J. Parallel Distrib. Comput. ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

J. Parallel Distrib. Comput.

journal homepage: www.elsevier.com/locate/jpdc

Operating cost and quality of service optimization for multi-vehicle-type timetabling for urban bus systems

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HIGHLIGHTS

- An integration of frequency setting and timetabling for multiple vehicle types.
- A multiobjective cellular evolutionary algorithm.
- Trade-off between the loss of quality of service and operational costs.
- A real-world case study based on the route 217 in Los Angles.

ARTICLE INFO

Article history:
Received 7 September 2017
Received in revised form 13 December 2017
Accepted 27 January 2018
Available online xxxx

Keywords:
Evolutionary algorithms
Metaheuristics
Multiple vehicle types
Multiobjective optimization
Public transport
Smart cities

ABSTRACT

In this paper, we propose a timetable optimization method based on a Multiobjective Cellular genetic algorithm to tackle the multiple vehicle-type problems. The objective is to determine bus assignment in each time period to optimize a quality of service and transport operating cost. The quality of service, represented by the unsatisfied user demand, guarantees a good experience in terms of comfort, safety, availability, improving effects on how passengers perceive wait times. The operational cost contributes to reducing the traffic jams, the flux of unfilled vehicles and fuel consumption, helping to diminish the negative environmental impact. With the operation data of Los Angeles bus route 217 northbound, at peak and off-peak hours, we obtain a set of non-dominated solutions that represent different assignments of vehicles covering a given set of trips in a defined route. The experimental analysis based on several quality indicators, like Hypervolume, Spread, ε-Indicator, and Set Coverage, indicates that our algorithm is a competitive technique comparing with well-known techniques presented in the literature.

1. Introduction

In increasingly interconnected and globalized world, more than half of the population (54%) live now in urban areas, as opposed to the 30% in 1950. This abrupt growth implies deep changes in a size and distribution of space (i.e., population density). This tendency will be accentuated in the coming years. An estimated 66% of the world population will live in cities in 2050 leading to a rise in demand for all infrastructures that interact directly with the people [31]. It involves several problems, for example, congestion, increased demand for a limited supply of resources, water, goods, energy, and services, including education, healthcare, and transportation.

Harrison et al. [16] define a smart city as an "instrumented, interconnected and intelligent city". Different areas like public administration, education, health services, energy, transportation, and logistics can be improved to make them more intelligent, interconnected and efficient by computing technologies. Smart cities can reduce living costs, make responsible use of resources,

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https://doi.org/10.1016/j.jpdc.2018.01.009
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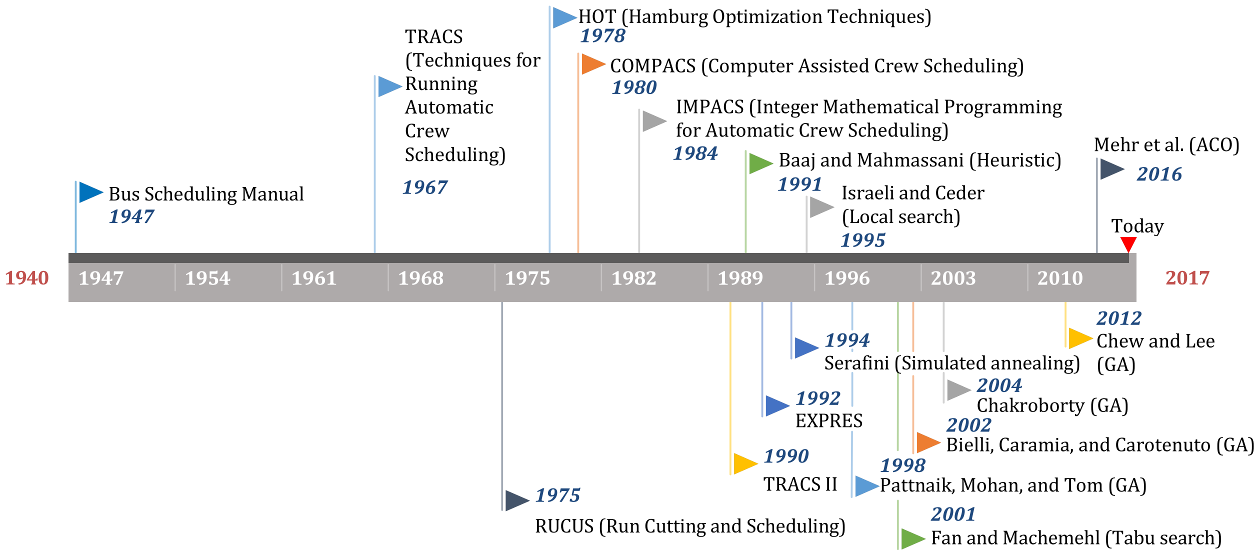


Fig. 1. Timeline of the research evolution of optimization problems associated with transportation systems.

and inspire the active citizens' participation in decision-making processes, to achieve a sustainable and inclusive city.

The main challenges of urban mobility are often related to the inability of public transport systems to satisfy needs of a growing number of users. Though each city has specific issues, local authorities and responsible mobility agencies share common objectives such as reducing congestion by improving traffic flow, sustainable and cleaner environment, increasing the use of public transport, and other greener options, like bikes and electric vehicles.

In 2010, the European Union defined the Intelligent Transport Systems (ITS) as an advanced set of applications, which provides innovative services relating to different modes of transport and traffic management. ITS integrates telecommunications, electronics and information technologies with transport engineering to plan, design, operate, maintain and manage transport systems [11]. Technological advances in computer science allow to collect a considerable amount of transport and mobility data, and design novel algorithms and mobile applications benefiting users, government organizations and operators (i.e., transport service providers) [2].

The main objectives of the ITS are as follows: improving capacity, efficiency, safety, reducing energy consumption and negative environmental impact. It enhances economic productivity for users and operators, improves personal mobility, convenience and comfort, and creating an environment, in which new ITS technologies can be applied.

The problems with more than one optimization criteria are known as Multiobjective Optimization Problems (MOPs), where a single solution that optimizes all objectives, at the same time, does not exist. The solution consists of a set of non-dominated solutions, called Pareto front or Pareto optimal set. In most cases, for NP-hard problems, its calculating is impractical. It can contain an infinite number of non-dominated solutions. Therefore, the goal is to obtain a good approximation of the real Pareto front, in a reasonable time. Heuristics and metaheuristics are a popular class of algorithms to find a high-quality solution for MOPs [25].

In this paper, we present a MultiObjective Cellular (MOCell) metaheuristic to solve the Multiple Vehicle-Types Timetabling Problem (MVTTP). We study two conflicting objectives due to the transport service providers want to minimize the operating cost, and users expect a better service. Hence, we propose solutions for a distribution of vehicles (proper frequency calculation), reducing the operating cost, and guaranteeing the quality of service.

The paper is structured as follows. The next section briefly reviews related works, models, and algorithms for transport problems. Section 3 describes the main approaches to multiobjective MVTTP. Section 4 provides descriptions of MOCell and other evolutionary algorithms. Section 5 presents simulation setup and experimental analysis. Section 6 highlights the conclusions of the paper and future work.

2. Related work

This section presents a brief overview of models and algorithms for transport problems, mainly, for urban public transport (see Fig. 1). Most of these works are based on computational intelligence techniques to improve approximate solutions, since, the problem is NP-hard [21].

2.1. Early methods

So-called early methods were not very advanced, as nowadays, because computers have not enough power to run complex mathematical solvers and use techniques for numerical models. Many approaches were reduced to the construction of an initial schedule by using heuristic process, and, then, attempting to improve this schedule by making limited changes.

One of the first systems named TRACS (Techniques for Running Automatic Crew Scheduling) was developed at the University of Leeds in 1967. The system assumes that a poor initial solution cannot turn into a good solution by heuristic improvements, which might be true since metaheuristics were not available at that time.

IMPACS (Integer Mathematical Programming for Automatic Crew Scheduling) was developed for bus operation in the late 1970s. Parker and Smith presented the prototype and Wren and Smith [30] gave a full description of the system. It was installed in London Transport in 1984 and Greater Manchester Buses in 1985. The vehicle or driver scheduling problem was formulated as a set covering problem and expressed as an integer linear programming problem.

2.2. Metaheuristic methods

Techniques for solving combinatorial problems can be classified into two main categories: exact and heuristic algorithms. The exact algorithms guarantee to find the global optimum. However, often, only small-sized instances can be practically solved. Heuristics and metaheuristics are more efficient and flexible, allowing approximate global optimum.

Israeli and Ceder [18] propose a local search algorithm for a multiobjective optimization of the size of the fleet, income that represents the number of passengers per hour, waiting time of passengers between each stop, and travel time when the bus is empty. The problem has been resolved in three stages. First, several sets of non-dominated solutions are generated, and frequencies for each vehicle are determined. Then, the local search method is used for exploring solutions. Finally, best solutions are selected from the Pareto-optimal set.

