Designing a Sustainable Bus Transport System with High QoS Using Computational Intelligence

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Abstract

Administrations are facing new challenges in the transformation of cities into sustainable smart cities caused by the environmental consciousness and fostered by advances in technology. Special attention is given to sustainable mobility solutions where not only sustainability is a must but also attractiveness for users, so that they prioritize it over other transportation alternatives that negatively contribute to the greenhouse emissions and traffic congestion. In this article, we propose a two-step optimization method based on computational intelligence tools that offers a holistic efficient sustainable solution for the design of public bus transportation systems. It minimizes transfers waiting times and optimizes the battery use of a heterogeneous fleet of electric buses. Results show significant achievements on the bus system in Montevideo.

Key Points

- Propose a holistic approach for sustainable urban bus system in smart cities.
- Use of Machine Learning techniques for fast optimization problem solving.
- Use of Evolutionary Algorithms for accurate optimization problem solving.
- Synchronization of bus lines to minimize transfer waiting times.
- Schedule of a heterogeneous fleet of electric buses to maximize energy use.
- Accurate mathematical formulation of the consumption of electric buses.
- Application of the method in a real world case of study.

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Introduction

The unstoppable global warming and high levels of pollution is pushing towards an urgent analysis in many critical sectors, especially in transportation which is one of the largest contributors to these negative environmental and societal effects (EPA, 2022).

Administrations are targeting the complete redesign of the current public transportation (PT) systems to a new one more sustainable and attractive to the users which will directly impact not only on the greenhouse emissions but also in the quality of life of citizens. This new system will face three main challenges: (1) sustainability to reduce the level of pollutants, (2) efficiency so it is a profitable business for the operators, and (3) high quality of service so it is attractive to users.

Rail and bus move over 95% of the passengers in USA (American Public Transportation Association, 2021), and according to the International Association of Public Transport (UITP) (UITP, 2019) transit buses are among the most popular modes of public transport worldwide (see **Fig. 1**(a)). Therefore, the importance of buses in urban mobility is self-evident.

The first but not the only step towards this new PT system redesign is moving to electric buses (eBuses hereinafter). Currently, it is estimated that 16% of buses worldwide are electric and it is expected that by 2050 all buses will be electric. China and Europe are leading the race to electro-mobility and they will continue leading in clean technologies Bloomberg, 2021. The estimation of the European program *Zero Emission Urban Bus System* Zeeus, 2018, led by UITP, is that by 2030 the most popular propulsion system used in public transport will be electric energy. This shift to electro-mobility raises important challenges, mainly motivated by the limited autonomy of eBuses with onboard batteries.

Moreover, in order to consider profitability and high quality of service in the new redesign of the PT systems, user demand and quality of service must be taken into account. For that, particular attention should be drawn into systems operated by heterogeneous bus fleets. Although traditional buses are predominant in most fleets (see **Fig. 1**(b)), other alternatives like mini or midi buses can offer lower consumption or occupy less space giving more flexibility and offering an optimal service. Developing intelligent transport systems (ITS) managing a heterogeneous fleet of eBuses for the varying demand of passengers can significantly reduce noise pollution, greenhouse gas emissions, road congestion and at the same time being more cost effective.

However, passengers are reluctant to use PT when the service is not good, buses are delayed or waiting times are long. They are expected to do the first and last mile on foot, transfer between vehicles if needed and wait for their arrival. The time spent walking, transferring and waiting at stations or stops is usually valued lower by passengers since it implies a much greater effort than time spent onboard a vehicle because the passenger has fewer opportunities to use their time effectively. It is shown in (Wardman, 2004), that passengers perceive between 1.4 and 2.5 times longer the time spent transferring than the time spent within the vehicle. The distance of the first and last mile of each user is out of our control, however, reducing the waiting time of passengers (especially when transferring) increases the perceived quality of service offered by the PT system. Therefore, timetable scheduling can maximize the number of synchronized transfers, thus minimizing the waiting time experienced by users during transfers.

This work tackles two of the most relevant phases of PT systems: timetable design and vehicle scheduling (Ceder, 2016) using a heterogeneous fleet of eBuses. The main goals are maximizing the battery use of the most suitable type of vehicles while minimizing the passengers' waiting time in transfers. We propose to tackle this problem in two different steps. First, the optimization of the timetables or frequency planning also called Bus Synchronization Problem (BSP) will be addressed, and the results obtained will be used as input for the optimization of the vehicle scheduling of the heterogeneous electric bus fleet (with diverse battery capacities), that we call the Heterogeneous eBuses Fleet Scheduling Problem (or HeFSP).

In this work we address the PT system in Montevideo, Uruguay. We use in the experimentation real data provided by the Metropolitan Transportation System of Montevideo about the passenger demand and bus lines in the city. Additionally, the set of eBuses chosen for composing the fleet have typical features that were selected from several brands in the electro-mobility market.

The main contributions of this article are given next. We propose a novel two-step optimization method to design an efficient and sustainable bus transportation system with high QoS; the system is a zero emissions solution that also enhances user

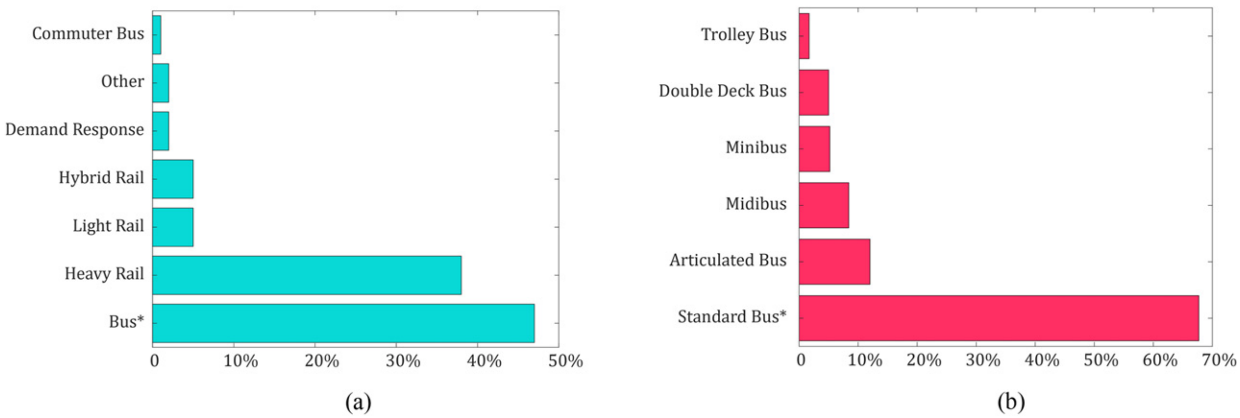


Fig. 1 Percentage of the use of the different means of transport and percentage of the use of the type of buses in a heterogeneous fleet. (a) Ratio of trips by different means of transport APTA2021.) (Types of vehicles that constitute bus fleets worldwide. Reproduced from UITP, 2019. The International Association for Public Transport (UITP). Global Bus Survey Report of International Association of Public Transport. Technical report.

experience reducing waiting times in transfers. Our preliminary approach in (Peña *et al.*, 2022a,b,c) was enhanced by using Virtual Savant (a novel machine learning-based optimization framework) in the first step to considerably improve bus transfer times in reduced computational time, compared to our previous method. Also, we generated a new more realistic scenario, composed by a higher number of longer real routes in the city of Montevideo (Uruguay). The passenger demand was calculated from real data obtained from the smartcards used in PT in Montevideo. Additionally, the topography of the routes was also considered for accurate estimations of the consumption of eBuses. Finally, an in depth analysis of the results was done, showing how solutions with similar quality offer a wide range of different possibilities to the operator.

