№1

$$P\{\xi = k\} = \frac{4}{k(k+1)(k+2)}$$

$$M(\xi) = \sum_{k=1}^{+\infty} \frac{4}{k(k+1)(k+2)} \cdot \xi_k = \sum_{k=1}^{+\infty} \frac{4}{\cancel{k}(k+1)(k+2)} \cdot \cancel{k} = \sum_{k=1}^{+\infty} \frac{4}{(k+1)(k+2)} =$$

$$= 4 \cdot \sum_{k=1}^{+\infty} \frac{1}{(k+1)(k+2)} = 4 \cdot \frac{1}{2} = \underline{2}$$

---

№2

$$p_\xi(x) = \begin{cases} \frac{3}{x^4}, & \text{при } x \geq 1 \\ 0, & \text{при } x < 1 \end{cases}$$

$$E\xi = \int_1^{+\infty} \frac{3}{x^4} \cdot x\,dx = \int_1^{+\infty} \frac{3}{x^3}\,dx = 3 \cdot \frac{1}{-2x^2}\Big|_1^{+\infty} = \left(0 + \frac{3}{2}\right) = \frac{3}{2}$$

$$D\xi = \int_1^{+\infty} \left(x - \frac{3}{2}\right)^2 \cdot \frac{3}{x^4}\,d = \int_1^{+\infty} \left(\frac{3}{x^2} - \frac{9}{x^3} + \frac{27}{4x^4}\right)dx = -\frac{3}{x}\Big|_1^{+\infty} + \frac{9}{2x^2}\Big|_1^{+\infty} - \frac{27}{3 \cdot 4x^3}\Big|_1^{+\infty} =$$

$$= \frac{3}{4}$$

$$E\left(\frac{1}{\xi}\right) = \int_1^{+\infty} \frac{3}{x^4} \cdot \frac{1}{x}\,dx = 3 \cdot \frac{1}{-4x^4}\Big|_1^{+\infty} = \frac{3}{4}$$

$$D\left(\frac{1}{\xi}\right) = \int_1^{+\infty} \left(\frac{1}{x} - \frac{3}{4}\right)^2 \cdot \frac{3}{x^4}\,dx = \frac{3}{16}\left(\cdot 9\int_1^{+\infty} \frac{1}{x^4}dx - 24\int_1^{+\infty} \frac{1}{x^5}dx + 16\int_1^{+\infty} \frac{1}{x^6}dx\right) = \frac{3}{80}$$

№3

$$P\{\xi \le x\} = (1 - e^{-x})\,\mathbb{I}\{x \ge 0\}$$

$$p_\xi(x) = \underset{\text{при } x \ge 0}{P'\{\xi \le x\}} = e^{-x}$$

$$E(\xi) = \int_0^{+\infty} e^{-x}\cdot x\,dx = -xe^{-x}\Big|_0^{+\infty} - \int_0^{+\infty} -e^{-x}\,dx = 1$$

$$D(\xi) = \int_0^{+\infty} (x-1)^2\cdot e^{-x}\,dx = -(x-1)^2 e^{-x}\Big|_0^{+\infty} - \int_0^{+\infty} -2(x-1)e^{-x}\,dx =$$

$$= -(x^2+1)e^{-x}\Big|_0^{+\infty} = 1$$

$$E(\xi(1-e^{-2\xi})) = \int_0^{+\infty} x(1-e^{-2x})\cdot e^{-x}\,dx = \frac{8}{9}$$

$$D(\xi(1-e^{-2\xi})) = \int_0^{+\infty} \left(x(1-e^{-2x}) - \frac{8}{9}\right)^2\cdot e^{-x}\,dx = \frac{10912}{10125}$$

№4

$$F(x) = \begin{cases} 0, & x < -2 \\ \frac{1}{5}, & -2 \le x < 1 \\ \frac{x^2}{4}, & 1 \le x < 2 \\ 0, & x \ge 2 \end{cases}; \quad \underset{1 \le x < 2}{p(x) = F'(x)} = \frac{2x}{4} = \frac{x}{2}$$

$$\cancel{E(x) = -2\cdot\frac{1}{5} + 1\cdot\left(\frac{1}{4}-\frac{1}{5}\right) + \int_1^2 \frac{2x}{2}\cdot x\,dx = \frac{x^3}{3}\Big|_1^2 + \frac{1}{20} - \frac{2}{5}}$$

$$\cancel{\frac{70+3-96}{60} = \frac{47}{60}}$$

$$E(x) = -2\cdot\frac{1}{5} + 1\cdot\left(\frac{1}{4} - \frac{1}{5}\right) + \int_1^2 x\cdot\frac{x}{2}\,dx =$$

$$= -\frac{2}{5}^{\backslash 4} + \frac{1}{20} + \int_1^2 x\cdot\frac{x}{2}\,dx = \frac{1-8}{20} + \frac{1}{2}\cdot\frac{x^3}{3}\Big|_1^2 = \frac{-7}{20} + \frac{1}{2}\left(\frac{8}{3}-\frac{1}{3}\right) =$$

$$= \frac{-7}{20}^{\backslash 3} + \frac{7}{2\cdot 3}^{\backslash 10} = \frac{70-21}{60} = \frac{49}{60}$$

$$D(x) = \left(-\overset{60}{2}-\frac{49}{60}\right)^2\cdot\frac{1}{5} + \left(\overset{60}{1}-\frac{49}{60}\right)^2\cdot\left(\frac{1}{4}-\frac{1}{5}\right) + \int_1^2\left(x-\frac{49}{60}\right)^2\cdot\frac{x}{2}\,dx = \left(\frac{-120-49}{60}\right)^2\cdot\frac{1}{5} + \left(\frac{60-49}{60}\right)^2\cdot$$

$$+ \int_1^2\left(\frac{60x-49}{60}\right)^2\cdot\frac{x}{2}\,dx \approx 2{,}058$$

