y^2+1;
```

Created file '/home/xren/Dropbox/numan/hw05/problem_2a1.m'.

In [5]: %%file problem_2a2.m
```matlab
        function [t,w] = rungekutta_a
        h = 0.05;
        t(1) = 0;
        w(1) = 1;
        fprintf(' Step 0: t = %12.8f, w = %12.8f\n', t, w);
        for i=1:20
            k1 = h*f(t(i),w(i));
            k2 = h*f(t(i)+h/2, w(i)+k1/2);
            k3 = h*f(t(i)+h/2, w(i)+k2/2);
            k4 = h*f(t(i)+h, w(i)+k3);
            w(i+1) = w(i) + (k1+2*k2+2*k3+k4)/6;
            t(i+1) = t(i) + h;
            fprintf('Step %d: t = %6.4f, w = %18.15f\n', i, t(i+1), w(i+1));
        end
        plot(t,w,'b--o')
        %%%%%%%%%%%%%%%%%%
        function v = f(t,y)
        v = y^2+1;
```

Created file '/home/xren/Dropbox/numan/hw05/problem_2a2.m'.

In [6]: [t3, w3] = problem_2a1

        [t4, w4] = problem_2a2

```
 Step 0: t =   0.00000000, w =   1.00000000
Step 1: t = 0.1000, w =  1.223048913836784
Step 2: t = 0.2000, w =  1.508496167191276
Step 3: t = 0.3000, w =  1.895754160233435
Step 4: t = 0.4000, w =  2.464899686957883
Step 5: t = 0.5000, w =  3.407820425151801
Step 6: t = 0.6000, w =  5.327896816590514
Step 7: t = 0.7000, w = 11.553932075720173
```

5

```
Step 8: t = 0.8000, w = 192.169924962969446
Step 9: t = 0.9000, w = 311985276702220224.000000000000000
Step 10: t = 1.0000, w = 327819201500964647600496870626046666110422866612952625313621390536680
```

t3 =

  Columns 1 through 7

         0   0.1000   0.2000   0.3000   0.4000   0.5000   0.6000

  Columns 8 through 11

    0.7000   0.8000   0.9000   1.0000

w3 =

  1.0e+260 *

  Columns 1 through 7

    0.0000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000

  Columns 8 through 11

    0.0000   0.0000   0.0000   3.2782

```
 Step 0: t =   0.00000000, w =   1.00000000
Step 1: t = 0.0500, w =   1.105355603267246
Step 2: t = 0.1000, w =   1.223048901982997
Step 3: t = 0.1500, w =   1.356087867029104
Step 4: t = 0.2000, w =   1.508497619150346
Step 5: t = 0.2500, w =   1.685796252646820
Step 6: t = 0.3000, w =   1.895764601593182
Step 7: t = 0.3500, w =   2.149746231412271
Step 8: t = 0.4000, w =   2.464959126034956
Step 9: t = 0.4500, w =   2.868874581383584
Step 10: t = 0.5000, w =   3.408197466764288
Step 11: t = 0.5500, w =   4.169284574052874
Step 12: t = 0.6000, w =   5.331563775771916
Step 13: t = 0.6500, w =   7.338975331134137
Step 14: t = 0.7000, w =  11.668014352923244
Step 15: t = 0.7500, w =  27.694702600289805
Step 16: t = 0.8000, w = 1323.673567972432011
Step 17: t = 0.8500, w = 1399640540896851618811640320.000000000000000
Step 18: t = 0.9000, w =                Inf
Step 19: t = 0.9500, w =                Inf
Step 20: t = 1.0000, w =                Inf
```

6

