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Let's define intermediate variables a and b such that

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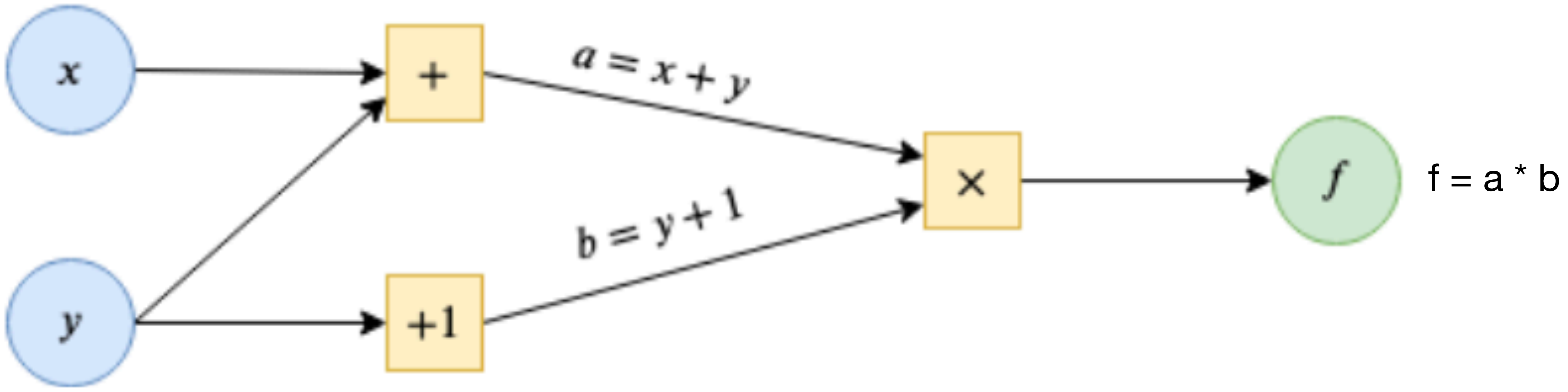
A computation graph for the "forward pass" through f is shown in Fig. 1.



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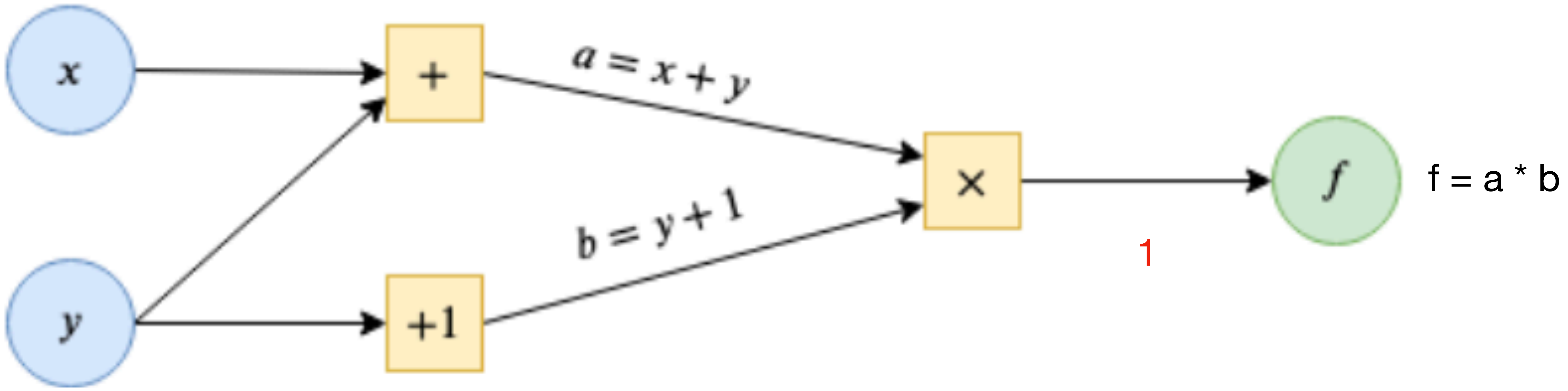

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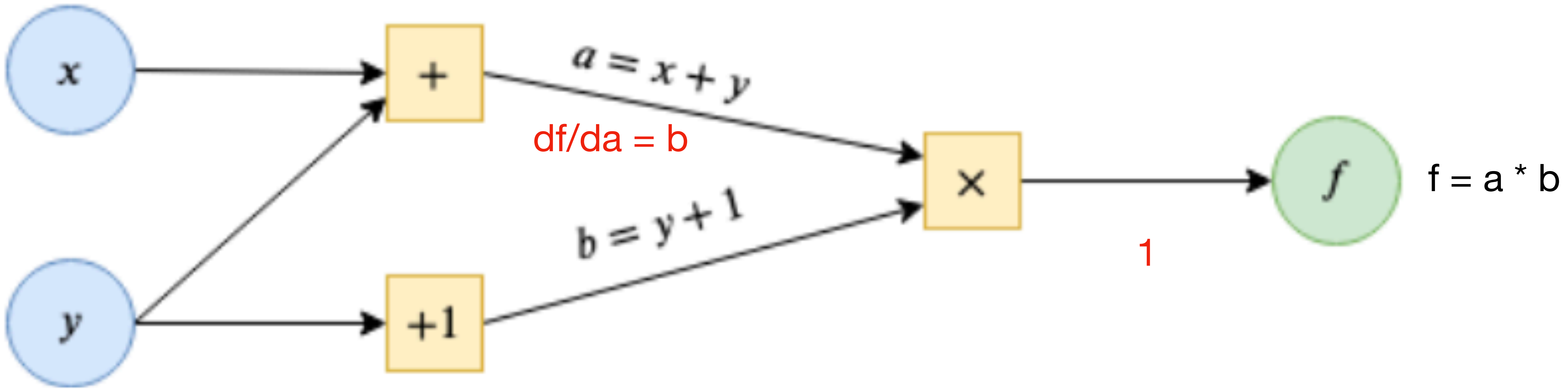

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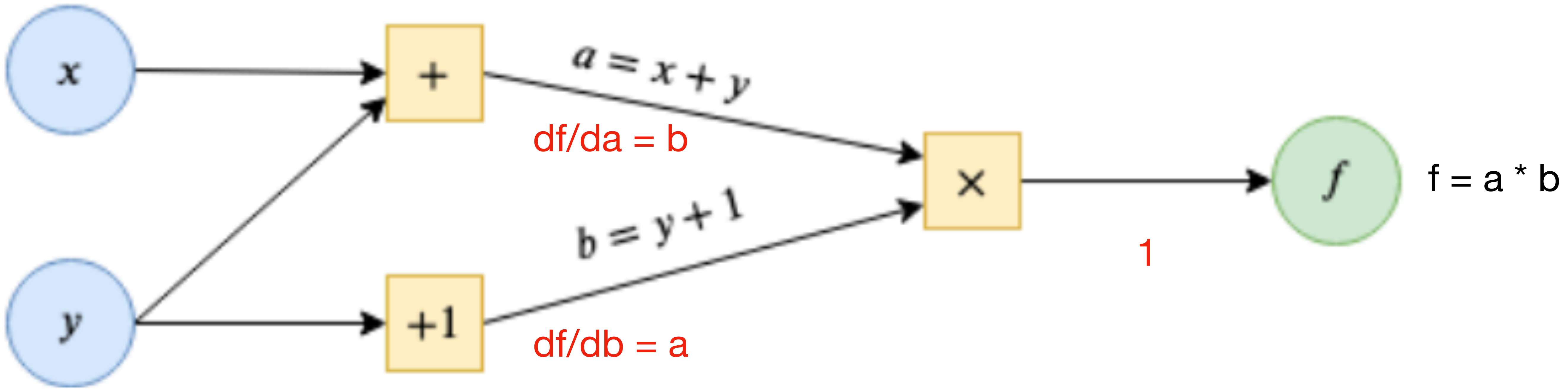

