# Efficient Black-box Checking of Snapshot Isolation in Databases

(Conference VLDB'2024)

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August 21, 2023

# Transaction and Isolation Level

A transaction is a *group* of operations that is executed atomically.

$x_1 \leftarrow \mathsf{R}(acct_1)$
$x_2 \leftarrow \mathsf{R}(acct_2)$
**if** $x_1 + x_2 > 100$
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  $\mathsf{W}(acct_2, x_2)$

$acct_1 = acct_2 = 60$

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Isolation Level (aka Transactional Consistency Model)

$acct_1 = acct_2 = 60$

The isolation levels specify how they are isolated from each other.

# Serializability (SER)

All transactions appear to execute in some total order.

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Serializability (SER)

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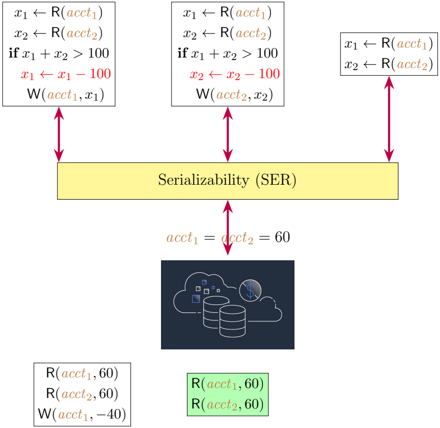

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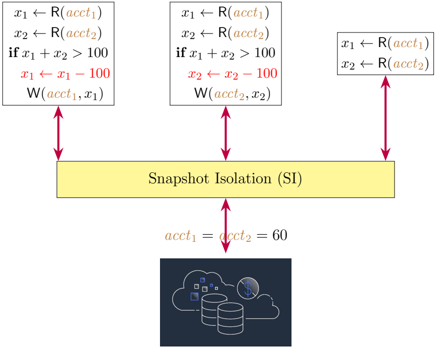
$\mathsf{R}(acct_1, 60)$
$\mathsf{R}(acct_2, 60)$

too expensive, especially for distributed transactions

# Snapshot Isolation (SI)



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Snapshot Isolation (SI)

$acct_1 = acct_2 = 60$

Write Skew

$\mathsf{R}(acct_1, 60)$
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Snapshot Isolation (SI)

$acct_1 = acct_2 = 60$

WRITE SKEW ✓

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Snapshot Read: Each transaction reads data from a *snapshot* of committed data valid as of the (logical) time the transaction started.

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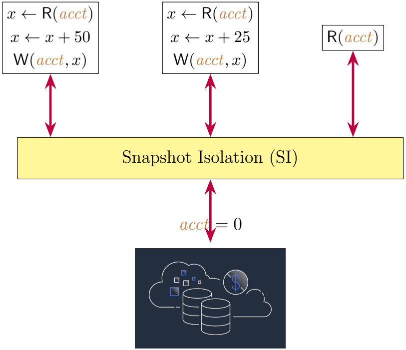

# Snapshot Isolation (SI)

$x \leftarrow \mathsf{R}(acct)$
$x \leftarrow x + 50$
$\mathsf{W}(acct, x)$

$x \leftarrow \mathsf{R}(acct)$
$x \leftarrow x + 25$
$\mathsf{W}(acct, x)$

$\mathsf{R}(acct)$

Snapshot Isolation (SI)

$acct = 0$

$\mathsf{R}(acct, 0)$
$\mathsf{W}(acct, 50)$

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$\mathsf{W}(acct, x)$

$\mathsf{R}(acct)$

Snapshot Isolation (SI)

$acct = 0$

$\mathsf{R}(acct, 0)$
$\mathsf{W}(acct, 50)$

$\mathsf{R}(acct, 0)$
$\mathsf{W}(acct, ?)$

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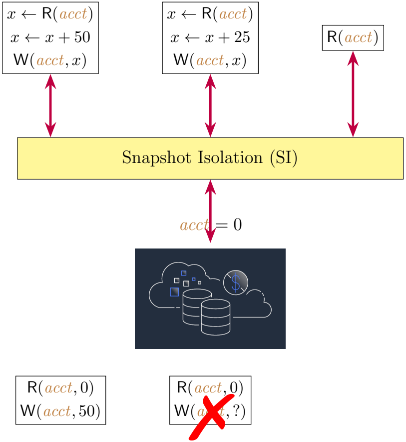


Snapshot Write: Concurrent transactions cannot write to the same key. One of them must be aborted.

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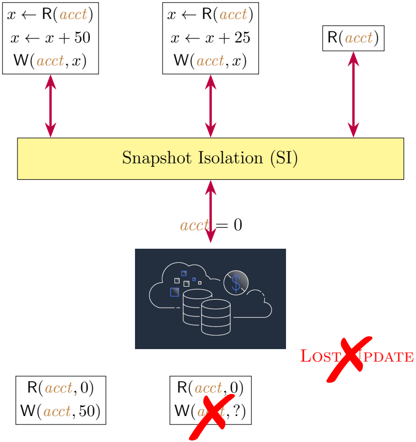


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$\mathsf{R}(acct)$

Snapshot Isolation (SI)

$acct = 0$

$\mathsf{R}(acct, 0)$
$\mathsf{W}(acct, 50)$

$\mathsf{R}(acct, 0)$
$\mathsf{W}(acct, 75)$

LOST UPDATE ✗

Snapshot Write: Concurrent transactions cannot write to the same key. One of them must be aborted.

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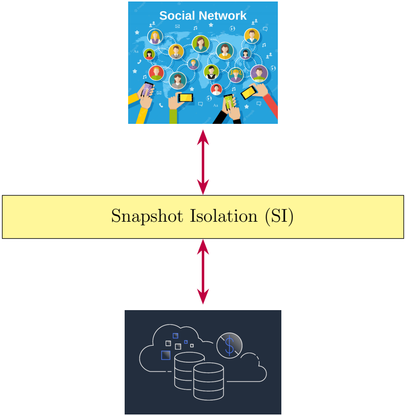


# Snapshot Isolation (SI)


Snapshot Isolation (SI)

$\boxed{\mathsf{W}(x, post)}$

# Snapshot Isolation (SI)


Snapshot Isolation (SI)

W($x$, $post$)

R($x$, $post$)
W($y$, $comment$)

# Snapshot Isolation (SI)


Snapshot Isolation (SI)

| | | |
|---|---|---|
| W($x$, *post*) | R($x$, *post*) W($y$, *comment*) | R($x$, *empty*) R($y$, *comment*) |

# Snapshot Isolation (SI)


Snapshot Isolation (SI)

CAUSALITY VIOLATION

| | | |
|---|---|---|
| $\mathsf{W}(x, post)$ | $\mathsf{R}(x, post)$<br>$\mathsf{W}(y, comment)$ | $\mathsf{R}(x, empty)$<br>$\mathsf{R}(y, comment)$ |

# Snapshot Isolation (SI)


Snapshot Isolation (SI)

CAUSALITY VIOLATION

| | | |
|---|---|---|
| W($x$, *post*) | R($x$, *post*)<br>W($y$, *comment*) | R($x$, *empty*)<br>R($y$, *comment*) |

# Databases and Snapshot Isolation

database logos

Many databases claim to support SI.

# Databases and Snapshot Isolation

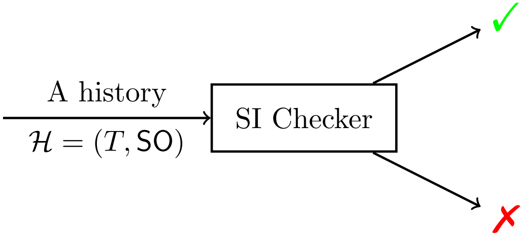
+papers

Databases may fail to provide SI as they claim.

# The SI Checking Problem

**Definition (The SI Checking Problem)**

The SI checking problem is the decision problem of determing whether a given history $\mathcal{H} = (T, \mathsf{SO})$ satisfies SI?



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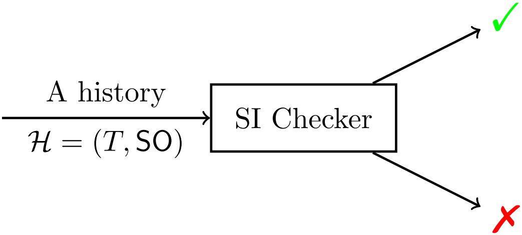
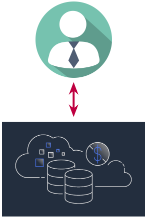



$\mathsf{SO}$ : *session order* among the set $T$ of transactions

# The SI Checking Problem

*Black-box checking:* do not rely on database internals



The histories are collected from database logs.

# The SI Checking Problem

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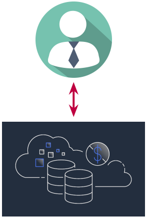

| | | |
|---|---|---|
| W($x$, *post*) | R($x$, *post*)<br>W($y$, *comment*) | R($x$, *empty*)<br>R($y$, *comment*) |

Lost Update

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# The SI Checking Problem

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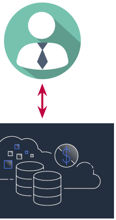

| | | | |
|---|---|---|---|
| W($x$, *post*) | R($x$, *post*)<br>W($y$, *comment*) | R($x$, *empty*)<br>R($y$, *comment*) | LOST UPDATE ✗ |

| | | | |
|---|---|---|---|
| R($acct_1$, 60)<br>R($acct_2$, 60)<br>W($acct_1$, $-40$) | R($acct_1$, 60)<br>R($acct_2$, 60)<br>W($acct_2$, $-40$) | R($acct_1$, $-40$)<br>R($acct_2$, $-40$) | WRITE SKEW ✓ |

The histories are collected from database logs.

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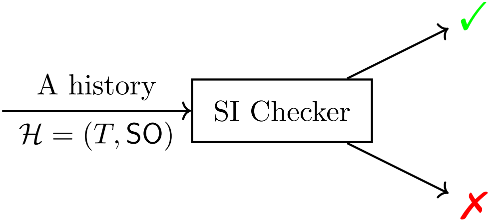

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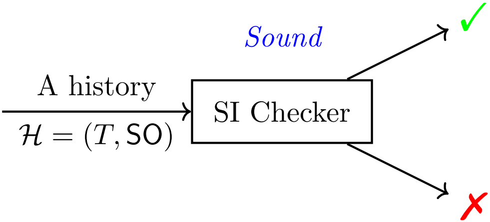
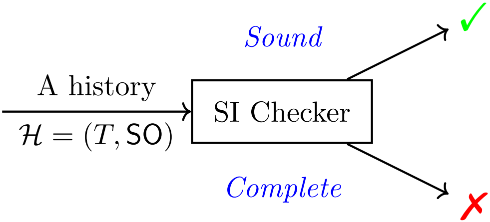


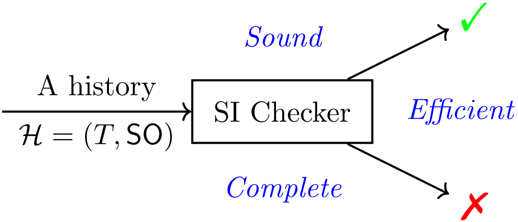

*Sound:* If the checker says ✗, then the history does *not* satisfy SI.

