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Date

1. a. $\int \frac{x}{\sqrt{1-x^2}} dx$ $\quad u = 1-x^2 \quad x\,dx = -\frac{1}{2} du$

$du = -2x$

$= \int \frac{1}{\sqrt{u}} \cdot -\frac{1}{2} du$

$= -\frac{1}{2} \int u^{2} du = -\frac{1}{2} \left( \frac{1}{\frac{1}{2}} u^{\frac{1}{2}} \right) + c$

$= -\frac{1}{2} 2 \sqrt{1-x^2} + C$

$= -\sqrt{1-x^2} + C$

b. $\int \frac{x^2}{1-x} dx$ $\quad u = 1-x \rightarrow x = -(u-1)$

$= \int \frac{x^2}{1-x} - 1 \; du$

$= \int \frac{(-(u-1))^2}{1-(-(u-1))} du = \int -\frac{(u-1)^2}{u} du = \int -\frac{u^2-2u+1}{u} du$

$= -\int \frac{u^2}{u} - \frac{2u}{u} + \frac{1}{u} \; du$

$= -\int u - 2 + \frac{1}{u} du = -\int u\,du - \int 2\,du - \int \frac{1}{t} du$

$= -\left( \frac{t^2}{2} - 2t + \ln(|t|) \right)$

$= -\left( \frac{(1-x^2)^2}{2} - 2(1-x) + \ln(|1-x|) \right) = \frac{-3+2x+x^2}{2} - \ln(|1-x|) + C$

c. $\int \frac{x^2}{\sqrt{1-x}} dx$ $\quad u = 1-x \quad -du = dx$

$du = -dx \quad x = 1-u$

$= \int \frac{(1-u)^2}{\sqrt{u}} (-du) = -\int \frac{1-2u+u^2}{\sqrt{u}} du = -\int \frac{1}{u^{\frac{1}{2}}} - \frac{2u}{u^{\frac{1}{2}}} + \frac{u^2}{u^{\frac{1}{2}}} du$

$= -\int (u^{\frac{1}{2}} - 2u^{\frac{1}{2}} + u^{\frac{3}{2}}) du = -\left( \frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} - 2 \cdot \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{u^{\frac{3}{2}+1}}{\frac{3}{2}+1} \right)$

$= -\frac{u^{\frac{1}{2}}}{\frac{1}{2}} + 2 \cdot \frac{u^{\frac{3}{2}}}{\frac{3}{2}} - \frac{u^{\frac{5}{2}}}{\frac{5}{2}} = -2\sqrt{1-x} + \frac{4}{3}\sqrt{(1+x)^3} - \frac{2}{5}\sqrt{(1-x)^5} + C$

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d. $\int \frac{x^2}{\sqrt{1-x^2}}\,dx =$ $x = \sin u$, $dx = \cos u\,du$

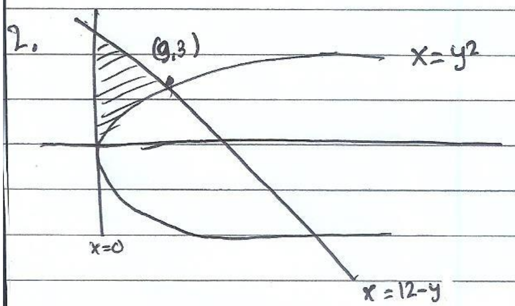
$= \int \frac{\sin^2(u)}{\sqrt{1-\sin^2(u)}} \cdot \cos(u)\,du = \int \frac{\sin^2(u)}{\cancel{\cos(u)}} \cdot \cancel{\cos u}\,du = \int \sin^2(u)\,du = \sin^2(x) = \int \frac{1}{2}(1-\cos 2u)\,du$

$= \frac{1}{2}\int 1\,du - \frac{1}{2}\cdot\frac{1}{2}\int 2\cos(2u)\,du = \frac{1}{2}u - \frac{1}{4}\sin(2u)$

$= \frac{1}{2}u - \frac{1}{4}\,2\sin(u)\cos(u) = \frac{1}{2}\arcsin x - \frac{1}{2}\underbrace{\sin(\arcsin(x))}_{x} \cdot \underbrace{\cos(\arcsin(x))}_{\sqrt{1-x^2}}$

$= \frac{1}{2}\arcsin(x) - \frac{1}{2}x\sqrt{1+x^2} + C$

2.




titik potong

$= y^2 + y - 12$

$= (y+4)(y-3) = y=3 \quad x=9 \rightarrow (9,3)$

$\therefore \int_9^0 12 - x - \sqrt{x} =$

$= \int 12x - \frac{1}{2}x^2 - \frac{2}{3}x\sqrt{x} \Big|_0^9$

$= 12.9 - \frac{9^2}{2} - \frac{2 \times 9\sqrt{9}}{3} - 0$

$= \frac{99}{2}$

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3. Dengan menggunakan Dareah di D nomor 2 maka

$V = \pi \int_b^a x^2 \, dy \rightarrow$

$= \pi \int_0^9 144 - 24x + x^2 \, dx$

$= 144x - 12x^2 + \frac{x^3}{3} \Big|_0^9$

$= 144 \times 9 - 12 \times 9^2 + \frac{9^3}{3} - (0)$

$= \pi \times 567$