The remainder of this work is organized as follows. Section "Overview of Relevant Approaches" briefly describes the most relevant works in the literature. The model definition is presented in Section "Problem Definition", and the methodology applied in Section "Methodology". The experiments accomplished are detailed in Section "Experimentation", while Section "Results and Discussion" presents the experimental results. Finally, Section "Conclusion and Future Works" concludes the work and presents future research lines.

Overview of Relevant Approaches

The deployment of a transit network is composed of several crucial steps. The planning process can be mainly divided into three categories: (1) the network design and line planning, (2) frequencies definition, (3) generating the timetables, and (4) vehicle scheduling (see (Kuo *et al.*, 2022)).

When a complete urban transport system is being deployed the impact of one step into the other is crucial and a holistic approach is essential to provide an optimal service. However, this is not very realistic as nowadays, most of the cities generally have an inefficient public transport system that needs to be redesigned and optimized, but already exists. Therefore, changing network design and line planning in a city is not very realistic because of the extra cost of placing/removing the infrastructure (bus stops, bus lanes), the existing familiarization of passengers to it, etc.

Regarding the second step, the frequencies definition targets two different conflicting objectives, i.e., the minimization of the operational cost and the maximization of customer service quality. Administrations and operators usually have already an agreement and changing it will surely incur in extra expenses. Therefore, in order to optimize the existing public transport system in a city, the most straightforward approach is to optimize the last two steps, i.e., timetable and vehicle scheduling problems. These are the problems addressed in this work.

There exist many different factors that influence the formulation of the timetabling problem like passenger transfer (at one or more stops), bus frequencies, even headway, passenger activity-based planning, etc., (Ibarra and Rios, 2012). However, one of the most important goals nowadays is the quality of service, to encourage the passenger to use the public transport. Frequent and convenient departure times, minimum number of transfers and waiting time as well as short trips are desirable. Therefore, coordination of schedule and capacity is crucial specially for rush-hours peaks. In large cities, many lines from many residential areas converge to the city center, and usually there is not a unique city center but several attractive areas, making interchanges a reality. In this situation, the bus synchronization problem comes into play. Unlike trains, the travel time of a bus is variable due to traffic conditions, weather, passenger's demand, etc., thus, stochastic models are usually applied to solve this problem. Section "Timetabling Problem" reviews some of the most relevant and recent research in the topic.

The last step of the planning process is the vehicle scheduling problem, i.e., the assignment of vehicles to the planned trips (the most relevant works are reviewed in Section "Vehicle Scheduling Problem"). At this stage different goals must be considered both on the side of the operators and the passengers, e.g., minimization of the operational cost and/or the number of vehicles and satisfying passenger's demand, among others. In the bus transport system, an appropriate heterogeneous fleet of buses provides more flexibility (in terms of capacity) and reduces cost (smaller buses are cheaper), however a thorough analysis of every specific scenario must be accomplished in order to obtain an optimal solution.

There exist a further step, ahead from the planning processes: the duty scheduling and crew rostering, also called the bus driver scheduling. For a complete overview of all the Transit Network Planning problems on bus transport systems and the most suitable strategies to that time please refer to (Ibarra-Rojas *et al.*, 2015).

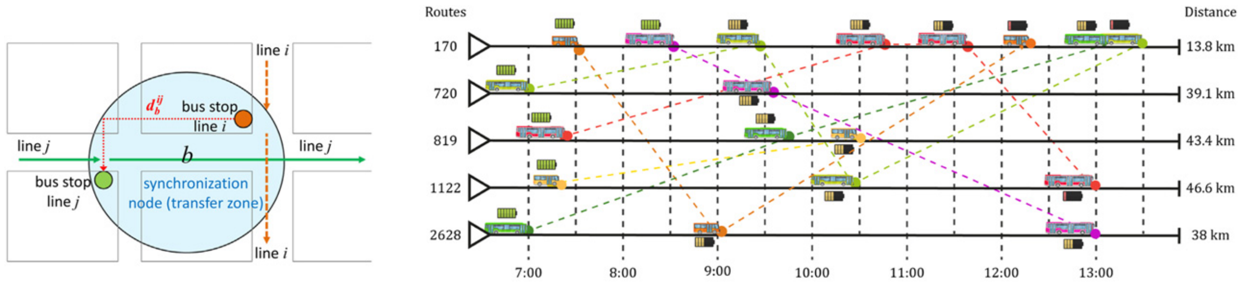
Recent advances in technology and more specific data availability is leading research into models that target to a single step of the planning process but several. In (Kuo *et al.*, 2022), an extensive review of the planing process problems, integrated planning models, as well as future challenges in metro, bus and train transport systems is given.

Timetabling Problem

Cities have been rapidly growing in the last decades, requiring the redesign of their public transport systems to their new requirements. The problem of planning public transport has been shown to be NP-hard (Lenstra and Kan, 1981), so computational intelligence techniques arise as useful tools for facing the problem.

Ceder *et al.* identify in (Ceder, 2016) the timetabling problem and the vehicle scheduling problem (VSP) to be among the critical problems in public transport planning. They are tightly related problems, often addressed simultaneously in the literature, where the timetable problem is typically first solved, and its result is used as an input for the VSP (Guihaire and Hao, 2008; Carosi *et al.*, 2019).

Timetable design is one of the main approaches applied for bus synchronization to reduce the waiting times for passengers when transferring between buses. The main goal in this approach is to determine headway values (i.e., the time between consecutive vehicles)



(a) (b)

Fig. 2 The BSP problem finds the optimal timetable to minimize passengers waiting times in bus transfers, and the HeFSP sets the best buses schedule for that timetable. (a) Transfer synchronization node in BSP (b)Vehicles assignment example for HeFSP.

for each bus line that maximize successful transfer synchronizations. Headways are constrained to a certain range defined according to the service offered to citizens, the economic viability of the bus service and restrictions imposed by the transport authority. Daduna and Voß (1995) addressed the timetable synchronization problem using Simulated Annealing and Tabu Search over randomly generated examples based on the Berlin Underground network and three real-world cases from different German cities. Ceder *et al.* (2001) proposed a greedy approach focused on transfers at shared stops which was evaluated on small instances. Ibarra and Rios (2012) studied the timetable synchronization problem in Monterrey, Mexico, considering a time buffer for travel times to account for delays. The problem was solved using a Multi-start Iterated Local Search (MILS) and a Branch and bound algorithm and evaluated over eight instances of varying size (15–200 bus lines, 3–40 synchronization points). The method was able to compute efficient solutions for medium-size instances but failed to converge in the larger ones. Later, Ibarra *et al.* (2016) extended their formulation to account for multiple trips of a given set of lines, and compared the original MILS to a Variable Neighborhood Search and a simple population-based approach. The three methods were able to compute solutions of similar quality over small synthetic instances. Methods based on timetable design can be extended or complemented with other techniques at the operation level to account for dynamic demand and unexpected events, e.g., by holding buses in stops (Hall *et al.*, 2001; Delgado *et al.*, 2013; Van Oort *et al.*, 2010).