```
t4 =

  Columns 1 through 7

        0    0.0500    0.1000    0.1500    0.2000    0.2500    0.3000

  Columns 8 through 14

   0.3500    0.4000    0.4500    0.5000    0.5500    0.6000    0.6500

  Columns 15 through 21

   0.7000    0.7500    0.8000    0.8500    0.9000    0.9500    1.0000


w4 =

   1.0e+26 *

  Columns 1 through 7

   0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000

  Columns 8 through 14

   0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000

  Columns 15 through 21

   0.0000    0.0000    0.0000    1.3996       Inf       Inf       Inf
```
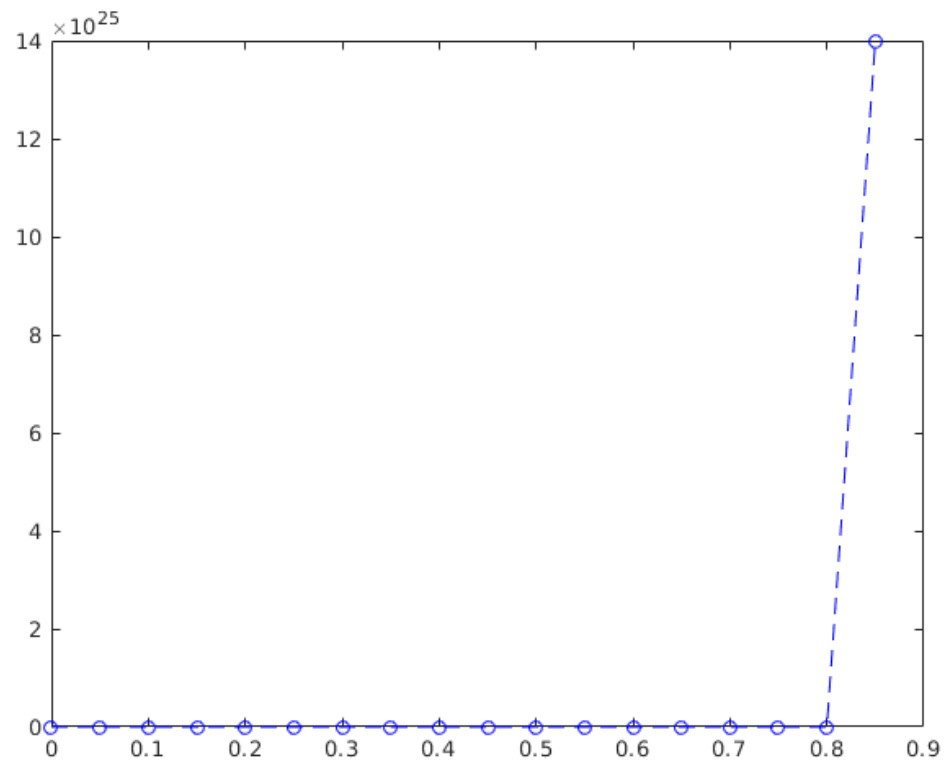
1.2.1 True y values
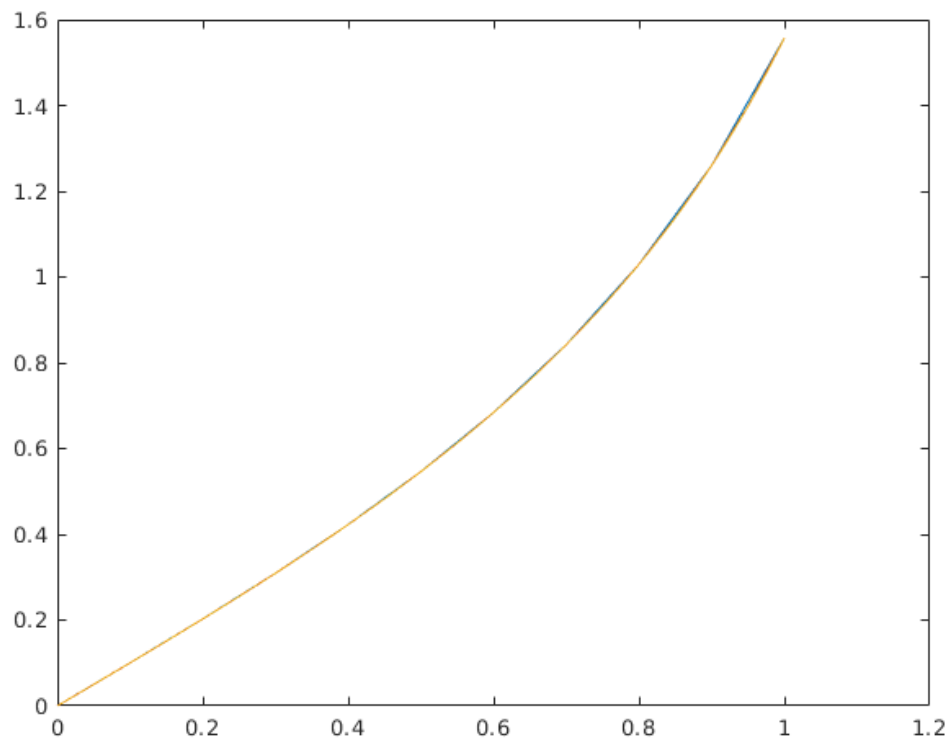
```
In [7]: tt = linspace(0, 1, 10^4);

In [8]: ww1 = tan(tt);

In [9]: ww2 = tan(tt+0.25*pi);
```
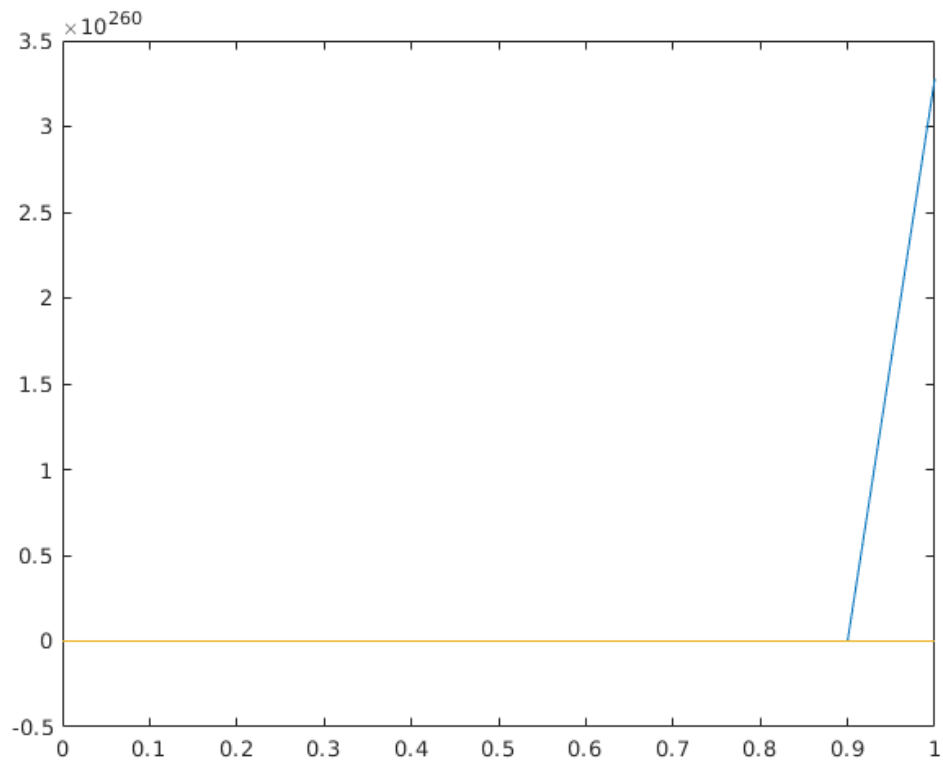
2 Ploting more stable case

