CS 7643	CGOH	1/26/21

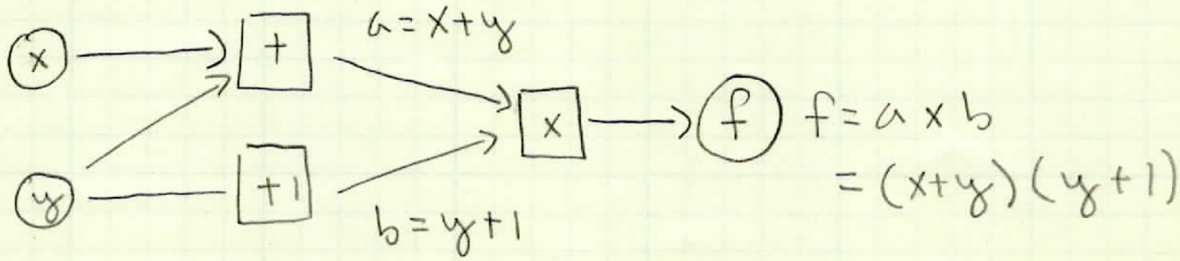
$$f(x,y) = (x+y)(y+1)$$

- Let a and b be intermediate vars:

$$a = x+y$$

$$b = y+1$$

$$f = a \times b$$

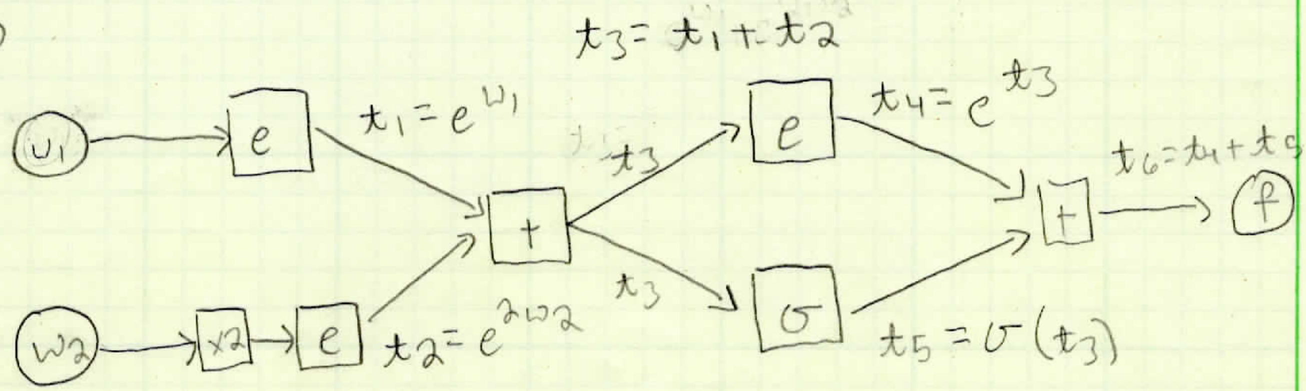


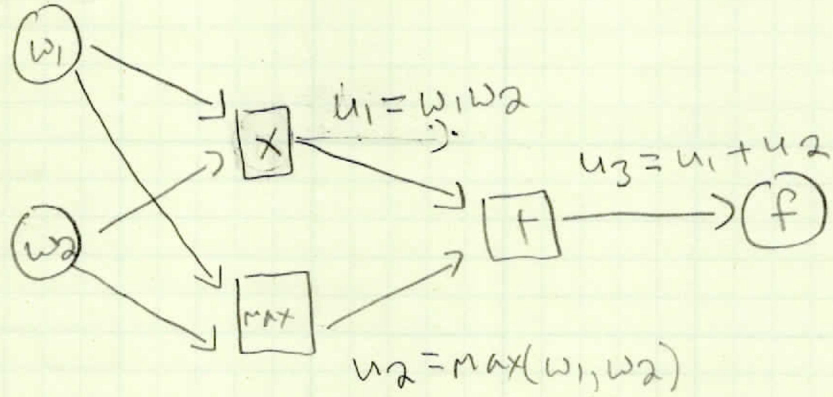
- CGs breakdown complex func. into intermediate steps.