Vehicle Scheduling Problem

The VSP deals with the choice of the vehicles to be used according to their capacity and the passengers demand for an efficient public transport system. Overloaded buses negatively impact the Quality of Service (QoS), while underused ones increase the cost and emissions. Making use of a heterogeneous fleet of vehicles gives the flexibility to choose the right bus for every situation, avoiding overloads in peak hours or an unprofitable and energy inefficient operation when the demand is low (Potter, 2003; Ceder, 2011). The operational cost was considered in the first articles in this area, but more recent works also consider the QoS of the system in a multi-objective formulation (Ceder, 2011; Ibarra-Rojas *et al.*, 2014; Pena *et al.*, 2019).

Besides cost operations and QoS, one important goal that is attracting more and more the attention of administrations and researchers is the reduction of greenhouse gas emissions in transportation. Some research works are considering it because it directly impacts on the detriment of the air quality (Yang and Liu, 2020; Peña *et al.*, 2022a,b,c). Urban bus fleets are gradually switching towards fully electric. This necessary change arises new challenges, motivated for the limited autonomy of electric buses (eBuses) and the long charging times. Yao *et al.* (2020) is one of the first works in this line, where authors propose a novel VSP formulation in which a heterogeneous fleet of eBuses is considered to reduce the annual operating cost.

In the context of electric buses the timetabling and VSP problems are still highly related problems but have some new constraints and thus, they have to be redefined. In PMT22, a preliminary work using two evolutionary algorithms is analyzed. The results showed promising solutions but with long computational times. In this work, sophisticated machine learning techniques giving much faster solutions and a new scenario including more and longer routes is proposed.

Problem Definition

This section presents the timetabling and VSP problem formulations used in this work. The timetabling problem considered is the Bus Synchronization Problem recently presented by Massobrio *et al.* (2022), while the VSP problem studied in this work is the Heterogeneous eBus Fleet Scheduling Problem (HeFSP) of (Peña *et al.*, 2022a,b,c).

The Bus Synchronization Problem

The Bus Synchronization Problem (BSP) focuses on improving the quality of service provided to passengers, specifically, on providing a better traveling experience for those users making bus transfers (see **Fig. 2**(a)). The problem consists in finding the headway of each bus line in a system in order to maximize the number of synchronized bus transfers. We consider that a bus transfer is synchronized when the waiting time at the stop is below a given threshold. Among all bus stops in the urban bus system of the city, the bus stops pairs most frequently used in transfers are considered for synchronization. The distance between two bus stops in such pairs is short, and the time to walk it is not considered as waiting time. Constraints are imposed in the possible headway values of lines reflecting the regulations of transport authorities and the economic viability for bus operators.

BSP formulation

The mathematical formulation for BSP considers the following elements:

- A set of bus lines $I = \{i_1, \ldots, i_n\}$ and a set of lines $J(i)$ that may synchronize with line i (in a synchronization node, see next item). Buses have a maximum capacity of C transfer passengers.
- A set of synchronization nodes $B = \{b_1, \ldots, b_m\}$. Each node $b \in B$ is a triplet $< i, j, d_b^{i,j} >$ indicating that lines i (inbound line) and j (outbound line) may synchronize in b, and the distance between bus stops for lines i and j is $d_b^{i,j}$.
- A planning period $[0,T]$, expressed in time units, and the number of trips f_i of each line i.
- A *traveling time function* $TT : I \times B \rightarrow Z$. $TT_b^i = TT(i, b)$ is the time for buses of line i to reach synchronization node b (from the origin of the line).
- A walking time function $WT : I \times I \times B \rightarrow \mathbb{N}$. $WT_b^{i,j} = WT(i, j, b) = d_b^{i,j}/ws$ indicates the time needed for a pedestrian to walk between bus stops at the synchronization node according to a given walking speed ws.
- A *demand function* $P : I \times I \times B \rightarrow Z$. $P_b^{i,j} = P(i, j, b)$ indicates the number of passengers that transfer from line i to line j in synchronization node b, within the planning period.
- A maximum waiting time $W_b^{i,j}$ stating how long passengers are willing to wait for line j, after alighting from line i and walking to the stop of line j, in synchronization node b.
- A valid range of headways, which define the separation (measured in time units), between consecutive trips of the same line. The range of valid headways for bus line i is defined by an interval $[h^i, H^i]$, where values of h^i and H^i are usually enforced by public transportation administrators.

The BSP proposes finding appropriate values of headways for each bus line to maximize the number of transfer synchronizations. The mathematical model is formulated in Eq. (1). X_r^i indicates the departure time of trip r of bus line i, and $Z_{r,s,b}^{i,j}$ indicates whether trip r of line i and trip s of line j are synchronized (1) or not (0) in node b. The objective function weights synchronizations by the number of passengers that transfer thus giving priority to synchronization nodes with larger transfer demands. The demand is split uniformly among the f_i trips of line i and the number of passengers on each synchronization node is bounded by the transfer capacity C.

$$2\,maximize \quad \sum_{b\in B}\sum_{i\in I}\sum_{j\in J(i)}\sum_{r=1}^{f_i}\sum_{s=1}^{f_j} Z_{r,s,b}^{i,j} \times \min\left(\frac{P_b^{i,j}}{f_i}, C\right) \tag{1}$$

$$subject\ \ to \quad X_1^i \leq H^i \tag{2}$$

$$T - H^i \leq X_{f_i}^i \leq T \tag{3}$$

$$h^i \leq X_{r+1}^i - X_r^i \leq H^i \tag{4}$$

$$(X_s^j + TT_b^j) - (X_r^i + TT_b^i) > WT_b^{i,j} \ \ if \ \ Z_{r,s,b}^{i,j} = 1 \tag{5}$$

$$(X_s^j + TT_b^j) - (X_r^i + TT_b^i) \leq W_b^{i,j} + WT_b^{i,j} \ \ if \ \ Z_{r,s,b}^{i,j} = 1 \tag{6}$$

$$X_r^i - X_{r-1}^i = X_s^i - X_{s-1}^i \forall r, s, r > 1, s > 1 \tag{7}$$

$$X_r^i \in \{0, \ldots, T\}, Z_{r,s,b}^{i,j} \in \{0, 1\} \tag{8}$$

Regarding constraints, Eq. (2) assures that the first trip of each line starts before the upper bound for headways for that line. Eq. (3) forces the last trip of each line to end before T. Eq. (4) guarantees that the computed headways of each line are bounded to the range of valid headways for that line. Eqs. (5) and (6) state that trip r of line i and trip s of line j are synchronized at node b if passengers are able to transfer, considering the walking time $WT_b^{i,j}$ and the maximum waiting time $W_b^{i,j}$. Eq. (7) states that all headways for a given bus line are constant throughout the planning period. Finally, Eq. (8) defines the domains for the decision variables of the problem.

The Heterogeneous eBus Fleet Scheduling Problem

Planning public transport using electric buses is considerably different from that of designing traditional diesel bus systems. The limited autonomy eBuses have, together with the high impact of several factors on it, makes it a highly challenge problem. Examples of these factors are the use of air conditioning or road slopes, which may considerably increase energy consumption.