```
In [10]: plot(t1, w1, t2, w2, tt, ww1)
```

3   Plotting Crazy Case

```
In [11]: plot(t3, w3, t4, w4, tt, ww2)
```

3.1 (7a)

The derivative of arctan(x) is $\frac{1}{x^2+1}$. With that in mind, consider the equation
$y = \tan(t + c)$.
Applying arctan to both sides yields $\arctan(y) = t + c$.
Implicit differentiation of both sides gives
$\frac{1}{y^2+1}dy = dt$
Hence $\frac{dy}{dt} = \frac{1}{y^2+1}$

3.2 (7b)

For $y = \tan(t + c)$, setting $y_0 = \tan(0 + c)$ gives $c = \arctan(y_0)$.

4 Problem 2

```
In [12]: %%file rk45_2a.m
         function rk45
         epsilon = 10^(-7);
         h = 0.5;
         hh(1) = h;
```

```matlab
t = 0;
w = 0.5;
tt(1) = t
ww(1) = w;
wer(1)=0;
i = 1;
ii(1) = 1;
fprintf('Step %d: t = %6.4f, w = %18.15f\n', i, t(1), w(1));
tmax = 6;
while t<tmax
h = min(h, tmax-t);
k1 = h*f(t,w);
k2 = h*f(t+h/4, w+k1/4);
k3 = h*f(t+3*h/8, w+3*k1/32+9*k2/32);
k4 = h*f(t+12*h/13, w+1932*k1/2197-7200*k2/2197+7296*k3/2197);
k5 = h*f(t+h, w+439*k1/216-8*k2+3680*k3/513-845*k4/4104);
k6 = h*f(t+h/2, w-8*k1/27+2*k2-3544*k3/2565+1859*k4/4104-11*k5/40);
w1 = w + 25*k1/216+1408*k3/2565+2197*k4/4104-k5/5;
w2 = w + 16*k1/135+6656*k3/12825+28561*k4/56430-9*k5/50+2*k6/55;
R = abs(w1-w2)/h;
delta = 0.84*(epsilon/R)^(1/4);
if R<=epsilon
t = t+h;
w = w1;
i = i+1;
ii(i) = i;
tt(i) = t;
ww(i)= w;
wer(i)= abs(-(1/2)*(exp(t)) + t^2 + 2*t + 1 - w);
hh(i-1) = tt(i)-tt(i-1);
%fprintf('Step %d: t = %6.4f, w = %18.15f\n', i, t, w);
fprintf('Step %d: t = %6.4f, w = %18.15f, ye = %18.15f\n', i, t, w, wer(i));
h = delta*h;
hh(i)=h;
else
h = delta*h;
end
end
subplot(1,3,1)
hold on
plot(tt, g(tt), 'r--x')
plot(tt,ww,'b--o')
hold off
subplot(1,3,2)
plot(tt,wer,'r--x')
subplot(1,3,3)
plot(ii,hh,'b--o')
%%%  ye = -(1/2)*(e^t) + t^2 + 2t + 1
```