# The SI Checking Problem



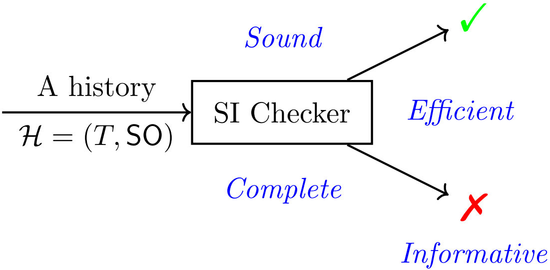

*Complete:* If the checker says ✓, then the history *satisfies* SI.

# The SI Checking Problem




*Efficient:* The checker should *scale* up to large workloads.

# The SI Checking Problem

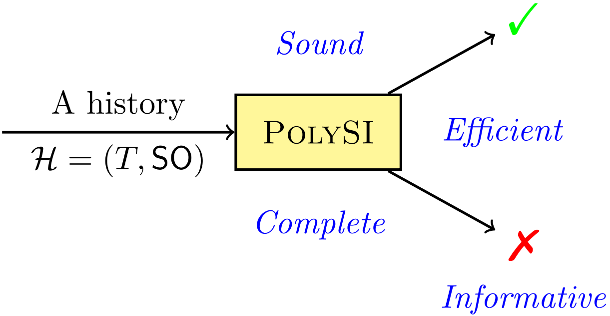



*Informative:* The checker should provide understandable *counterexamples* if it says ✗.

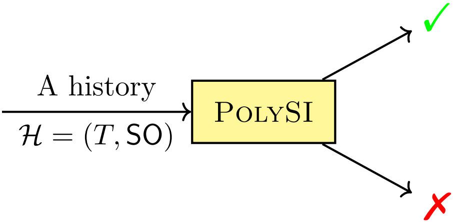
# The SI Checking Problem

related-work

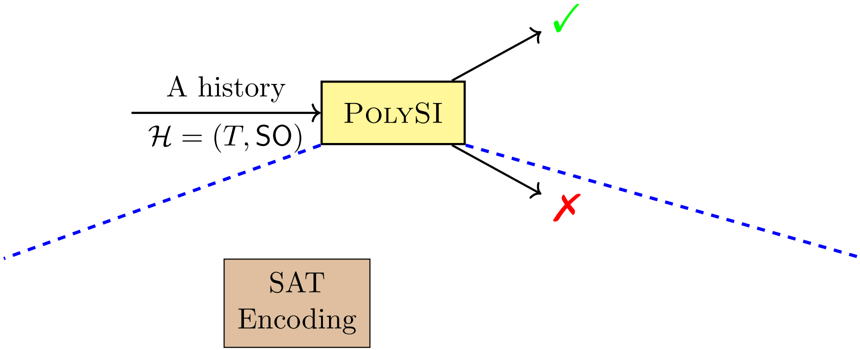
# Contribution: the PolySI Checker



# Contribution: the POLYSI Checker



# Contribution: the PolySI Checker




*Sound & Complete:* polygraph-based characterization of SI

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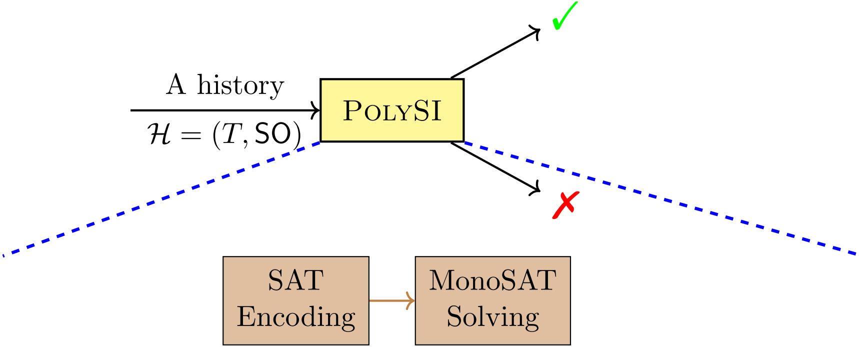
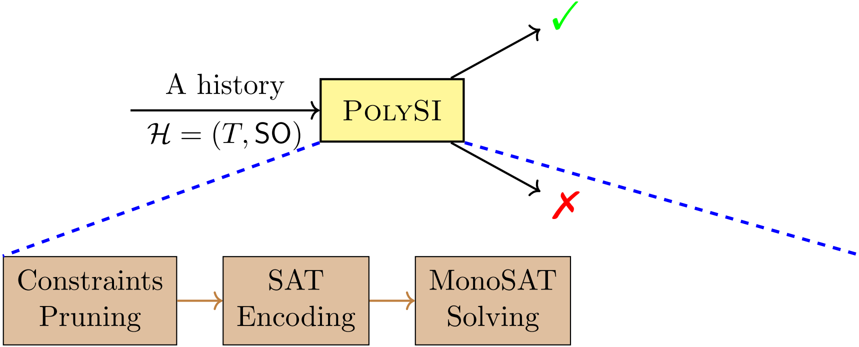


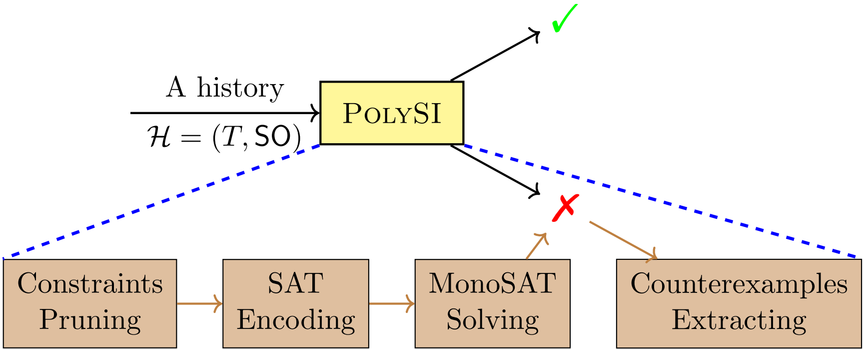

*Efficient:* utilizing MonoSAT solver optimized for graph problems

# Contribution: the PolySI Checker




*Efficient:* domain-specific pruning before encoding

# Contribution: the PolySI Checker




*Informative:* extract counterexamples from the unsatisifiable core

# Dependency Graph-based Characterization of SI

$T_A$

$\mathsf{R}(acct, 0)$
$\mathsf{W}(acct, 50)$

$T_0$

$\mathsf{W}(acct, 0)$

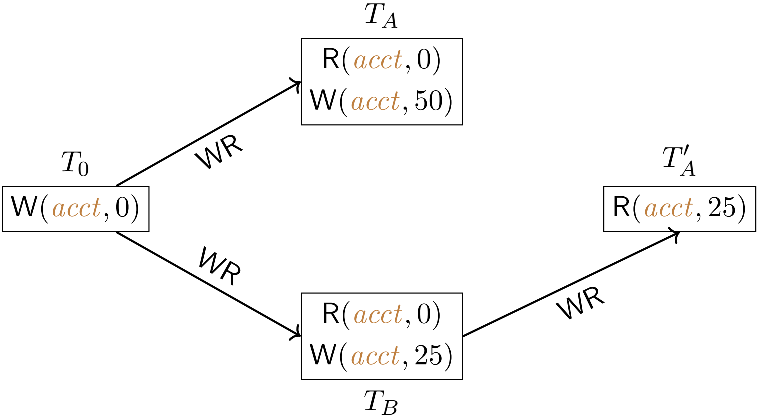
$T'_A$

$\mathsf{R}(acct, 25)$

$\mathsf{R}(acct, 0)$
$\mathsf{W}(acct, 25)$

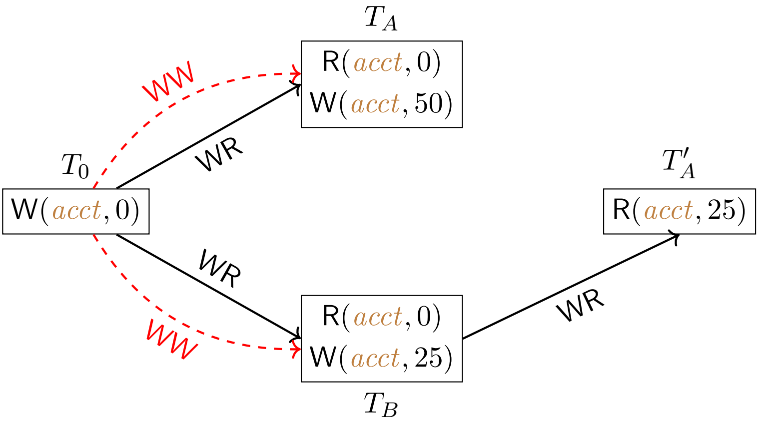
$T_B$

# Dependency Graph-based Characterization of SI




WR: "write-read" dependency capturing the "read-from" relation

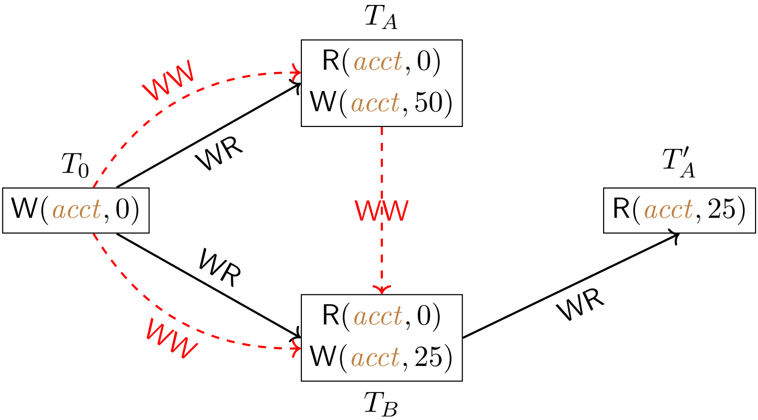
# Dependency Graph-based Characterization of SI




WW: "write-write" dependency capturing the version order

# Dependency Graph-based Characterization of SI

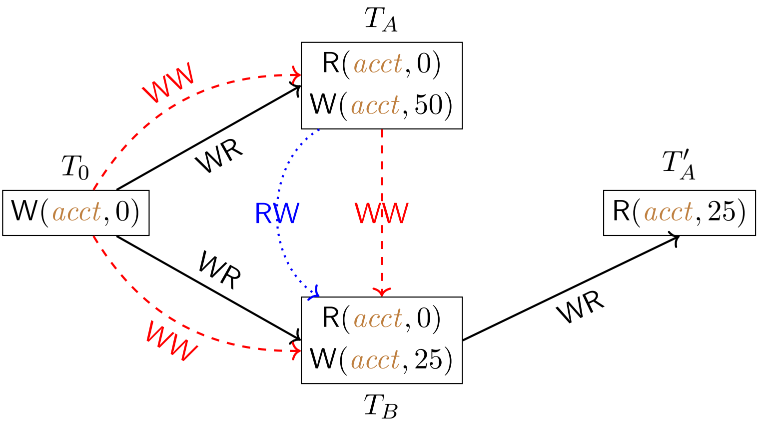
*Suppose that* $T_A \xrightarrow{\text{WW}} T_B$