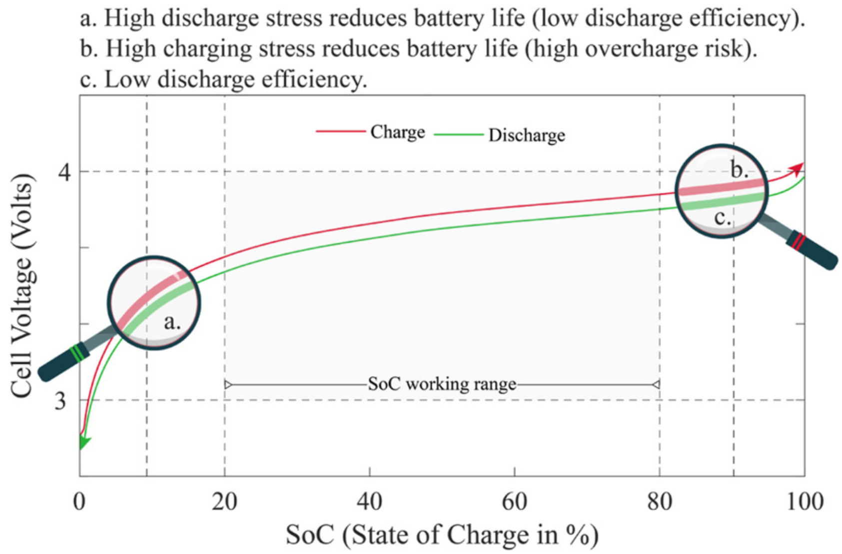


Fig. 3 Typical charge and discharge profiles for batteries in electric vehicles. The battery charge must always be between 20% and 80% to preserve battery SoH.

Therefore, the problem definition must consider the recharging requirements and the route elevation profile, as well as the passengers demand. Also, additional capacity restrictions could be considered in order to guarantee a security distance between people to mitigate some disease propagation, as it is the case of the pandemic produced by the SARS-CoV-2 virus.

The Heterogeneous eBus Fleet Scheduling Problem considers a fleet composed of various types of electric buses, in terms of capacity, driving range, and recharging conditions, and it lies in finding the best assignment of buses to each trip to minimize the number of buses required and optimize their energy use, as shown in **Fig. 2**(b). The elevation profile of the road (grade) must be taken into account for the energy consumption estimation, and the regenerative braking operation when circulating downhill must be considered too.

It is well known that fast charging methods may damage the battery life of vehicles (know as SoH, or State of Health). Therefore, HeFSP considers that vehicles are charged at overnight or at out of service periods using slow recharging methods in order to extend their SoH (Lipu *et al.*, 2018). For that, vehicles must be scheduled so that their overall remaining energy is reduced at the end of the day, this way maximizing the battery use of the fleet and minimizing the number of required vehicles.

HeFSP formulation

The HeFSP takes as an input a timetable of bus trips $T = t_1, \ldots, t_q$ for a given set of routes R and a fleet $B = \{B_1, \ldots, B_N\}$ of N different bus types, where B_n denotes the available number of eBuses of type n. The energy consumption of the whole fleet, F_C, is defined as follows:

$$F_C = \frac{1}{|N|} \sum_{n=1}^{|N|} SoC_n, \tag{9}$$

where SoC_n is the average state of charge of the onboard batteries of all eBuses of type n at the end of the day, measured in ampere-hour (Ah). This value represents the percentage of battery utilization, which is to be maximized.

The goal is to minimize the overall remaining energy in the fleet after the service is done, namely F_C. Therefore, the use of electrical energy from the batteries is maximized, and the number of eBuses required is reduced. This will allow using slow charging methods to recharge eBuses to preserve their batteries SoH, enlarging their lives.

SoC_n is defined as:

$$SoC_n = \frac{1}{|B_n|} \sum_{m=1}^{|B_n|} SoC_m, \tag{10}$$

$$SoC_m = SoC_m^{ini} - \frac{E_m + E_{aux}}{V_m}, \tag{11}$$

$$0.2 \times SoC_m^{max} \le SoC_m \le 0.8 \times SoC_m^{max}, \tag{12}$$

$$E_m = \frac{3600}{\eta_m^{bd}} \times \left(\sum_{p=1}^{|T_m|} PC_p - \sum_{c=1}^{|C_m|} PR_c \right), \tag{13}$$

where SoC_m is the state of charge of each bus m in B_n, being SoC_m^{ini} its initial charge. SoC_m^{max} is the maximum battery capacity of eBus m in Ah (different for each kind of bus). With the purpose of preserving the battery SoH, SoC_m level must be above 20% of SoC_m^{max}

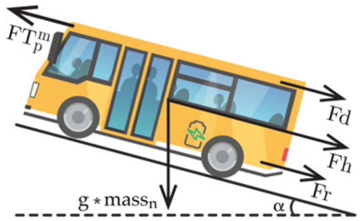


Fig. 4 The forces acting on an electric bus in an uphill slope as defined in Eq. (16).

to avoid the zone where the discharge efficiency gets worse, as depicted in **Fig. 3**. Additionally, batteries are not charged further than 80%, because otherwise battery stress levels would affect their useful life. E_m is the total energy-related to the operation of bus m per day, in watts hour (kWh). E_{aux} is the energy used for the auxiliary elements like air conditioner, lights, radio or doors opening/closing. V_m is the bus battery voltage in volts (v). PC_p is the total power consumption in watts (W) used to cover the trips T_m (set of trips assigned of a m bus). It is mathematically defined in Eq. (14). PR_c is the electrical generator power that considers recharging cycles during the operation or recapturing kinetic energy from braking (or transiting downhill) with a generator efficiency of 60%. The discharge efficiency of the eBus is η_m^{bd}.

$$PC_p = \frac{FT_p^m \times \overline{v}_p^m}{\eta_m^{total}}, \tag{14}$$

$$\eta_m^{total} = \eta_m^{motor} \times \eta_m^{trans} \times \eta_m^{DC-DC}, \tag{15}$$

$$FT_p^m = Fd_p^m + Fr_p^m + Fh_p^m + Fa_p^m. \tag{16}$$

In Eq. (14), η_m^{total} is the total drive-train efficiency (η_m^{total}) including the motor, transmission, and DC-DC converter efficiencies, and it is computed as defined in Eq. (15). $\overline{v}_p^m$ is the average speed (in $\frac{m}{s}$), and FT_p^m is the total traction force in Newtons (N), computed as the sum of the aerodynamic drag force (Fd_p^m), the rolling friction force (Fr_p^m), the hill-climbing (slope) force (Fh_p^m), and the acceleration resistance force (Fa_p^m). They are modeled in Eqs. (17) to (20).

$$Fd_p^m = \frac{\rho_{air} \times A_n \times C_{df} \times (\overline{v}_p^m)^2}{2}, \tag{17}$$

$$Fr_p^m = g \times C_{rr} \times \cos\alpha \times mass_n, \tag{18}$$

$$Fh_p^m = g \times \sin\alpha \times mass_n, \tag{19}$$

$$Fa_p^m = a_p^m \times mass_n. \tag{20}$$

Constant g is the gravity, a_p^m is the bus acceleration (in $\frac{m}{s^2}$), α is the angle of inclination in degrees, and C_{rr} is the coefficient of rolling resistance, which depends on the tires air pressure, their wear and tear, and road roughness. ρ_{air} represents the ambient air density, C_{df} is the drag coefficient, A_n means the frontal area in m^2 and $mass_n$ is the accumulated mass in kg of the vehicle of type n. In **Fig. 4**, we can see the forces acting on the e Bus in this problem.