(1) $$f_1(w_1, w_2) = e^{e^{w_1} + e^{2w_2}} + \sigma(e^{w_1} + e^{2w_2})$$

$$f_2(w_1, w_2) = w_1 w_2 + \max(w_1, w_2)$$

(a)





$$\vec{w} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \Rightarrow \vec{f} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} e^{e + e^{-2}} + \sigma(e^{e} + e^{-2}) \\ (1)(-1) + \max(1, -1) \end{bmatrix}$$

$$= \begin{bmatrix} 18.296 \\ 0 \end{bmatrix}$$

b $\quad \dfrac{\partial \vec{f}}{\partial \vec{w}} = \begin{bmatrix} \dfrac{\partial f_1}{\partial w_1} & \dfrac{\partial f_1}{\partial w_2} \\ \dfrac{\partial f_2}{\partial w_1} & \dfrac{\partial f_1}{\partial w_2} \end{bmatrix}$

$$\frac{\partial f_1}{\partial w_1} = \frac{f_1(w_1 + \Delta w, w_2) - f_1(w_1, w_2)}{\Delta w} = \frac{18.778 - 18.296}{0.01} = 48.192$$

$$f(1 + 0.01, -1) = e^{e^{1.01} + e^{-2}} + \sigma(e^{1.01} + e^{-2})$$

$$= 24.051$$

$$\frac{\partial f_1}{\partial w_2} = \frac{f_1(w_1, w_2 + \Delta w) - f_1(w_1, w_2)}{\Delta w_2} = \frac{18.344 - 18.296}{0.01} = 4.764$$

$$f_1(1, -0.99) = e^{e + e^{2 \cdot -0.99}} + \sigma(e + e^{2 \cdot -0.99})$$

$$= 18.344$$

$$\frac{\partial f_2}{\partial w_1} = \frac{f_2(w_1+\Delta w, w_2) - f_2(w_1, w_2)}{\Delta w} = \frac{f_2(w_1+\Delta w, w_2)}{\Delta w} = 0$$

$$f_2(1.01, -1) = (1.01)(-1) + \max(1.01, -1) = 0$$

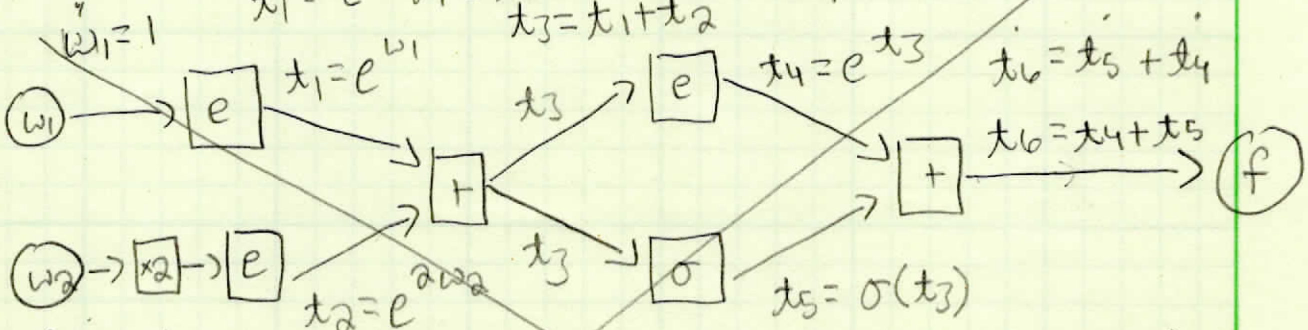
$$\frac{\partial f_2}{\partial w_2} = \frac{f_2(w_1, w_2+\Delta w)}{\Delta w} = \frac{(1)(-0.99) + \max(1, -0.99)}{0.01}$$

$$= 1$$

$$\frac{\partial \vec{f}}{\partial \vec{w}} = \begin{bmatrix} 48.192 & 4.764 \\ 0 & 1 \end{bmatrix}$$

$\dot t_1 = e^{w_1}\dot w_1 = t_1$ $\qquad \dot t_4 = e^{t_3}\dot t_3$

(c) $\dot w_1 = 1$ $\qquad t_3 = t_1 + t_2$ $\qquad \dot t_6 = \dot t_5 + \dot t_4$