WW: "write-write" dependency capturing the version order

# Dependency Graph-based Characterization of SI

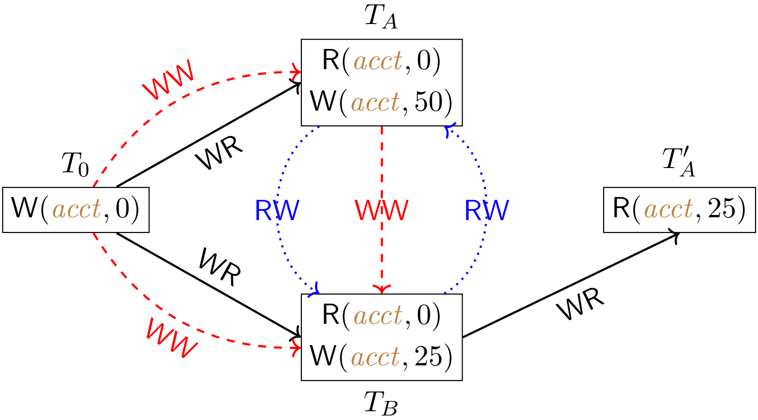
$$T_0 \xrightarrow{\mathsf{WR}} T_A \wedge T_0 \xrightarrow{\mathsf{WW}} T_B \implies T_A \xrightarrow{\mathsf{RW}} T_B$$




RW: "read-write" dependency capturing the overwritten relation

# Dependency Graph-based Characterization of SI

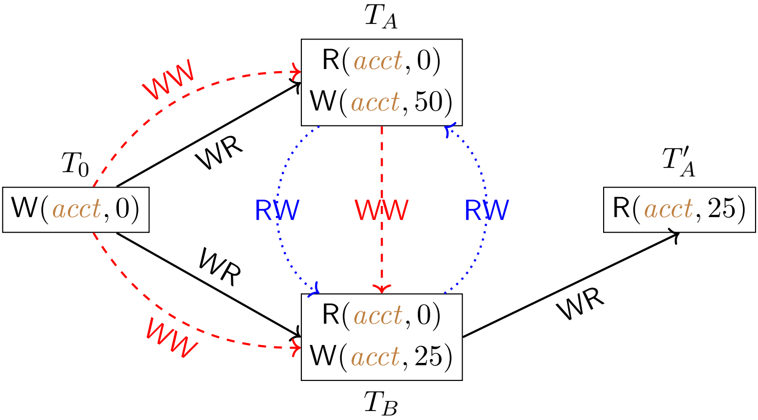
$$T_0 \xrightarrow{\text{WR}} T_B \wedge T_0 \xrightarrow{\text{WW}} T_A \implies T_A \xrightarrow{\text{RW}} T_A$$




RW: "read-write" dependency capturing the overwritten relation

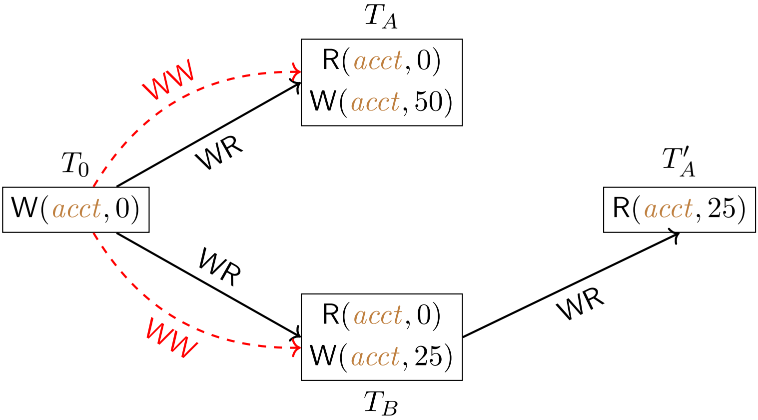
# Dependency Graph-based Characterization of SI

$\boxed{\textit{Suppose that } T_A \xrightarrow{\textsf{WW}} T_B}$



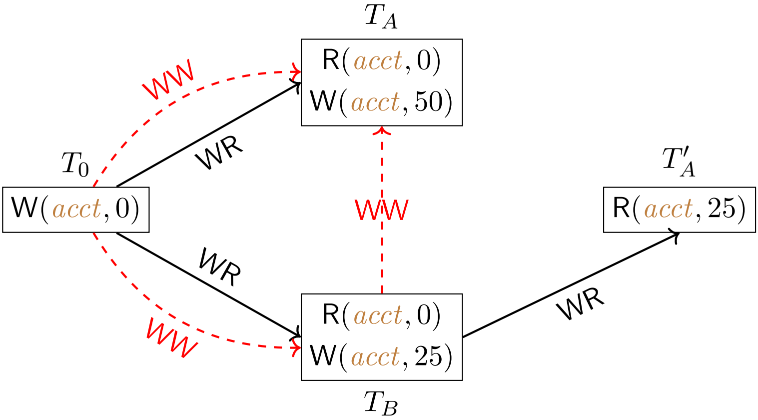

undesired cycle: $T_A \xrightarrow{\textsf{WW}} T_B \xrightarrow{\textsf{RW}} T_A$

# Dependency Graph-based Characterization of SI



# Dependency Graph-based Characterization of SI

*Suppose that* $T_B \xrightarrow{\text{WW}} T_A$



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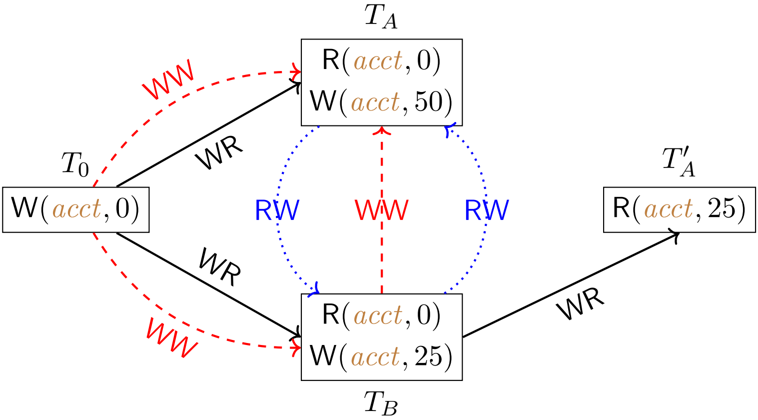

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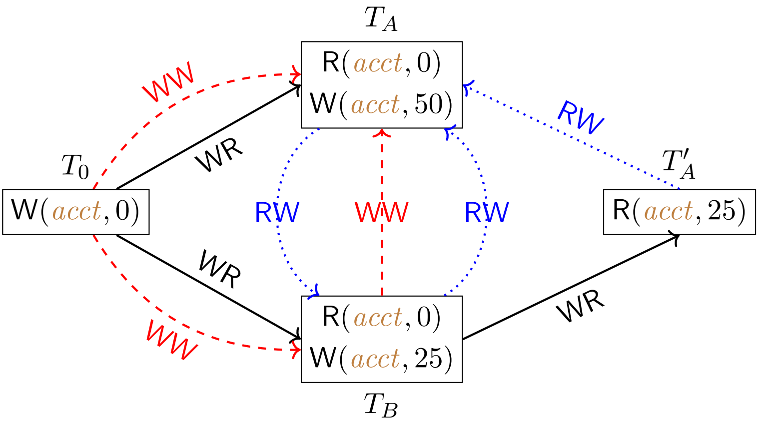
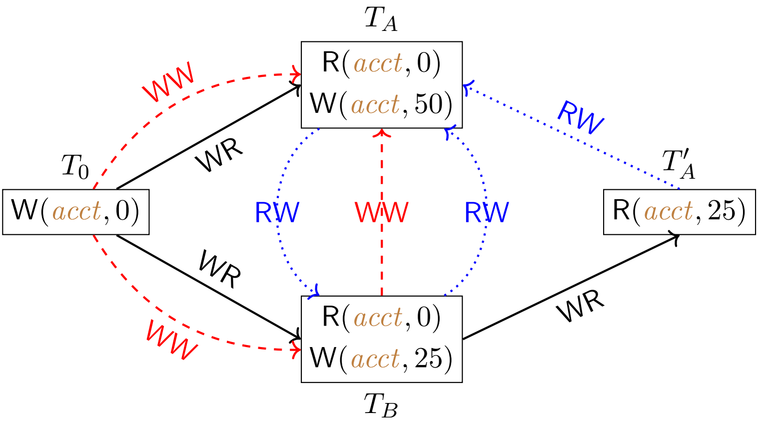


# Dependency Graph-based Characterization of SI

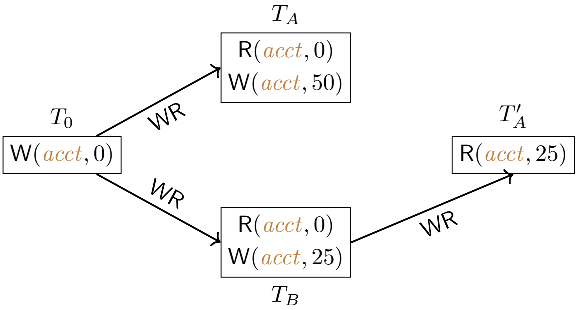
$\boxed{\textit{Suppose that } T_B \xrightarrow{\text{WW}} T_A}$




undesired cycle: $T_B \xrightarrow{\text{WW}} T_A \xrightarrow{\text{RW}} T_B$

# Dependency Graph-based Characterization of SI

We have considered both bases $T_A \xrightarrow{\mathsf{WW}} T_B$ and $T_B \xrightarrow{\mathsf{WW}} T_A$.




Either case leads to an undesired cycle.

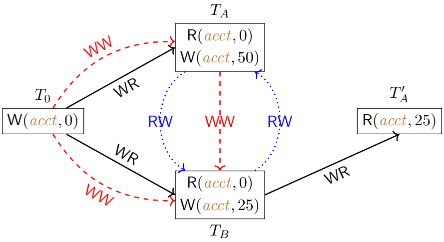
Therefore, it does not satisfy SI.

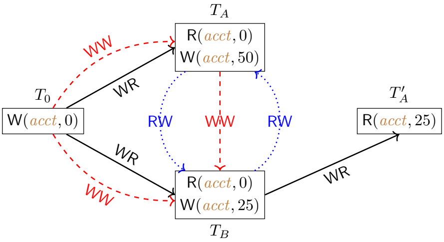
# Dependency Graph-based Characterization of SI

**Theorem (Theorem 4.1 of [AnalysingSI:JACM2018])**

*Informally, a history satisfies SI if only if*
*there exists a dependency graph for it that contains*
*only cycles (if any) with at least two adjacent* RW *edges.*

# Dependency Graph-based Characterization of SI




Every possible dependency graph contains an undesired  cycle.