```
%%%%%%%%%%%%%%%%%
function v = f(t,y)
v = y-t^2+1;
%%%%%%%%%%%%%%%%%%%
function y = g(t)
y = -0.5*exp(t) + t*2 + 2*t + 1;
```

Created file '/home/xren/Dropbox/numan/hw05/rk45_2a.m'.

In [13]: rk45_2a

tt =

     0

Step 1: t = 0.0000, w = 0.500000000000000
Step 2: t = 0.0768, w = 0.619559576818489, ye = 0.000000004356310
Step 3: t = 0.1503, w = 0.742030657459462, ye = 0.000000008095116
Step 4: t = 0.2243, w = 0.873106716319940, ye = 0.000000012136066
Step 5: t = 0.2988, w = 1.012834751096177, ye = 0.000000016509430
Step 6: t = 0.3741, w = 1.161276934094872, ye = 0.000000021248797
Step 7: t = 0.4501, w = 1.318522682964899, ye = 0.000000026392578
Step 8: t = 0.5270, w = 1.484698491059141, ye = 0.000000031985084
Step 9: t = 0.6048, w = 1.659982171228079, ye = 0.000000038077944
Step 10: t = 0.6839, w = 1.844624091975887, ye = 0.000000044732044
Step 11: t = 0.7643, w = 2.038979977526234, ye = 0.000000052020233
Step 12: t = 0.8464, w = 2.243563750534847, ye = 0.000000060031217
Step 13: t = 0.9306, w = 2.459137436638073, ye = 0.000000068875341
Step 14: t = 1.0174, w = 2.686875238646822, ye = 0.000000078693486
Step 15: t = 1.1077, w = 2.928692429807854, ye = 0.000000089671059
Step 16: t = 1.2029, w = 3.187999343857812, ye = 0.000000102059106
Step 17: t = 1.3060, w = 3.471831256998807, ye = 0.000000116190241
Step 18: t = 1.4243, w = 3.799712034857807, ye = 0.000000132217372
Step 19: t = 1.5434, w = 4.128677161592079, ye = 0.000000144236457
Step 20: t = 1.6399, w = 4.391627451499825, ye = 0.000000154580586
Step 21: t = 1.7361, w = 4.648733274368589, ye = 0.000000163732831
Step 22: t = 1.8230, w = 4.874237636240196, ye = 0.000000173163049
Step 23: t = 1.9033, w = 5.075225006330212, ye = 0.000000182940904
Step 24: t = 1.9790, w = 5.256600974355428, ye = 0.000000193052399
Step 25: t = 2.0510, w = 5.420816492929601, ye = 0.000000203510513
Step 26: t = 2.1202, w = 5.569190978379273, ye = 0.000000214334375
Step 27: t = 2.1869, w = 5.702478047407885, ye = 0.000000225543157
Step 28: t = 2.2514, w = 5.821108283124221, ye = 0.000000237154989
Step 29: t = 2.3141, w = 5.925309945590516, ye = 0.000000249186939
Step 30: t = 2.3750, w = 6.015176007347172, ye = 0.000000261655212
Step 31: t = 2.4345, w = 6.090704585452213, ye = 0.000000274575355
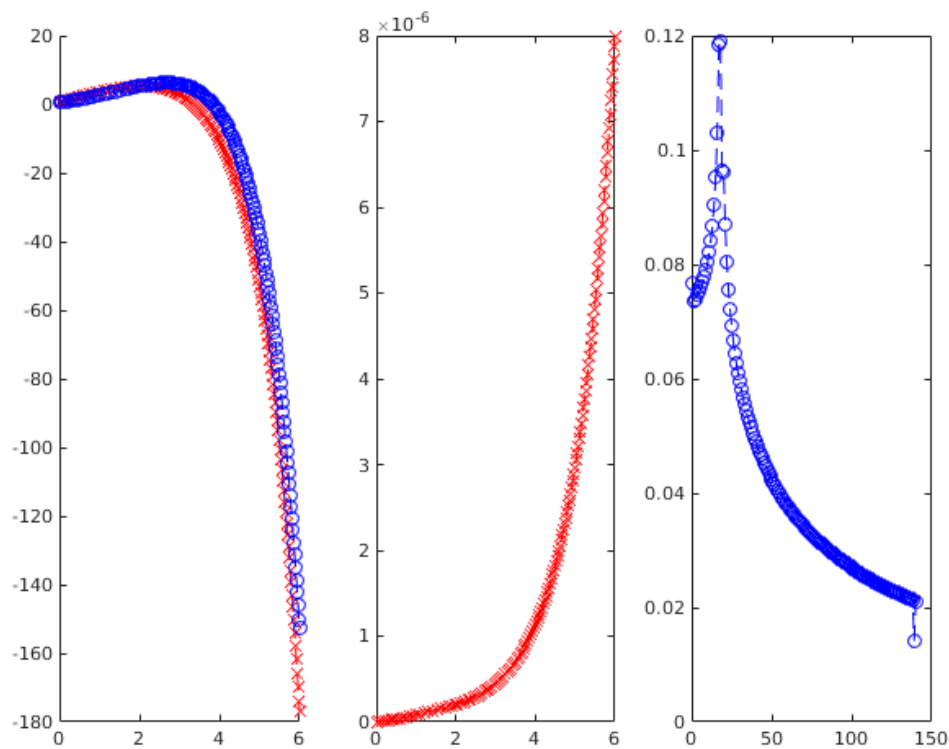```

```
Step 32: t = 2.4925, w =  6.151824626708730, ye =  0.000000287962417
Step 33: t = 2.5492, w =  6.198413140406506, ye =  0.000000301831083
Step 34: t = 2.6048, w =  6.230306992097271, ye =  0.000000316195784
Step 35: t = 2.6592, w =  6.247311371013497, ye =  0.000000331070776
Step 36: t = 2.7125, w =  6.249205903295523, ye =  0.000000346470188
Step 37: t = 2.7649, w =  6.235749217335863, ye =  0.000000362408083
Step 38: t = 2.8163, w =  6.206682398321990, ye =  0.000000378898496
Step 39: t = 2.8669, w =  6.161731659586748, ye =  0.000000395955452
Step 40: t = 2.9166, w =  6.100610404531749, ye =  0.000000413593006
Step 41: t = 2.9655, w =  6.023020840953785, ye =  0.000000431825250
Step 42: t = 3.0136, w =  5.928655298682089, ye =  0.000000450666335
Step 43: t = 3.0610, w =  5.817197281328158, ye =  0.000000470130486
Step 44: t = 3.1077, w =  5.688322288968972, ye =  0.000000490231998
Step 45: t = 3.1538, w =  5.541698545089001, ye =  0.000000510985273
Step 46: t = 3.1992, w =  5.376987489724163, ye =  0.000000532404799
Step 47: t = 3.2439, w =  5.193844343833650, ye =  0.000000554505173
Step 48: t = 3.2881, w =  4.991918489906511, ye =  0.000000577301091
Step 49: t = 3.3317, w =  4.770853739193912, ye =  0.000000600807380
Step 50: t = 3.3747, w =  4.530288588030608, ye =  0.000000625038975
Step 51: t = 3.4172, w =  4.269856633335513, ye =  0.000000650010929
Step 52: t = 3.4591, w =  3.989186454858280, ye =  0.000000675738425
Step 53: t = 3.5006, w =  3.687902169149752, ye =  0.000000702236764
Step 54: t = 3.5415, w =  3.365623250101212, ye =  0.000000729521389
Step 55: t = 3.5820, w =  3.021964778795005, ye =  0.000000757607858
Step 56: t = 3.6220, w =  2.656537537867888, ye =  0.000000786511867
Step 57: t = 3.6616, w =  2.268948111871892, ye =  0.000000816249256
Step 58: t = 3.7007, w =  1.858798977545424, ye =  0.000000846835980
Step 59: t = 3.7394, w =  1.425688515925411, ye =  0.000000878288137
Step 60: t = 3.7777, w =  0.969211131711374, ye =  0.000000910621970
Step 61: t = 3.8155, w =  0.488957184825513, ye =  0.000000943853850
Step 62: t = 3.8530, w = -0.015486778772707, ye =  0.000000978000291
Step 63: t = 3.8901, w = -0.544538148429651, ye =  0.000001013077935
Step 64: t = 3.9268, w = -1.098618149897599, ye =  0.000001049103582
Step 65: t = 3.9631, w = -1.678151837542889, ye =  0.000001086094158
Step 66: t = 3.9991, w = -2.283568032544637, ye =  0.000001124066726
Step 67: t = 4.0347, w = -2.915299410092598, ye =  0.000001163038507
Step 68: t = 4.0700, w = -3.573782256841131, ye =  0.000001203026859
Step 69: t = 4.1050, w = -4.259456744002757, ye =  0.000001244049263
Step 70: t = 4.1396, w = -4.972766656590424, ye =  0.000001286123365
Step 71: t = 4.1739, w = -5.714159515333062, ye =  0.000001329266924
Step 72: t = 4.2078, w = -6.484086856112243, ye =  0.000001373497876
Step 73: t = 4.2415, w = -7.283003484518762, ye =  0.000001418834293
Step 74: t = 4.2749, w = -8.111368045093142, ye =  0.000001465294359
Step 75: t = 4.3079, w = -8.969642575580551, ye =  0.000001512896423
Step 76: t = 4.3407, w = -9.858293931009030, ye =  0.000001561659015
Step 77: t = 4.3732, w = -10.777790885586340, ye =  0.000001611600705
Step 78: t = 4.4054, w = -11.728606961765948, ye =  0.000001662740313
Step 79: t = 4.4374, w = -12.711220117075376, ye =  0.000001715096788
```

