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Topological analysis of public transport networks' recoverability

Extended Abstract

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Abstract We present a topological approach to assess recoverability of public transport networks. The approach is based on previous works dealing with optical networks and adapted to the context of public transport. Two failure and three recovery strategies are defined and evaluated for the metro networks of Berlin and Paris using two performance metrics. Preliminary results suggest that simple greedy recovery strategies are able to quickly bounce back from the loss of performance inflicted during the failure process and that the proposed methodology is a promising approach to assess recoverability in public transport networks.

Keywords recoverability · public transport networks · network topology

1 Introduction

Reducing the impact of disruptions is critical for providing reliable and attractive public transport services. A large body of research has been devoted to study Public Transport Networks (PTN) vulnerability and robustness, i.e., the extent to which systems can withstand disruptions. Particularly, many works have studied robustness from a topological point of view, e.g., assessing the impact on connectivity properties of damaged/missing nodes (Angeloudis and Fisk, 2006; Berche et al, 2009) and links (Berche et al, 2012; von Ferber et al, 2012), studying the effect of cycles (Derrible and Kennedy, 2010),

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and increasing the capacity of certain links to improve robustness (Cats and Jenelius, 2015). Next to network's ability to withstand disruptions, its ability to recover back to its original level of performance is critical. While there is a wealth of literature related to assessing and improving the robustness of PTNs, little is known on the topological aspects of the recovery process once these failures occur.

We study the notion of *recoverability*. In doing so, we are inspired by an analysis performed in another context, namely for optical networks, where Sun et al (2021) proposed a topological approach to assess recoverability and recovery strategies. Unlike optical networks, we are interested in explicitly accounting for travel impedance.

2 Topological approach for measuring PTN's recoverability

A mathematical description of key components is provided, followed by a description of performance metrics and failure/recovery strategies used in the experiments.

2.1 Model overview

Let $G_0(N, L_0)$ be the **L**-space representation of a PTN comprised of N nodes and L_0 links. M_{G_0} denotes the original performance of that network. The failure process consists of K steps where, in each step, a given link of the network is removed. Conversely, the recovery process corresponds to the iterative addition of the K links previously removed. The order in which links are removed/added to the network depends on the failure/recovery strategy employed.

We define the retained performance ratio R_{G_i} as the performance at step i normalized by the performance of the original network: $R_{G_i} = \frac{M_{G_i}}{M_{G_0}}$. Thus, $R_{G_0} = R_{G_{2K}} = 1$. The cumulative performance loss during the failure process is given by $S_f = \sum_{i=0}^{i=K} (1 - R_{G_i})$. Conversely, the cumulative performance gain during the recovery process is given by $S_r = \sum_{i=K+1}^{i=2K} R_{G_i} - R_{G_K}$. The rebound ratio is given by $\eta = \frac{S_r}{S_f}$. A rebound ratio $\eta > 1$ indicates a high network recoverability, where the recovery measures are able to bounce back faster than the initially inflicted degrading of performance experienced during the failure process. In contrast, a rebound ratio $\eta < 1$ suggests poor recoverability.

2.2 Performance metrics

Several performance metrics have been defined in the literature (Bešinović, 2020). In this paper, we extend the two metrics used by Sun et al (2021) to assess performance in PTNs.

2.2.1 Connectivity

We define network connectivity (C_G) as the fraction of pairs of nodes (i.e., stations) which have at least one path connecting them. A fully connected PTN has a $C_G = 1$.

$$C_G = \frac{\sum_{i \neq j \in G} 1_{\text{exists a path (i,j)}}}{N \times (N-1)}$$

2.2.2 Efficiency

We define network efficiency (E_G) as the mean of the reciprocals of travel times between each pair of nodes based on the shortest path in a labeled graph and $\text{travel\_time}(i, j) = \infty$ if no path exists between i and j.

$$E_G = \frac{\sum_{i \neq j \in G} 1/\text{travel\_time}(i, j)}{N \times (N-1)}$$

2.3 Failure and recovery strategies

Two failure strategies are considered:

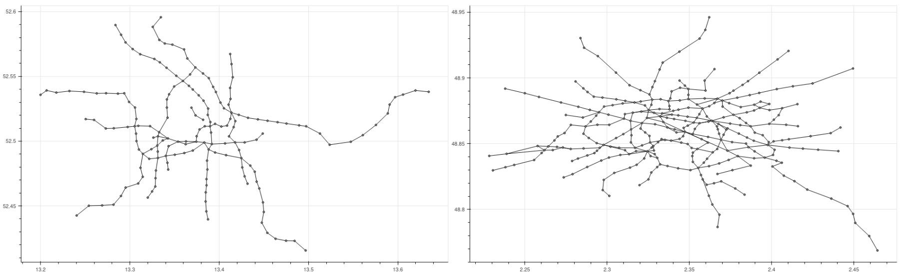
1. *Random*: links are iteratively removed following a uniform probability distribution.
2. *Greedy*: in each step of the failure process each link in the network is considered for removal and the one that results in the largest decrease in R_{G_i} is selected. We consider the C_G metric for this failure strategy, representing a targeted attack.

Analogously, three recovery strategies are considered:

1. *Random:* links are iteratively added back to the graph randomly and uniformly.
2. *Greedy (C_G)*: selects the link among those removed during failure that, if added, would render the largest R_{G_i} improvement for the C_G performance metric.
3. *Greedy (E_G)*: same as Greedy (C_G) but considering the E_G performance metric.