The speed of the buses was set to 12 km/h (considered the average speed of a standard bus on Labor Day), according to the Spanish association of managing companies for urban collective transport (Cortés *et al.*, 2011). We are using a constant speed profile for the e Buses, the acceleration is therefore set to 0. As stated in Yu *et al.* (2016), the passenger mass only influences the bus energy consumption for speeds over 30 km/h. Because the stop-and-go pattern of buses all along their routes prevent them from reaching such high velocities, the passenger overweight is disregarded in this work. Several control points are selected on the eBus routes and their slopes are computed, giving also information of whether the eBus moves uphill or downhill. In case it moves downhill the regenerative brakes system will recover energy.

Methodology

This section describes the approach proposed to solve the considered problem. It is designed as a two-steps optimization procedure, as it is shown in **Fig. 5**. In the first step, the BSP problem is addressed with the aim of synchronizing bus lines in the city to minimize overall passenger transfer waiting time. This is done here using Virtual Savant (VS) PDB18, a fast optimizer based on machine learning. This first optimization step is described in Section "Virtual Savant for BSP".

The output of the first step is taken as the input for the second optimization step targeting the HeFSP problem. An evolutionary algorithm (EA) is used to find a suitable schedule of the available vehicles so that their battery use is maximized. This is explained in Section "Evolutionary Algorithm for HeFSP".

Route	Time	eBus	
720	07:00	MidiBus_01	
2628	07:00	MidiBus_02	
1122	07:15	MiniBus_01	
819	07:20	Standard_01	
170	07:30	MiniBus_02	
170	08:30	Standard_02	
2628	09:00	MiniBus_02	
170	09:28	MidiBus_01	
720	09:32	Standard_02	
819	09:45	MidiBus_02	
⋮	⋮	⋮	⋮

Fig. 5 The timetable with minimum waiting times in transfers (second column) is computed by VS in a first step. In the second step, an EA is used to assign vehicles to each trip (third column), so that the number of buses is minimized and the battery use is optimized.

Virtual Savant for BSP

Virtual Savant is a novel paradigm inspired by the Savant Syndrome, a rare mental condition where patients excel at certain abilities far above the average. In analogy to the Savant Syndrome, VS uses machine learning and parallel computing to find patterns from a set of previously-solved instances (called reference solutions) to solve the problem at hand. For that, VS makes use of a Machine Learning (ML) classifier for every problem variable. This classifier learns from the reference solutions the value the variable should take, according to a number of defined features, specific to every problem.

In the case of BSP, the algorithm needs to decide, for every synchronization node, what is the headway for both the inbound and the outbound bus lines. Therefore we make use of two classifiers to predict the headway of the inbound and outbound lines in every transfer node.

Three different sets of features are chosen for VS to learn to solve BSP. The first set is composed of synchronization node features, such as walking time between the pair of bus stops, the transfer demand, and the maximum waiting time. The second set aggregates inbound line features, namely travel time to synchronization node, minimum allowed headway, and maximum allowed headway. Finally, the third set comprises outbound line features, specifically, travel time to synchronization node, minimum allowed headway, and maximum allowed headway. The classifier that predicts the inbound headway uses the synchronization node and inbound line features, while the other one relies on the synchronization node and outbound line features.

We propose in Massobrio *et al.* (2022) how VS can be trained for the BSP. In that work, the set of instances solved by Nesmachnow *et al.* (2020) with an EA were used as reference solutions. In the learning process, each synchronization node is considered independently, so the method can scale to larger instances without the need for further training. The model aims to predict the headway of each pair of bus lines in a synchronization node based on features of the inbound and outbound lines and the node itself. The predicted solution is then refined using a local search to improve its quality. Several different headways can be proposed for the same line by different classifiers (when the same line is the inbound/outbound in different transfer nodes). The local search step generates different solutions taking into account combinations of all these values, and a refinement step is further applied to improve each of them. It tries to improve the solution by iteratively selecting a bus line and randomly change its assigned headway (according to a uniform distribution in the range of valid headways for the line). The change is accepted if the quality of the solution improves, otherwise it is discarded. The best solution found is reported as the result of VS.

Evolutionary Algorithm for HeFSP

Evolutionary Algorithms (EA), are well-known search and optimization techniques based on the Darwinian theory of evolution (Goldberg,1989). It consists of an iterative method that evolves a panmictic or distributed population of individuals which represent potential solutions of a predefined problem. The quality of each individual is directly related to its performance in the targeted problem and it is calculated by the fitness function.

In this work we are using as a second step optimizer, a cellular genetic algorithm (cGA) AD08, a type of EA with a decentralized population where individuals are arranged in a toroidal grid (see **Fig. 6**). In each iteration of the cGA, the population evolves using biologically inspired operators such as recombination, mutation and selection. These new individuals are evaluated and given a fitness value representing their performance to solve the problem.

The solution obtained by VS establishes the starting time of each bus route (order) as well as the bus headways. This determines the length of the chromosome of the individual of the cGA (the total number of trips the fleet has to do in time sequence). Individuals are encoded as arrays of integers, representing the bus types assigned to cover the timetabled trips, whose departure time is specified in the

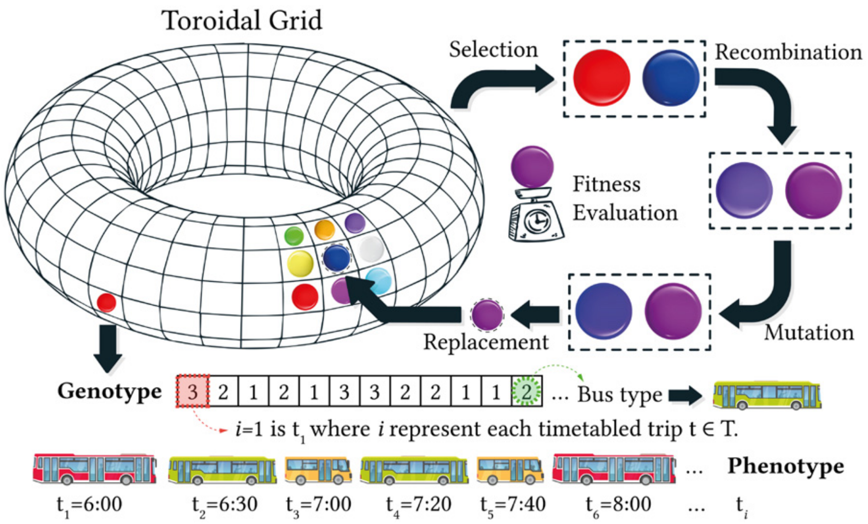


Fig. 6 Representation and encoding of the cGA for HeFSP problem.

sequence. See **Fig. 6** for a detailed representation of the problem encoding. The cGA will find the assignment of the heterogeneous eBus fleet that maximizes the batteries utilization after the operation time is completed, i.e., minimizing the remaining energy of the onboard batteries in the eBuses (see Eq. 9 where F_C refers to the average of the available power at the end of the day).

Fitness function

In order to evaluate the fitness value of the individual, a greedy algorithm is used for assigning the programmed trips to the minimum number of required vehicles to compose the heterogeneous fleet. This algorithm defines the eBus fleet size, i.e., the number of buses of different types, and it can be easily adapted to meet the requirements of the operator.

As stated before, in this work we consider preventive measures to preserve the SoH of batteries: slow charging and a limited operation range [20%, 80%] (see Fig. 3), for preventing stress levels and inefficiencies (Lipu *et al.*, 2018). Therefore, best solutions found by the cGA are those with an average SoC as close as possible to 20%.

