6.0 Language Modeling

References: 1. 11.2.2, 11.3, 11.4 of Huang or

2. 6.1- 6.8 of Becchetti, or

3. 4.1- 4.5, 8.3 of Jelinek

Some Fundamentals of Information Theory

- **Examples for Languages**

  $0 \leq H(S) \leq \log M$

  – Source of English text generation

  $\boxed{S} \longrightarrow$ this course is about speech.....

  - the random variable is the character $\Rightarrow$ 26*2+.....<64=2^6
    $H(S) < 6$ bits (of information) per character
  - the random variable is the word $\Rightarrow$ assume total number of words=30,000<10^{15}
    $H(S) < 15$ bits (of information) per word

  – Source of speech for Mandarin Chinese

  $\boxed{S} \longrightarrow$ 這一門課有關語音.....

  - the random variable is the syllable (including the tone) $\Rightarrow$ $1300 < 2^{11}$
    $H(S) < 11$ bits (of information) per syllable (including the tone)
  - the random variable is the syllable (ignoring the tone) $\Rightarrow$ $400 < 2^9$
    $H(S) < 9$ bits (of information) per syllable (ignoring the tone)
  - the random variable is the character $\Rightarrow$ $8{,}000 < 2^{13}$
    $H(S) < 13$ bits (of information) per character

  – Comparison: speech— 語音, girl— 女孩, computer— 計算機

Perplexity

- **Perplexity of A Language Source S**

$$H(S) = -\sum_i p(x_i) \log\left[p(x_i)\right]$$

$$PP(S) = 2^{H(S)}$$

  - size of a "virtual vocabulary" in which all words (or units) are equally probable
    - e.g. 1024 words each with probability $\frac{1}{1024}$, $I(x_i) = 10$ bits (of information)

      $H(S) = 10$ bits (of information), $PP(S) = 1024$
  - branching factor estimate for the language
- **A Language Model**
  - assigning a probability $P(w_i|c_i)$ for the next possible word w_i given a condition c_i

    e.g. $P(W=w_1,w_2,w_3,w_4....w_n)=\underset{\uparrow\, c_1=\Phi}{P(w_1)}\,\underset{\uparrow\, c_2}{P(w_2|w_1)}\prod_{i=3}^{n} P(w_i|\underbrace{w_{i-2},w_{i-1}}_{c_i})$
- **A Test Corpus D of N sentences, with the i-th sentence W_i has n_i words and total words N_D**

$$D = [W_1,W_2,....,W_N], \qquad W_i = w_1,w_2,w_3,....w_{n_i}$$

$$N_D = \sum_{i=1}^{N} n_i$$

Perplexity

- **Perplexity of A Language Model $P(w_i|c_i)$ with respect to a Test Corpus D**
  – $H(P;D) = -\frac{1}{N_D}\sum_{i=1}^{N}\left[\sum_{j=1}^{n_i}\log P\left(w_j|c_j\right)\right]$, average of all log $P(w_j|c_j)$ over the whole corpus D

  $= -\sum_{i=1}^{N}\sum_{j=1}^{n_j}\log\left[P\left(w_i|c_i\right)^{\frac{1}{N_D}}\right]$, logarithm of geometric mean of $P(w_i|c_i)$

  – pp (P ; D) $=2^{H(P;D)}$

  average branching factor (in the sense of geometrical mean of reciprocals)

  e.g. $P(W=w_1w_2...w_n)=P(w_1)\ P(w_2|w_1)\ P(w_3|w_1,w_2)\ P(w_4|w_2,w_3)\ P(w_5|w_3,w_4)\ .....$

  $\frac{1}{1024} \quad \frac{1}{512} \quad \frac{1}{256} \quad \frac{1}{128} \quad \frac{1}{256}$

  $\Rightarrow \left(\left[\left(\frac{1}{1024}\right)\left(\frac{1}{512}\right)\left(\frac{1}{256}\right)\left(\frac{1}{128}\right)\left(\frac{1}{256}\right)\cdots\right]^{\frac{1}{n}}\right)^{-1} = 312$

  – the capabilities of the language model in predicting the next word given the linguistic constraints extracted from the training corpus
  – the smaller the better, performance measure for a language model with respect to a test corpus
  – a function of a language model P and text corpus D

