Math 414 Homework 05

<u>(1.) Text (2nd edition), 6.4 Computer Problem, #5.</u>

5. Plot the fourth-order Runge–Kutta Method approximate solution on $[0, 1]$ for the differential equation $y' = 1 + y^2$ and initial condition (a) $y_0 = 0$ (b) $y_0 = 1$, along with the exact solution (see Exercise 6.1.7). Use step sizes $h = 0.1$ and 0.05.

7. (a) Show that $y = \tan(t + c)$ is a solution of the differential equation $y' = 1 + y^2$ for each c. (b) For each real number y_0, find c in the interval $(-\pi/2, \pi/2)$ such that the initial value problem $y' = 1 + y^2$, $y(0) = y_0$ has a solution $y = \tan(t + c)$.

<u>(2.) Using the Runge-Kutta-Fehlberg 4/5 method to solve</u>

a.) $\frac{dy}{dt} = y - t^2 + 1 \quad with\ y(0) = 0.5\ tol = 10^{-7}\ on\ [0\ \ 6]\ start\ with\ \ h = 0.5$

For comparison and errors use $y_e = -\frac{1}{2}\, e^t + t^2 + 2\,t + 1$

b.) $\frac{dy}{dt} = 10(1 - y) \quad with\ y(0) = 0.5\ tol = 10^{-7}\ on\ [0\ \ 10]\ start\ with\ \ h = 0.1$

For comparison and errors use $y_e = 1 - \frac{1}{2}\, e^{-10t}$

Make plots to see how the step size changes and to see how the actual error grows.

<u>(3.) Use the text code “predcorr” on page 342 (edition 2) to solve the same DE as (2a.) above.</u>

$$\frac{dy}{dt} = y - t^2 + 1 \qquad with\ y(0) = 0.5 \quad on\ [0\ \ 6]$$

Run the code with n=20 and s=2. Compare the value of y(6) obtained here with the one obtained in (2a.) above.
