2_QubitQubit

May 31, 2021

1 Qubit-qubit systems

Initialize the RepLAB toolbox (be in the /replab directory or use **run path/replab/replab_init.m**)

[1]:
```
replab_init
replab.globals.useReconstruction(1); % use new algorithms for decomposition
```

```
Adding RepLAB to the path
Initializing dependency vpi
Initializing dependency YALMIP
Initializing dependency sdpt3
Adding embedded SDPT3 solver to the path
Initializing dependency MOcov
Initializing dependency MOxUnit
Initializing dependency cyclolab
```

Werner states and isotropic states are transformed by unitary matrices $\mathcal{U}(d)$.

[2]:
```
d = 2;
Ud = replab.U(d)
```

```
Ud =

Unitary group U(2)
  algebra: 'C'
 identity: [1, 0; 0, 1]
isSpecial: false
        n: 2
```

We take a random sample. A unitary group element is … a unitary matrix. For groups in RepLAB, the random samples are actually taken from a uniform measure (the Haar measure).

[3]:
```
Ud.sample
```

```
ans =

  -0.288776 - 0.930226i  -0.211616 - 0.080667i
```

```
  -0.062935 - 0.217550i   0.903233 + 0.364529i
```

If we represent group elements by … unitary matrices, it's a quite straightforward representation called the "defining representation".

[4]:
```
rep = Ud.definingRep
```

```
rep =

Unitary fully nontrivial irreducible representation
          dimension: 2
divisionAlgebraName: []
              field: 'C'
              group: Unitary group U(2)
          isUnitary: true
```

This representation is irreducible, i.e. it doesn't have proper invariant subspaces.

[5]:
```
rep.isIrreducible
```

```
ans = 1
```

1.1 Werner states

Werner states are invariant when applying the same unitary to the first and second subsystem.

[6]:
```
rho_werner = kron(rep, rep)
```

```
rho_werner =

Unitary tensor representation
          dimension: 4
divisionAlgebraName: []
              field: 'C'
              group: Unitary group U(2)
          isUnitary: true
          factor(1): Unitary fully nontrivial irreducible representation
          factor(2): Unitary fully nontrivial irreducible representation
```

What are the invariant subspaces?

[7]:
```
dec_werner = rho_werner.decomposition
```

```
dec_werner =

Unitary reducible representation
          dimension: 4
divisionAlgebraName: []
```

```
              field: 'C'
              group: Unitary group U(2)
 injection_internal: 4 x 4 double
       isSimilarRep: true
          isUnitary: true
     mapsAreAdjoint: true
             parent: Unitary tensor representation
projection_internal: 4 x 4 double
  basis(1,'double'): 4 x 1 double
  basis(2,'double'): [-0.17958-0.19368i; -0.47792+0.48653i; -0.47792+0.48653i…
  basis(3,'double'): [0; 0; 0; 1]
  basis(4,'double'): [-0.33664-0.90383i; 0.17179-0.073284i; 0.17179-0.073284i…
       component(1): Isotypic component C(1) (nontrivial)
       component(2): Isotypic component C(3) (nontrivial)
```

[8]:
```
dec_werner.basis
```

```
ans =

  -0.0000 + 0.0000i  -0.1796 - 0.1937i        0 +      0i  -0.3366 - 0.9038i
  -0.7070 - 0.0120i  -0.4779 + 0.4865i        0 +      0i   0.1718 - 0.0733i
   0.7070 + 0.0120i  -0.4779 + 0.4865i        0 +      0i   0.1718 - 0.0733i
        0 +      0i        0 +      0i   1.0000 +      0i        0 +      0i
```

1.2 Isotropic states

Isotropic states are invariant when applying a unitary matrix to the first subsystem, and its conjugate to the second.

[9]:
```
rho_iso = kron(rep, conj(rep))
dec_iso = rho_iso.decomposition
dec_iso.basis
```

```
rho_iso =

Unitary tensor representation
          dimension: 4
divisionAlgebraName: []
              field: 'C'
              group: Unitary group U(2)
          isUnitary: true
          factor(1): Unitary fully nontrivial irreducible representation
          factor(2): Unitary fully nontrivial derived representation (conjugate)

dec_iso =
```

```
Unitary reducible representation
           dimension: 4
divisionAlgebraName: []
               field: 'C'
               group: Unitary group U(2)
 injection_internal: 4 x 4 double
       isSimilarRep: true
          isUnitary: true
     mapsAreAdjoint: true
              parent: Unitary tensor representation
projection_internal: 4 x 4 double
  basis(1,'double'): 4 x 1 double
  basis(2,'double'): 4 x 1 double
  basis(3,'double'): 4 x 1 double
  basis(4,'double'): 4 x 1 double
       component(1): Isotypic component C(1) (trivial)
       component(2): Isotypic component C(3) (nontrivial)

ans =

   0.7071 - 0.0000i        0 + 0.0000i  -0.0000 - 0.0000i   0.7071 + 0.0000i
  -0.0000 + 0.0000i   1.0000 - 0.0000i   0.0000 - 0.0000i        0 - 0.0000i
  -0.0000 - 0.0000i  -0.0000 - 0.0000i   1.0000 + 0.0000i   0.0000 + 0.0000i
   0.7071 - 0.0000i  -0.0000 + 0.0000i  -0.0000 - 0.0000i  -0.7071 - 0.0000i
```

2 Generalizations (exercices)

2.1 Higher dimensions

Both cases split in invariant subspaces of dimensions 1+3.

What about dimension 3? dimension 4?

2.2 Cloning machines

The Choi state of an isotropic cloning machine would transform as:

`kron(rep, rep, ..., rep, conj(rep), conj(rep), ..., conj(rep))`

where the number of `rep` is the number of output copies, and the number of `conj(rep)` is the number of input copies.

In 2018, the decomposition was made analytically for the case of a single `conj(rep)` copy: http://stacks.iop.org/1751-8121/51/i=12/a=125202