$\dot w_2 = 0$ $\qquad \dot t_2 = 0$ $\qquad \dot t_5 = \sigma(1-\sigma(t_3))\dot t_3$

$$\dot t_3 = \dot t_1 + \dot t_2 = \dot t_1$$

$$\frac{\partial f_1}{\partial w_1} = \dot t_6 = \dot t_4 + \dot t_5 = e^{t_3}\dot t_3 + \sigma(1-\sigma(t_3))\dot t_3$$

$$= e^{t_3}\dot t_3 + \sigma(1-\sigma(t_3))\dot t_3$$

$$= e^{t_1+t_2}t_1 + \sigma(1-\sigma(t_1))t_1$$

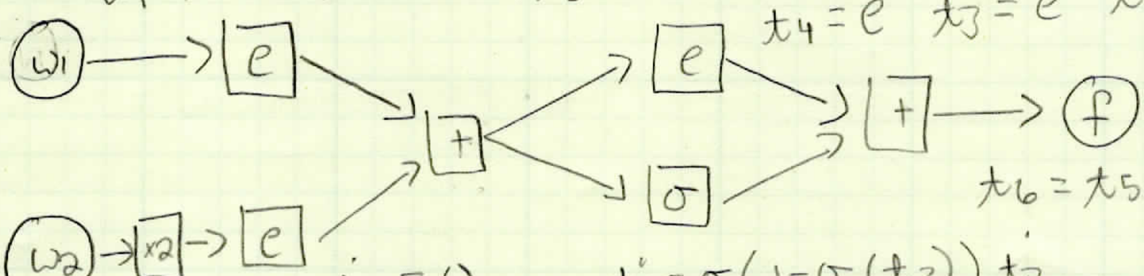
$$= e^{e^{w_1}+e^{2w_2}}e^{w_1} + \sigma(1-\sigma(e^{w_1}))e^{w_1}$$

$$\frac{\partial f_1}{\partial w_1} = e^{e^{w_1}+e^{2w_2}} e^{w_1} + \sigma(1-\sigma(e^{w_1}))e^{w_1}$$

$$= e^{e+e^{-2}} e + \sigma(1-\sigma(e))e$$

$=$

$\dot{w}_1 = 1$ $\quad \dot{t}_1 = \overbrace{e^{w_1}}^{t_1}$ $\quad \dot{t}_3 = \dot{t}_1 = t_1$ $\quad \dot{t}_4 = e^{t_3}\dot{t}_3 = e^{t_1} t_1$



$t_6 = t_5 + t_4$

$\dot{t}_2 = 0$ $\quad \dot{t}_5 = \sigma(1-\sigma(t_3))\dot{t}_3$

$\dot{w}_2 = 0$

$$\frac{\partial w_1}{\partial w_1} = 1 \qquad \frac{\partial t_1}{\partial w_1} = e^{w_1} = t_1 \qquad \frac{\partial t_3}{\partial w_1} = \frac{\partial t_3}{\partial t_1}\frac{\partial t_1}{\partial w_1} = t_1$$

$$\frac{\partial w_1}{\partial w_2} = 0 \qquad \frac{\partial t_2}{\partial w_1} = 0 \qquad \frac{\partial t_4}{\partial w_1} = \frac{\partial t_4}{\partial t_3}\frac{\partial t_3}{\partial w_1} = t_4 t_1$$

$$\frac{\partial t_5}{\partial w_1} = \frac{\partial t_5}{\partial t_3}\frac{\partial t_3}{\partial w_1} = \sigma(t_3)(1-\sigma(t_3))\cdot t_1 = t_5(1-t_5)t_1$$

$$= \sigma(t_3)(1-t_5)t_1$$

$$\frac{\partial t_6}{\partial w_1} = \frac{\partial t_5}{\partial w_1} + \frac{\partial t_4}{\partial w_1}$$

$$= \sigma(t_3)(1-t_5)t_1 + t_4 t_1$$

$$= \sigma(t_3)(1-\sigma(e^{w_1}+e^{2w_2}))e^{w_1} + e^{e^{w_1}+e^{2w_2}} e^{w_1}$$

$$= \sigma(t_3)(1-\sigma(e+e^{-2}))e + e^{e+e^{-2}}\cdot e$$

$=$

$$= \sigma(e^{w_1}+e^{2w_2})(1-\sigma(e^{w_1}+e^{2w_2}))e^{w_1} + e^{e^{w_1}+e^{2w_2}+w_1}$$

$$= \sigma(e+e^{-2})(1-\sigma(e+e^{-2})e + e^{e+e^{-2}+1}$$

$$= 47.303$$

$$\frac{\partial w_1}{\partial w_2} = 0 \qquad \frac{\partial t_1}{\partial w_2} = 0 \qquad \frac{\partial t_3}{\partial w_2} = \cancel{\frac{\partial t_3}{\partial t_2}} \frac{\partial t_2}{\partial w_2} = 2t_2$$