Genetic operators

The evolution of the population relies on the genetic operators chosen. The selection operator decides which (normally two) individuals will be selected for mating. The recombination operator, or crossover, generates one or more new solutions by exchanging the information in the parents. Then, the mutation operator introduces slight changes in the chromosome of the newly generated individuals with low probability. Finally, the population is updated with (some of) the new individuals following a given replacement policy.

Experimentation

As already mentioned, this work consists of a two steps problem computing first the timetables and finding later the best eBuses schedule so that the transfer waiting times and the global energy required of a heterogeneous fleet of eBuses are minimum, satisfying the passengers demand.

Realistic instances were created using different sources of real data from the public transportation system in Montevideo (Uruguay). We are using open data and ticket sales from smart cards transactions, as well as open data from the city transportation authorities (bus lines, bus stops and timetables) for the entire public transportation system. The transfers were previously estimated using real data from 2015 of the ticket sales using smartcards (Massobrio and Nesmachnow, 2020).

From all possible transfers, the 171 most demanded were randomly chosen with uniform probability as synchronization nodes, and their passengers' demand was assigned in terms of the real data registered by smartcard transactions in May 2015. The real data of the ticket sales is shown in **Fig. 7**. As it can be observed, the peak of passengers' demand occurs around 13:00 h. Therefore, in this work Virtual Savant is used to maximize the number of synchronized trips in the most critical hours, i.e., from 12:00 h to 14:00 h.

The transfer capacity for buses was fixed to $C = 5$. The walking time between bus stops $WT_b^{i,j}$ corresponds to the real distance between the bus stops and assuming a constant walking speed of 6 km/h. The travel time of the bus lines to the synchronization nodes and the range of allowed headways for each line $[h^i, H^i]$ were defined according to publicly available timetables.

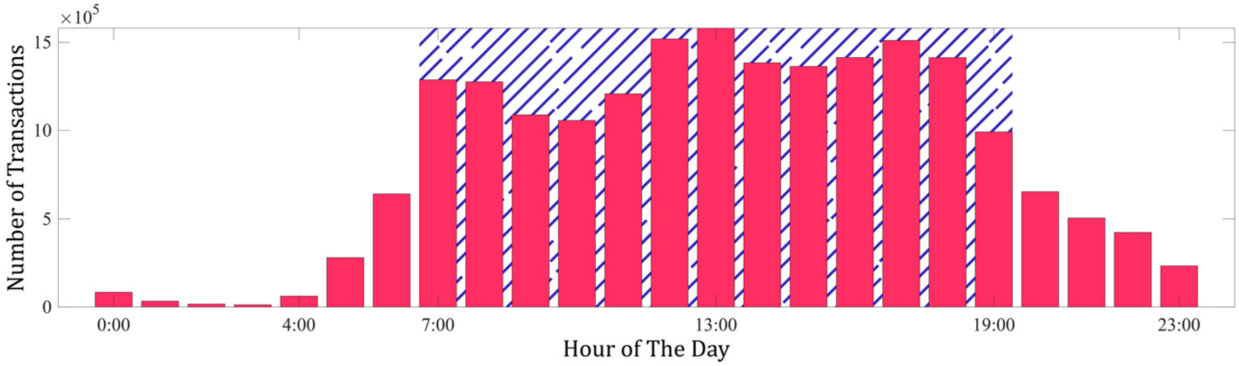


Fig. 7 Smartcard transactions per hour for May 2015.

Table 1 Parameters of the three selected vehicle types (microbus, midibus and standard bus) according to their brochures

Bus type	*Vehicle Parameters*				
	Range	*Area*	*Weight*	*Battery capacity*	*Passengers*
Microbus	208 km	8.37 m^2	13800 kg	250 kWh	20
Midibus	260 km	8.66 m^2	15600 kg	380 kWh	31
Standard	290 km	9.26 m^2	19300 kg	440 kWh	54

Table 2 Parameters of the algorithms used

Virtual Savant	
Classifier	*Random forest*
Local search iterations	5000
Independent runs	10
Cellular Genetic Algorithm	
Length of individuals	353
Population size	100
Population shape	10×10
Population initialization	Random
Neighborhood	C9
Parents selection	Current individual + Binary Tournament
Recombination	Uniform Crossover
Recombination probability	$\rho_c = 0.9$
Mutation	Random Value
Individual mutation probability	$\rho_0 = 0.1$
Gene mutation probability	$\rho_{rr} = 0.05$
Replacement	Replace if non worse
Independent runs	10

The parameter configuration of VS is given in **Table 1**. Random Forest Bre01 is used as classifiers to solve BSP, in particular its scikit-learn implementation. The local search algorithm was applied for 5000 iterations. The VS is executed 10 independent times and the best overall solution is taken as the input for the HeFSP.

In the second step, the cellular genetic algorithm is applied for obtaining the best schedule of the heterogeneous eBus fleet with minimum global energy consumption. From the set of bus routes in Montevideo, we have selected five of them with close starting points, so that buses ending one route could be reused for another. These route lines are 170, 720, 819, 1122 and 2628, which connect neighborhoods like Old Town, Pocitos, Buceo, la Teja, Cerro, Los Bulevares or Bella Vista among other locations (see **Fig. 8**).

The output of VS is the headway of the buses for the peak hours (12:00–14:00 h). We assume that the best result found for the peak hour will also be suitable when the demand of passengers is lower. Therefore, we create the timetable from 7:00 h to 19:00 h according to the headways obtained with the first step of the optimization process.