Shen and Kwan [30] develop an approach based on a Tabu Search for the driver scheduling problem called Heuristics for Automatic Crew Scheduling (HACS). The HACS is based on a representation of the problem involving sequences of links. The links and its associated active relief opportunities compose a solution space.

Costa et al. [8] present an algorithm for high-speed trains. After random initialization, it uses classical techniques to improve cooling solution and escape from local optimums. The difference with classical simulated annealing algorithms of a single objective is the use of weighted aggregation rules of objective values.

Mehr et al. [23] implement a metaheuristic based on ant colony systems for solving the problem of design lines of light rail and bus rapid transit in the Mashhad city in Iran.

Agrawal and Mathew [1] develop a parallel Genetic Algorithm (GA) to solve the transit network design problem. They use some constraints related to the load factor and demand satisfaction with bounded frequency. The objective function is the operational cost based on travel times and trip distances. The performance of the GA is improved by increasing the number of processors.

Chen et al. [6] propose a prediction scheme for improving the driving safety and efficiency in the transportation systems using GAs and neural networks in the Internet of Vehicles.

The previously presented literature shows different transport systems problems. In our approach, we tackle the tactical planning with the goal to optimize bus timetables based on the frequency setting using different vehicle types. In this area, researchers find solutions with several approaches such as heuristics, analytical and mathematical methods.

Newell [26] proposes to minimize the waiting time of the passengers by comparing a set of vehicles that have departure time assigned with a smoothed passenger arrival function. He shows that the optimal frequency for large vehicles varies over time, approximately as the square root of the arrival rate of passengers.

Furth and Wilson [13] propose a mathematical programming model that maximizes the ridership benefits due to saved waiting time. The model incorporates restrictions related to the fleet size, operating intervals (maximum headway case as the inverse to the minimum frequency) and the total budget. This model was extended by Koutsopoulos et al. [19] simplifying the problem by dividing it into equidistant operating intervals and solving it by linear programming. They propose a formulation based on social implications, divided into three components: operating costs, waiting times, and inconveniences of passengers overcrowding.

Ceder compiles and defines four different methods for calculating the frequency that depends on the load profile of passenger demand and restrictions stipulated by regulators entities. He shows how to obtain optimal timetables when selecting the maximum passenger load as a reference point [5].

Hadas and Shnaiderman [15] address the minimization of total cost based on empty space (e.g., unoccupied seats) and unfulfilled demand. The authors explain probability distributions for travel times and passenger demand with geolocation tools. Based on this information, they describe an analytical optimization methodology that determines the frequencies and size of vehicles. The implementation of the proposed approach shows that the most significant cost reduction is obtained in cases with a low level of service.

For the case of multiobjective techniques, Li et al. [22] consider stochastic parameters such as demand, waiting times and travel times. The authors define a hybrid stochastic optimization method to find the frequency that minimizes the waiting time for passengers and maximizes operator gains. They develop a GA and compare it with the traditional (analytical) models for frequency adjustment proposed by Newell and Ceder. The authors state that the intervals obtained are larger than those used in the Newell approach and shorter than those using by Ceder. Also, they mention that these reasonable intervals provide a better balance between bus operating costs and passenger satisfaction.

Kwan and Chang [20] present a formulation of the scheduling problem with two conflicting objectives: to minimize the cost of the number of transfers required and to minimize the cost caused by deviations from an initial schedule. The authors implement Non-dominated Sorting Genetic Algorithm II (NSGA-II) [9], a classic GA to address multiobjective problems, combined with other methods (e.g., local search).

Hassold and Ceder [17] study scheduling issues to minimize waiting time for passengers (quality of service) and a penalty based on the unoccupied space (operating efficiency). The main idea of the study is to be able to combine different types of vehicles based on the concept proposed by Potter [28], avoiding overloads by improving the vehicles utilization. The authors implement a heuristic based on graphs to combine different schedules in search of the optimal Pareto set. Numerical results of a case study in New Zealand show significant savings in passenger waiting times, but also an acceptable passenger load on all vehicles.

3. The multiobjective MVTTP

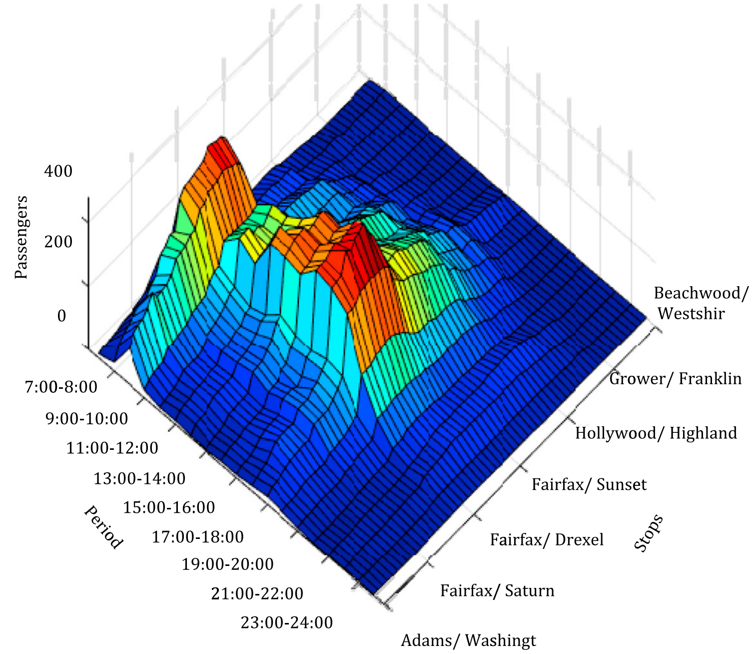
Smart city issues imply the development and implementation of computational techniques for planning mobility. The ITSs include three primary participants:

- *Citizens* or public transport users, which are looking for an efficient, economical, safe, comfortable and friendly multimodal system.
- *Companies* of transportation services, which are seeking to reduce operating costs and maximize profits, focusing efforts on economic subjects under the regulations of government authorities.
- *Governments*, whose policies pursue to ensure a high-quality life for citizens, setting mobility rules that satisfy their needs and ensuring the proper functioning of mobility systems.

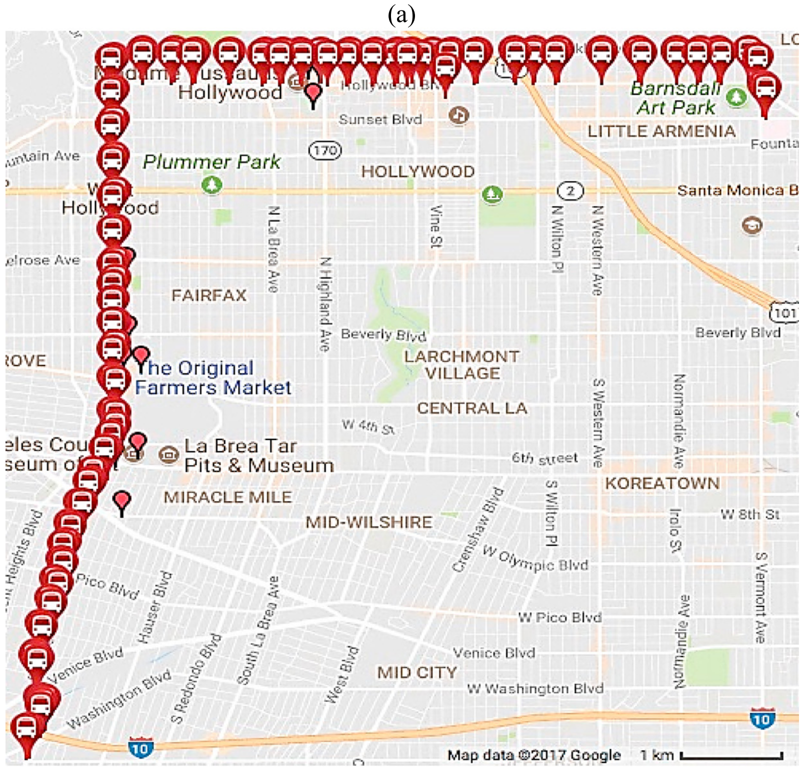
3.1. Problem description

The MVTTP models a realistic scenario, where vehicles of different types are assigned for covering the trips in a defined route. The MOP is to find an appropriate distribution of multiple vehicle-types, with the goal of simultaneously minimizing two essential objectives: the operating cost for providers f_1 and unsatisfied user demand f_2 (see Eqs. (1) and (2)).

The quality of service, represented by the unsatisfied user demand function (f_2), should guarantee a good experience in terms of comfort, safety, availability and affects the perceived delay to board the vehicle (waiting time). The comfort perception is associated to a load factor, i.e., number of passengers on board.