3 Preliminary results

We use graphs corresponding to the Paris and Berlin metro networks based on the dataset in Kujala et al (2018). Links are labeled with the average travel time between stations. Platforms located within a single interchange station are merged to ensure a one-to-one correspondence between stations and nodes in the graph. Figure 1 shows the resulting networks.



(a) Berlin (172 nodes, 183 links) (b) Paris (302 nodes, 359 links)

Fig. 1: Metro networks used in the experiments

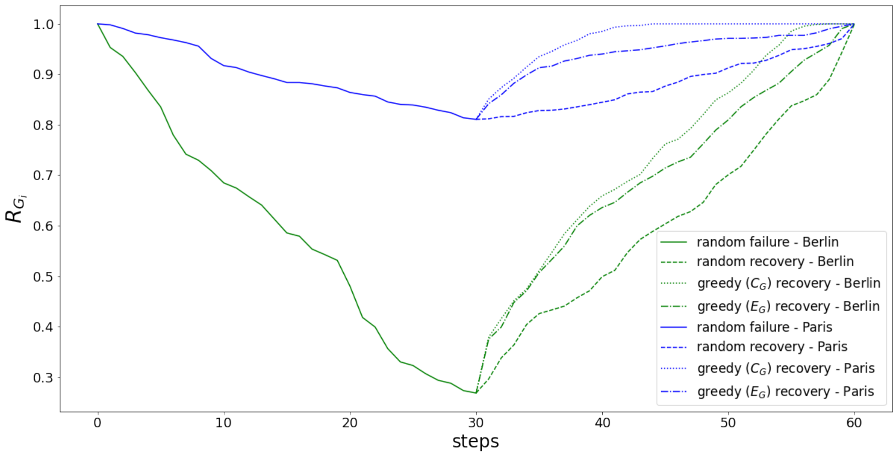
We simulate a random failure process of $K = 30$ steps for each city. Fig. 2 shows the average R_{G_i} obtained in ten independent executions for Berlin (green) and Paris (blue) for each recovery strategy. Fig. 2a displays the R_{G_i} for the C_G performance metric while Fig. 2b for the E_G metric. The average performance loss (S_f), gain (S_r), and rebound ratio (η), are reported in Table 1.

Regarding the failure process, results show a much larger decrease of both performance metrics for Berlin metro network compared to Paris. The disparity in R_{G_i} values between both cities may be attributed to the difference in size of the networks, with Paris PTN having almost twice the number of links as Berlin.

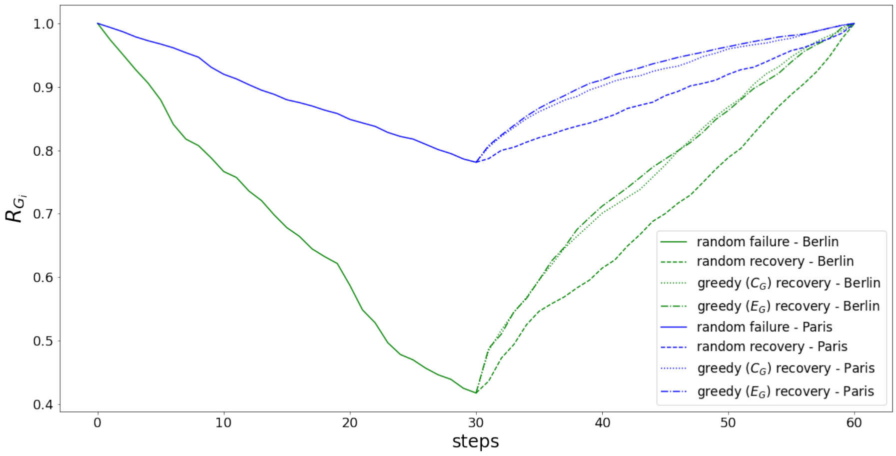
When looking at the recovery process, results show that both greedy approaches outperform the random recovery strategy, which achieved rebound ratios $\eta < 1$ in all cases. In contrast, both greedy algorithms achieved ratios $\eta > 1$, indicating that the gains obtained in the recovery process occur faster than the losses inflicted during the failure process. As expected, each greedy strategy performs better when measured with the respective performance metric they aim to optimize. Nevertheless, Greedy (C_G) performs really well for both metrics.

4 Outlook

We presented a novel topological approach to measure recoverability in PTNs inspired by previous works in the field of optical networks. Preliminary results for the metro networks of Berlin and Paris showed that greedy recovery heuristics are able to rebound quickly from the loss of performance during the failure process of the network. As part of on-going work we devise other failure and recoverability strategies, incorporate new performance metrics, and assess the proposed approach for a large number of PTNs worldwide to control for network size.



(a) C_G metric



(b) E_G metric

Fig. 2: Average retained performance ratios (R_{G_i}) for Berlin and Paris using a random failure and different recovery strategies

city	*metric*	*recovery strategy*		
		random	*greedy* C_G	*greedy* E_G
Berlin	C_G	0.833 (12.748;10.619)	1.141 (12.748;14.545)	1.060 (12.748;13.500)
	E_G	0.904 (9.897;8.943)	1.098 (9.897;10.869)	1.102(9.897;10.910)
Paris	C_G	0.725 (3.121;2.263)	1.591 (3.121;4.968)	1.321 (3.121;4.122)
	E_G	0.938 (3.474;3.260)	1.240 (3.474;4.307)	1.293 (3.474;4.491)

Table 1: Average rebound ratio (η), performance loss (S_f), and gain (S_r) with random failure and varying recovery strategies. Notation: η (S_f;S_r).

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