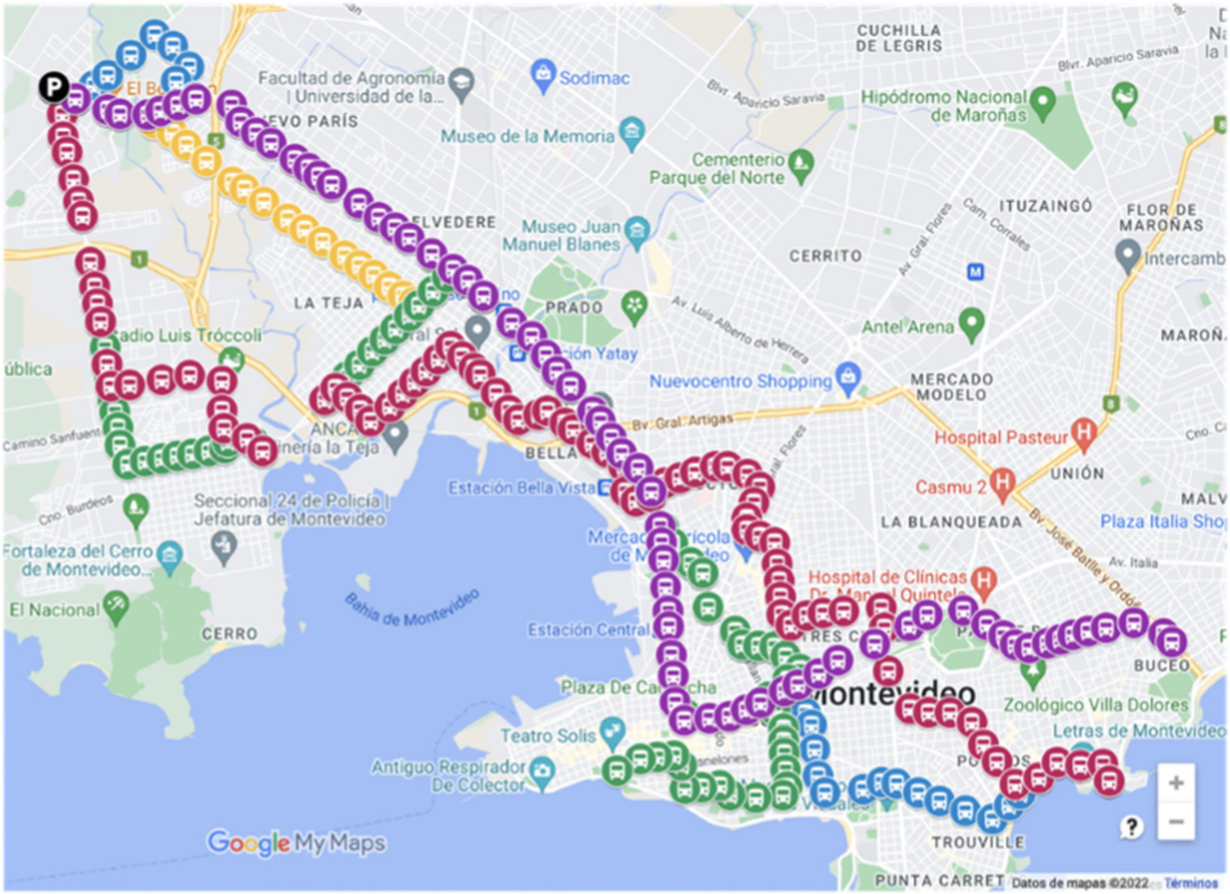


Fig. 8 A real scenario composed by five routes in Montevideo (Uruguay) is studied in this work.

Three different commercial buses were considered in this work, microbus, midibus, and standard bus. In **Table 1** some of the most relevant characteristics are shown, taken from the bus brochures (including battery capacity, weight, frontal area or comfort capacity).

Moreover, in terms of the energy consumption, we are considering regenerative breaking when circulating downhill (see Eq. 13) with an average speed of 12 km/h (Cortés *et al.,* 2011) and an overnight recharging strategy.

The parameters of the cGA are all summarized in **Table 2**. It implements a population of 100 individuals arranged in a square lattice. A Moore neighborhood is used, composed by the individual itself and its eight closest neighbors in the lattice. Therefore, the neighborhoods of adjacent individuals are overlapped, allowing a slow diffusion of genetic information across the population.

The size of individuals' chromosome is 353 variables, matching the total number of trips that must be done in the selected scenario. The initial population is randomly generated, i.e., the cGA randomly assigns different types of vehicles to each departure. The algorithm updates every individual one after the other. One parent is the current individual itself, while the other one is selected by binary tournament from within the neighborhood. Based on our previous work (Peña *et al.,* 2022a,b,c) the selected recombination method is uniform crossover: one offspring is generated taking each variable value from one of the two parents with equal probability. It is applied with probability $\rho_c = 0.9$. The mutation operator overwrites some randomly selected variables (with probability $\rho_{rr} = 0.05$) to a random value (within the allowed range of values). This mutation operator is applied with probability $\rho_m = 0.1$. The resulting individual replaces the current one in the population only if its fitness value is better. Because the cGA is a non-deterministic algorithm, we perform ten independent runs of the algorithm and evaluate the results, in order to get concluding results.

Results and Discussion

The main results obtained in this work are summarized in this section. The first step was solving the BSP with VS. In the training phase of VS, results show an overall good prediction accuracy for both classifiers. The median prediction accuracy was 0.67 for the inbound line classifier and 0.72 for the outbound line classifier, when compared to the solutions computed by the EA used as reference for learning. The solution computed by VS before applying the local search operator is already 11.5% better than the real solution (implemented by city authorities according to the published schedule), 12.6% better than a solution that sets the headway values randomly, and 3.0% better than a solution that sets the minimum allowed headways (which is very expensive to

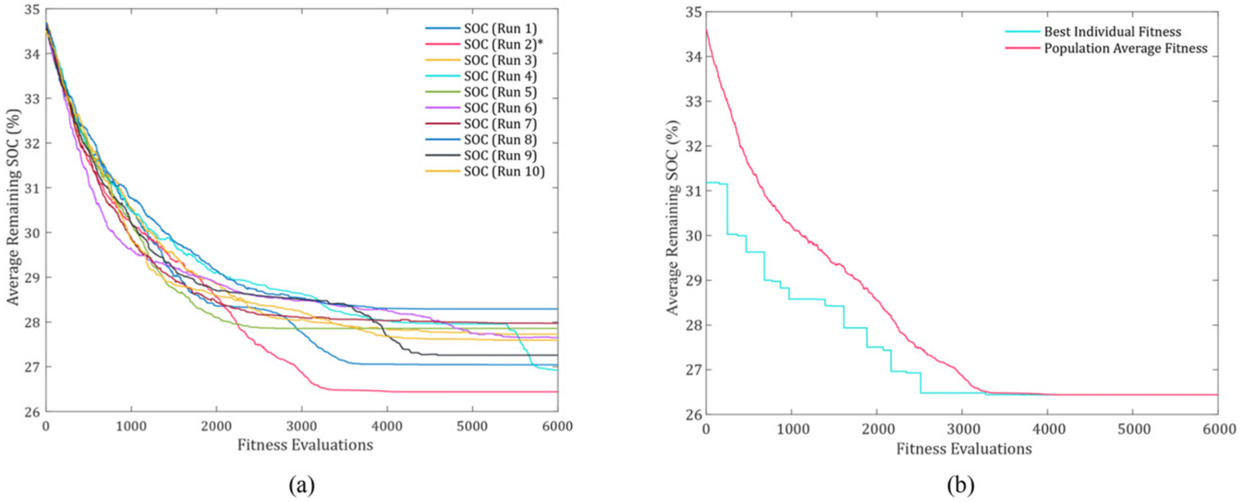


Fig. 9 Convergence performance study of the cGA for HeFSP. (a) Evolution of the average solution in the population for the 10 run. (b) Comparison of best and average solution in the population in a sample run.