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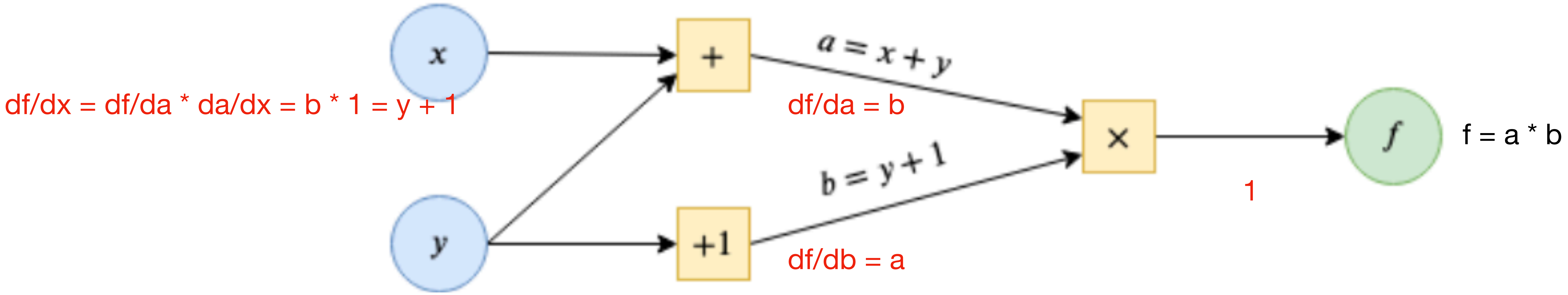

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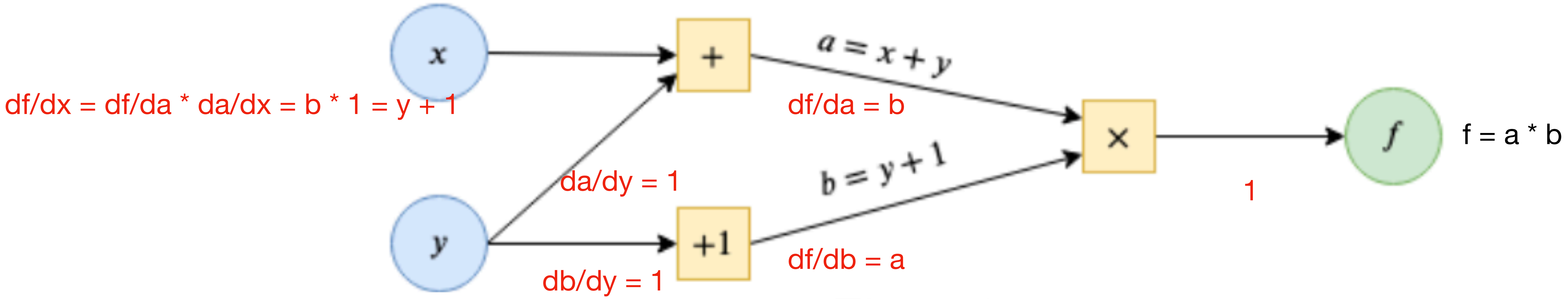

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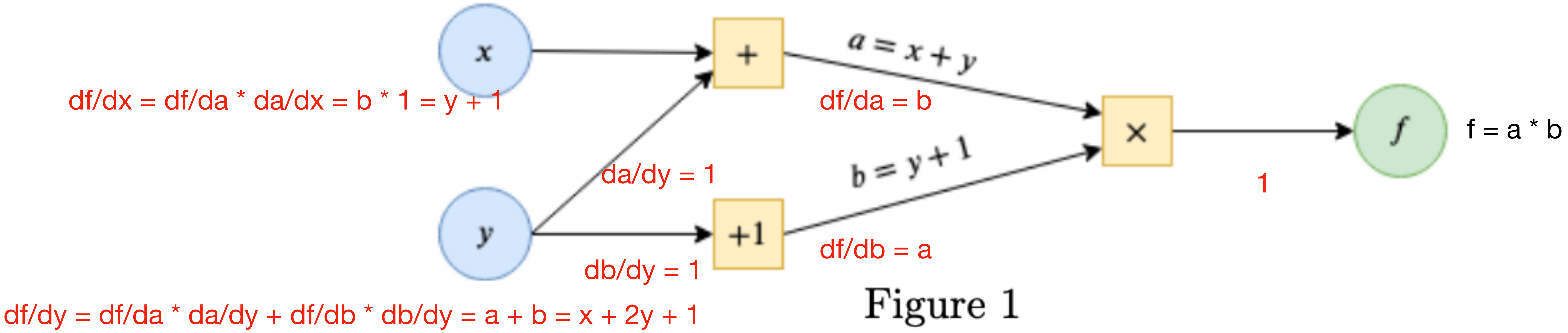

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(a) Draw the computation graph. Compute the value of f at $\vec{w} = (1, -1)$.

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$$f = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

$$f(w_1, w_2) = \begin{bmatrix} f_1(w_1, w_2) \\ f_2(w_1, w_2) \end{bmatrix}$$

$$f(1,-1) = \begin{bmatrix} 18.296 \\ 0.0 \end{bmatrix}$$

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$$\frac{\partial \mathbf{f}}{\partial \mathbf{w}} \approx \left[\frac{\mathbf{f}(1+\Delta w, -1) - \mathbf{f}(1, -1)}{\Delta w}, \quad \frac{\mathbf{f}(1, -1+\Delta w) - \mathbf{f}(1, -1)}{\Delta w} \right]$$

$$\frac{\partial \mathbf{f}}{\partial \mathbf{w}} \approx \begin{bmatrix} 48.192 & 4.764 \\ 0.00 & 1.00 \end{bmatrix}$$

Or, using central differences:

$$\frac{\partial \mathbf{f}}{\partial \mathbf{w}} \approx \left[\frac{\mathbf{f}(1+\Delta w, -1) - \mathbf{f}(1-\Delta w, -1)}{2\Delta w}, \quad \frac{\mathbf{f}(1, -1+\Delta w) - \mathbf{f}(1, -1-\Delta w)}{2\Delta w} \right]$$

$$\frac{\partial \mathbf{f}}{\partial \mathbf{w}} \approx \begin{bmatrix} 47.316 & 4.711 \\ 0.0 & 1.00 \end{bmatrix}$$

