

This is brief explanation of variable pairing with NWAOBDDs - specifically which incoming edge is matched with a specific outgoing edge.

For the purposes of this definition the edges are numbered from 0 to  $v - 1$  where  $v$  is the number of variables represented by the NWAOBDD.

(1) For a given outgoing edge representing the variable  $i$  at level  $k$ , its paired incoming edge represents the variable  $i + 2^{k+1} - 3$

(2) If  $x$  is an outgoing edge at level  $k + 1$ , then  $x + 1$  and  $(x + 1) + 2^{k+1} - 2$  are represented by outgoing edges at level  $k$

(3) If the NWAOBDD has  $n$  levels total, then the outgoing edges at level  $n$  are 0 and  $2^{n+1} - 2$

From 1,2,and 3, we can get the level and pairings of every variable represented by the NWAOBDD. For example, a 3-level NWAOBDD has outgoing edges represented by 0 and 14 at level 3 (from def. 3). At level 2, it has outgoing edges 1,7,15,21 (from def. 2) and at level 1, it has outgoing edges 2,4,8,10,16,18,22,24.

Using definition 2, we can pair incoming and outgoing edge to get the following pairings (0,13),(14,27) for level 3, (1,6)(7,12)(15,20)(21,26) at level 2, and (2,3)(4,5)(8,9)(10,11)(16,17)(18,29)(22,23)(24,25) for level 1.

Is variable  $i$  an outgoing-edge variable at level  $k$ , given the value of offset accumulated from maxLevel?

```
bool P(offset, i, k) {
    unsigned a = offset;
    unsigned b = offset + 2^{k+1} - 3;
    unsigned c = offset + 2^{k+1} - 2;
    unsigned d = offset + 2^{k+1} - 5;

    if (i==a || i==c) return true;
    else if (i==b || i==d) return false;
    else if (i<b)                // a < i && i < b
        return P(a+1, i, k-1);
    else                          // c < i && i < d
        return (c+1, i, k-1);
}
```