```
Step 80: t = 4.4690, w = -13.726110304650929, ye =  0.000001768689165
Step 81: t = 4.5004, w = -14.773761671814258, ye =  0.000001823536614
Step 82: t = 4.5316, w = -15.854662441182883, ye =  0.000001879658539
Step 83: t = 4.5625, w = -16.969304766206434, ye =  0.000001937074440
Step 84: t = 4.5931, w = -18.118183253243835, ye =  0.000001995803920
Step 85: t = 4.6235, w = -19.301797251281549, ye =  0.000002055866794
Step 86: t = 4.6537, w = -20.520649820413635, ye =  0.000002117283021
Step 87: t = 4.6836, w = -21.775246929074875, ye =  0.000002180072602
Step 88: t = 4.7133, w = -23.066098622413651, ye =  0.000002244255800
Step 89: t = 4.7427, w = -24.393718503061720, ye =  0.000002309852977
Step 90: t = 4.7719, w = -25.758623821487912, ye =  0.000002376884638
Step 91: t = 4.8009, w = -27.161335681918271, ye =  0.000002445371432
Step 92: t = 4.8297, w = -28.602377658396083, ye =  0.000002515334131
Step 93: t = 4.8583, w = -30.082280164247329, ye =  0.000002586793741
Step 94: t = 4.8866, w = -31.601573840771177, ye =  0.000002659771315
Step 95: t = 4.9148, w = -33.160795292148826, ye =  0.000002734288088
Step 96: t = 4.9427, w = -34.760479681365304, ye =  0.000002810365309
Step 97: t = 4.9705, w = -36.401176800229315, ye =  0.000002888024774
Step 98: t = 4.9980, w = -38.083428753477264, ye =  0.000002967287962
Step 99: t = 5.0254, w = -39.807787833331204, ye =  0.000003048176758
Step 100: t = 5.0525, w = -41.574806637113191, ye =  0.000003130713061
Step 101: t = 5.0795, w = -43.385044623160574, ye =  0.000003214919005
Step 102: t = 5.1063, w = -45.239058410710932, ye =  0.000003300816715
Step 103: t = 5.1328, w = -47.137417694772374, ye =  0.000003388428688
Step 104: t = 5.1593, w = -49.080689860183995, ye =  0.000003477777504
Step 105: t = 5.1855, w = -51.069443990862048, ye =  0.000003568885660
Step 106: t = 5.2115, w = -53.104262472460924, ye =  0.000003661776226
Step 107: t = 5.2374, w = -55.185721103652234, ye =  0.000003756472026
Step 108: t = 5.2631, w = -57.314400831100230, ye =  0.000003852996109
Step 109: t = 5.2887, w = -59.490890165865522, ye =  0.000003951371738
Step 110: t = 5.3140, w = -61.715779358499006, ye =  0.000004051622369
Step 111: t = 5.3392, w = -63.989667750330881, ye =  0.000004153771712
Step 112: t = 5.3643, w = -66.313148359246682, ye =  0.000004257843230
Step 113: t = 5.3891, w = -68.686829230441148, ye =  0.000004363861095
Step 114: t = 5.4139, w = -71.111295532837005, ye =  0.000004471848428
Step 115: t = 5.4384, w = -73.587171906989212, ye =  0.000004581829884
Step 116: t = 5.4628, w = -76.115070465490447, ye =  0.000004693829851
Step 117: t = 5.4871, w = -78.695612500703291, ye =  0.000004807872912
Step 118: t = 5.5112, w = -81.329406383699435, ye =  0.000004923983127
Step 119: t = 5.5352, w = -84.017086957007066, ye =  0.000005042185705
Step 120: t = 5.5590, w = -86.759283042311523, ye =  0.000005162505545
Step 121: t = 5.5827, w = -89.556620381398645, ye =  0.000005284967358
Step 122: t = 5.6062, w = -92.409726289955103, ye =  0.000005409596085
Step 123: t = 5.6296, w = -95.319245267927528, ye =  0.000005536417120
Step 124: t = 5.6528, w = -98.285825217719051, ye =  0.000005665456257
Step 125: t = 5.6759, w = -101.310106710949583, ye =  0.000005796738847
Step 126: t = 5.6989, w = -104.392736258035697, ye =  0.000005930290456
Step 127: t = 5.7217, w = -107.534356463182363, ye =  0.000006066136464
```