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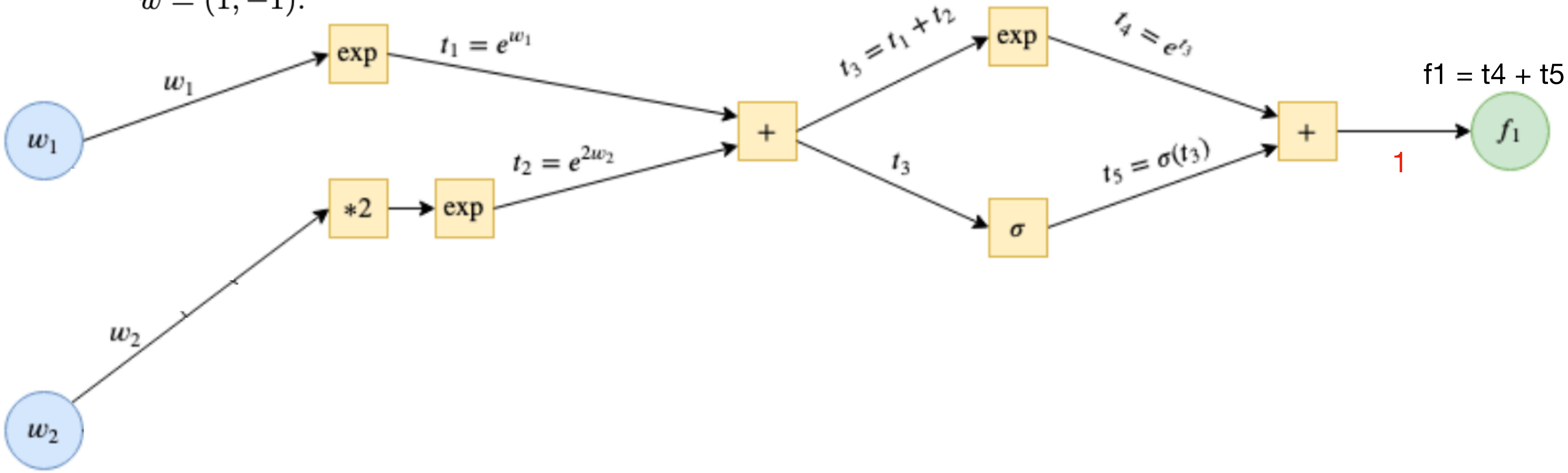

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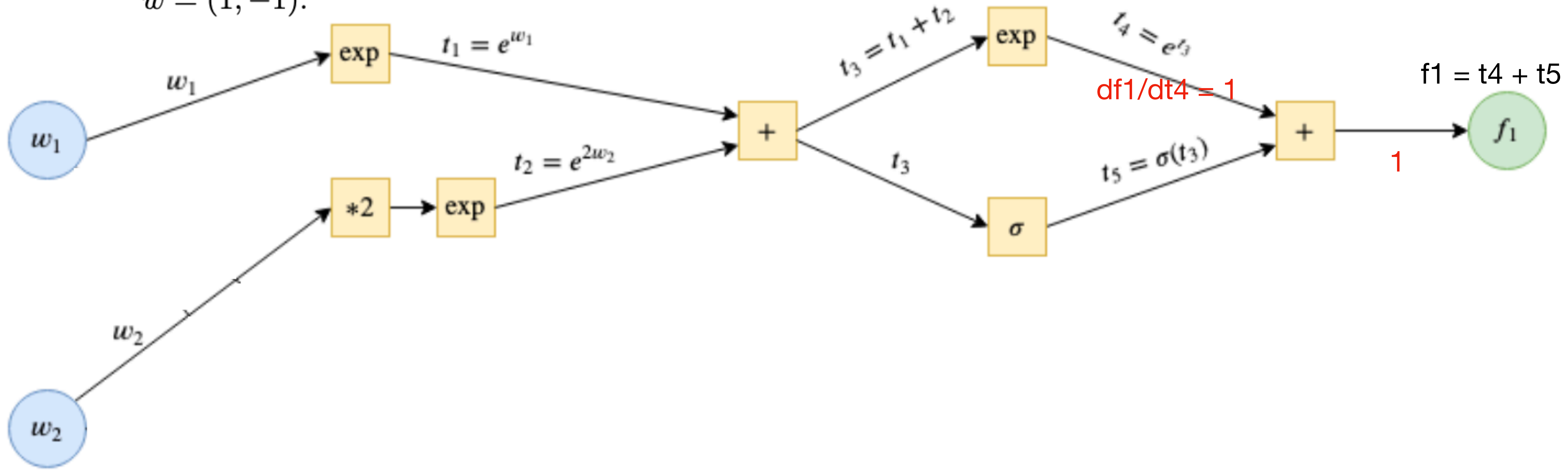

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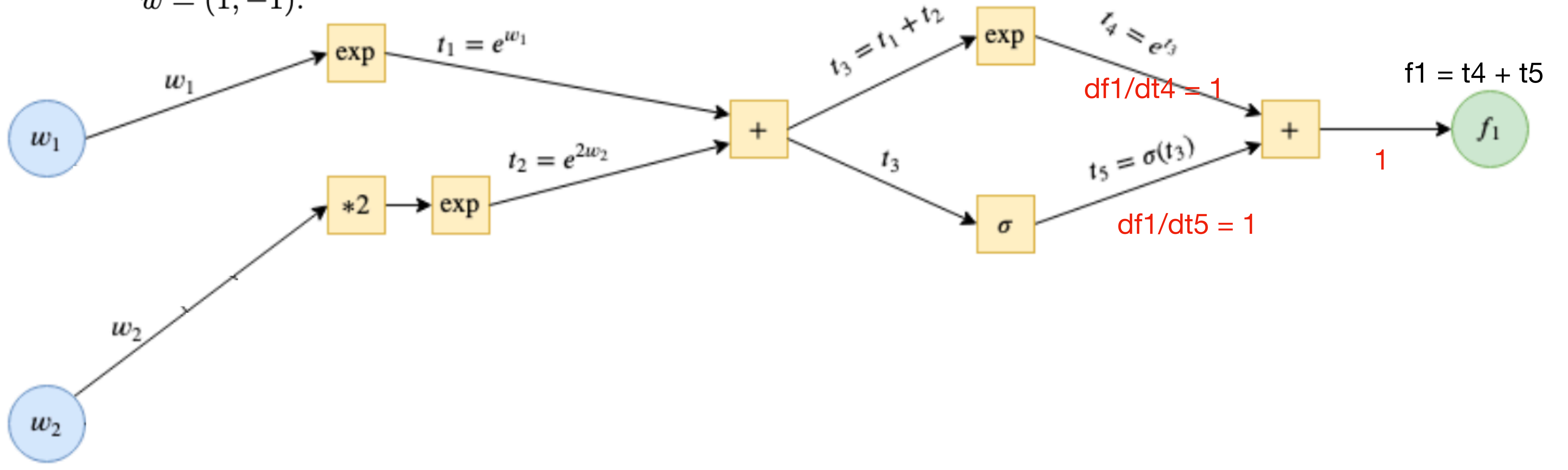