Perplexity

- **Cross-Entropy**

$$D\left[p(x)\|q(x)\right]=\sum_i p(x_i)\log\left[\frac{p(x_i)}{q(x_i)}\right]\geq 0$$

  – Jensen's Inequality

$$-\sum_i p(x_i)\log\left[p(x_i)\right]\leq -\sum_i \underbrace{p(x_i)\log\left[q(x_i)\right]}$$

Someone call this "cross-entropy" = X[p(x) || q(x)]

  – entropy when p(x) is incorrectly estimated as q(x) (leads to some entropy increase)

- **The True Probabilities $\bar{P}(w_i|c_1)$ incorrectly estimated as $P(w_i|c_i)$ by the language model**

$$\lim_{N\to\infty}\frac{1}{N}\sum_{k=1}^{N}\log\left[q(x_k)\right]=\sum_i p(x_i)\log\left[q(x_i)\right]$$

(averaging by all samples) ⇧ (averaging if $p(x_i)$ is known)

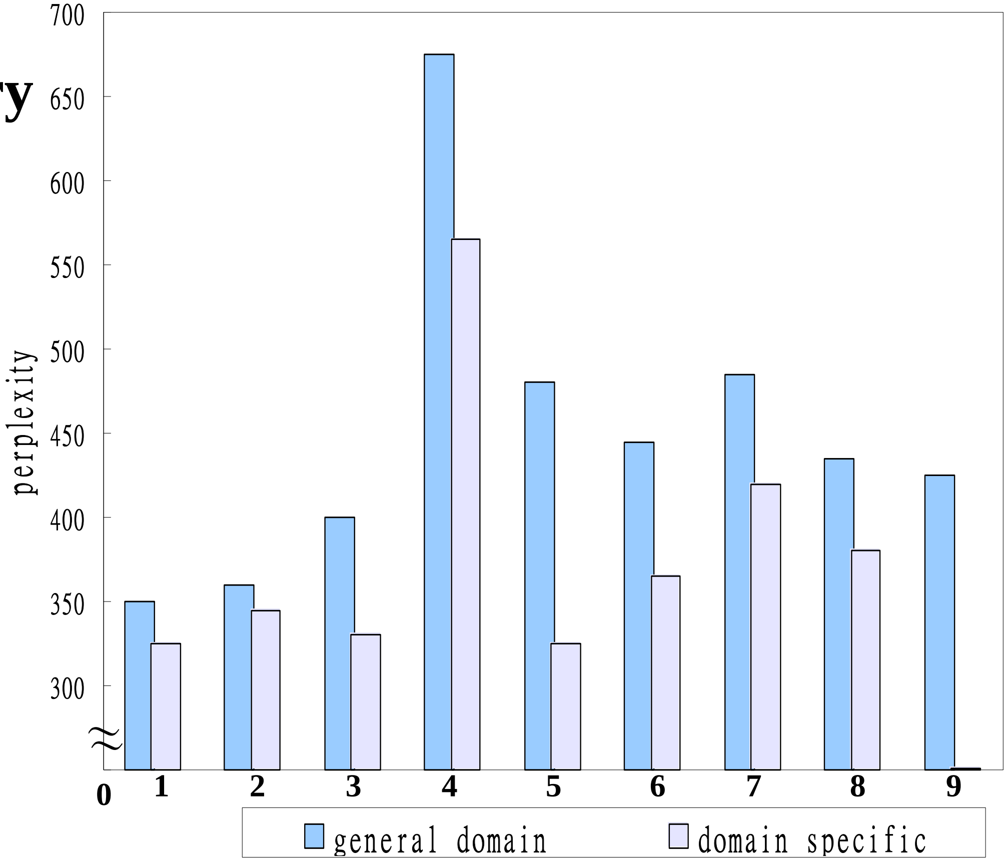
law of large numbers

- **The Perplexity is a kind "Cross-Entropy" when the true statistical characteristics of the test corpus D is incorrectly estimated as $p(w_i|c_i)$ by the language model**
  – H (P ; D) = X (D || P)
  – the larger the worse