№5

$$E(3\xi-4\eta-1)^2 = E(9\xi^2-12\xi\eta-3\xi-12\eta\xi+16\eta^2+4\eta-3\xi+4\eta+1) =$$

$$= 9E(\xi^2) - 24\cdot E(\xi)\cdot E(\eta) - 6E(\xi) + 8E(\eta) + 16E(\eta^2) + E(1) = 143$$

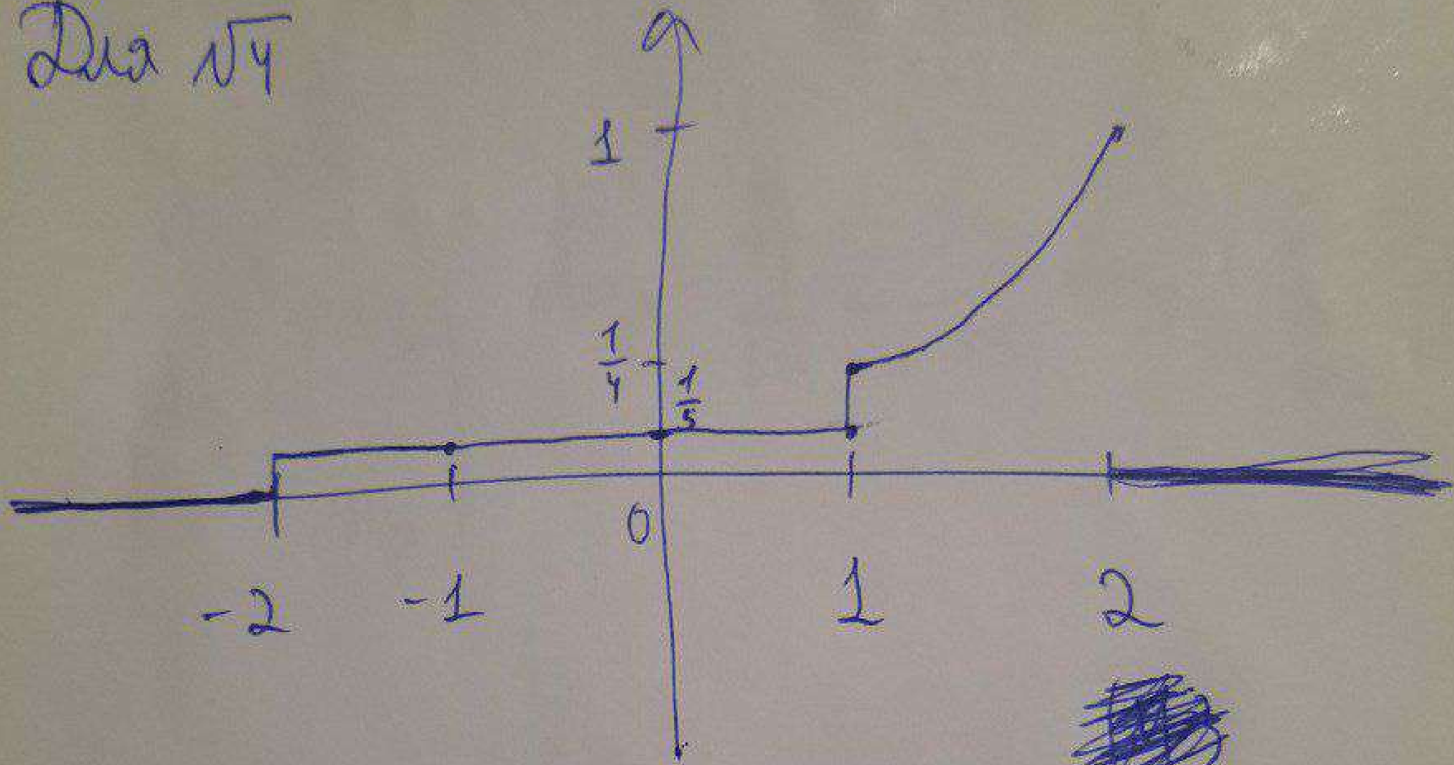
$$D(\xi) = E(\xi^2) - (E(\xi))^2;\quad E(\xi^2) = D(\xi) + (E(\xi))^2 = 3+1 = 4$$

$$E(\eta^2) = D(\eta) + (E(\eta))^2 = 1+4 = 5$$

$$= \frac{1}{4} + \frac{1}{12} + \frac{1}{24} + \frac{1}{40} + \frac{1}{60}$$

$4 \quad 4\cdot 3 \quad 4\cdot 6 \quad 4\cdot 10 \quad 4\cdot 15$

Для №4




$F(X<x)$