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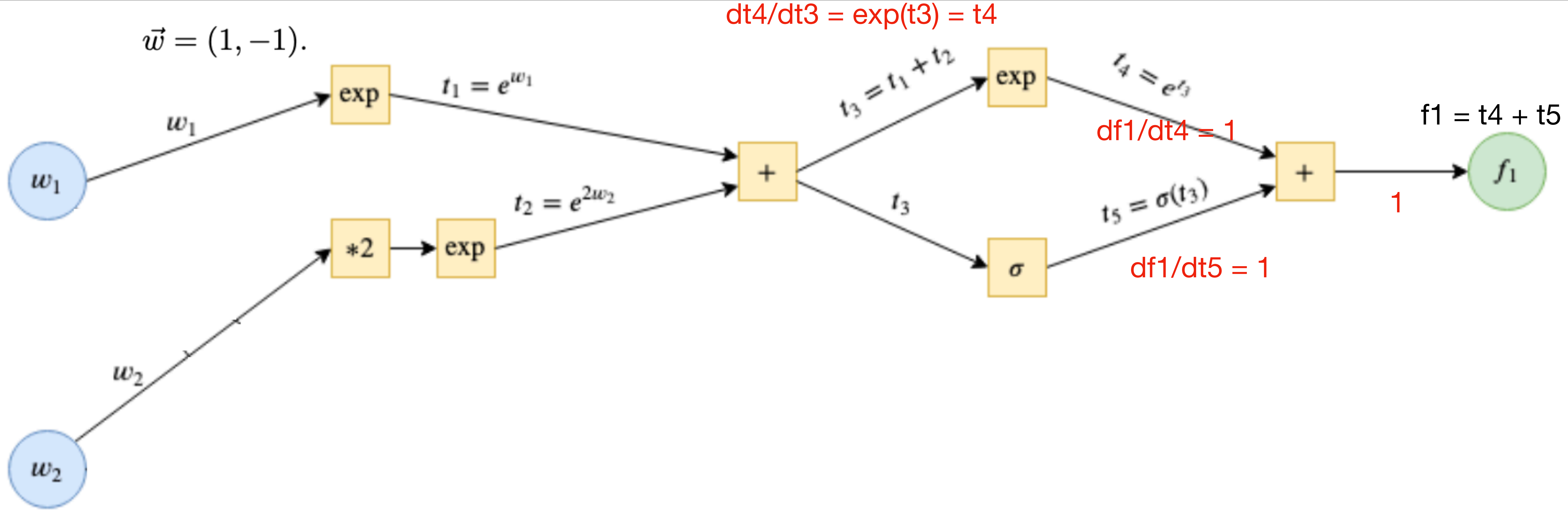

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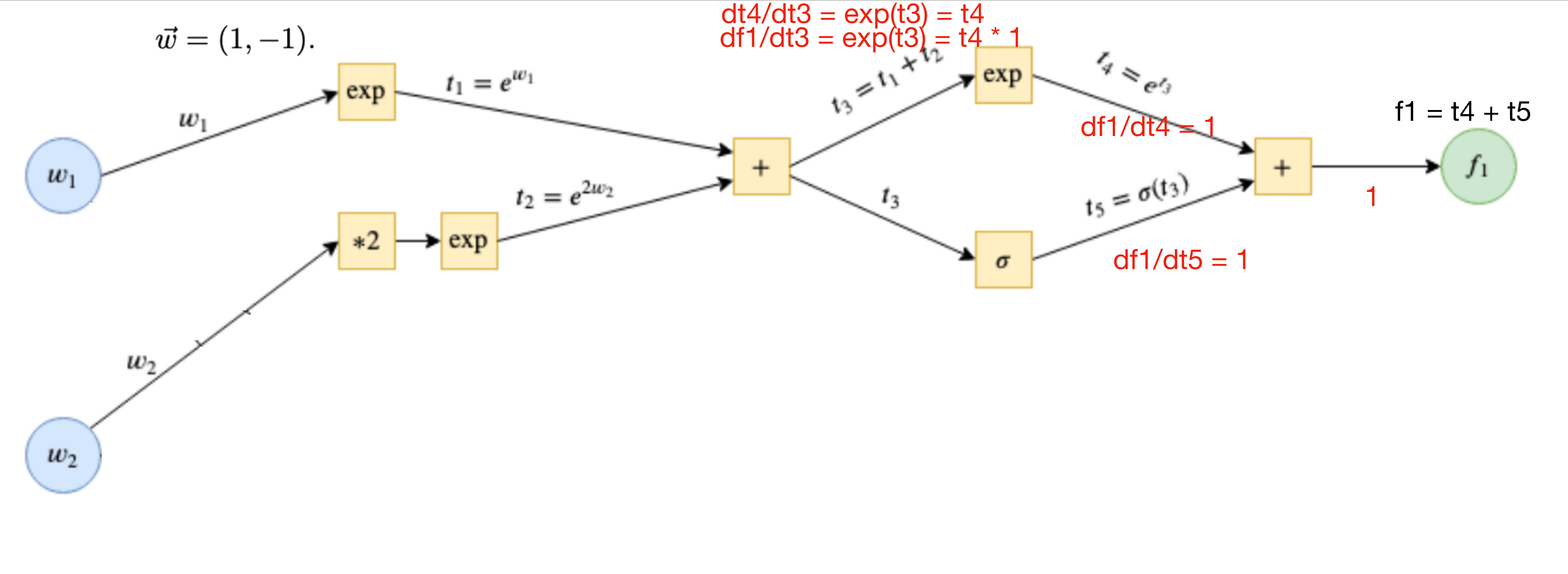

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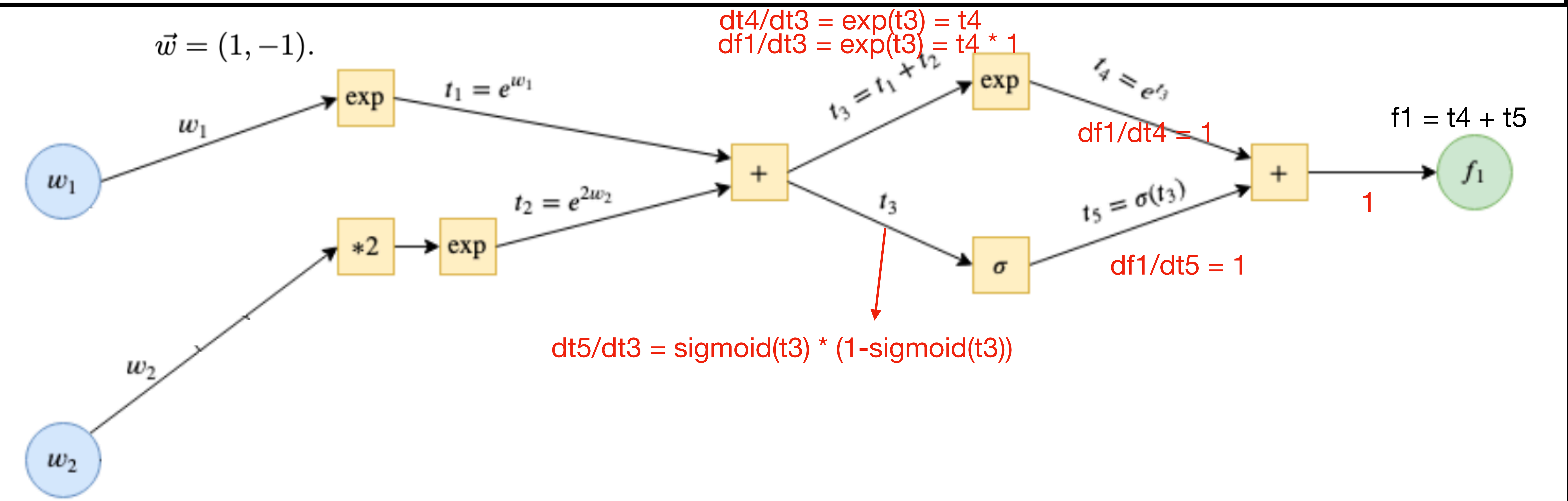