An Perplexity Analysis Example with Respect to Different Subject Domains

- **Domain-specific Language Models Trained with Domain Specific Corpus of Much Smaller Size very often Perform Better than a General Domain Model**
  - Training corpus: Internet news in Chinese language

1	politics	19.6 M
2	congress	2.7 M
3	business	8.9 M
4	culture	4.3 M
5	sports	2.1 M
6	transportation	1.6 M
7	society	10.8 M
8	local	8.1 M
9	general(average)	58.1 M



  - Sports section gives the lowest perplexity even with very small training corpus

Smoothing of Language Models

- **Data Sparseness**
  - many events never occur in the training data

    e.g. Prob [Jason immediately stands up]=0 because Prob [immediately| Jason]=0
  - smoothing: trying to assign some non-zero probabilities to all events even if they never occur in the training data
- **Add-one Smoothing**
  - assuming all events occur once more than it actually does

    e.g. bigram

$$p(w^j|w^k) = \frac{N(<w^k, w^j>)}{N(w^k)} = \frac{N(<w^k, w^j>)}{\sum_j N(<w^k, w^j>)} \Rightarrow \frac{N(<w^k, w^j>)+1}{\sum_j N(<w^k, w^j>)+V}$$

V: total number of distinct words in the vocabulary

- **Back-off Smoothing**

$$\overline{P}(w_i|w_{i-n+1}, w_{i-n+2},\dots w_{i-1}) = \begin{cases} P(w_i|w_{i-n+1}, w_{i-n+2},\dots w_{i-1}), & \text{if } N(<w_{i-n+1}, \dots w_{i-1}, w_i>)>0 \\ a(w_{i-n+1},\dots w_{i-1})\,\overline{P}(w_i|w_{i-n+2},\dots w_{i-1}), & \text{if } N(<w_{i-n+1},\dots, w_{i-1}, w_i>)=0 \end{cases}$$

  - back-off to lower-order if the count is zero, prob (you| see)>prob (thou| see)
- **Interpolation Smoothing**

$$\overline{P}(w_i|w_{i-n+1}, w_{i-n+2},\dots w_{i-1}) = b(w_{i-n+1},\dots w_{i-1})P(w_i|w_{i-n+1},\dots w_{i-1}) + (1-b(w_{i-n+1},\dots w_{i-1}))\overline{P}(w_i|w_{i-n+2},\dots w_{i-1})$$

  - interpolated with lower-order model even for events with non-zero counts

Smoothing of Language Models

- **Good-Turing Smoothing**
  - Good-Turning Estimates: properly decreasing relative frequencies for observed events and allocate some frequencies to unseen events
  - Assuming a total of K events {1,2,3...,k,.....K}
    number of observed occurrences for event k: n(k),

    N: total number of observations, $N = \sum_{k=1}^{K} n(k)$

    n_r: number of distinct events that occur r times (number of different events k such that n(k) = r)

    $$N = \sum_r r\, n_r$$
  - Good-Turning Estimates:

    an event occurring r times is assumed to have occurred r^* times,

    $$r^* = (r+1)\frac{n_{r+1}}{n_r}$$

    ⇨
    - $r^* \neq 0$ for unseen events (r = 0)
    - $\sum_r r^* n_r = \sum_r (r+1)\frac{n_{r+1}}{n_r} n_r = \sum_r (r+1)\, n_{r+1} = N$
    - total occurrences for events having occurred r times: $rn_r \rightarrow (r+1)n_{r+1}$
    - total counts assigned to unseen events = $n_0\left(\frac{n_1}{n_0}\right) = n_1$

Smoothing of Language Models

- **Good-Turing Smoothing**
  - An analogy: during fishing, getting each kind of fish is an event

    an example: n(1)=10, n(2)=3, n(3)=2, n(4)= n(5)= n(6)=1, N=18

    prob (next fish got is of a new kind) = prob (those occurring only once) = $\frac{3}{18}$

    prob (next fish is 6) = $\frac{1^*}{18} = \frac{(1+1)\cdot 1/3}{18} = \frac{1}{27}$
- **Katz Smoothing**
  - large counts are reliable, so unchanged
  - small counts are discounted, with total reduced counts assigned to unseen events, based on Good-Turning estimates

    $$\sum_{r=1}^{r_0} n_r (1 - d_r)\, r = n_1 \quad , \quad d_r\text{: discount ratio for events with r times}$$
  - distribution of counts among unseen events based on next-lower-order model: back off
  - an example for bigram:

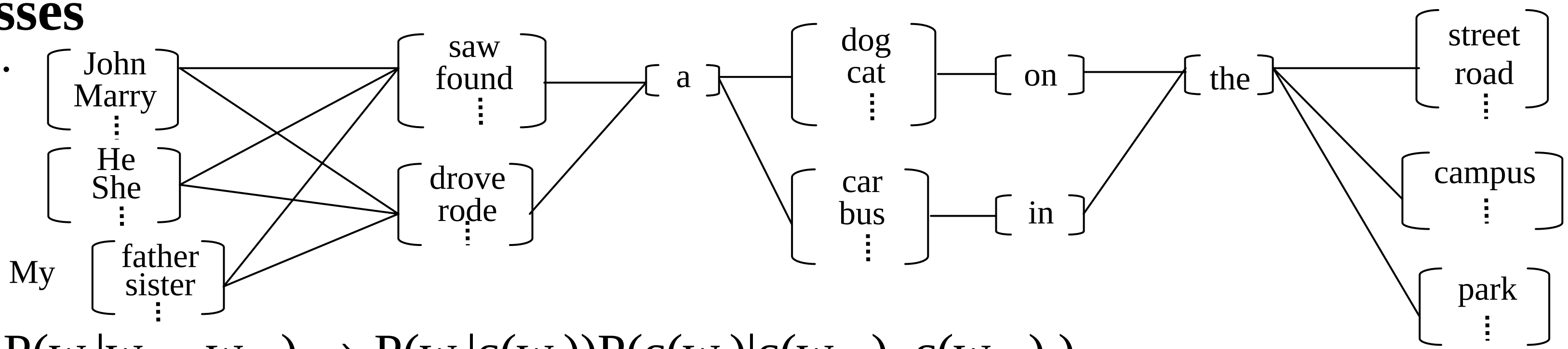
$$\overline{P}(w_i | w_{i-1}) = \begin{cases} N(<w_{i-1},w_i>) / N(w_i) & , r > r_0 \\ d_r \cdot N(<w_{i-1},w_i>) / N(w_i) & , r_0 \geq r > 0 \\ a(w_{i-1},w_i)\, P(w_i) & , r = 0 \end{cases}$$

a (w_{i-1},w_i): such that the total counts equal to those assigned

Class-based Language Modeling

- **Clustering Words with Similar Semantic/Grammatic Behavior into Classes**

  e.g.



  – $P(w_i|w_{i-2}, w_{i-1}) \Rightarrow P(w_i|c(w_i))P(c(w_i)|c(w_{i-2}), c(w_{i-1}))$

  $c(w_j)$: the class including w_j

  – Smoothing effect: classes complementing the lower order models

  – parameter size reduced

- **Limited Domain Applications: Rule-based Clustering by Human Knowledge**

  e.g. Tell me all flights of [United, China Airline, Eva Air, …] from [Taipei, …] to [Los Angeles, …] on [Sunday, …]

  – new items can be easily added without training data

- **General Domain Applications: Data-driven Clustering (probably aided by rule-based knowledge)**

Class-based Language Modeling

- **Data-driven Word Clustering Algorithm Examples**
  - Example 1:
    - initially each word belongs to a different cluster
    - in each iteration a pair of clusters was identified and merged into a cluster which minimizes the overall perplexity
    - stops when no further (significant) reduction in perplexity can be achieved