(a)



(b)

Fig. 2. Passenger demand. (a) Ride-check data for Los Angeles bus route 217 northbound, 19 time-period of one hour and 59 stops, maximum load of passengers 481 in Fairfax/Rosewood between 17:00 and 18:00 (peak hour) [4,27]. (b) Route map with the stops of 217 Metro Local Line—LA Metro Bus (southbound and northbound) placed.

On the other hand, the transport companies strive to reduce the operational cost, minimizing the cost function f_1. Additionally, it contributes to reducing the traffic jams, the flux of unfilled vehicles and fuel consumption, helping to diminish the negative environmental impact.

Ceder [4] proposed four basic steps for a transit planning problem: (1) Network route design; (2) Timetable development; (3) Vehicle scheduling, and (4) Crew scheduling. We divide the second step into two activities: frequency determination and timetable design. Therefore, we assume the following:

- The network route design problem is solved; hence, the topology of the route is defined, with stops located in a specific place, with a particular distance between them (ℓ_s).
- A passenger demand (load profile) is provided for each time-period in every stop (e.g., the number of passengers in the Los Angeles bus route showed in Fig. 2).

Unsatisfied demand is the amount of passenger that cannot be moved satisfactory, which implies more waiting time and overcrowding in the selected set of vehicles to cover the route in this time-period. *The operating cost* for each vehicle-trip includes driver salary, fuel consumption, and vehicle maintenance.

3.2. Mathematical formulation

The following elements are given: A set of vehicles $B = \{b_1, \ldots, b_n\}$, where b_i is the number of vehicles of type i, n is the number of types of vehicles, and $\sum_{i=1}^{n} b_i$ is the total fleet. T is a set of required trips $T = \{t_1, \ldots, t_m\}$ of a defined route R. The MVTTP is based on two objective functions f_1 and f_2:

Minimize

$$f_1 = \sum_{i=1}^{n} \omega_i \tag{1}$$

$$f_2 = \sum_{j=1}^{Lp} \sum_{s \in R} LQ_j^s \tag{2}$$

Subject to:

$$c_i = c_i^{bus} + c_i^{gas} + c_i^{driver} \tag{3}$$

$$\omega_i = c_i m_i \tag{4}$$

$$LF_j \leq LF_j^{max} \tag{5}$$

$$f_j \geq f_j^{min} \tag{6}$$

$$LQ_j^s = max\left(P_j^s - LF_j \sum_{i \in M_j} CAP_i, 0\right) \tag{7}$$

c_i^{gas} is the fuel cost for each vehicle; c_i^{driver} is the hourly pay cost (salary) of the bus driver; c_i^{bus} is the vehicle maintenance and other expenses; c_i is the cost involved in using a vehicle of type i; m_i is the number of i kind of vehicles to service all trips in T; ω_i is the total cost involved in using m_i vehicles of type i; P_j^s is the passengers number on a s stop in the route R; f_j is the frequency for period j; f_j^{min} is the minimum required frequency; LF_j is the load factor during period j; LF_j^{max} is the maximum load factor; M_j is the set of vehicles used during the period j; CAP_i is the passenger capacity of a vehicle of type i; LQ_j^s is the passengers demand at stop s that exceeds the capacity of the vehicles $i \in M_j$ during the period j and Lp is the last period.

4. Multiobjective evolutionary algorithms

The problem tackled in this paper is composed of two conflicting objectives that must be optimized at the same time. The general formulation of a MOP is the following:

Find a vector $x^* = [x^*{}_1, x^*{}_2, \ldots, x^*{}_n]^T$, which satisfies the m inequality constraints $g_i(x) \geq 0, i = 1, 2, \ldots, m$, p equality constraints $h_i(x) = 0, i = 1, 2, \ldots, p$, and minimizes the vector function $f(x) = [f_1(x), f_2(x), \ldots, f_k(x)]^T$, where $x = [x_1, x_2, \ldots, x_n]^T$ is the vector of decision variables.

A MOP consists of k objectives reflected in the k objective functions, $m + p$ constraints on the objective functions and n decision variables. The set of all the values satisfying the constraints defines the feasible region (or solution space) S. Any point $x \in S$ is a feasible solution.

A key concept in multiobjective optimization is *Pareto dominance* [7], which is defined as follows. Given two vectors $u = (u_1, \ldots, u_k)$ and $v = (v_1, \ldots, v_k)$, we say that u dominates v (denote by $u \prec v$), if and only if u is partially less than v, i.e., $\forall i \in \{1, \ldots, k\}$, $u_i \leq v_i$ and $\exists i \in \{1, \ldots, k\} : u_i < v_i$.

Solving a MOP can be viewed as the process of finding the set of solutions that dominate every other point in the solution space. This means that the solutions in this set are *Pareto optimal* for the problem. A set composed of all the Pareto optimal solutions is known as the *Pareto Optimal Set*, or simply the *Pareto set*. Each vector in the Pareto set has a correspondence in objective function space, getting the so-called *Pareto front*. Formally:

Pareto Optimality: A solution $x^* \in S$ is Pareto optimal if and only if it is non-dominated by any other solution $x' \in S$.

Pareto Optimal Set: For a given MOP, $f(x)$, the Pareto Optimal Set, $\mathcal{P}_S$, is defined as follows:

$$\mathcal{P}_S = \{x \in S | \nexists x' \in S\, f(x') \prec f(x)\} \tag{8}$$

Pareto Front: For a given MOP, $f(x)$, and Pareto Optimal Set, $\mathcal{P}_S$, the Pareto Front $\mathcal{P}_F$ is defined as follows:

$$\mathcal{P}_F = \{f(x) \in \mathbb{R}^k | x \in \mathcal{P}^*\} \tag{9}$$

As discussed before, MOPs can have a Pareto front composed of a possibly infinite number of solutions. When stochastic techniques, such as metaheuristics (e.g., Evolutionary Algorithms (EA), Simulated Annealing or Tabu Search) are used, the goal is to obtain a $\mathcal{P}_F$ approximation (also called approximation set), i.e., a subset of solutions that represents the true Pareto front.

4.1. Evolutionary algorithms

EAs are nature-inspired search methods that emulate the evolution process of species to solve optimization problems. The evolution is the result of the interaction between the creation of new genetic information and evaluation its behavior for future selection. When an individual exhibits a better performance, it has a higher opportunity to live for a longer and generate a genetic inheritance (mutated) by mating.

The non-deterministic nature of reproduction leads to a permanent production of new genetic information and, therefore, to the creation of different offspring [3].

The MultiObjective EAs (MOEAs) have been applied to solve many MOPs. They work with a set of possible solutions simultaneously, obtaining accurate results in several research areas (e.g., bioinformatics, transport, structural and mechanical engineering, robotics, scheduling, finance or manufacture process). Due to their population nature, they can find a set of assorted solutions that approximate to the whole $\mathcal{P}_F$ of a MOP in one single run.

MOEAs are designed considering two features at the same time: satisfactory convergence and diversity properties. It means that they not only look for finding the approximate $\mathcal{P}_F$, with a high degree of convergence (be as close as possible to the $\mathcal{P}_F$), but also the Pareto optimal solutions must be uniformly spread along the $\mathcal{P}_F$.

These techniques apply iterative and stochastic processes on a set of individuals (population), where everyone represents a potential solution to the problem. They use fitness values to measure the solution quality to guide the search.

Most EAs use a single population (panmixia) of individuals and apply the operators as a whole. If we think on the population of an EA in terms of graphs, a panmictic EA is an entirely connected graph. On the other hand, in the case of distributed EAs or cellular EAs, the individuals in a population can just interact with a reduced number of individuals partitioned into a set of islands or located in a nearby neighborhood (see Fig. 3).

In this work, we focus on cellular GAs, particularly, on MOCell [24]. The main feature of this type of algorithms is that the population is distributed in a two-dimensional toroidal grid, where each individual belongs to a cell and can only be recombined with the surrounding cells (neighboring cells). The main idea of this limitation is to perform a greater exploration of the search space because the overlapped neighborhoods induce a slow diffusion of solutions through the population, while a kind of exploitation takes place inside each neighborhood by genetic operations.

The algorithm (see Algorithm 1) maintains an external file to store non-dominated solutions using the crowding distance of NSGA-II [9] to maintain a diverse set of solutions and a feedback mechanism to replace individuals in the population after each iteration.