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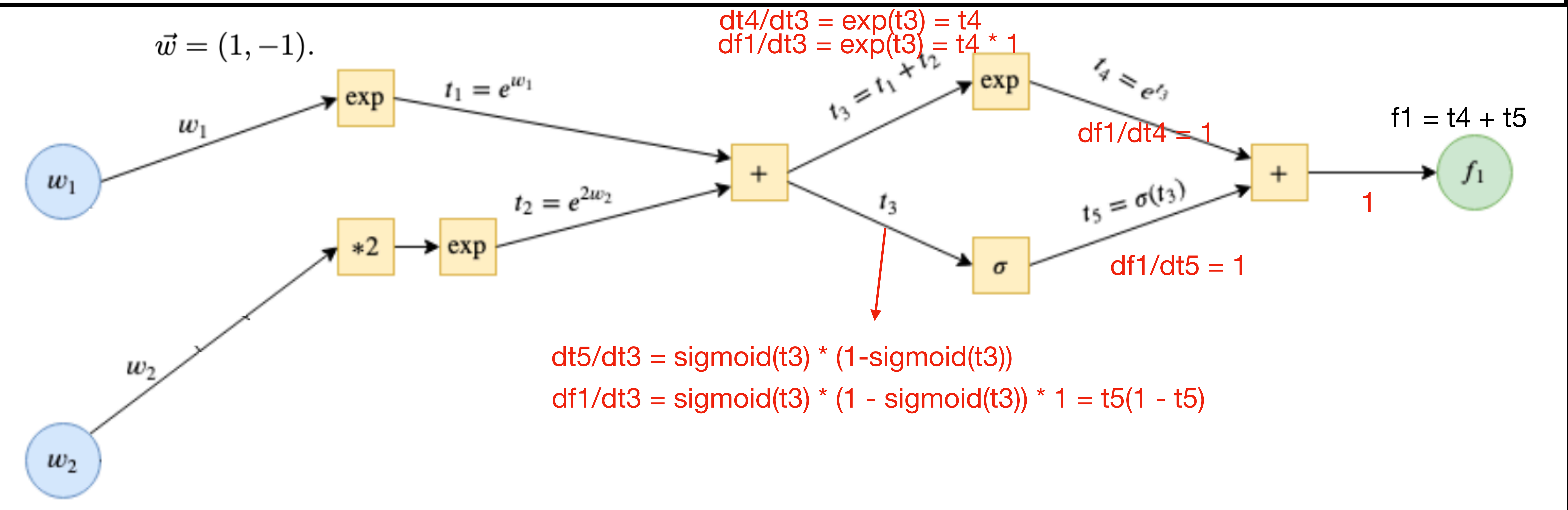

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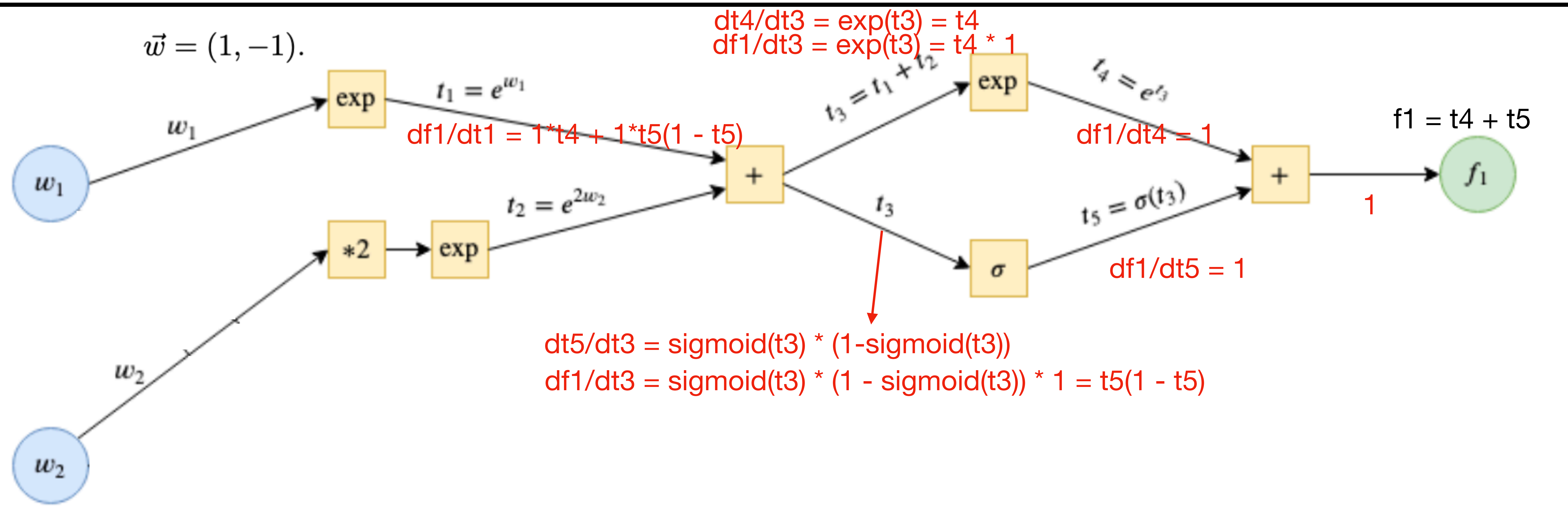