    Reference: "Cluster-based N-gram Models of Natural Language", Computational Linguistics, 1992 (4), pp. 467-479
  - Example 2:

$$\text{Prob}\,[W=w_1w_2w_3....w_n]=\prod_{i=1}^{n}\text{Prob}(w_i|w_1,w_2....w_{i-1})=\prod_{i=1}^{n}\text{Prob}(w_i|h_i)$$

h_i: $w_1,w_2,...w_{i-1}$, history of w_i

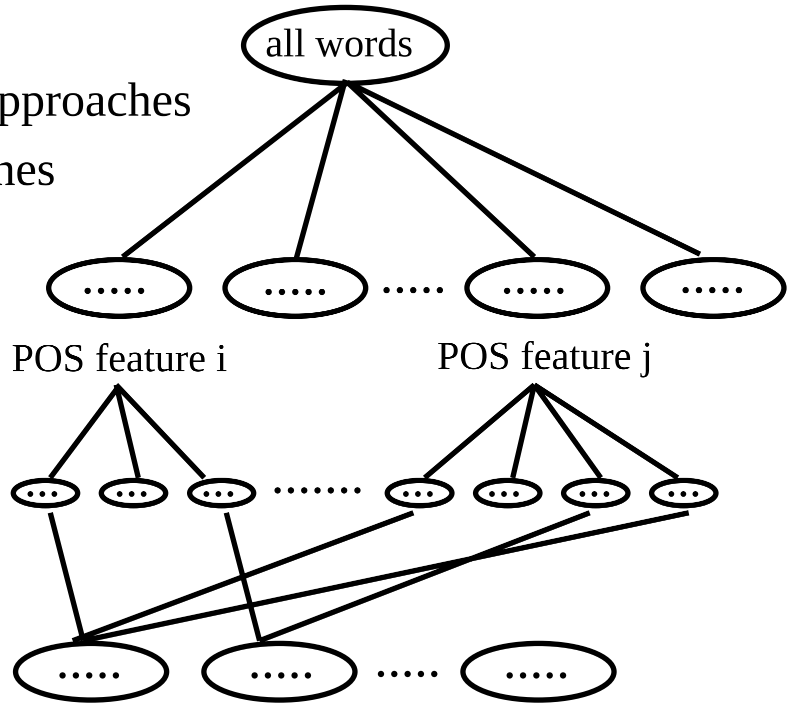
    - clustering the histories into classes by decision trees (CART)
    - developing a question set, entropy as a criterion
    - may include both grammatic and statistical knowledge, both local and long-distance relationship

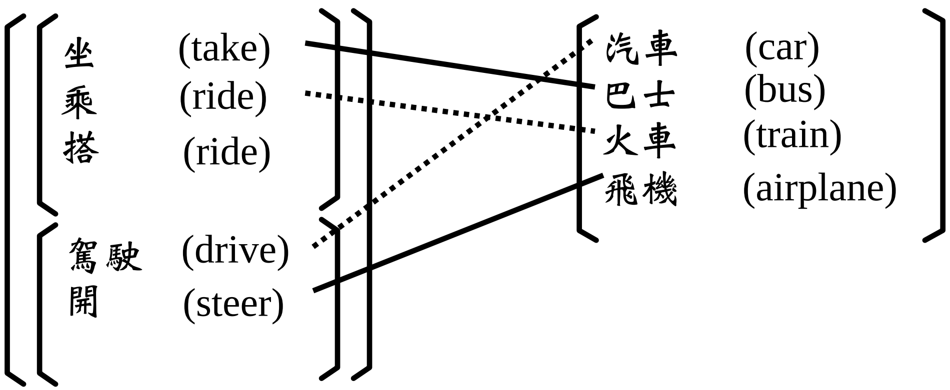
    Reference: "A Tree-based Statistical Language Model for Natural Language Speech Recognition", IEEE Trans. Acoustics, Speech and Signal Processing, 1989, 37 (7), pp. 1001-1008

An Example Class-based Chinese Language Model

- **A Three-stage Hierarchical Word Classification Algorithm**
  - **stage 1** : classification by 198

    POS features (syntactic & semantic)
    - *each word belonging to one class only*
    - *each class characterized by a set of POS's*
  - **stage 2** : further classification with data-driven approaches
  - **stage 3** : final merging with data-driven approaches





  - rarely used words classified by human knowledge
  - both data-driven and human-knowledge-driven

An Example of Class-based Chinese Language Model

- **An Improved Approach for the Three-stage Hierarchical Classification Algorithm**
  - words with multiple features assigned to more than one classes