Algorithm 1 Pseudocode of metaheuristic based on MOCell

[data]=**Setup** ( ); */Algorithm parameters and data input*
pop=[**popGen**( )&**neighborhood**( )]; */Creates an initial population and distribute in a toroidal grid*
$\mathcal{P}_F$= [ ]; */Creates an empty Pareto front*
while (terminationCondition==false) **do**
  for k=1**to** popSize **do** */individual = k*
    ndPop=**getNeighborhood**(pop,k);
    parents=**selection**(ndPop);
    offspring=**recombination**(data, parents);
    offspring=**mutation**(data, offspring);
    pop(k)=**replacement**(ndPop, offspring);
    $\mathcal{P}_F$=**insertPF**(pop(k), $\mathcal{P}_F$);
  end
pop=**feedback**(paretoFront);
end

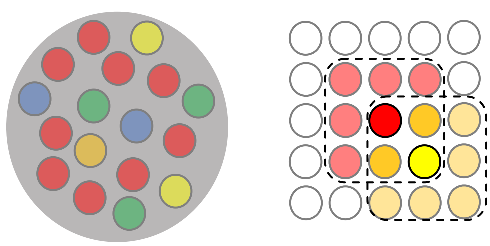


Fig. 3. Example of the individuals' distribution in a population for panmictic EA (left) and cellular GAs (right).

4.2. Encoding and solution representation

Solutions are encoded as arrays of integers, representing the type of vehicle assigned to cover one trip of T. Zeros mark new time-periods. The order of departures is specified in the sequence. Fig. 4 shows an example of solution encoding for a problem instance with three different types of vehicles, six trips $t_k \in T$, and three periods of one hour.

The array size is taken from prior demand study and preliminary frequency determination with a method based on a load profile. A lower-bound level on the frequency (F_j) for each time-period, given the same vehicle-capacity constraint, and average capacities of different vehicle-types ($\overline{CAP}$) is calculated as follows.

$$F_j = max\left[\frac{A_j}{LF_j \cdot \overline{CAP} \cdot L}, \frac{P_j^{max}}{\overline{CAP}}, f_j^{min}\right] \tag{10}$$

$$A_j = \sum_{s \in R} P_j^s \cdot \ell_s \tag{11}$$

$$L = \sum_{s \in R} \ell_s \tag{12}$$

where A_j is the area in passenger-km under load profile during the time period j, P_j^{max} is the maximum number of passengers at any stop, ℓ_s is the distance between the stops s and $s+1$, and L is the route length [4]. The other notations are previously defined in Eqs. (6) and (8). The load profile is the average of passengers at every stop $s \in R$ (see Fig. 2(a)) during a specific period. One way to calculate the vehicles frequency is to take the bus stop with maximum ridership as the reference. The main problem is the empty space, if the load profile is concentrated in a few stops. To solve this issue, one option is to use the load average $\overline{P_j^s}$ per period. However, this solution can produce overcrowding of passengers at the critic bus-stops.

Ceder [5] propose the ratio A_j/L as an average representative of the load P_j^s (regardless of its statistical definition), divided by the product $LF_j \cdot \overline{CAP}$, the desired occupancy on the vehicles at period j. The load at off-peak hours normally is smaller. F_j depends of the P_j^{max} and maximum capacity of the vehicles. In the worst case, F_j is equal to the minimum permitted frequency, usually defined by the government entities.

The method to compute F_j is important because directly affects the number of departures of the timetable. We select this method based on the possibility of combining different vehicles capacities to cover the load P_j^s avoiding overruns. The chromosome size is not very long or too short. It allows the genetic operators to work on an individual that represents a timetable designed with F_j. If the average P_j^s and distance between the stops are used, it guarantees that the onboard passengers at the max load route segment will not experience crowding above the given vehicle capacity CAP [5].

The distribution of zeros can be changed but cannot be consecutive, and every time-period j has a same length and depends on total travel time to cover the route R.

4.3. Objective functions and fitness evaluation

The optimization problem is formulated with two objective functions f_1 and f_2. The first one is to minimize the operating cost that covers all departures for a specific route. The second one is to minimize the perceived loss of quality of service, when the user demand is not satisfied.

The f_1 function, Eq. (1), consists of the summation of the all costs for each vehicle assigned to cover a trip t_k. It is calculated according to the schedule (timetable) and variable cost depending on the vehicle type.

The f_2 function defined by Eq. (2) indicates the number of passenger-km that cannot be transported by the fleet assigned for each trip t_k. For example, in Fig. 5, the dotted line represents the capacity of the vehicles assigned in a period j. Unsatisfied demand f_2 defines the number of passengers that cannot be moved satisfactorily, which implies more waiting time and overload in the selected vehicles to cover the route.

The values of the functions are normalized since we know the highest cost of the operation by choosing all the most expensive type of vehicle and maximum loss of passengers when vehicles with minor capacity are selected.

4.4. Evolutionary operators

4.4.1. Population initialization

The population is generated by randomly assigning different types of vehicles to each departure taking into account the size of

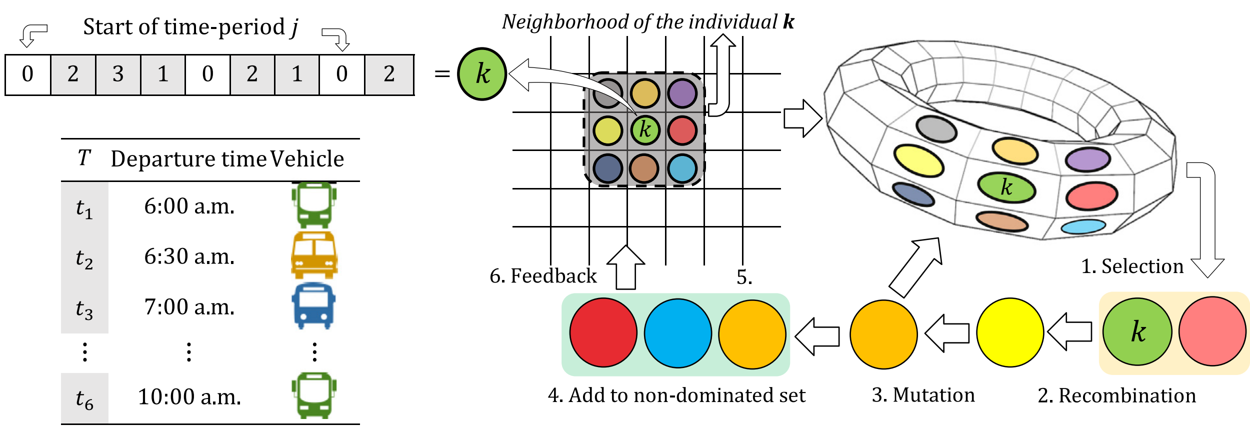


Fig. 4. Example of solution representation for the MVTTP and reproduction steps in MOCell.

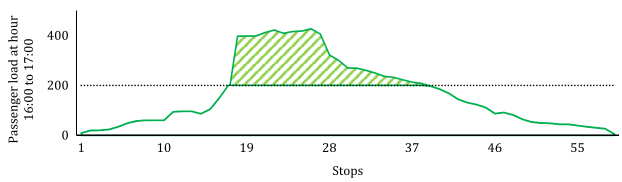


Fig. 5. Example of unsatisfied demand due to selected vehicles.

schedule and zeros distribution based on the previously frequency calculation F_j (see Eq. (10)). All individuals are distributed in a toroidal grid (see Fig. 4).

4.4.2. Selection

A tournament selection (tournament size: 9 individuals) is chosen to select the parents (two individuals survive) in the neighborhood of the studied individual.

4.4.3. Recombination

The recombination or crossover generates new vehicles assignment by combining two other chromosomes with probability *Pc*. Five operators known in the literature are considered: SPX, TPX, DX, UX, and HUX.

Single point crossover (SPX): A variant of the classic SPX technique consists in selecting one random zero position as crossover point, time periods from the beginning of chromosome to the crossover point is copied from one parent, the rest is copied from the second parent.

Two-point crossover (TPX): Two crossover points are selected (random zero positions), elements from the beginning of chromosome to the first crossover point is copied from parent one. The part from the first to the second crossover point is copied from the second parent and the rest is copied from the first parent.

Discrete crossover (DX): In this method, for each position in the offspring chromosome, the parent who contributes its gene value (vehicle-type) is chosen randomly with equal probability. The second offspring is computed analogously.

Uniform crossover (UX): The uniform crossover allows the parent chromosomes to contribute the gene level rather than the segment level, unlike SPX or TPX. Each gene value is randomly copied from the first or the second parent with some probability p_{UX}. If p_{UX} is less that some threshold (typically 0.5) the value of the first parent's gene is assigned to the first offspring and the value of the second parent's gene is to the second offspring. When p_{UX} is major or equal to the threshold value of the first parent's gene is assigned to the second offspring and the value of the second parent's gene is to the first offspring.

Half uniform crossover (HUX): In this operator, exactly half of the information in the parents' chromosomes are assigned to offspring. The first step is to calculate the Hamming distance (the number of different values). This number is divided by two. The resulting number is how many of the genes that do not match between the two parents will be assigned to offspring using the same methodology of UX.

4.4.4. Mutation

This genetic operator makes random changes in one or more chromosome's genes. The mutation is carried out according to the mutation probability *Pm*. Three mutation operators are proposed: UM, OGPPM and RPM.