$$\frac{\partial w_2}{\partial w_2} = 1 \qquad \frac{\partial t_2}{\partial w_2} = 2t_2 \qquad \frac{\partial t_4}{\partial w_2} = \frac{\partial t_4}{\partial t_3}\frac{\partial t_3}{\partial w_2}$$

$$e^{2w_2} \qquad e^{2w_2} \cdot 2 \qquad t_2 \cdot 2 \qquad = 2 t_2 t_4$$

$$\frac{\partial t_5}{\partial w_2} = \frac{\partial t_5}{\partial t_3}\frac{\partial t_3}{\partial w_2} = t_5(1-t_5)\cdot 2t_2$$

$$\frac{\partial f_1}{\partial w_2} = \frac{\partial t_6}{\partial w_2} = \frac{\partial t_5}{\partial w_2} + \frac{\partial t_4}{\partial w_2} = \overset{0.946}{\cancel{t_5}}(1-\overset{0.946}{t_5})\cdot 2\overset{0.135}{\cancel{t_2}} + 2\overset{0.135}{\cancel{t_2}}\overset{17.35}{\cancel{t_4}}$$

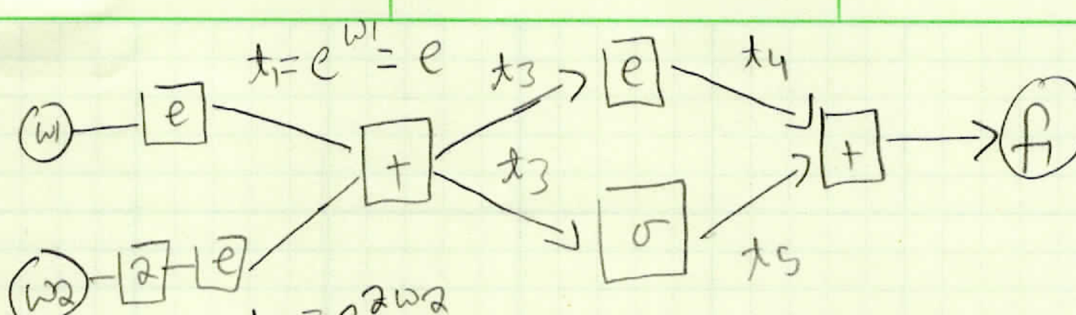
$$= \sigma(t_3)(1-\sigma(t_3))2t_2 + 2t_2 e^{t_3}$$

$$= \sigma(t_1+t_2)(1-\sigma(t_1+t_2))2t_2 + 2t_2 e^{t_1+t_2}$$

$$= \sigma(e^{w_1}+e^{2w_2})(1-\sigma(e^{w_1}+e^{2w_2}))2e^{w_2} + 2e^{w_2} e^{e^{w_1}+e^{2w_2}}$$

$$= \sigma(e+e^{-2})(1-\sigma(e+e^{-2}))2e^{-2} + 2e^{e+e^{-2}-1}$$

$$=$$



$t_2 = e^{2w_2}$

$= e^{-2}$

$t_3 = e + e^{-2} = 2.854$

$t_4 = e^{t_3} = e^{2.854} = 17.350$

$t_5 = \sigma(t_3) = \dfrac{1}{1+e^{-2.854}} = 0.946$

$f_1 = t_4 + t_5 = 18.296$

$\dfrac{\partial w_1}{\partial w_2} = 0 \quad \dfrac{\partial t_1}{\partial w_2} = 0 \qquad \dfrac{\partial t_3}{\partial w_2} = \cancel{\dfrac{\partial t_1}{\partial w_2}} + \cancel{\dfrac{\partial t_2}{\partial w_2}}^{\,2t_2} = 2t_2$

$\dfrac{\partial w_2}{\partial w_2} = 1 \quad \dfrac{\partial t_2}{\partial w_2} = e^{2w_2}\cdot 2$

$= 2t_2$

$\dfrac{\partial t_4}{\partial w_2} = \dfrac{\partial t_4}{\partial t_3}\;\dfrac{\partial t_3}{\partial w_2}$