```
Step 128: t = 5.7444, w = -110.735651531879753, ye =  0.000006204303972
Step 129: t = 5.7670, w = -113.997249005291977, ye =  0.000006344817905
Step 130: t = 5.7894, w = -117.319861424490099, ye =  0.000006487706486
Step 131: t = 5.8118, w = -120.704103500112083, ye =  0.000006632993902
Step 132: t = 5.8340, w = -124.150669819190682, ye =  0.000006780707835
Step 133: t = 5.8560, w = -127.660251841869069, ye =  0.000006930875600
Step 134: t = 5.8780, w = -131.233533440913618, ye =  0.000007083524338
Step 135: t = 5.8998, w = -134.871149948171194, ye =  0.000007238679046
Step 136: t = 5.9215, w = -138.573799482605637, ye =  0.000007396367437
Step 137: t = 5.9431, w = -142.342203539885475, ye =  0.000007556618073
Step 138: t = 5.9645, w = -146.177037309262147, ye =  0.000007719457699
Step 139: t = 5.9859, w = -150.078986355652944, ye =  0.000007884913401
Step 140: t = 6.0000, w = -152.714388749267499, ye =  0.000007997100056
```


In [14]: ```%%file rk45_2b.m```
```matlab
function rk45
epsilon = 10^(-7);
h = 0.1;
hh(1) = h;
t = 0;
```

```matlab
w = 0.5;
tt(1) = t
ww(1) = w;
wer(1)=0;
i = 1;
ii(1) = 1;
fprintf('Step %d: t = %6.4f, w = %18.15f\n', i, t(1), w(1));
tmax = 10;
while t<tmax
    h = min(h, tmax-t);
    k1 = h*f(t,w);
    k2 = h*f(t+h/4, w+k1/4);
    k3 = h*f(t+3*h/8, w+3*k1/32+9*k2/32);
    k4 = h*f(t+12*h/13, w+1932*k1/2197-7200*k2/2197+7296*k3/2197);
    k5 = h*f(t+h, w+439*k1/216-8*k2+3680*k3/513-845*k4/4104);
    k6 = h*f(t+h/2, w-8*k1/27+2*k2-3544*k3/2565+1859*k4/4104-11*k5/40);
    w1 = w + 25*k1/216+1408*k3/2565+2197*k4/4104-k5/5;
    w2 = w + 16*k1/135+6656*k3/12825+28561*k4/56430-9*k5/50+2*k6/55;
    R = abs(w1-w2)/h;
    delta = 0.84*(epsilon/R)^(1/4);
    if R<=epsilon
        t = t+h;
        w = w1;
        i = i+1;
        ii(i) = i;
        tt(i) = t;
        ww(i)= w;
        wer(i)= abs(1-0.5*exp(-10*t) - w);
        hh(i-1) = tt(i)-tt(i-1);
        %fprintf('Step %d: t = %6.4f, w = %18.15f\n', i, t, w);
        fprintf('Step %d: t = %6.4f, w = %18.15f, ye = %18.15f\n', i, t, w, wer(i));
        h = delta*h;
        hh(i)=h;
    else
        h = delta*h;
    end
end
subplot(1,3,1)
hold on
plot(tt,ww, 'b--o')
plot(tt, g(tt), 'r')
hold off
subplot(1,3,2)
plot(tt,wer,'r--x')
subplot(1,3,3)
plot(ii,hh,'b--o')
%%%  ye = -(1/2)*(e^t) + t^2 + 2t + 1
%%%%%%%%%%%%%%%%%
```