Uniform mutation (UM): In this operator, from the entire chromosome, a subset of genes are randomly chosen, and their values are swapped for other random value from the set of allowable values (set of different types of vehicles).

One gene per period mutation (OGPPM): This method can select one gene of every time-period on the chromosome at random and their values are swapped for other random value, i.e., another different vehicle type.

Reset period mutation (RPM): This mutation operator selects all genes of the same time-period on the chromosome, which will be replaced by a new set of random values (vehicle types) with the same size of the selected time-period.

5. Experimental results

This section details the experimentation methodology used in our study. First, we describe indicators used to assess the quality of the computed Pareto front approximation. Second, we report the parameter settings and experimental analysis of the proposed algorithm for a given passengers' demand case of the route 217 in Los Angeles city (see Fig. 2) used in the literature for an approach using analytical and heuristic methods [4,5].

The structure of data set represents the average of passengers arriving at a stop on a specific route. Some parameters are obtained from the website of the bus company, which include vehicles' capacity and operational costs.

5.1. Quality indicator

Different metrics have been proposed in the literature to evaluate MOEAs in terms of convergence and uniform distribution along the Pareto front. We have chosen the Hypervolume (I_{HV}) to assess these two criteria together, *Spread* (I_{Δ}) to evaluate the dispersion of a non-dominated solution set, Two Set Coverage (I_{TSC}) to provide a relative coverage comparison between two sets, and ε-Indicator

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to induce a relation of how much one set of solutions is worse than another one.

Instead the optimal Pareto front, which is not known for the considered problem, we use an *artificial* Pareto front ($\mathcal{P}_a$). $\mathcal{P}_a$ is built by merging all the Pareto front approximations computed by the two tested algorithms in every independent run into one single front as suggested by Dorronsoro et al. [10].

To avoid biases due to the differences in the order of magnitude of the different objectives, the evaluated approximation sets are normalized by the maximum values for every objective function.

Hypervolume (I_{HV}) [33]. The I_{HV} calculates the multi-dimensional region enclosed between the individuals in the computed approximation to the Pareto front and a reference point in the objective function space. The closer the approximation is to the Pareto optimal front, the higher the value of this indicator. On the other hand, if the spread of the individuals along the Pareto front is good (desirably uniform), the higher the value of this indicator. Hence, a solution that produces the higher value as possible of this indicator is desirable.

I_{HV} can be calculated as follows. First, given an approximation set $\mathcal{P}$ placed in objective space and reference point W, a hypercube (Hc_k) for each solution ($k \in \mathcal{P}$) is constructed, taking as corners the reference point and the solution point. After that the union of all these hypercubes is equal to the hypervolume:

$$I_{HV} = volume\left(\bigcup_{k=1}^{|\mathcal{P}|} Hc_k\right) \tag{13}$$

Spread (I_Δ) [9]. I_Δ measures the extent of spread achieved among the computed solutions. It is defined as follows:

$$I_\Delta = \frac{d_f + d_l + \sum_{i=1}^{(|\mathcal{P}|-1)} \left|d_i - \overline{d}\right|}{d_f + d_l + \overline{d}(|\mathcal{P}| - 1)}, \tag{14}$$

where d_i is the Euclidean distance between consecutive solutions in the computed non-dominated set of solutions $\mathcal{P}$. $\overline{d}$ is the average of these distances. d_i and d_f denote the Euclidean distances between the extreme solutions of the optimal Pareto front and the closest solution of $\mathcal{P}$, respectively. This indicator takes a zero value for an ideal distribution with the existence of the extreme solutions in $\mathcal{P}$.

Two Set Coverage (I_{TSC}) [32]. I_{TSC} is used to show that the results (i.e., the computed non-dominated set of solutions) of an algorithm dominate the results of another algorithm. This metric provides a relative comparison between two solutions sets X' and X'' based on the dominance relationship.

I_{TSC} provides the ratio of points in X'' that are dominated by X', mapping the ordered pair (X', X'') to the interval [0, 1] according to the following equation:

$$I_{TSC}(X', X'') = \frac{\left|\left\{a'' \in X''; \exists a' \in X' : a' \succcurlyeq a''\right\}\right|}{|X''|} \tag{15}$$

$I_{TSC}\left(X', X''\right) = 1$ means that all solutions in X'' are dominated by or equal to solutions in X'. Otherwise, $I_{TSC}\left(X', X''\right) = 0$ represents that none of the solutions in X'' are covered by the set X'. Both cases $I_{TSC}\left(X', X''\right)$ and $I_{TSC}\left(X'', X'\right)$ should be considered because $I_{TSC}\left(X', X''\right)$ is not necessarily equal to $1 - I_{TSC}\left(X'', X'\right)$.

ε-Indicator (I_ε) [34]. This indicator provides a factor showing how far one approximation set is worse than another one taking into account all objectives. We use this metric to calculate how much a solution set with m objective vectors $X = \left(x^1, \ldots, x^m\right)$ is worse than the *artificial* Pareto front $\mathcal{P}_a$. Generally speaking, $I_\varepsilon\,(X, \mathcal{P}_a)$ is the minimum factor ε such that any objective vector in $\mathcal{P}_a$ multiplied by ε is dominated by at least one objective vector in X. More formally, for a minimization problem with two objectives, an objective vector $x = (x_1, x_2) \in X$ dominates with ε factor ($x \preccurlyeq_\varepsilon \rho$) an objective vector $\rho = (\rho_1, \rho_2) \in \mathcal{P}_a$, if and only if, $x_1 \leq \varepsilon \cdot \rho_1 \wedge x_2 \leq \varepsilon \cdot \rho_2$, for a given $\varepsilon > 0$. $I_\varepsilon = (X, \mathcal{P}_a)$ is defined by Eq. (16).

$$I_\varepsilon\,(X, \mathcal{P}_a) = \inf_{\varepsilon \in \mathbb{R}} \{\forall \rho \in \mathcal{P}_a \exists x \in X : x \preccurlyeq_\varepsilon \rho\} \tag{16}$$

$I_\varepsilon\,(X, \mathcal{P}_a) = 1$ represents that approximation set is equal to the *artificial* Pareto front. It means that a solution that produces the value close to one is desirable.

Table 1
Calibration parameters.

Parameter	Values
Population size	100
Neighborhood size	8
Selection method:	binary tournament (9 to 2)
Crossover operators:	SPX, TPX, DX, UX, HUX
Crossover probability (Pc):	0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1
Mutation operators:	UM, OGPPM, RPM
Mutation probability (Pm):	0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1

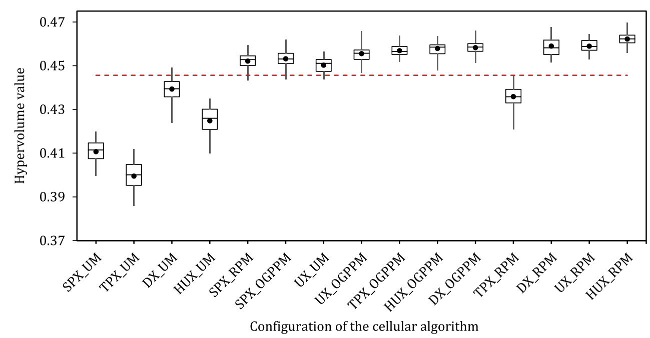

Fig. 6. Boxplot representation of the hypervolume values obtained for the studied case with different configurations of the algorithm.

5.2. Experimentation methodology

In this section, we describe tuning and evaluation of the proposed cellular genetic algorithm focusing on quality of solutions and performance.

The following parameters were set for the calibration: 30 independent executions of 100 generations with a population size of 100 individuals (10×10 toroidal grid and 10 000 fitness evaluations) (Table 1).

Hence, $5 \times 10 \times 3 \times 10 = 1500$ different setups were considered to calibrate our algorithm.

For assessing the statistical significance of the experimental results, and observe the effect of different parameters on the quality of the outcome, we apply a Shapiro–Wilk test [29], which shows that 90% of the experimental results demonstrate normal distribution.

Then, we apply the non-parametric statistical Friedman test [12]. The repeated analysis of variance by ranks is used to determine which parameters combination have a significant effect on the algorithm performance and produce better results in terms of hypervolume.

There is a statistically significant difference in hypervolume values. Depending on mutation or recombination probabilities, 15 algorithm configurations were selected for, $\chi^2(14) = 397.93$, $p = 2.2\epsilon^{-16}$. These results are summarized in Table 2, where the possible parameter combinations are placed based on their Friedman ranking values for the crossover and mutation probability combinations (Pc and Pm).

Fig. 6 shows hypervolume values obtained from 30 independent MOCell runs in the form of boxplots. It is important to keep in mind

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Table 2
Friedman rank of the algorithms with different genetic operators and their probabilities.