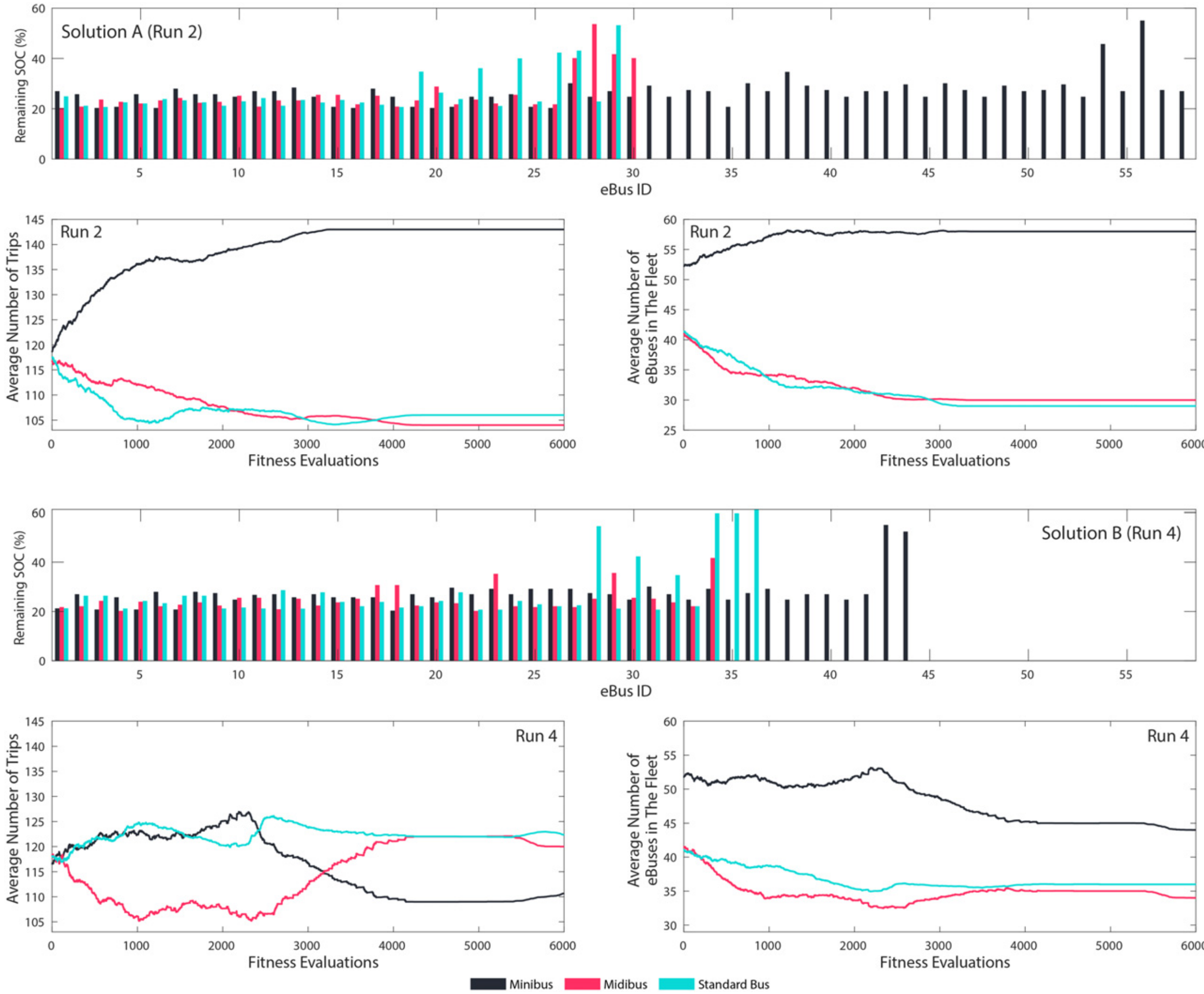


Fig. 10 Distribution of the final SoC of the eBuses after the daily service for two different solutions. Solution A in the top of the figure is the best-found result. Solution B in the bottom of the figure has less than 0.5% worst performance than Solution A. The evolution of the number of trips assigned to each type of vehicle and the fleet composition during the optimization process for each solution is displayed.

put into practice). After including the 5000-step local search operator, VS is able to outperform the reference EA for over 70% of the problem instances studied.

The best solution found by VS was used as an input to solve the HeFSP with the cGA in the second optimization step. The final results obtained with the cGA are presented in **Fig. 9**. On the left side, **Fig. 8** shows the evolution of the average fitness value of the population of 100 individuals during $6,000$ generations, for each of the 10 independent runs. As it can be seen, the population of the algorithm always converges to a stable solution whose percentage of average SoC is in all cases under 28.3% (the worst scenario being Run 1). The best execution obtained proposes a eBuses schedule where the average remaining SoC for the complete fleet is lower than 26.5% (26.44% –Run 2).

On the right side, **Fig. 8** displays, for the execution that found the best result (Run 2), the evolution of the best solution in the population, together with the average SoC of the complete population. It can be observed, that the population converges to the best solution, however the diversity is kept over half of the generations executed. After that point, the algorithm will hardly improve its best solution because information diversity is lost in the population AD04.

Results also show that different solutions, i.e., different scheduling of eBuses for the proposed timetable for each route along the day, are feasible and nearly optimal. To elucidate it, **Fig. 10** highlights the importance of the vehicle assignment. We can see the resulting eBus scheduling of two different solutions with very similar performance (less than 0.5% of remaining SoC of difference). The top of **Fig. 10** represents the performance of Solution A, i.e., the vehicle assignment, the evolution of the number of trips and the number of eBuses of each type along the algorithm execution of the best individual found (corresponding to Run 2 with 26.44% of remaining SoC). The bottom part of the same figure, shows the same metrics of the performance for Solution B (corresponding to the best individual found in Run 4 with 26.92% of remaining SoC).

The best solution found by the algorithm (Solution A) proposes a fleet of 117 eBuses, while Solution B suggests a shorter one of 114 vehicles. The former is composed of a higher number of minibuses (58) and lower number of both midi and standard buses (30 and 29 respectively). The fleet of the latter is more homogeneous with 44 minibuses, 34 midibuses and 36 standard buses. **Fig. 10** also shows the evolution of the average number of trips during the optimization process for each type of eBus for Solution A and Solution B. The total number of trips is defined by the problem instance, thus all solutions have 353 trips in our case. We can observe that Solution A promotes the trips of minibuses (over 140) while the number of midi and standard buses keep around 105 trips. However, in Solution B the trips accomplished by minibuses are lower (111) and the number of midi and standard buses moves around 120 trips.

In the comparison analysis of these two solutions, we can see that, with almost same energy consumption, Solution A proposes a slightly larger fleet (3 more ebuses) but with a larger number of minibuses, what will suppose lower cost for the operator. Solution B proposes a smaller fleet with a larger number of midi and standard buses accomplishing a higher number of trips what will suppose a larger bus capacity in their trips.

As it can be inferred, those results represent certain level of flexibility and adaptation to different scenarios. It can easily attend to other constraints established by operators (quality of service, operating costs, etc.) or tailored assess companies with a pre-existing electrical fleet that look to complement or optimize their service without fully renewing their fleets. Moreover, those constrains could be included in the fitness function so that the problem is solved specifically for the operator necessities.

Conclusion and Future Works

In this work, we propose the optimization of the holistic problem of the design of an efficient, competitive and sustainable public transport system with high quality of service in the city of Montevideo, Uruguay. Real data from the public transportation system of Montevideo and from the city transportation authorities is used to tackle all these criteria. The work is organized in two different optimization steps. The first one, optimizes the headways of the eBuses so that the transfer waiting times of passengers is minimized at peak hours using Virtual Savant. This result is used as input for the second optimization step, where a Cellular Genetic Algorithm optimizes the schedule of a heterogeneous fleet of electric buses so that the global energy used is minimized.

Results show that the average remaining battery levels obtained are close to the minimum (only 6.44% over) what indicates the solution provided is efficient. Moreover, an in depth analysis of the solutions obtained showed that different configurations are possible with almost similar performance. This opens two possible new research lines. First, the flexibility of the bus assignment problem, a key aspect for the operator because it directly affects on the operational cost or fleet capacity among other aspects. Second, the possibility to customize solutions to existing small electric fleets just by including tailored constraints to the problem.

As future work, we are planing to consider the passenger demand at every bus stop as well as to redesign the bus assignment algorithm with a more advanced mechanism that could make better use of the unused available resources of buses nearby. Also it would be interesting to investigate a new formulation of the problem including all bus lines in Montevideo and not only the bus lines with a close first/last stop. That would lead to a high dimensional problem that might require more advanced intelligent algorithms to tackle it.

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