```
        function v = f(t,y)
        v = 10*(1-y);
        %%%%%%%%%%%%%%%%%%%
        function y = g(t)
        y = 1-0.5.*exp(-10*t);
```

Created file '/home/xren/Dropbox/numan/hw05/rk45_2b.m'.

In [15]: rk45_2b

tt =

     0

Step 1: t = 0.0000, w = 0.500000000000000
Step 2: t = 0.0049, w = 0.523790939913550, ye = 0.000000000185781
Step 3: t = 0.0101, w = 0.548170695974093, ye = 0.000000000434818
Step 4: t = 0.0154, w = 0.571570520699691, ye = 0.000000000672735
Step 5: t = 0.0208, w = 0.594042832515929, ye = 0.000000000901273
Step 6: t = 0.0263, w = 0.615612328800150, ye = 0.000000001120662
Step 7: t = 0.0318, w = 0.636303827540361, ye = 0.000000001331155
Step 8: t = 0.0374, w = 0.656141776321454, ye = 0.000000001533000
Step 9: t = 0.0431, w = 0.675150269795652, ye = 0.000000001726441
Step 10: t = 0.0489, w = 0.693353057174084, ye = 0.000000001911722
Step 11: t = 0.0547, w = 0.710773524002345, ye = 0.000000002089080
Step 12: t = 0.0607, w = 0.727434701126421, ye = 0.000000002258748
Step 13: t = 0.0667, w = 0.743359273813985, ye = 0.000000002420958
Step 14: t = 0.0728, w = 0.758569573754015, ye = 0.000000002575936
Step 15: t = 0.0790, w = 0.773087573239183, ye = 0.000000002723907
Step 16: t = 0.0853, w = 0.786934898033899, ye = 0.000000002865091
Step 17: t = 0.0917, w = 0.800132812518449, ye = 0.000000002999704
Step 18: t = 0.0982, w = 0.812702238700887, ye = 0.000000003127959
Step 19: t = 0.1048, w = 0.824663738452186, ye = 0.000000003250066
Step 20: t = 0.1115, w = 0.836037523620202, ye = 0.000000003366231
Step 21: t = 0.1183, w = 0.846843454481907, ye = 0.000000003476655
Step 22: t = 0.1252, w = 0.857101040285490, ye = 0.000000003581539
Step 23: t = 0.1323, w = 0.866829436798987, ye = 0.000000003681077
Step 24: t = 0.1395, w = 0.876047448506330, ye = 0.000000003775462
Step 25: t = 0.1468, w = 0.884773528305975, ye = 0.000000003864882
Step 26: t = 0.1542, w = 0.893025779382877, ye = 0.000000003949521
Step 27: t = 0.1618, w = 0.900821955294666, ye = 0.000000004029561
Step 28: t = 0.1695, w = 0.908179456090357, ye = 0.000000004105181
Step 29: t = 0.1773, w = 0.915115334215812, ye = 0.000000004176554
Step 30: t = 0.1853, w = 0.921646289574434, ye = 0.000000004243852
Step 31: t = 0.1935, w = 0.927788674184406, ye = 0.000000004307242
Step 32: t = 0.2018, w = 0.933558490501059, ye = 0.000000004366889
```
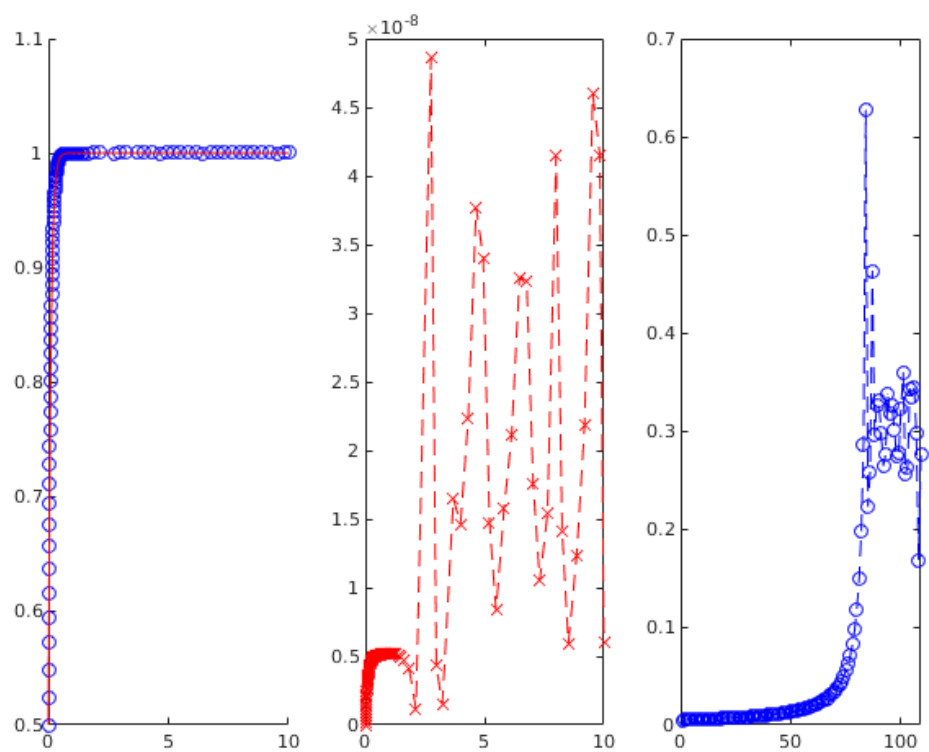
```
Step 33: t = 0.2103, w = 0.938971391798039, ye = 0.000000004422953
Step 34: t = 0.2190, w = 0.944042682192620, ye = 0.000000004475591
Step 35: t = 0.2279, w = 0.948787318883637, ye = 0.000000004524958
Step 36: t = 0.2369, w = 0.953219908837791, ye = 0.000000004571204
Step 37: t = 0.2462, w = 0.957354712635627, ye = 0.000000004614476
Step 38: t = 0.2556, w = 0.961205643403216, ye = 0.000000004654918
Step 39: t = 0.2653, w = 0.964786267339758, ye = 0.000000004692671
Step 40: t = 0.2752, w = 0.968109803567630, ye = 0.000000004727871
Step 41: t = 0.2854, w = 0.971189126728376, ye = 0.000000004760653
Step 42: t = 0.2958, w = 0.974036763778448, ye = 0.000000004791147
Step 43: t = 0.3065, w = 0.976664898373155, ye = 0.000000004819481
Step 44: t = 0.3174, w = 0.979085368466177, ye = 0.000000004845779
Step 45: t = 0.3287, w = 0.981309668766688, ye = 0.000000004870162
Step 46: t = 0.3402, w = 0.983348949889608, ye = 0.000000004892748
Step 47: t = 0.3521, w = 0.985214019919137, ye = 0.000000004913651
Step 48: t = 0.3643, w = 0.986915344546162, ye = 0.000000004932984
Step 49: t = 0.3769, w = 0.988463047916150, ye = 0.000000004950854
Step 50: t = 0.3899, w = 0.989866913295000, ye = 0.000000004967368
Step 51: t = 0.4033, w = 0.991136383934446, ye = 0.000000004982629
Step 52: t = 0.4171, w = 0.992280563769648, ye = 0.000000004996736
Step 53: t = 0.4314, w = 0.993308218081120, ye = 0.000000005009787
Step 54: t = 0.4462, w = 0.994227774940667, ye = 0.000000005021874
Step 55: t = 0.4615, w = 0.995047325838735, ye = 0.000000005033091
Step 56: t = 0.4773, w = 0.995774626742699, ye = 0.000000005043525