		UM										OGPPM										RPM									
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
SPX	0.1	57	475	1324	2145	2382	2740	2930	2643	2917	2935	30	99	312	708	1266	2015	2036	1975	2172	2215	168	205	283	180	113	88	66	233	142	194
	0.2	266	669	1764	2304	2624	2577	2718	2858	2875	2828	69	392	781	1303	2471	2472	2293	2260	1643	2643	506	542	410	405	574	338	491	558	460	490
	0.3	300	736	1717	1833	2240	2446	2618	2456	2747	2809	134	881	1490	2312	2318	2726	2398	2566	1959	2439	631	779	737	935	860	806	919	648	955	953
	0.4	187	636	1420	1693	1971	2168	2502	2514	2538	2652	225	948	2111	2787	2132	2289	2941	2833	2503	1522	1091	1264	934	1026	1052	1126	1704	1130	1228	1197
	0.5	220	636	1002	1647	2002	1961	2329	2125	2233	2109	822	1085	1292	2026	2821	2527	2625	2125	1377	1656	1644	1398	1008	1682	1199	1516	1529	1445	1484	1250
	0.6	290	641	1224	1565	1422	2119	2018	2030	2000	2157	1033	2060	1756	2835	2541	2084	2474	1912	1750	1329	1901	1904	2040	1844	1430	2392	1884	2284	1984	1692
	0.7	189	568	1036	1197	1232	1568	1887	1692	2017	1746	513	1364	2151	2078	1842	2155	2012	1575	1191	1116	1615	2026	2387	2043	1919	2091	2364	2781	1971	1779
	0.8	156	353	610	1092	1102	1644	1381	1698	1305	1388	847	1472	1744	1548	1823	1555	1010	914	565	468	2501	2318	2304	2006	2692	2616	2470	2348	2383	2502
	0.9	97	243	488	730	841	1049	1074	1234	1387	1087	1154	1221	1475	1668	1754	1169	822	327	679	294	2670	2489	2419	1549	2127	1876	2448	2067	2574	1907
	1	38	364	478	571	1050	892	799	1246	1154	873	1090	1335	808	773	554	402	451	285	258	239	2372	2867	2424	1891	2356	2744	2221	2323	2331	2798
TPX	0.1	1062	868	751	720	648	524	392	382	328	32	30	137	437	711	1685	2151	1815	2422	1825	2393	897	954	799	736	646	523	419	372	333	35
	0.2	1539	1496	1270	1155	1065	921	703	511	408	70	65	772	1200	1927	2684	2534	1888	1936	2578	2177	1604	1471	1353	1183	1025	965	676	534	376	71
	0.3	2032	1914	1753	1669	1338	1192	976	620	444	92	162	1581	2348	2219	2409	2529	2896	1911	1633	2016	2044	1864	1713	1547	1426	1226	939	626	452	88
	0.4	2367	2122	2022	1851	1666	1435	1190	887	500	139	310	675	2180	2649	2240	2321	2293	2035	1790	1621	2329	2226	2021	1911	1662	1449	1154	820	485	178
	0.5	2646	2389	2206	2032	1788	1615	1388	988	569	175	817	1515	1940	2612	2782	2603	2744	2129	1894	1553	2628	2402	2360	1920	1878	1654	1220	1071	584	208
	0.6	2740	2572	2450	2205	2045	1661	1432	1116	652	197	625	1761	2575	2361	2228	2462	2868	1604	1336	1418	2725	2600	2421	2141	2084	1671	1331	1084	676	216
	0.7	2855	2680	2515	2321	2074	1891	1529	1226	804	219	865	951	1704	2134	2327	1529	1524	1170	772	663	2891	2726	2531	2320	2196	1787	1692	1251	821	162
	0.8	2933	2830	2616	2463	2326	1972	1667	1352	845	285	846	952	2037	2082	1656	1888	1379	1686	734	317	2896	2810	2676	2558	2212	1981	1515	1282	699	251
	0.9	2948	2885	2736	2464	2335	2123	1749	1273	805	228	1056	795	1292	1478	1226	709	1182	898	515	308	2962	2837	2709	2531	2350	2032	1802	1478	978	193
	1	2993	2879	2775	2651	2572	2220	1885	1465	976	215	1274	1199	661	579	822	298	455	252	180	93	2999	2934	2821	2534	2358	2231	1858	1416	966	248
DX	0.1	123	705	1987	2080	2716	2400	2288	1531	1029	498	95	447	1482	2234	2481	2671	2089	2657	2541	2019	74	440	1390	1446	2702	2148	2694	2223	2469	2584
	0.2	263	1049	1810	2081	2150	2413	1933	1486	890	521	408	1318	2099	2262	2314	2602	1529	2338	1884	1833	158	967	1959	2672	2265	2696	1810	1966	1939	1418
	0.3	99	707	1367	2796	1939	2687	2461	2283	925	716	729	1909	2339	2690	2639	1984	2125	1479	1491	1110	562	2046	2160	2610	2613	2262	1811	1462	1048	851
	0.4	219	509	1977	2413	2088	2725	2197	1311	1402	456	889	2748	2484	2644	2693	2561	1683	1597	1210	930	829	1874	1874	2705	2621	1951	1802	1294	1223	949
	0.5	126	752	1511	2714	2310	2620	1876	1115	1402	458	1040	2466	2470	2438	1981	2304	1579	981	1026	637	1375	1874	2020	2478	2539	1932	1274	1241	878	698
	0.6	141	1588	1189	1573	2248	2493	2386	1442	794	725	1174	1947	2451	2305	1667	1506	1712	904	735	394	2313	2741	2020	2049	2195	1941	1284	683	810	349
	0.7	183	641	1318	2048	2758	1850	2278	1253	1248	924	2297	1811	2213	2223	1392	1470	1230	729	420	297	1783	2247	2485	2682	1402	1169	878	552	466	343
	0.8	161	925	1863	2501	2209	2169	2457	1395	1012	527	1445	2535	1857	1460	1867	977	889	364	322	202	2168	2392	2847	1675	1764	1096	678	541	332	176
	0.9	195	829	1436	2543	2645	2276	2687	1623	1338	493	2267	2154	1614	1167	1076	751	483	375	208	102	2070	1843	2103	1871	953	881	562	420	221	134
	1	376	655	1634	1956	1980	2323	2131	1820	1502	645	2506	1590	1414	848	849	520	351	156	85	30	2209	2177	1611	999	1017	744	374	258	83	33
UX	0.1	873	830	827	683	663	576	484	387	315	31	1960	2392	2336	2177	1366	1288	1108	809	432	266	734	769	762	716	536	456	446	369	315	79
	0.2	1757	1552	1259	1220	1105	972	754	515	367	139	2776	2728	2067	2157	1548	1293	1260	763	513	159	1182	1254	1144	1225	1110	830	793	567	387	48
	0.3	2012	2030	1785	1506	1362	1124	950	695	436	120	2295	2608	2219	1870	1606	1457	835	834	432	130	1641	1847	1574	1409	1211	1322	915	700	455	90
	0.4	2399	2143	1970	1729	1699	1433	1164	835	481	134	2629	2496	2429	2324	1720	1399	1119	753	342	161	2074	2024	1899	1752	1757	1461	1315	855	601	164
	0.5	2557	2450	2245	2035	1819	1594	1395	1042	491	95	2156	2574	2578	1837	1605	1052	911	664	499	152	2244	2258	2092	2087	1825	1656	1261	1027	549	126
	0.6	2801	2622	2393	2322	1982	1751	1361	1076	618	210	2838	2474	2533	2436	1757	1575	1098	943	388	253	2290	2296	2458	2387	1996	1972	1356	1344	725	247
	0.7	2766	2804	2550	2264	2150	1859	1524	1264	669	228	2480	2678	2013	2172	1993	1743	1244	726	516	165	2514	2691	2605	2582	2252	1818	1621	1244	800	180
	0.8	2964	2791	2634	2483	2273	2084	1608	1302	857	211	2656	2344	2178	2459	1841	1497	1205	807	530	141	2715	2798	2620	2584	2303	1942	1824	1486	1018	253
	0.9	2951	2862	2684	2481	2285	2042	1684	1363	954	222	2520	2557	2601	2056	1649	1368	984	780	505	155	2924	2924	2760	2733	2737	2308	2004	1545	1030	193
	1	2970	2893	2822	2632	2466	2157	1830	1450	1017	275	2734	2516	2110	2477	1501	1594	1199	817	542	68	2793	2963	2725	2856	2577	2479	2170	1679	980	286
HUX	0.1	30	111	492	1204	1448	2087	1843	2653	1934	2027	36	626	1341	2511	2382	2776	2581	2819	2290	2710	30	99	312	708	1266	2015	2036	1975	2172	2215
	0.2	61	344	1114	1529	2276	2527	2730	2276	1689	2123	121	1100	2310	2608	2695	2306	2166	2055	2268	1961	69	392	781	1303	2471	2472	2293	2260	1643	2643
	0.3	148	884	1584	2336	2347	2806	2491	2210	1470	1965	319	1226	1892	2390	2904	2517	2339	2233	2080	1760	134	881	1490	2312	2318	2726	2398	2566	1959	2439
	0.4	419	1376	2779	2776	2249	2730	2369	1796	1712	1678	316	1294	2487	2601	2754	2420	2358	2146	1603	1198	225	948	2111	2787	2132	2289	2941	2833	2503	1522
	0.5	357	1815	1622	2335	2549	2850	2701	2424	2202	1222	288	1119	2196	2528	2765	2300	2578	1306	1115	1071	822	1085	1292	2026	2821	2527	2625	2125	1377	1656
	0.6	455	1499	2383	2083	2722	2583	1287	1837	1942	1862	270	1362	2106	2224	2618	2031	1824	1956	1597	812	1033	2060	1756	2835	2541	2084	2474	1912	1750	1329
	0.7	675	1959	2122	2181	2524	1622	1616	2232	1095	1011	264	1005	1724	2022	1746	1882	1666	1267	1069	406	513	1364	2151	2078	1842	2155	2012	1575	1191	1116
	0.8	1017	841	1406	2005	1259	1919	1504	1197	561	562	147	787	1275	1463	1747	2007	1121	791	808	471	847	1472	1744	1548	1823	1555	1010	914	565	468
	0.9	647	1364	877	1182	1578	1273	1230	688	429	318	94	647	896	1215	1617	1140	1150	562	648	340	1154	1221	1475	1668	1754	1169	822	327	679	294
	1	769	546	701	1037	522	488	370	517	127	176	89	321	758	1099	1419	1054	775	669	534	240	1090	1335	808	773	554	402	451	285	258	239