# Dependency Graph-based Characterization of SI

**Theorem (Theorem 4.1 of [AnalysingSI:JACM2018])**

*For a history* $\mathcal{H} = (T, \mathsf{SO})$,

$$\mathcal{H} \models \mathrm{SI} \iff \mathcal{H} \models \textsc{Int} \wedge$$
$$\exists\, \mathsf{WR}, \mathsf{WW}, \mathsf{RW}.\ \mathcal{G} = (\mathcal{H}, \mathsf{WR}, \mathsf{WW}, \mathsf{RW}) \wedge$$
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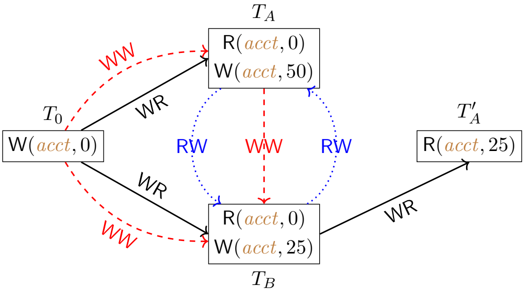

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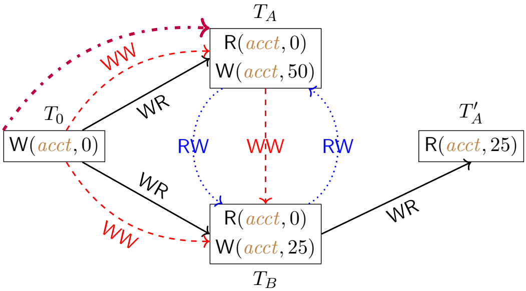

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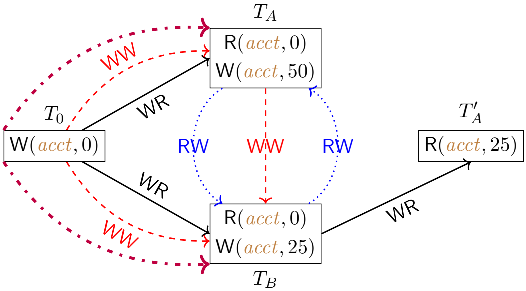

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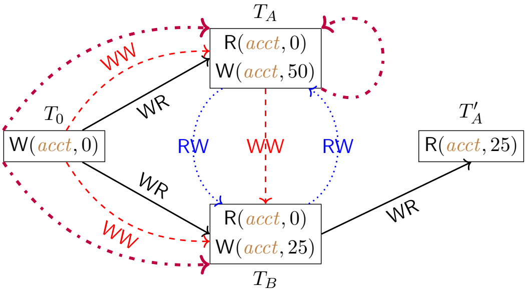

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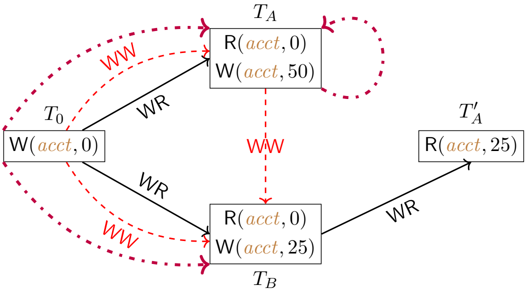
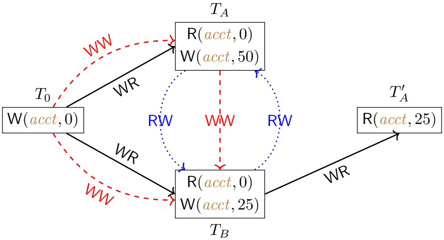
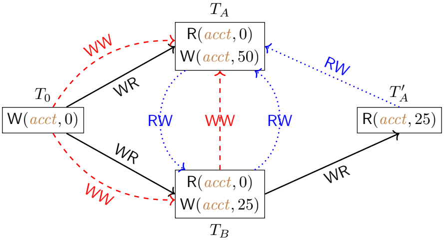


# Dependency Graph-based Characterization of SI

$\mathcal{Q}$ : How to capture and resolve all possible WW dependencies?



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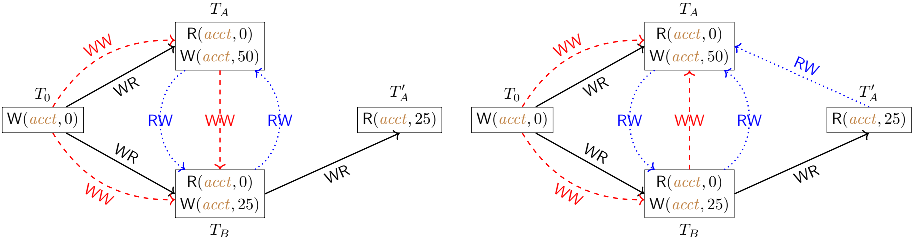
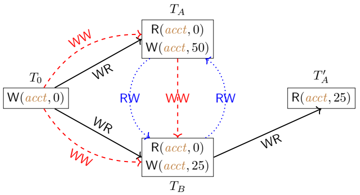
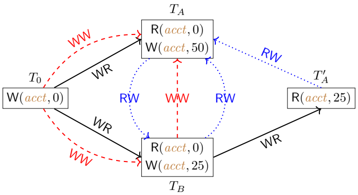



$\mathcal{A}$ : encode them into SAT formulas based on (generalized) polygraphs and solve them using SAT solvers.

# Polygraphs: A Family of Dependency Graphs

Consider the two cases of WW dependencies between $T_A$ and $T_B$.



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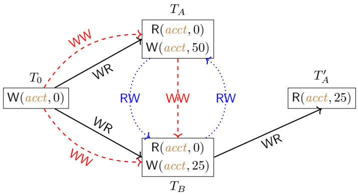


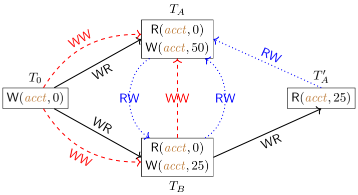


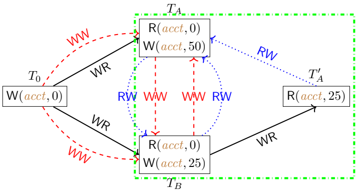


generalized polygraph:

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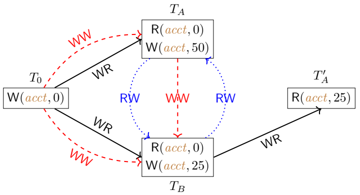
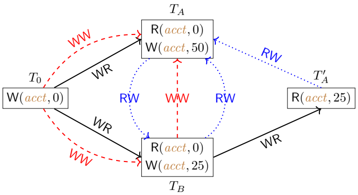
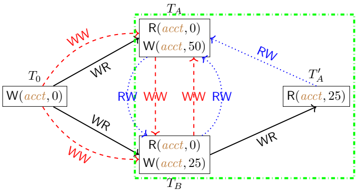


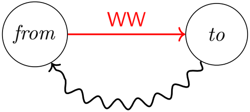



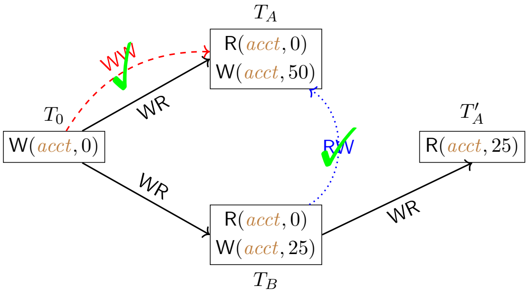




generalized polygraph:

$$\langle either \triangleq \{T_A \xrightarrow{\mathsf{WW}} T_B\}, or \triangleq \{T_B \xrightarrow{\mathsf{WW}} T_A, T'_A \xrightarrow{\mathsf{RW}} T_A\}\rangle$$

# PolySI: Pruning before Encoding (the WW case)



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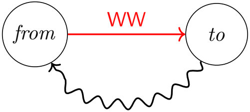


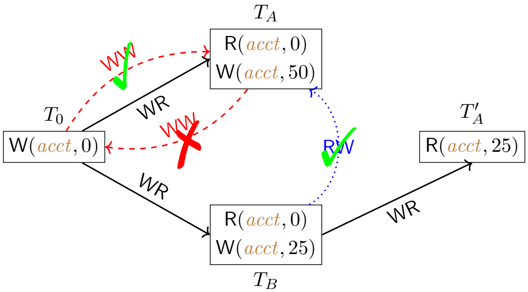


$T_A \xrightarrow{\mathsf{WW}} T_0$ can be pruned due to the $T_A \xrightarrow{\mathsf{WW}} T_0 \xrightarrow{\mathsf{WR}} T_A$ cycle.

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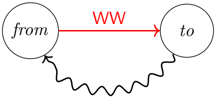


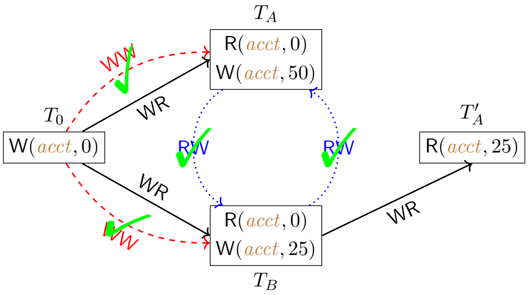

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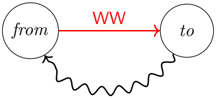
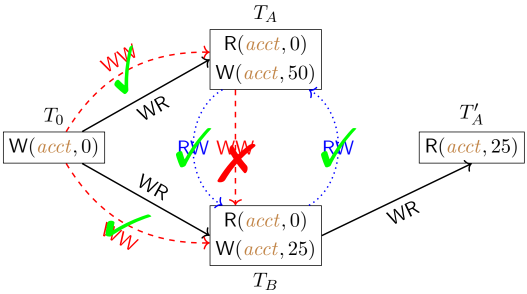
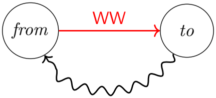
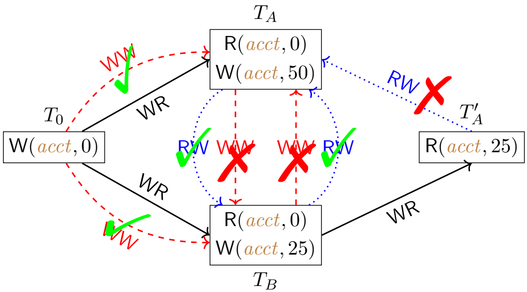



$T_A \xrightarrow{\text{WW}} T_B$ is pruned due to the $T_A \xrightarrow{\text{WW}} T_B \xrightarrow{\text{RW}} T_A$ cycle.