$= e^{t_3}\cdot 2t_2$

$= 2t_2 t_4$

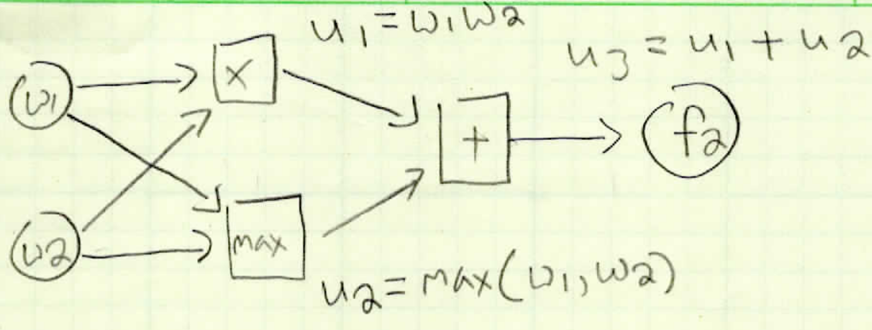
$\dfrac{\partial t_5}{\partial w_2} = \dfrac{\partial t_5}{\partial t_3}\;\dfrac{\partial t_3}{\partial w_1}$

$= t_5(1-t_5)\,2t_2$

$\dfrac{\partial f_1}{\partial w_2} = \dfrac{\partial t_4}{\partial w_2} + \dfrac{\partial t_5}{\partial w_2} = 2t_2 t_4 + t_5(1-t_5)\,2t_2$

$= 2(e^{-2})(17.35) + 0.946\,(1-0.946)\,2(e^{-2})$

$= 4.71$



$u_1 = (1)(-1) = -1$

$u_2 = \max(1, -1) = 1$

$u_3 = -1 + 1 = 0$

w_1: $\dfrac{\partial w_1}{\partial w_1} = 1$ $\quad \dfrac{\partial u_1}{\partial w_1} = w_2$

$\dfrac{\partial w_2}{\partial w_1} = 0$ $\quad \dfrac{\partial u_2}{\partial w_1} = \dfrac{\partial}{\partial w_1} w_1 = 1$

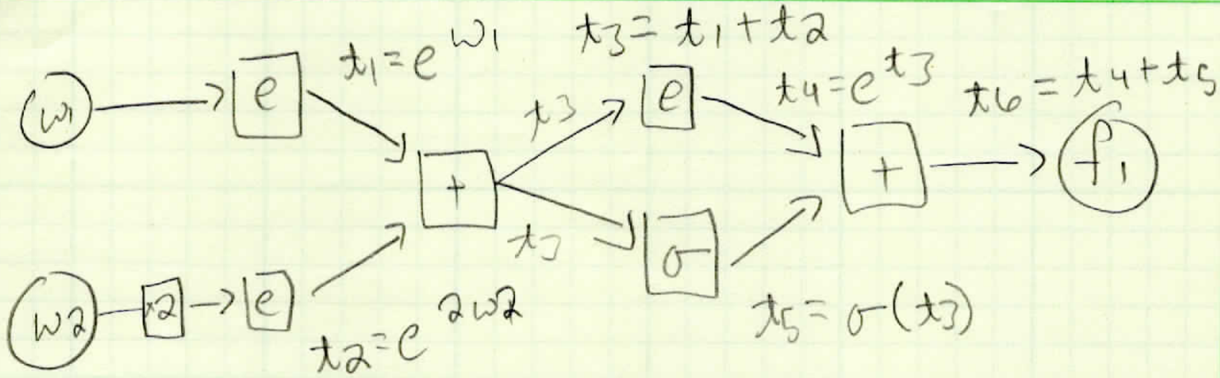
$$\frac{\partial u_3}{\partial w_1} = \frac{\partial u_1}{\partial w_1} + \frac{\partial u_2}{\partial w_1}$$
$$= w_2 + 1$$
$$= -1 + 1$$
$$= 0$$

w_2: $\dfrac{\partial w_1}{\partial w_2} = 0$ $\quad \dfrac{\partial u_1}{\partial w_2} = w_1$

$\dfrac{\partial w_2}{\partial w_2} = 1$ $\quad \dfrac{\partial u_2}{\partial w_2} = \dfrac{\partial}{\partial w_2} w_1 = 0$

$$\frac{\partial u_3}{\partial w_2} = \frac{\partial u_1}{\partial w_2} + \frac{\partial u_2}{\partial w_2} = w_1 + 0 = 1$$