that the bigger the hypervolume better performance in terms of convergence and diversity.

Better instances of the combinations between crossover and mutation methods are highlighted in Table 2. To choose only one best configuration, we applied Friedman test. The results are summarized in Table 3.

García et al. [14] propose a Friedman test to determine the existence of significant differences among all the mean values and a post-hoc statistical analysis. The null hypothesis of Friedman test supposes a condition of equality between the medians of the samples. The alternative hypothesis is the negation of the previous one: at least two of the samples differ in their medians.

Table 4 shows the p values computed by three post-hoc tests (Nemenyi, Holm and Shaffer) using 0.05 and 0.10 as levels of statistical significance (α), which confirm the validity of the alternative hypothesis.

To determine the generation number as stop criteria and the population size, we run 30 different executions using the following parameters: Population size: 49, 100, 225 and 400 individuals

Table 3
Friedman rank of the algorithms with different parameters configuration (crossover and mutation operators).

Configuration	Pc	Pm	Mean	Rank
HUX_RPM	0.4	0.7	0.4622	446
UX_RPM	1.0	0.2	0.4589	388
DX_RPM	0.8	0.3	0.4590	368
TPX_RPM	1.0	0.1	0.4358	360
DX_OGPPM	0.4	0.2	0.4582	345
HUX_OGPPM	0.3	0.5	0.4578	328
TPX_OGPPM	0.3	0.7	0.4568	268
UX_OGPPM	0.6	0.1	0.4555	241
UX_UM	0.6	0.1	0.4502	218
SPX_OGPPM	0.4	0.7	0.4531	181
SPX_RPM	0.2	1.0	0.4521	157
HUX_UM	0.3	0.5	0.4248	118
DX_UM	0.4	0.2	0.4393	91
TPX_UM	0.3	0.7	0.3994	61
SPX_UM	0.4	0.7	0.4106	30

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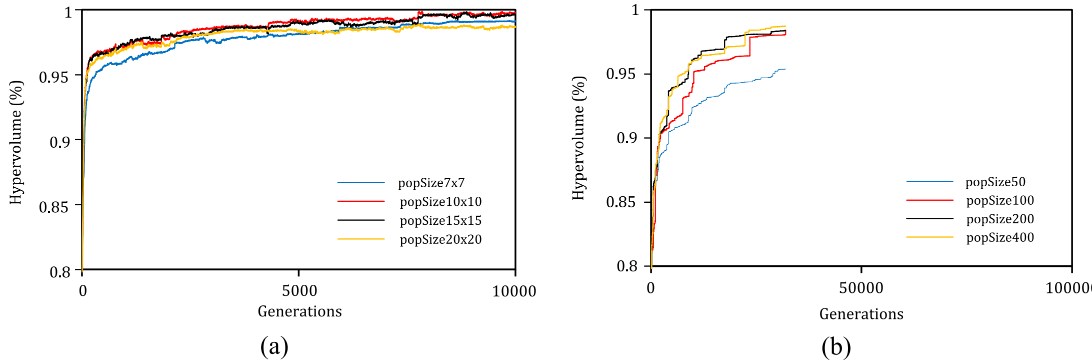


Fig. 7. Average performance of the hypervolume (a) MOCell and (b) NSGA-II.

Table 4
p values of the post-hoc tests.

α	p_{Nemeny}	p_{Holm}	p_{Shaffer}
0.05	0.000476	0.000962	0.000476
0.10	0.000952	0.002041	0.000952

over 10 000 generations (i.e., look the toroidal grid individual per individual). We use the following parameters obtained during the previous calibration step (see Table 3): Recombination operator: HUX; Recombination probability (Pc): 0.7; Mutation operator: RPM and Mutation probability (Pm): 0.7.

The results are summarized in Fig. 7(a). The performance of the population size of 100 and 225 are similar. However, the 10 × 10 toroidal grid achieves 98.303% of the best hypervolume at 2500 generations, using 55% and 75% fewer fitness evaluations than 15 × 15 and 20 × 20, respectively, representing a significant reduction of execution time.

After an exhaustive experimentation, the best-chosen instance is the one in which the recombination probability is 0.4 with the HUX operator, and the mutation probability is 0.7 using the RPM operator. The adopted parameter settings are presented in Table 5.

5.3. NSGA-II

To compare the proposed algorithm with other techniques for multiobjective combinatorial optimization, we implement a classical evolutionary algorithm NSGA-II, proposed by Deb et al. in 2002 [9]. The same parameters are used in recombination and mutation operators (see Table 5).

Fig. 7(b) shows hypervolume (average of 30 executions) used to calibrate the population size and generation number as stop criteria. The population with 200 individuals has the best performance about the number of generations. After 17 476 generations, it achieves 98% of the best hypervolume found using NSGA-II, while 50,100 and 400 keep at 94%, 96%, and 97%, respectively. In consequence, a population size of 200 individuals is profitable due to it extends the diversity of the initial population without a considerable increasing the execution time.

18 000 generations are used in each run of NSGA-II, assessing the performance by using the I_{HV}, I_{Δ} and I_{ε} as quality indicators for both algorithms.

When comparing results of two algorithms, the value of I_{TSC} is calculated by applying this metric $I_{TSC}\left(X', X''\right)$ and $I_{TSC}\left(X'', X'\right)$ to the sets of non-dominated solutions computed by each heuristic.

Table 6
Multiobjective optimization metrics computed for MOCell and NSGA-II.

Metric		MOCell	NSGA-II
Non-dominated points	max	31	46
	mean	25.533	38.210
	σ	2.2854	4.6930
Spread (I_{Δ})	min.	0.5076	0.5853
	mean	0.6341	0.7307
	σ	0.0676	0.0883
Hypervolume (I_{HV})	max	0.4706	0.4262
	mean	0.4687	0.4242
	σ	0.0011	0.0010

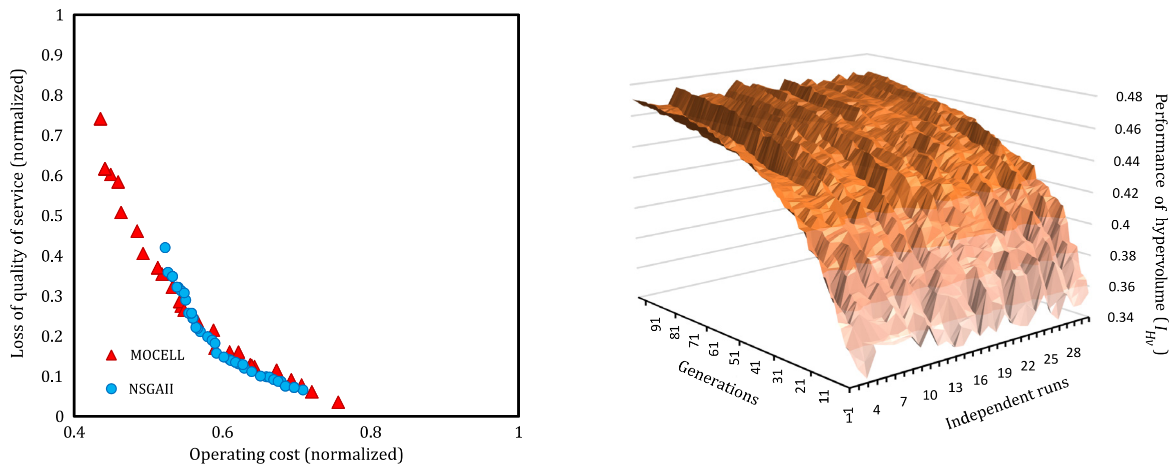
5.4. Results and discussion

I_{HV} obtained after applying to each front, collected from 30 independent runs, shows a good performance of the proposed algorithm with maximum I_{HV} value of 0.4706 (0.4687 on average in the last generation) (see Table 6).