# PolySI: Pruning before Encoding (the WW case)

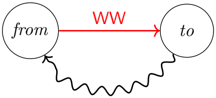


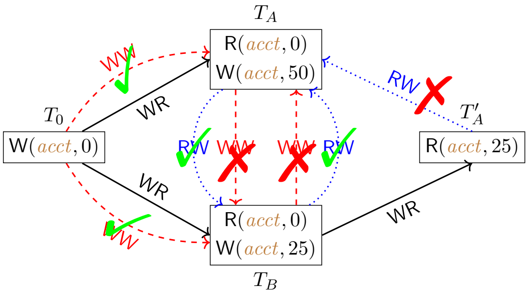




$$T_A \xrightarrow{\mathsf{WW}} T_B \text{ is pruned due to the } T_A \xrightarrow{\mathsf{WW}} T_B \xrightarrow{\mathsf{RW}} T_A \text{ cycle.}$$

$$T_B \xrightarrow{\mathsf{WW}} T_A \text{ is pruned due to the } T_B \xrightarrow{\mathsf{WW}} T_A \xrightarrow{\mathsf{RW}} T_B \text{ cycle.}$$

# PolySI: Pruning before Encoding (the WW case)




$$T_A \xrightarrow{\text{WW}} T_B \text{ is pruned due to the } T_A \xrightarrow{\text{WW}} T_B \xrightarrow{\text{RW}} T_A \text{ cycle.}$$

$$T_B \xrightarrow{\text{WW}} T_A \text{ is pruned due to the } T_B \xrightarrow{\text{WW}} T_A \xrightarrow{\text{RW}} T_B \text{ cycle.}$$

Therefore, we are sure that the history does *not* satisfy SI.

# PolySI: Pruning before Encoding (the RW case)

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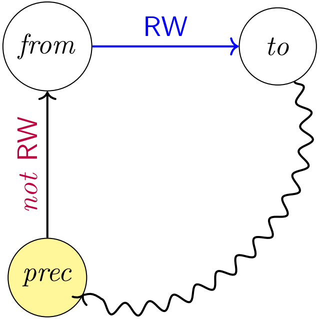
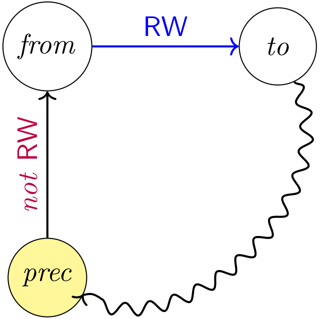


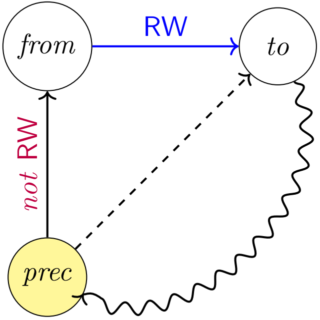
# POLYSI: Pruning before Encoding (the RW case)




Theorem (Theorem 4.1 of [**AnalysingSI:JACM2018**])

*Informally, a history satisfies SI if only if there exists a dependency graph for it that contains only cycles (if any) with at least two adjacent* RW *edges.*

# PolySI: Pruning before Encoding (the RW case)




Theorem (Theorem 4.1 of [AnalysingSI:JACM2018])

*Informally, a history satisfies SI if only if there exists a dependency graph for it that contains only cycles (if any) with at least two adjacent RW edges.*

# PolySI: An Illustrating Example of "Long Fork"

$T_0$ $\boxed{\mathsf{W}(x, 0)\ \mathsf{W}(y, 0)}$

# PolySI: An Illustrating Example of "Long Fork"

$T_1$

$\boxed{\mathsf{W}(x, 1)}$

$T_0$ $\boxed{\mathsf{W}(x, 0)\ \mathsf{W}(y, 0)}$

# PolySI: An Illustrating Example of "Long Fork"

$T_1$

$\boxed{\mathsf{W}(x, 1)}$

$T_0$ $\boxed{\mathsf{W}(x, 0)\ \mathsf{W}(y, 0)}$

$\boxed{\mathsf{W}(y, 1)}$

$T_2$

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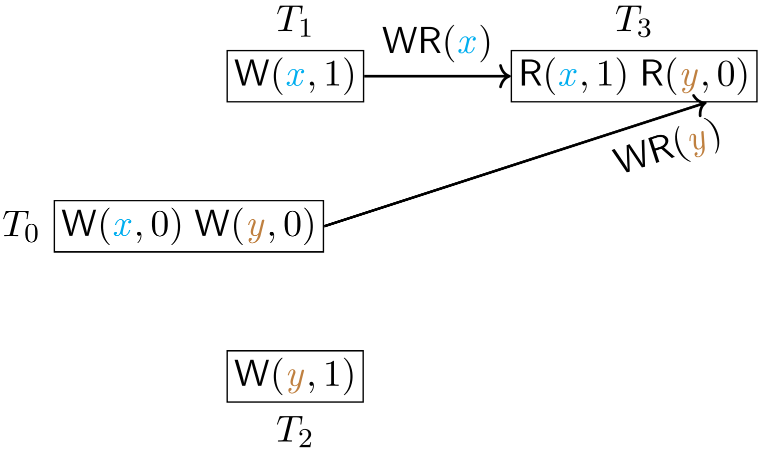

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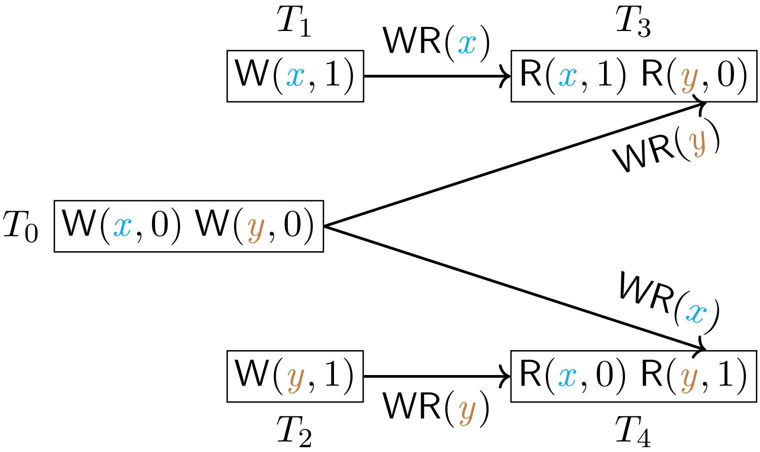

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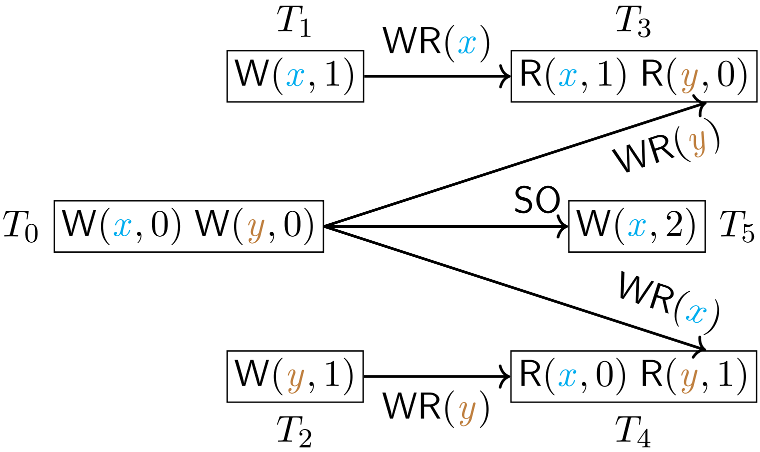


order between $T_0$, $T_1$, and $T_5$ (on $x$) and between $T_0$ and $T_2$ (on $y$)

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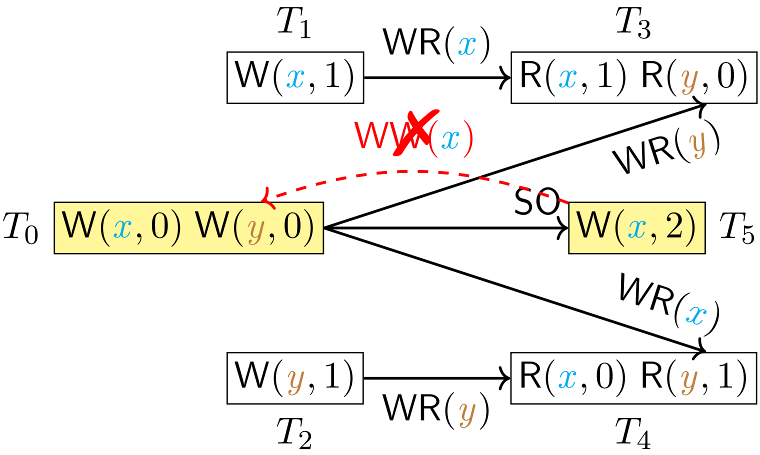


The $T_5 \xrightarrow{\mathsf{WW}(x)} T_0$ case is pruned due to $T_0 \xrightarrow{\mathsf{SO}} T_5 \xrightarrow{\mathsf{WW}(x)} T_0$.

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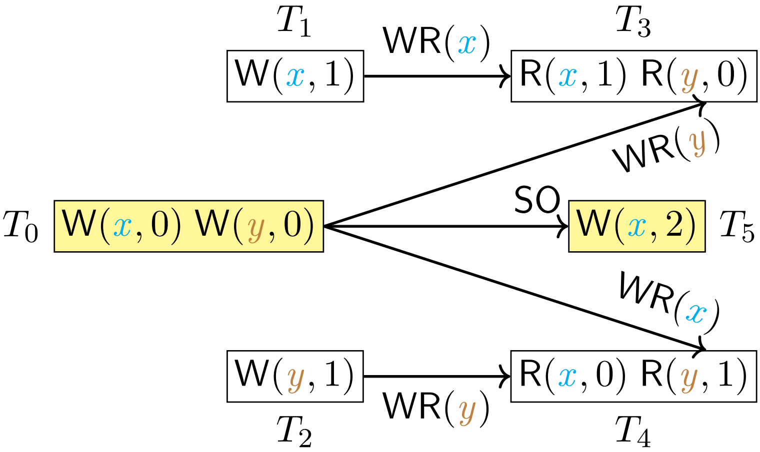

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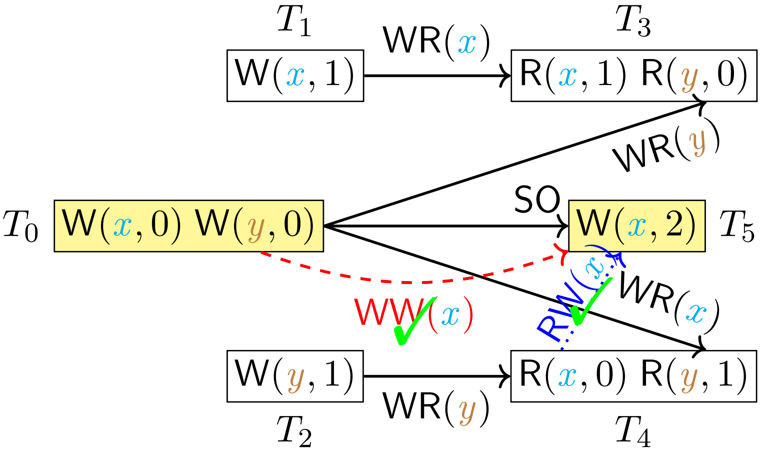
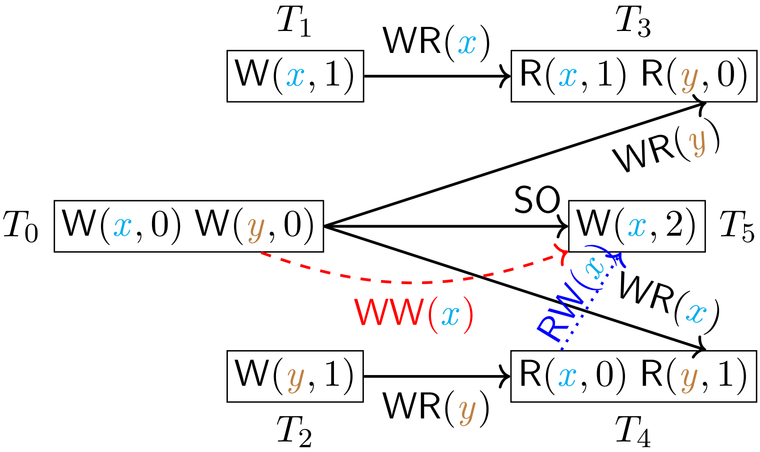



The $T_0 \xrightarrow{WW(x)} T_5$ case becomes known.