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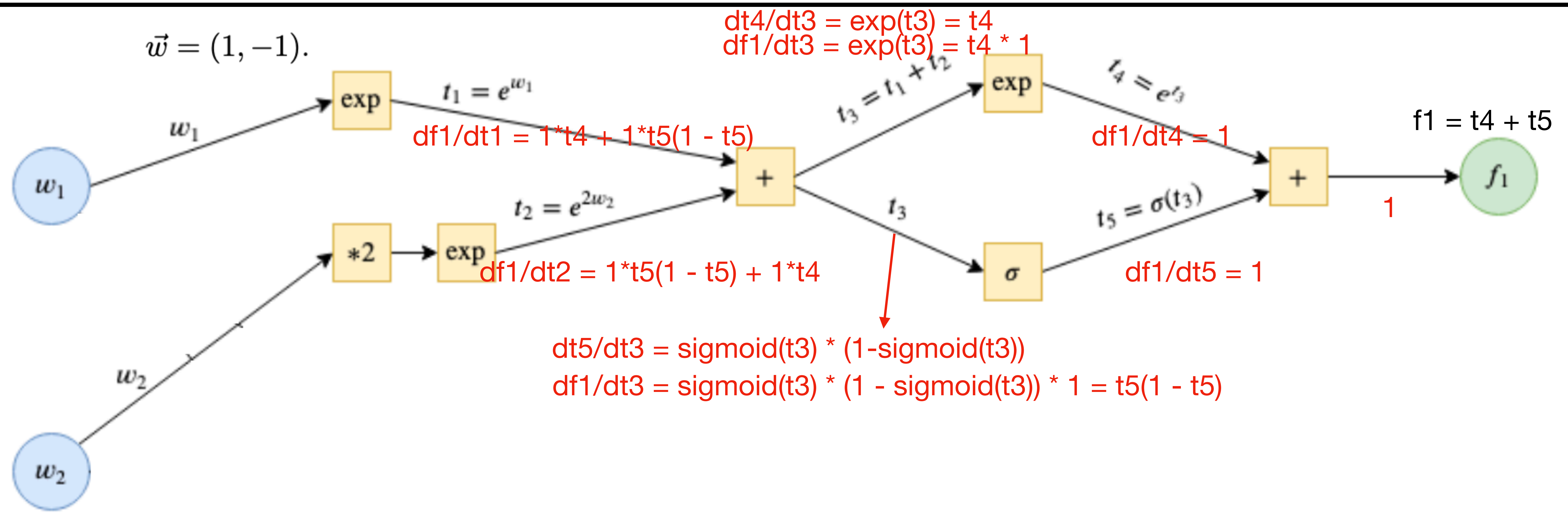

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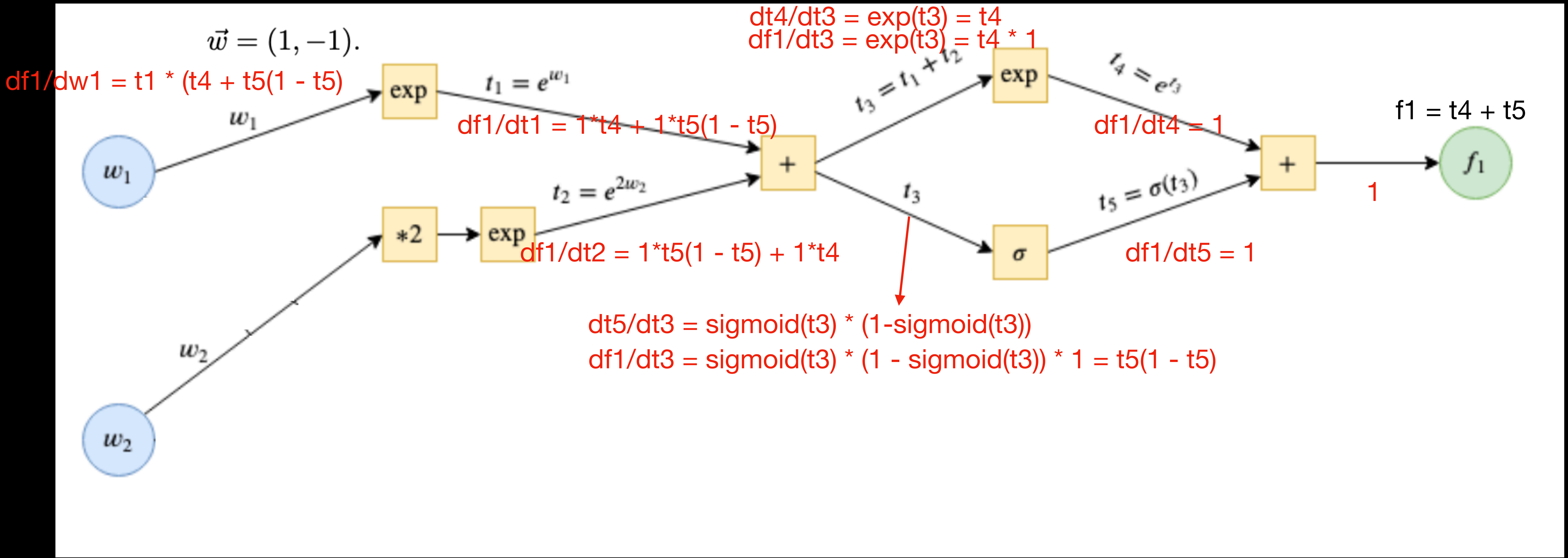

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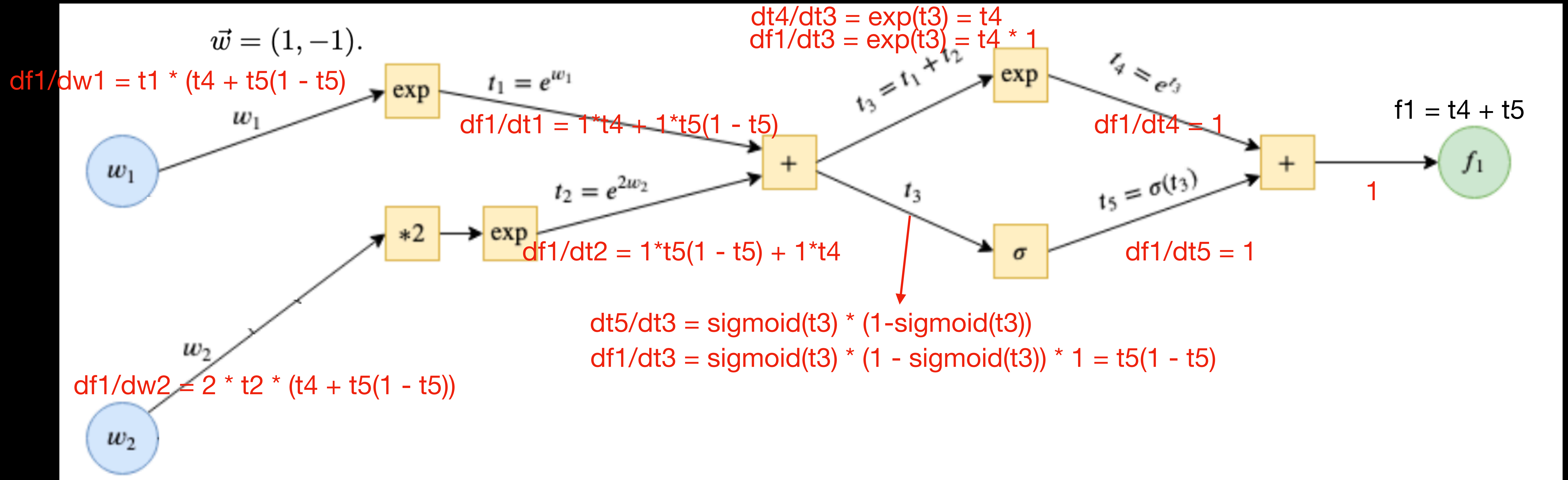