In Figs. 8–9, we observe that each run similarly improves the hypervolume providing a stable behavior.

Table 5
Parameterization of the multiobjective cellular algorithm.

Parameter	MOCell	NSGA-II
Stopping condition	2500 generations	18000 generations
Population size	100 individuals (10 × 10)	200 individuals
Neighborhood	8 surrounding neighbors (C9)	–
Selection parents	Binary tournament	Binary tournament
Recombination	HUX	HUX
Probability of recombination	0.4	0.4
Mutation	RPM	RPM
Probability of mutation	0.7	0.7
Replacement	Replace if better ranking and crowding	Replace if better ranking and crowding
Density estimator	Crowding distance	Crowding distance
Feedback	20 individuals	–



(a) (b)

Fig. 8. (a) Hypervolume constructed by the best solutions computed by both algorithms and reference point (1, 1) and (b) performance of hypervolume of 30 independent runs of MOCell from 100 generations.

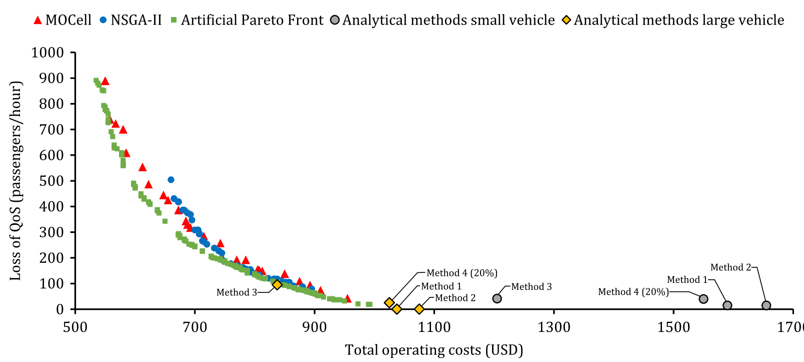


Fig. 9. Comparison of best case for MOCell, NSGA-II and the *artificial* Pareto front built using the best individuals of the 1500 algorithm executions, also, the four different methods proposed by Ceder [5] using only one kind of vehicle, small or large.

They show the $\mathcal{P}_F$ approximation obtained in a better run for NSGAII ($I_{HV} = 0.4242$) and our algorithm ($I_{HV} = 0.4706$) for the same instance (LA route 217).

Fig. 9 highlights the elitism and exploration process of MOCell and feedback from the better individuals based on the crowding distance, which influences on increasing the spreading along the front, hence, a greater I_{HV}.

Furthermore, we create the non-dominated set (*artificial* Pareto front) formed by the 45 000 approximation sets, 30 for each independent run.

Fig. 9 presents the Pareto front approximation in the best case computed by the two algorithms, $\mathcal{P}_a$ and four analytical methods with the same vehicle-capacity constraints (i.e., small or large vehicles).

We observe that NSGA-II has difficulties in diversity; hence, it has poor I_{HV} value and bigger I_Δ in comparison with MOCell. On the other hand, I_{TSC} (see Table 7) reports that the solutions set provided by NSGA-II covers (dominates) 46% of the objective vectors from Pareto front approximations computed by MOCell. NSGA-II gives 33% solutions with good performance. However, we can observe that starting from the same initial populations, MOCell explores the solution space better than NSGA-II. In Table 8, we see that average value of I_ε of MOCell demonstrates a better performance close to $I_\varepsilon = 1$.

The comparison shows that MOCell outperforms NSGA-II achieving better I_{HV} values in average and best cases. Moreover, its solutions have 48% better performance. Our approach provides a good approximation to the optimal Pareto front and solutions that preserve diversity (I_Δ is 13% better than NSGA-II).

The analytical methods present a good performance in terms of QoS. The maximum number of lost passengers per period is 12% of the metaheuristics results. On the other hand, the total cost is up to

Table 7
Comparison of results with $I_{TSC}\left(X', X''\right)$ where X' and X'' represent the algorithm in the row and the column, respectively.

		NSGA-II	MOCell
NSGA-II	mean	–	0.3158
	max	–	0.4615
MOCell	mean	0.1478	–
	max	0.4117	–

Table 8
Comparison of results with $I_\varepsilon\,(X, \mathcal{P}_a)$ where X represents the approximation sets provided by the algorithm in the rows and $\mathcal{P}_a$ is the *artificial* Pareto front.

		$\mathcal{P}_a$
NSGA-II	mean	6.2351
	max	7.6418
MOCell	mean	3.2983
	max	4.7815

the double of the maximum total cost in the artificial Pareto front (extreme solutions).

Method 1 uses the daily max load point. Method 2 uses the hourly max load point. Method 3 uses the area in passenger-km under the load profile at every period as an average representative of the load profile taking into account the distance between stops. Method 4 uses a constant to control when the number of passengers on board is above of the desired occupancy for an extended distance [5].

Those methods were evaluated using the larger and smaller vehicles to compare not only the performance for the two objective functions but to study the impact of a heterogeneous fleet. It shows that using the largest vehicles the cost can be reduced but implies a waste of fuel in off-peak hours, where using small vehicles can be moved the same number of passengers, avoiding congestion on the roads, reducing costs and adverse environmental impact.

On the other hand, combining several types of vehicles implies more frequency, allowing save time for passengers to board the vehicle.

In real-world applications, the decision maker is normally interested in certain types of trade-offs based on regulations and restrictions usually framed by ITS. From this point of view, the approximation sets produced by our algorithm repeat some values for the fitness functions (Fig. 9). It happens because buses have the same timetable but the order of the vehicles is not the same. The decision maker can choose one of the solutions based on a detailed study (e.g., local search) at specific times of the day (e.g., peak hours).

6. Conclusions

In this paper, we address the multiobjective multiple vehicle-type timetabling problem and propose a cellular genetic algorithm to solve it. The objective is to determine timetables using various bus types with different capacity, operating cost, gas consumption, size, weight, etc. to optimize a quality of service, represented by the unsatisfied user demand, and total operating cost.

With the operation data of Los Angeles bus route 217 northbound, we obtain a set of non-dominated solutions that represent different assignments of vehicles covering a given set of trips in a defined route. To provide effective guidance in choosing a good strategy, we use several quality indicators: Hypervolume, Spread, ε-Indicator, and Set Coverage.

Our experimental analysis of case studies results in several contributions:

(a) Proposed algorithm based on MOCell is very competitive technique comparing with known in the literature. It demonstrates the capacity to find a set of different vehicle types to cover a specific route with a reduced cost in comparison to other methods. It also guarantees the quality of service under the regulations of government authorities.
(b) We show that it is a useful tool to solve problems associated with frequency determination and timetabling design. It provides a wide range of trade-off timetables applicable in different test cases considering characteristics and regulation of the ITS.
(c) When we examine overall performance based on real data, we find an improvement about 26% in operating cost without degradation of the quality of service.

However, further study of the problem with additional optimization criteria, for instance, the greenhouse gas emission (GHG) levels due to wasted fuel that has a negative environmental impact on the global climate change, is required to assess its actual efficiency and effectiveness.

Transportation can be improved to make them more intelligent, interconnected and efficient by computing technologies considering more restrictions of government agencies for public transport systems in the context of smart cities. A real-time condition of transport networks such as current traffic, accidents, traffic jam and other traffic congestions are important to improve the vehicles assignment and produce adaptive solutions. These solutions can be dynamically adjusted to cope with the passenger demand and traffic situation. To this end, the information about traffic and passenger behavior must be analyzed for a certain time interval to determine appropriate factors for next interval. This will be a subject of future work for better understanding of multi-objective transport optimization in real environments.

Acknowledgments

The work is partially supported by RFBR, project no. 16-07-00931, 18-07-01224-a, Federal Agency of Scientific Organizations (FASO), project no. 0348-2017-0010, and CONACYT, México, grant no. 178415.

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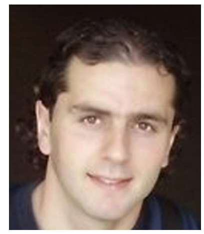

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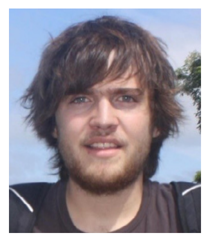

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