    我不會去開那個會

  - the same word belongs to different classes characterized by different features
  - language model trained by "tagged corpus" only
  - a modification in stage 1
- **POS Tagging**
  - tagging POS for each word in the sentence
  - automatic POS tagging algorithm based on a lexicon of words with possible POS's and POS N-grams trained with human-tagged corpus
- **Integrated Class-based and Word-based Models**
  - word-based models are more precise for frequently used words
  - back-off to class-based models for events with inadequate counts
  - each single word is a class if frequent enough

Structural Features of Chinese Language

- **Almost Each Character with Its Own Meaning, thus Playing Some Linguistic Role Independently**
- **No Natural Word Boundaries in a Chinese Sentence**

  電腦科技的進步改變了人類的生活和工作方式

  - word segmentation not unique
  - words not well defined
  - commonly accepted lexicon not existing
- **Open (Essentially Unlimited) Vocabulary with Flexible Wording Structure**
  - new words easily created everyday 電(electricity)+腦(brain)→電腦(computer)
  - long word arbitrarily abbreviated 臺灣大學 (Taiwan University) →臺大
  - name/title 李登輝前總統 (former President T.H. Lee) →李前總統登輝
  - unlimited number of compound words 高 (high) + 速 (speed) + 公路 (highway)→高速公路(freeway)
- **Difficult for Word-based Approaches Popularly Used in Alphabetic Languages**
  - serious out of vocabulary(OOV) problem

Word-based and Character-based Chinese Language Models

- **Word-based and Class-based Language Modeling**
  - words are the primary building blocks of sentences
  - more information may be added
  - lexicon plays the key role
  - flexible wording structure makes it difficult to have a good enough lexicon
  - accurate word segmentation needed for training corpus
  - serious "out-of -vocabulary(OOV)" problem in many cases
  - all characters included as " mono-character words"
- **Character-based Language Modeling**
  - avoiding the difficult problem of flexible wording structure and undefined word boundaries
  - relatively weak without word-level information
  - higher order N-gram needed for good performance, which is relatively difficult to realize
- **Integration of Class-based/Word-based/Character-based Models**

Segment Pattern Lexicon for Chinese

- **Segment Patterns Replacing the Words in the Lexicon**
  - segments of a few characters often appear together : one or a few words
  - regardless of the flexible wording structure
  - automatically extracted from the training corpus (or network information) statistically
  - including all important patterns by minimizing the perplexity
- **Advantages**
  - bypassing the problem that the word is not well-defined
  - new words or special phrases can be automatically included as long as they appear frequently in the corpus (or network information)
  - can construct multiple lexicons for different task domains as long as the corpora are given(or available via the network)

Example Segment Patterns Extracted from Network News Outside of A Standard Lexicon

- **Patterns with 2 Characters**
  - 一套，他很，再往，在向，但從，苗市，記在
    深表，這篇，單就，無權，開低，蜂炮，暫不
- **Patterns with 3 Characters**
  - 今年初，反六輕，半年後，必要時，在七月
    次微米，卻只有，副主委，第五次，陳水扁，開發中
- **Patterns with 4 Characters**
  - 大受影響，交易價格，在現階段，省民政廳，專責警力
    通盤檢討，造成不少，進行了解，暫停通話，擴大臨檢

Word/Segment Pattern Segmentation Samples

- **With Extracted Segment Pattern**

交通部 考慮 禁止 民眾 *開車* 時
使用 大哥大
已 *委由* 逢甲大學 *研究中*
預計 *六月底* 完成
至於 實施 *時程*
因涉及 交通 處罰 條例 *的修正*
必須 *經立法院* 三讀通過
交通部 *無法確定*
交通部 *官員表示*
世界 *各國對* 應否 立法 禁止 民眾
開車 時 打 大哥大
意見 相當 分岐

- **With A Standard Lexicon**

交通部 考慮 禁止 民眾 開 車 時
使用 大哥大
已 委 由 逢甲大學 研究 中
預計 六月 底 完成
至於 實施 時 程
因 涉及 交通 處罰 條例 的 修
正
必須 經 立法院 三讀通過
交通部 無法 確定
交通部 官員 表示
世界 各 國 對 應否 立法 禁止
民眾 開 車 時 打 大哥大
意見 相當 分岐

- **Percentage of Patterns outside of the Standard Lexicon : 28%**