(d)



$$\bar{t}_6 = \frac{\partial f}{\partial t_6} = 1$$

$$\bar{t}_5 = \frac{\partial f}{\partial t_5} = \frac{\partial f}{\partial t_6}\frac{\partial t_6}{\partial t_5} = \bar{t}_6 \qquad \bar{t}_4 = \frac{\partial f}{\partial t_4} = \frac{\partial f}{\partial t_6}\frac{\partial t_6}{\partial t_4} = \bar{t}_6$$

$$\bar{t}_{3_1} = \frac{\partial f}{\partial t_{3_1}} = \frac{\partial f}{\partial t_4}\frac{\partial t_4}{\partial t_{3_1}} = \bar{t}_6\, e^{t_3}$$

$$\bar{t}_{3_2} = \frac{\partial f}{t_5}\frac{\partial t_5}{\partial t_3} = \bar{t}_6 \cdot \sigma(t_3)(1-\sigma(t_3))$$

$$\bar{t}_3 = \bar{t}_6\left(e^{t_3} + \sigma(t_3)(1-\sigma(t_3))\right)$$

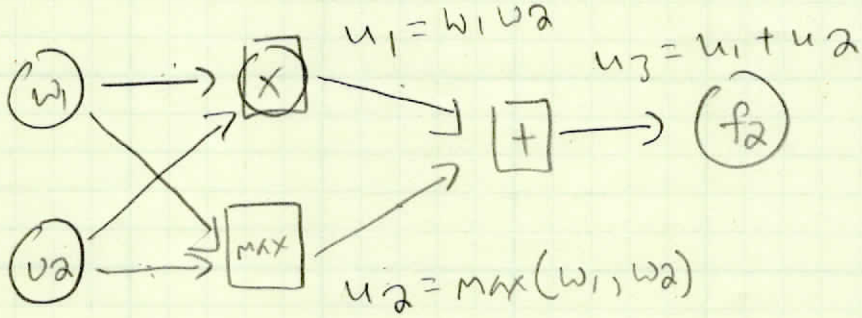
$$\bar{t}_2 = \frac{\partial f}{\partial t_2} = \frac{\partial f}{\partial t_3}\frac{\partial t_3}{\partial t_2} = \bar{t}_3 \cdot 1 = \bar{t}_3$$

$$\bar{t}_1 = \frac{\partial f}{\partial t_1} = \frac{\partial f}{\partial t_3}\frac{\partial t_3}{\partial t_1} = \bar{t}_3$$

$$\bar{w}_2 = \frac{\partial f}{\partial w_2} = \frac{\partial f}{\partial t_2}\frac{\partial t_2}{\partial w_2} = \bar{t}_3 \cdot 2t_2$$

$$\bar{w}_1 = \frac{\partial f}{\partial w_1} = \frac{\partial f}{\partial t_1}\frac{\partial t_1}{\partial w_1} = \bar{t}_3\, t_1$$

$$\bar{t}_3 = t_4 + t_5(1-t_5) = 17.35 + 0.946(1-0.946)$$
$$= 17.4$$
$$\bar{w}_2 = \bar{t}_3 \cdot 2t_2 = 17.4 \cdot 2 \cdot e^{-2} = 4.71$$
$$\bar{w}_1 = \bar{t}_3\, t_1 = 17.4 \cdot e = 47.3$$



$$\bar{u}_3 = \frac{\partial f_2}{\partial u_3} = 1$$

$$\bar{u}_2 = \frac{\partial f}{\partial u_2} = \frac{\partial f}{\partial u_3}\frac{\partial u_3}{\partial u_2} = \bar{u}_3$$

$$\bar{u}_1 = \frac{\partial f}{\partial u_1} = \frac{\partial f}{\partial u_3}\frac{\partial u_3}{\partial u_1} = \bar{u}_3$$

$$\bar{w}_{2_1} = \frac{\partial f_2}{\partial w_2} = \frac{\partial f}{\partial u_2}\frac{\partial u_2}{\partial w_2} = \bar{u}_3 \cdot 0 = 0$$

$$\bar{w}_{2_2} = \frac{\partial f_2}{\partial w_2} = \frac{\partial f_2}{\partial u_1}\frac{\partial u_1}{\partial w_2} = \bar{u}_3 \cdot w_1 = 1 \cdot 1 = 1$$

$$\bar{w}_2 = \bar{w}_{2_1} + \bar{w}_{2_2} = 0 + 1 = 1$$

$$\bar{w}_{1_1} = \frac{\partial f_2}{\partial w_1} = \frac{\partial f}{\partial u_2} \cdot \frac{\partial u_2}{\partial w_1} = \bar{u}_3 \cdot 1 = 1 \qquad \bar{w}_1 = 1 - 1 = 0$$

$$\bar{w}_{1_2} = \frac{\partial f_2}{\partial w_1} = \frac{\partial f}{\partial u_1} \cdot \frac{\partial u_1}{\partial w_1} = \bar{u}_3 \cdot w_2 = -1$$