# PolySI: An Illustrating Example of "Long Fork"



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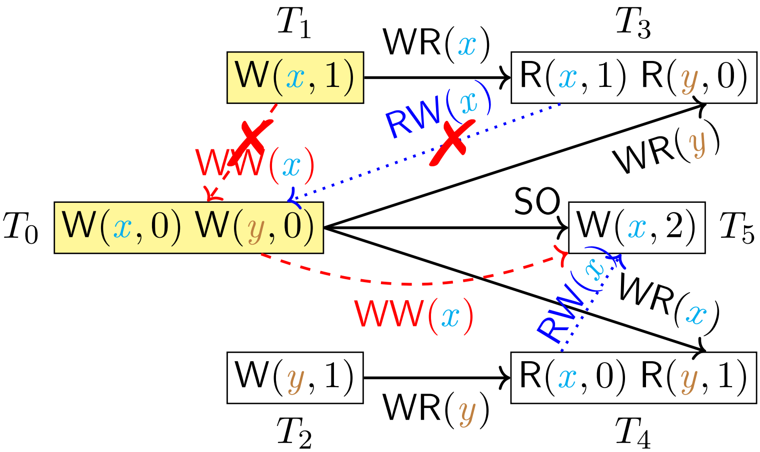
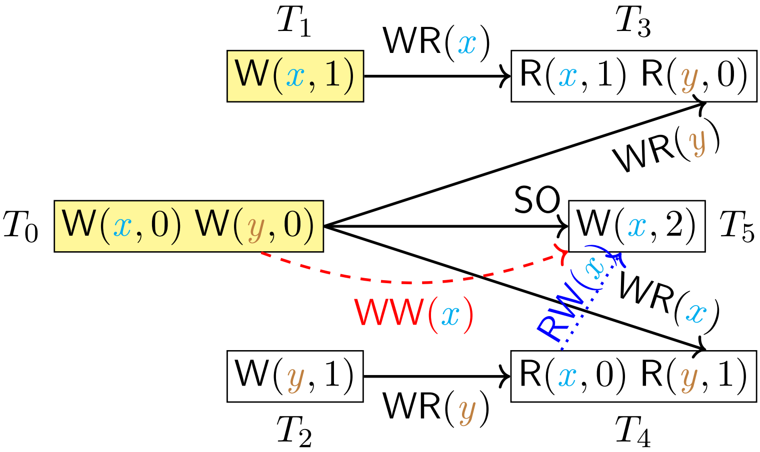



The $T_1 \xrightarrow{\mathsf{WW}(x)} T_0$ case is pruned due to $T_3 \xrightarrow{\mathsf{RW}(x)} T_0 \xrightarrow{\mathsf{WR}(y)} T_3$.

# PolySI: An Illustrating Example of "Long Fork"



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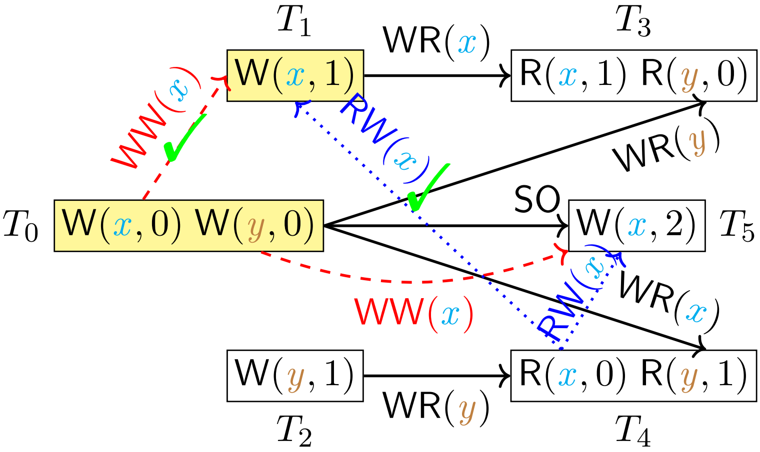
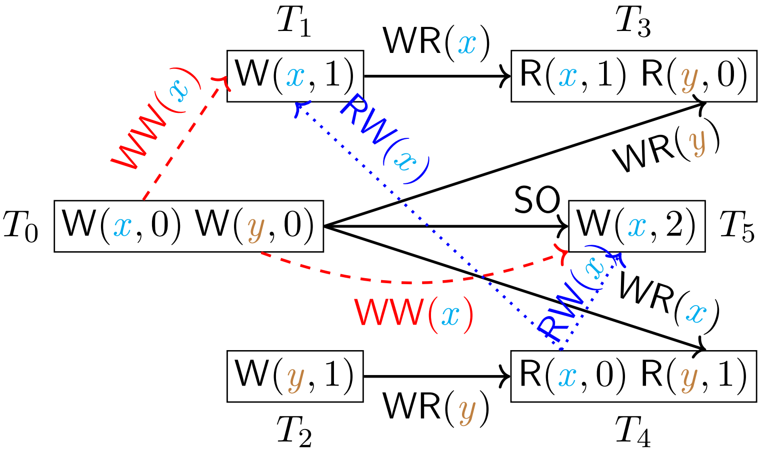



The $T_0 \xrightarrow{\mathsf{WW}(x)} T_1$ case becomes known.

# PolySI: An Illustrating Example of "Long Fork"



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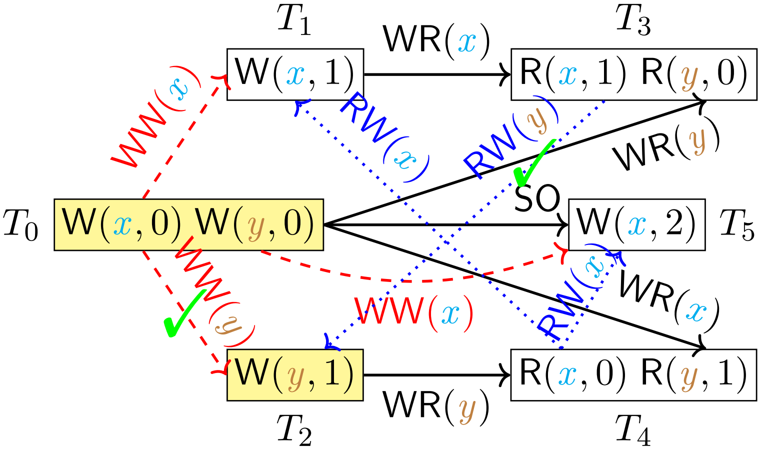
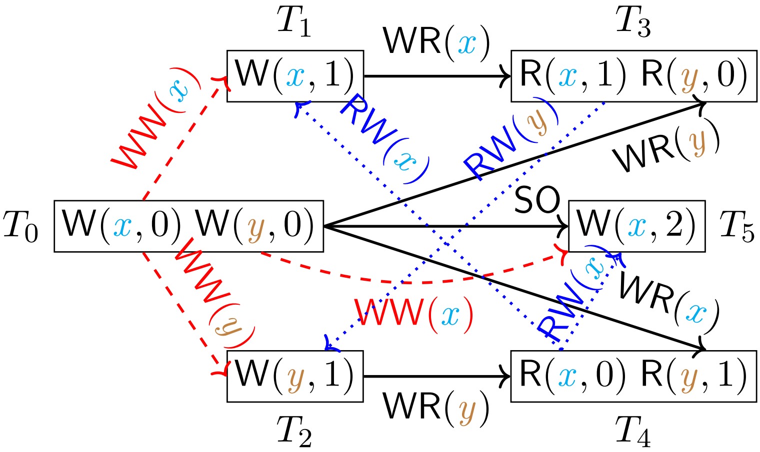


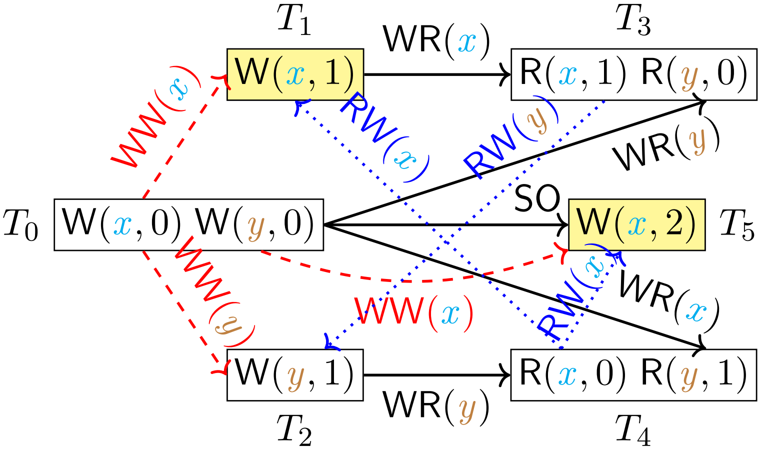

The $T_2 \xrightarrow{\mathsf{WW}(y)} T_0$ case is pruned,

while the $T_0 \xrightarrow{\mathsf{WW}(y)} T_2$ case becomes known.