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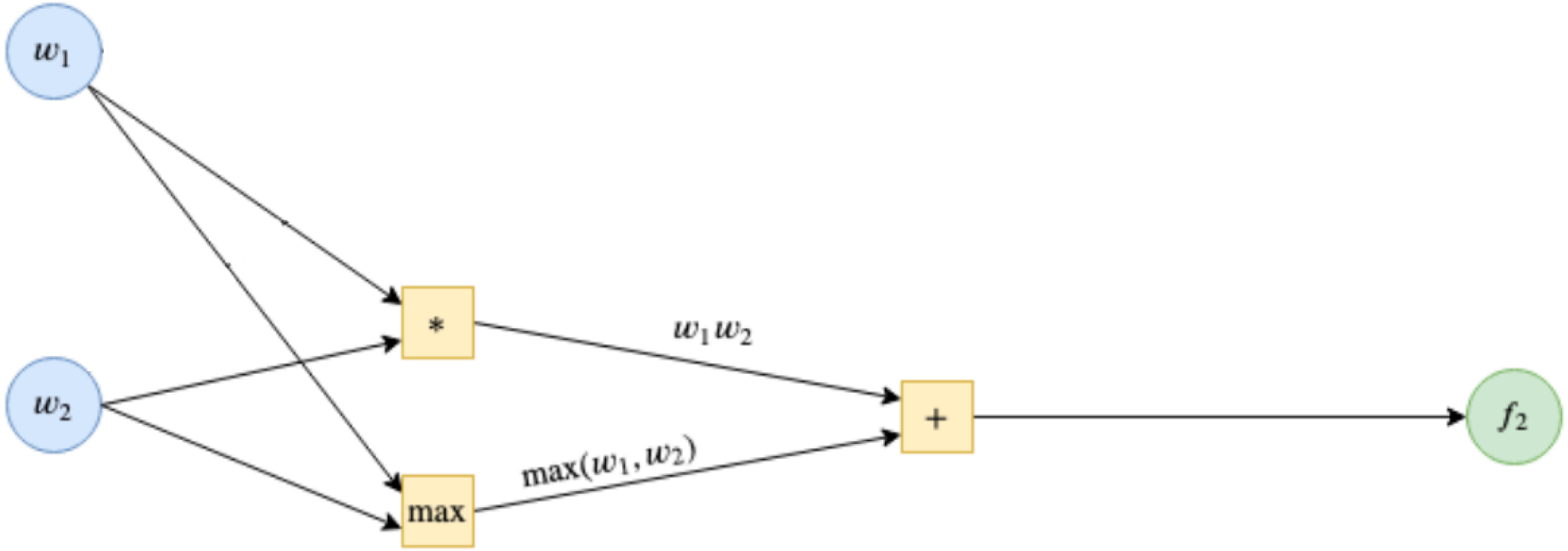

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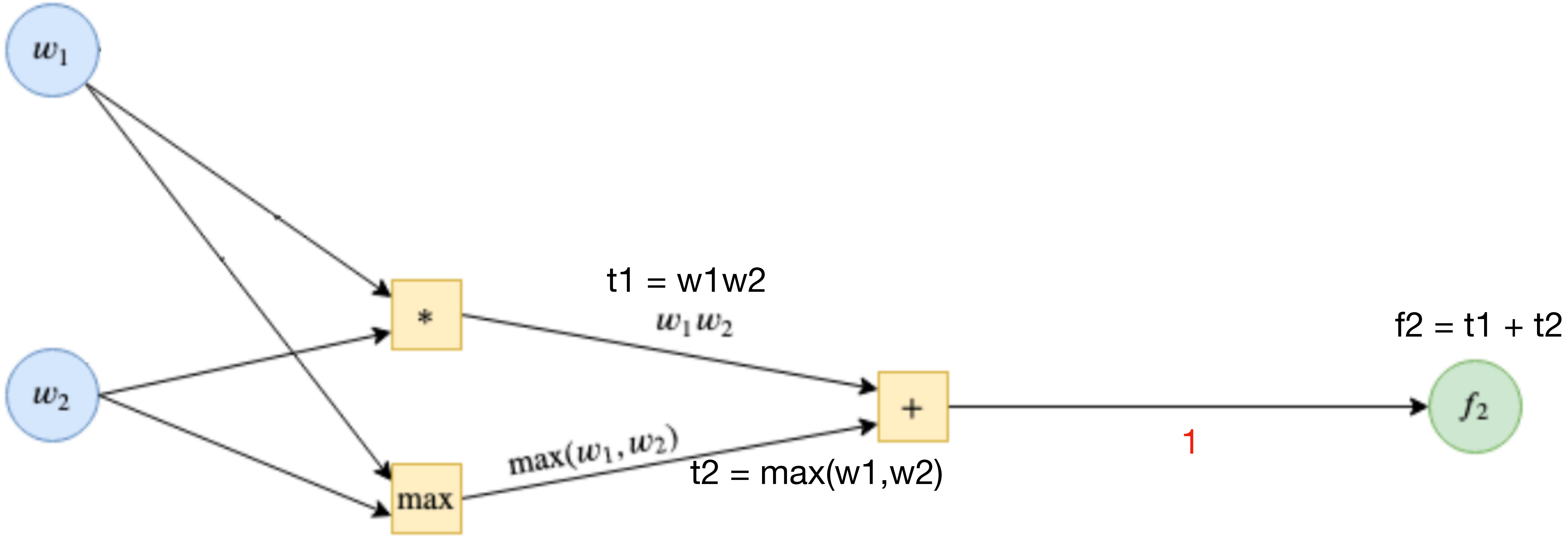