Step 57: t = 0.4938, w = 0.996417099190086, ye = 0.000000005053263
Step 58: t = 0.5110, w = 0.996981831790425, ye = 0.000000005062387
Step 59: t = 0.5289, w = 0.997475581254671, ye = 0.000000005070978
Step 60: t = 0.5475, w = 0.997904773972868, ye = 0.000000005079113
Step 61: t = 0.5670, w = 0.998275507573829, ye = 0.000000005086867
Step 62: t = 0.5874, w = 0.998593552589514, ye = 0.000000005094310
Step 63: t = 0.6087, w = 0.998864354353570, ye = 0.000000005101509
Step 64: t = 0.6312, w = 0.999093035031104, ye = 0.000000005108526
Step 65: t = 0.6549, w = 0.999284395892887, ye = 0.000000005115417
Step 66: t = 0.6800, w = 0.999442919781327, ye = 0.000000005122230
Step 67: t = 0.7065, w = 0.999572773947459, ye = 0.000000005129001
Step 68: t = 0.7347, w = 0.999677813111077, ye = 0.000000005135751
Step 69: t = 0.7648, w = 0.999761582982752, ye = 0.000000005142479
Step 70: t = 0.7971, w = 0.999827324190532, ye = 0.000000005149149
Step 71: t = 0.8318, w = 0.999877976768548, ye = 0.000000005155675
Step 72: t = 0.8694, w = 0.999916185284723, ye = 0.000000005161894
Step 73: t = 0.9102, w = 0.999944304732928, ye = 0.000000005167521
Step 74: t = 0.9550, w = 0.999964407385593, ye = 0.000000005172078
Step 75: t = 1.0044, w = 0.999978290786367, ye = 0.000000005174772
Step 76: t = 1.0595, w = 0.999987487175098, ye = 0.000000005174285
Step 77: t = 1.1216, w = 0.999993274676745, ye = 0.000000005168399
Step 78: t = 1.1924, w = 0.999996690694255, ye = 0.000000005153280
Step 79: t = 1.2746, w = 0.999998548041758, ye = 0.000000005122067
Step 80: t = 1.3719, w = 0.999999454469896, ye = 0.000000005061717
```

```
Step 81: t = 1.4902, w =  0.999999836289636, ye =  0.000000004944760
Step 82: t = 1.6392, w =  0.999999966696102, ye =  0.000000004700552
Step 83: t = 1.8368, w =  0.999999998789815, ye =  0.000000004062914
Step 84: t = 2.1228, w =  1.000000000874583, ye =  0.000000001176298
Step 85: t = 2.7501, w =  0.999999951372859, ye =  0.000000048626572
Step 86: t = 2.9725, w =  1.000000004294052, ye =  0.000000004294113
Step 87: t = 3.2294, w =  0.999999998472349, ye =  0.000000001527646
Step 88: t = 3.6927, w =  1.000000016497900, ye =  0.000000016497900
Step 89: t = 3.9886, w =  0.999999985378871, ye =  0.000000014621129
Step 90: t = 4.3136, w =  1.000000022343216, ye =  0.000000022343216
Step 91: t = 4.6444, w =  0.999999962276556, ye =  0.000000037723444
Step 92: t = 4.9412, w =  1.000000034023922, ye =  0.000000034023922
Step 93: t = 5.2054, w =  0.999999985263471, ye =  0.000000014736529
Step 94: t = 5.4805, w =  1.000000008347482, ye =  0.000000008347482
Step 95: t = 5.8180, w =  0.999999984221276, ye =  0.000000015778724
Step 96: t = 6.1356, w =  1.000000021129223, ye =  0.000000021129223
Step 97: t = 6.4611, w =  0.999999967399092, ye =  0.000000032600908
Step 98: t = 6.7627, w =  1.000000032355722, ye =  0.000000032355722
Step 99: t = 7.0362, w =  0.999999982404992, ye =  0.000000017595008
Step 100: t = 7.3136, w =  1.000000010504941, ye =  0.000000010504941
Step 101: t = 7.6361, w =  0.999999984640606, ye =  0.000000015359394
Step 102: t = 7.9958, w =  1.000000041429368, ye =  0.000000041429368
Step 103: t = 8.2510, w =  0.999999985922614, ye =  0.000000014077386
Step 104: t = 8.5141, w =  1.000000005920939, ye =  0.000000005920939
Step 105: t = 8.8574, w =  0.999999987681823, ye =  0.000000012318177
Step 106: t = 9.1911, w =  1.000000021849427, ye =  0.000000021849427
Step 107: t = 9.5351, w =  0.999999954007997, ye =  0.000000045992003
Step 108: t = 9.8319, w =  1.000000041509288, ye =  0.000000041509288
Step 109: t = 10.0000, w =  1.000000005967068, ye =  0.000000005967068
```

5   Problem 3

```
In [16]: predcorr([0 6], 0.5, 20, 2)

ans =

  Columns 1 through 7

        0    0.3000    0.6000    0.9000    1.2000    1.5000    1.8000

  Columns 8 through 14

   2.1000    2.4000    2.7000    3.0000    3.3000    3.6000    3.9000

  Columns 15 through 21

   4.2000    4.5000    4.8000    5.1000    5.4000    5.7000    6.0000
```

MultiStep HW05 prob 3