# PolySI: An Illustrating Example of “Long Fork”

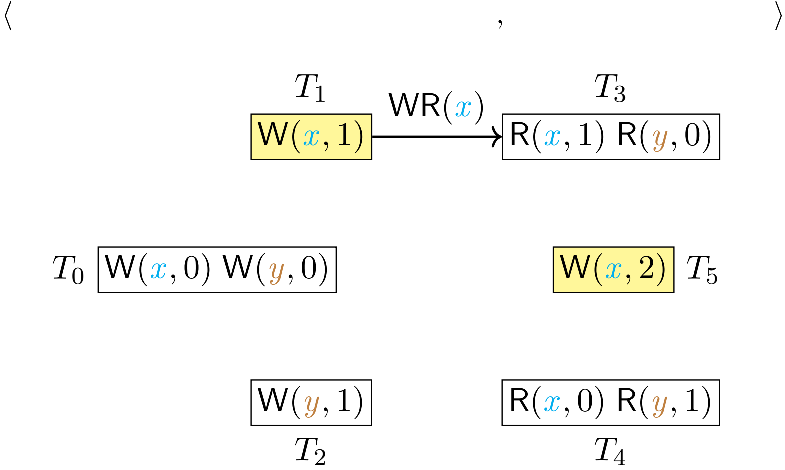


# PolySI: An Illustrating Example of "Long Fork"



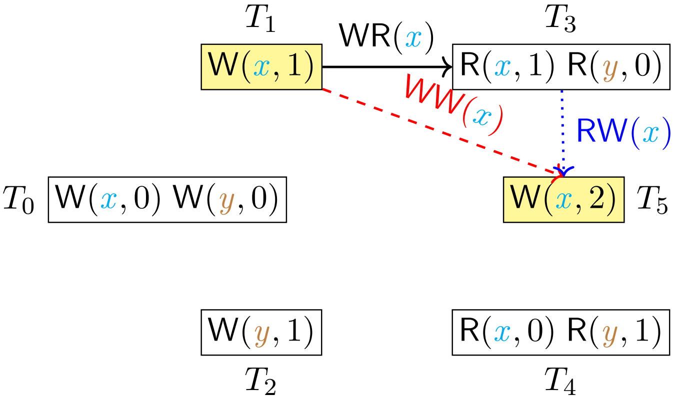

The order between $T_1$ and $T_5$ is still uncertain after pruning.

# PolySI: An Illustrating Example of “Long Fork”



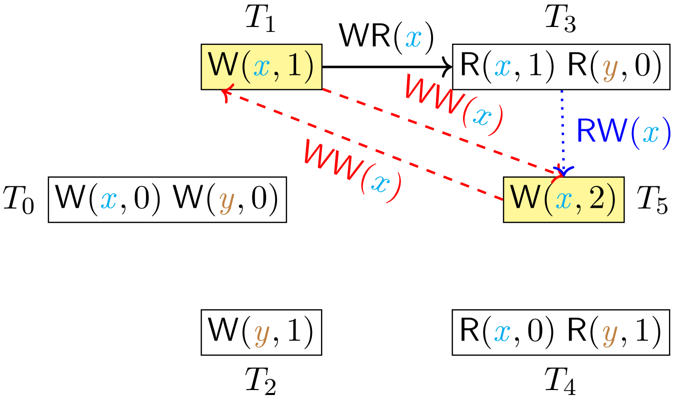
# PolySI: An Illustrating Example of "Long Fork"

$$\langle \textit{either} = \{T_1 \xrightarrow{\mathsf{WW}(x)} T_5, T_3 \xrightarrow{\mathsf{RW}(x)} T_5\}, \quad \rangle$$



# PolySI: An Illustrating Example of “Long Fork”

$$\langle \textit{either} = \{T_1 \xrightarrow{\mathsf{WW}(x)} T_5, T_3 \xrightarrow{\mathsf{RW}(x)} T_5\}, \textit{or} = \{T_5 \xrightarrow{\mathsf{WW}(x)} T_1\} \rangle$$



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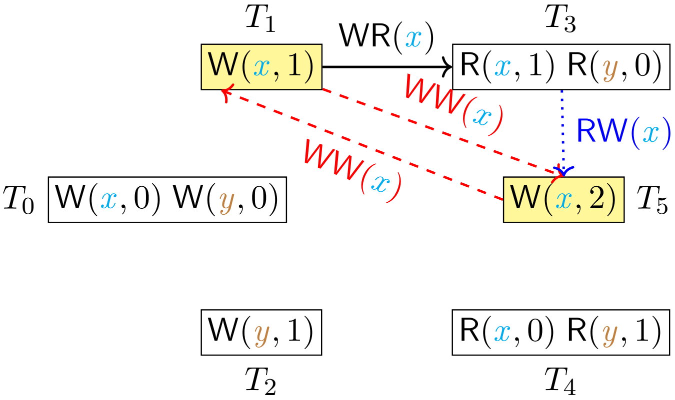
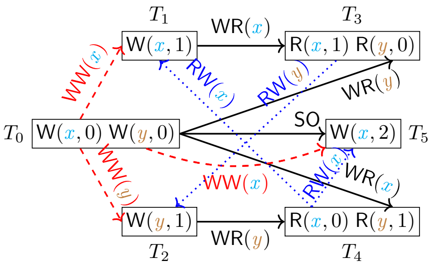
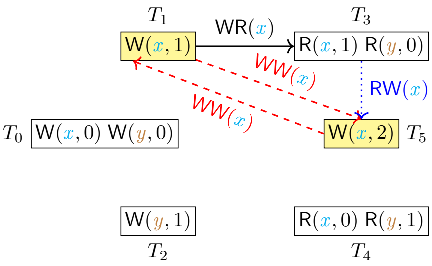



$$(\mathsf{BV}_{1,5} \wedge \mathsf{BV}_{3,5} \wedge \neg \mathsf{BV}_{5,1}) \vee (\mathsf{BV}_{5,1} \wedge \neg \mathsf{BV}_{1,5} \wedge \neg \mathsf{BV}_{3,5})$$

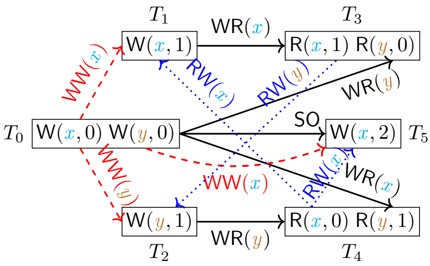
# PolySI: An Illustrating Example of "Long Fork"

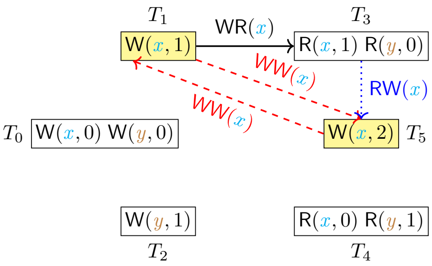


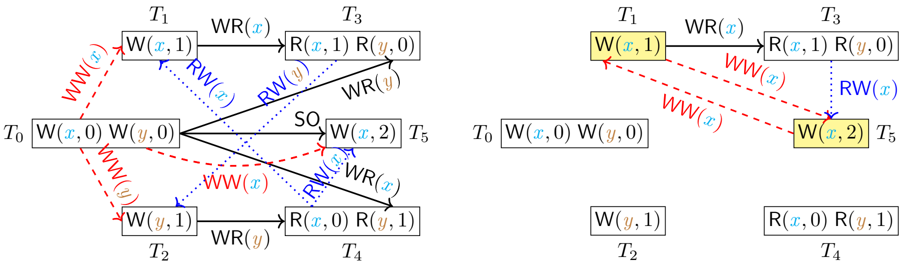
# PolySI: An Illustrating Example of "Long Fork"

$\boxed{((\mathsf{SO}_{\mathcal{G}} \cup \mathsf{WR}_{\mathcal{G}} \cup \mathsf{WW}_{\mathcal{G}}) \; ; \; \mathsf{RW}_{\mathcal{G}}?)}$ *is acyclic.*



$$\boxed{((\mathsf{SO}_{\mathcal{G}} \cup \mathsf{WR}_{\mathcal{G}} \cup \mathsf{WW}_{\mathcal{G}})\ ;\ \mathsf{RW}_{\mathcal{G}}?)}\quad \textit{is acyclic.}$$




We need to encode the "composition ( ; )" of dependency edges.

# PolySI: An Illustrating Example of "Long Fork"

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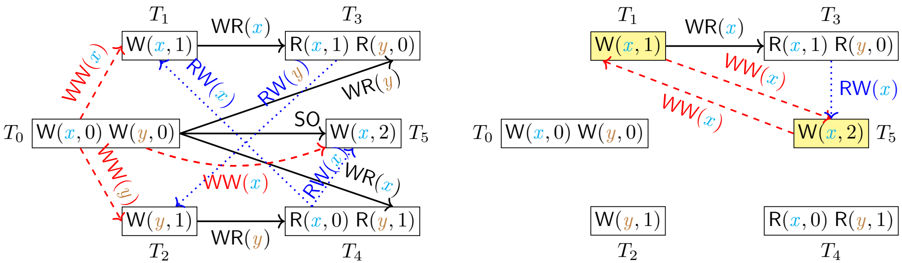


We need to encode the "composition ( ; )" of dependency edges.

$$T_1 \xrightarrow{\mathsf{WR}} T_3 \xrightarrow{\mathsf{RW}} T_2 : \ \mathsf{BV}^{I}_{1,2} = \mathsf{BV}_{1,3} \wedge \mathsf{BV}_{3,2} \quad (I \text{ for the induced graph})$$

# PolySI: An Illustrating Example of "Long Fork"

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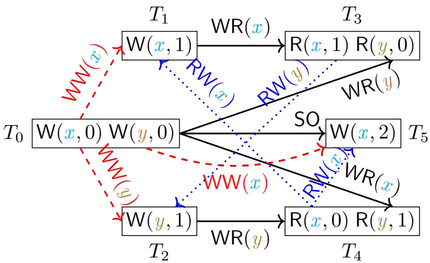


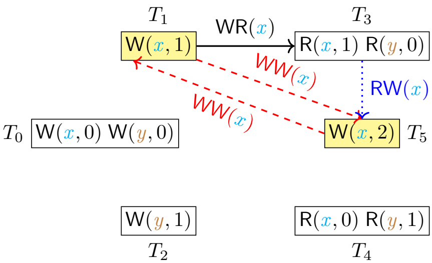


We need to encode the "composition ( ; )" of dependency edges.

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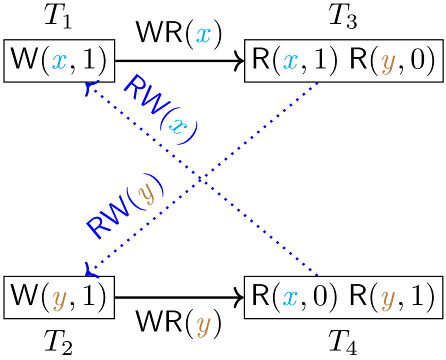
$$T_1 \xrightarrow{\mathsf{WR}} T_3 \xrightarrow{\mathsf{RW}} T_5 : \; \mathsf{BV}^{I}_{1,5} = \mathsf{BV}_{1,3} \wedge \mathsf{BV}_{3,5} \quad (I \text{ for the induced graph})$$

# PolySI: An Illustrating Example of "Long Fork"

Feed the SAT formula into the MonoSAT solver [**MonoSAT:AAAI2015**] optimized for *cycle detection*


Assert that the induced graph $I$ is acyclic.

# PolySI: An Illustrating Example of "Long Fork"




The undesired cycle for "long fork" found by MonoSAT.

# Experimental Evaluation

(1) *Effective:* Can PolySI find SI violations in production databases?

(2) *Informative:* Can PolySI provide understandable counterexamples for SI violations?

(3) *Efficient:* How efficient is PolySI? Is it scalable?