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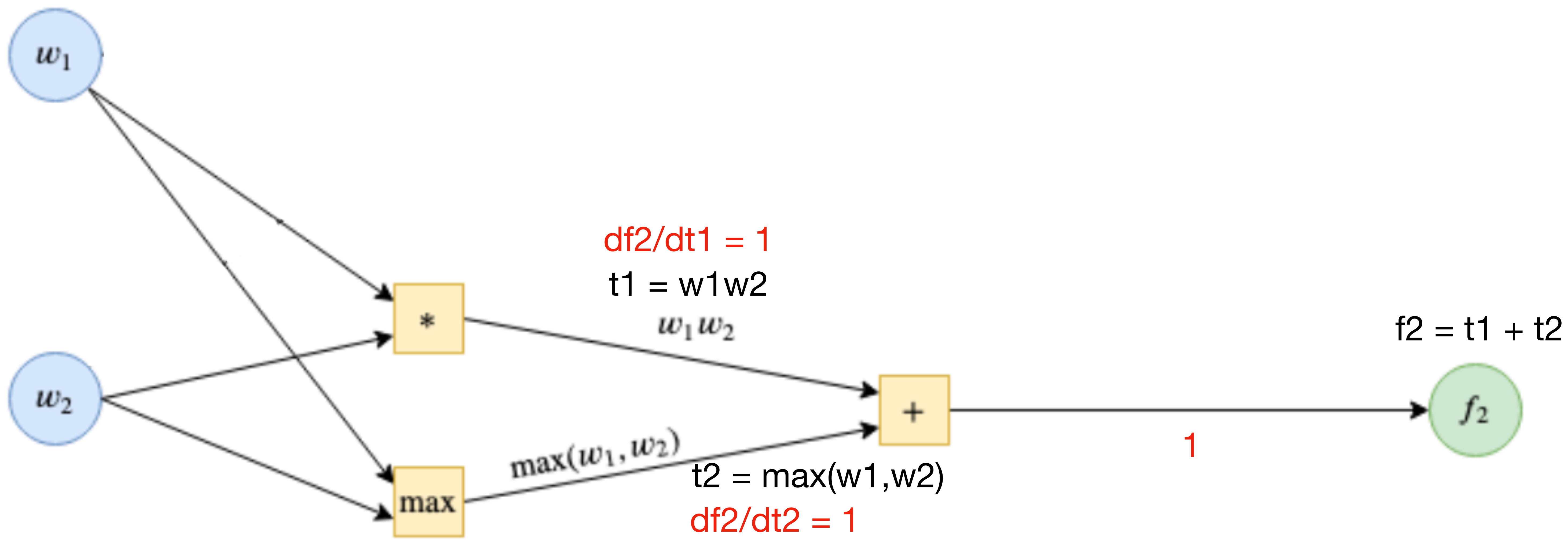

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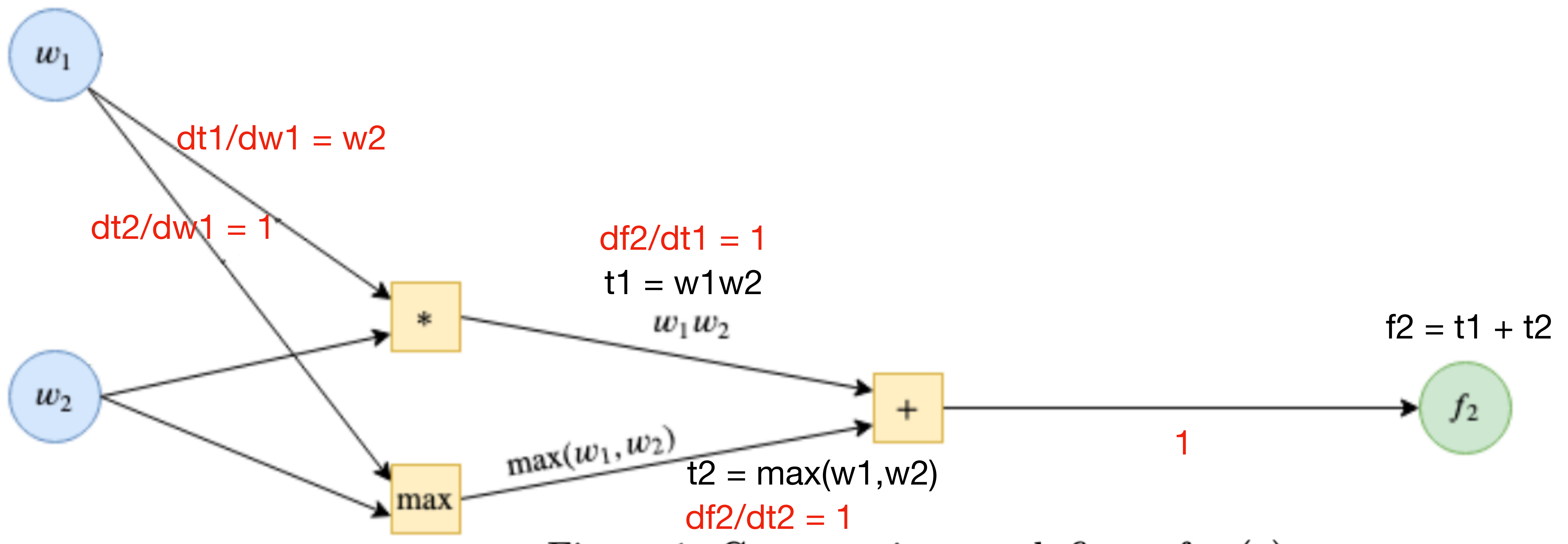

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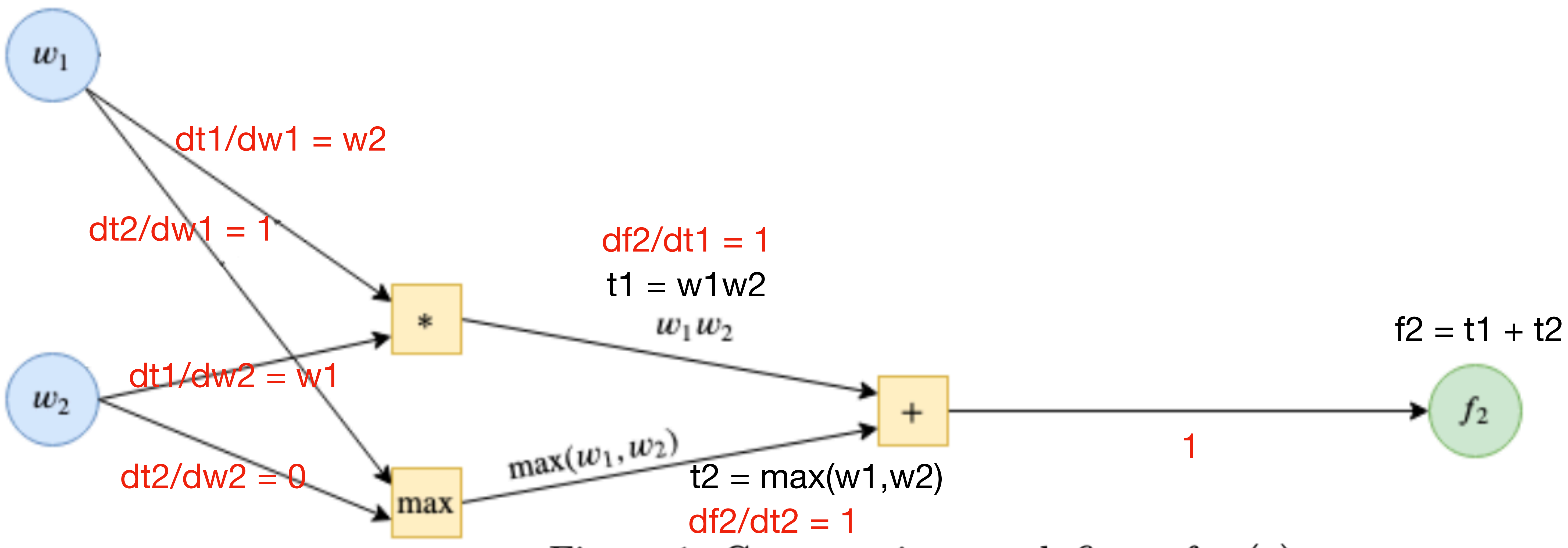

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