`https://github.com/hengxin/PolySI-PVLDB2023-Artifacts`

# Workloads

Table: Workload parameters and their default values.

| Parameter | Default Value |
|---|---|
| #sess | 20 |
| #txns/sess | 100 |
| #ops/txn | 15 |
| #keys | 10, 000 |
| %reads | 50% |
| distribution | zipfian |

## Benchmarks

RuBis: an eBay-like bidding system

TPC-C: an open standard for OLTP benchmarking

C-Twitter: a Twitter clone

GeneralRH: read-heavy workloads with 95% reads

GeneralRW: medium workloads with 50% reads

GeneralWH: write-heavy workloads with 30% reads

Use a simple database schema of a *two-column table* storing keys and values.

# Finding SI Violations

Table: Reproducing known SI violations.

| **Database** | **GitHub Stars** | **Kind** | **Release** |
|---|---|---|---|
| CockroachDB | 25.1k | Relational | v2.1.0, v2.1.6 |
| MySQL-Galera | 381 | Relational | v25.3.26 |
| YugabyteDB | 6.7k | Multi-model | v1.1.10.0 |

An extensive collection of 2477 anomalous histories

[**Complexity:OOPSLA2019**; **CockroachDB-bug**; **YugabyteDB-bug**]

# Finding SI Violations

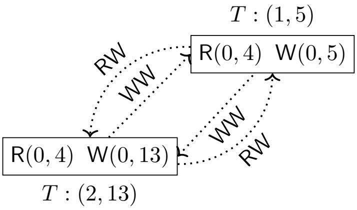
Dgraph: helped the Dgraph team confirm some of their suspicions about their latest release

Table: Detecting new violations.

| Database | GitHub Stars | Kind | Release |
|---|---|---|---|
| Dgraph | 18.2k | Graph | v21.12.0 |
| MariaDB-Galera | 4.4k | Relational | v10.7.3 |
| YugabyteDB | 6.7k | Multi-model | v2.11.1.0 |

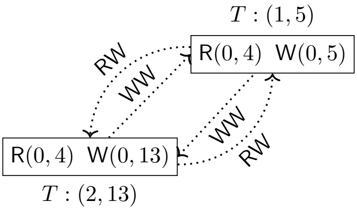
Galera: confirmed the incorrect claim on preventing "lost updates" for transactions issued on different cluster nodes

# Understanding Violations (Lost Update)

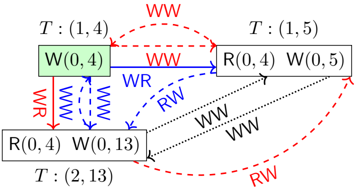



(a) Original output

# Understanding Violations (Lost Update)

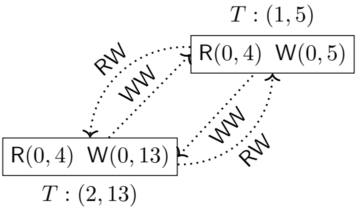



(a) Original output




(b) Missing participants

# Understanding Violations (Lost Update)

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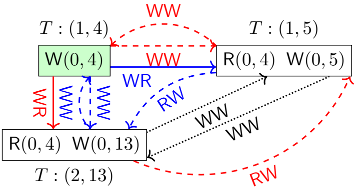
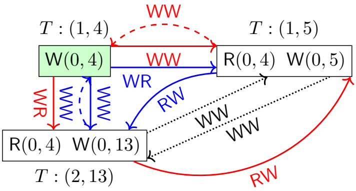
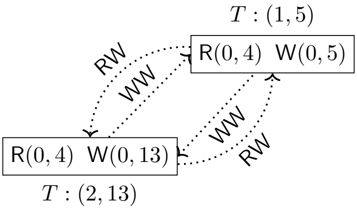



(b) Missing participants




(c) Recovered scenario

# Understanding Violations (Lost Update)

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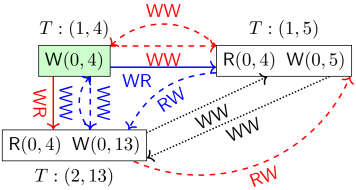
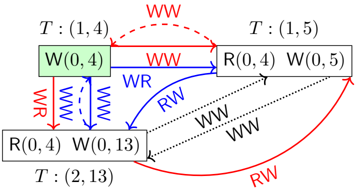



(b) Missing participants

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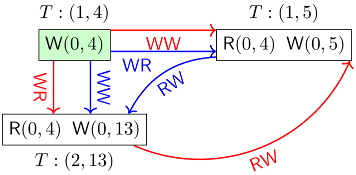



(d) Finalized scenario

# Performance Evaluation

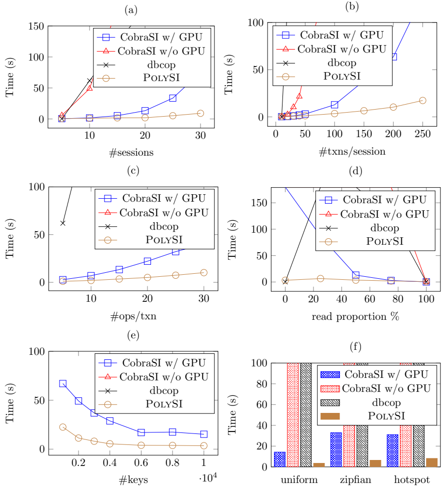
dbcop [**Complexity:OOPSLA2019**]: the state-of-the-art SI checker without using SAT solvers

Cobra [**Cobra:OSDI2020**]: the state-of-the-art SER checker using both MonoSAT and GPU; as a baseline

CobraSI: reducing SI checking to SER checking [**Complexity:OOPSLA2019**] to leverage Cobra with/without GPU

# Performance Evaluation: Runtime

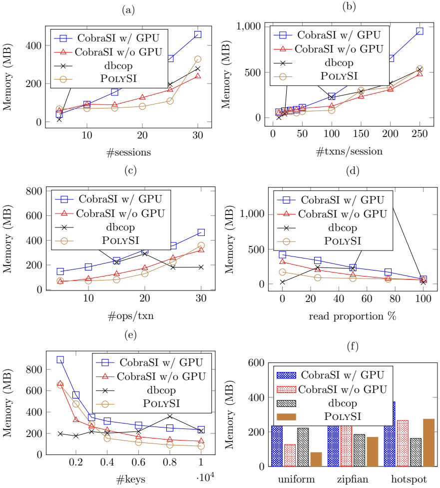
PolySI significantly outperforms the competitors.




All the input histories extracted from PostgreSQL satisfy SI.

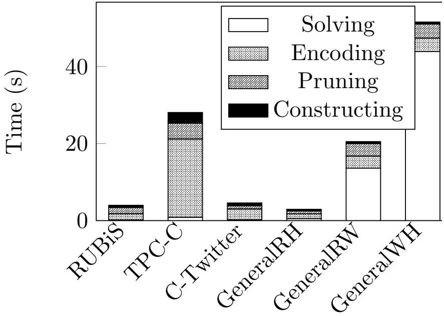
# Performance Evaluation: Memory

PolySI consumes less memory.



# Performance Evaluation: Decomposition

TPC-C incurs more overhead in *encoding* as the number of operations in total is 5x more than the others.




The solving time depends on the remaining constraints and unknown dependencies *after pruning*.

## Performance Evaluation: Pruning

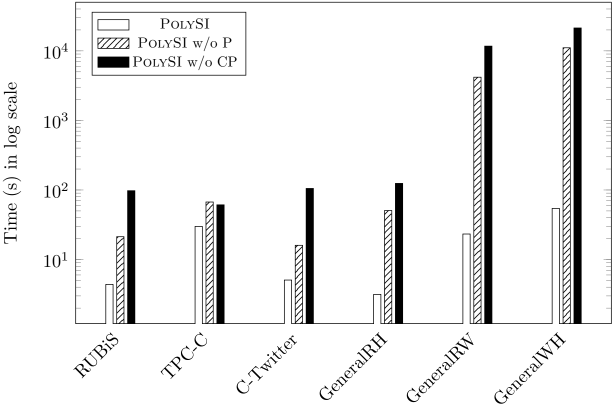
PolySI can effectively prune a huge number of constraints.

| Benchmark | #cons. before P | #cons. after P | #unk. dep. before P | #unk. dep. after P |
|---|---|---|---|---|
| TPC-C | 386k | 0 | 3628k | 0 |
| GeneralRH | 4k | 29 | 39k | 77 |
| RUBiS | 14k | 149 | 171k | 839 |
| C-Twitter | 59k | 277 | 307k | 776 |
| GeneralRW | 90k | 2565 | 401k | 5435 |
| GeneralWH | 167k | 6962 | 468k | 14376 |

TPC-C: read-only transactions + RMW transactions

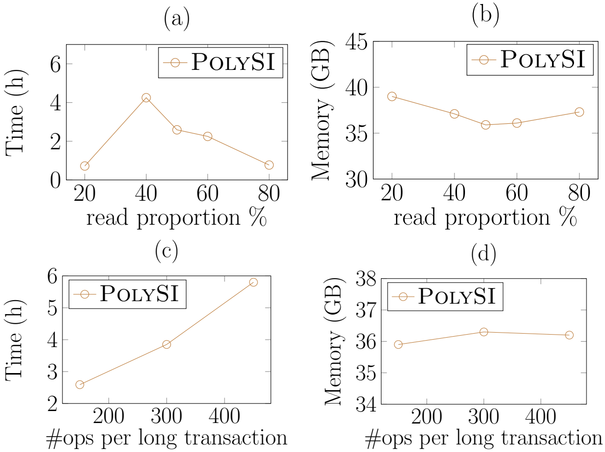
# Performance Evaluation: Differential Analysis

Pruning is crucial to the efficiency of PolySI.

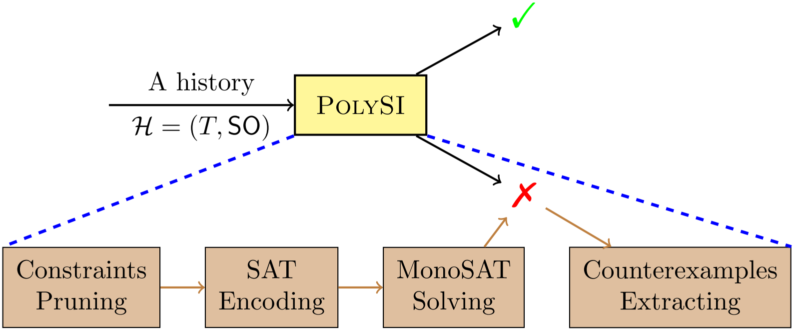


# Performance Evaluation: Scalability

several hours and 35 ∼ 40GB memory for checking 1M transactions



# Conclusion



A history
$\mathcal{H} = (T, \mathsf{SO})$
PolySI
✓
✗
Constraints Pruning
SAT Encoding
MonoSAT Solving
Counterexamples Extracting

# Future Work

POLYSI uses MonoSAT as a black-box.

Working on a **theory solver** dedicated to isolation level checking, which is deeply integrated with SAT solvers [**Zord:PLDI2021**].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