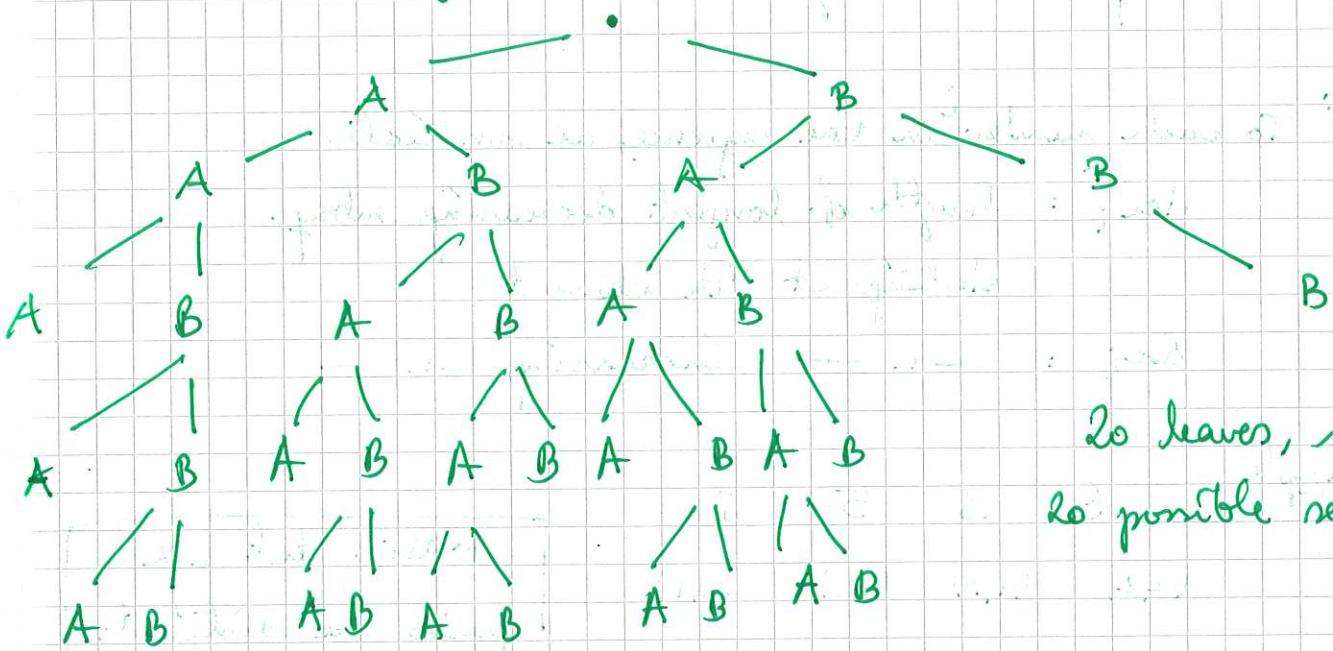


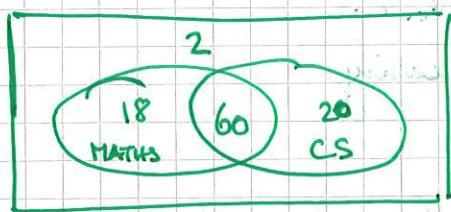
Example (Tree diagrams)



Poll 1

$$|\{0\} \times \{0, 1\}^6 \times \{1\}| + |\{1\} \times \{0, 1\}^6 \times \{0\}| = 2^6 + 2^6 = 128.$$

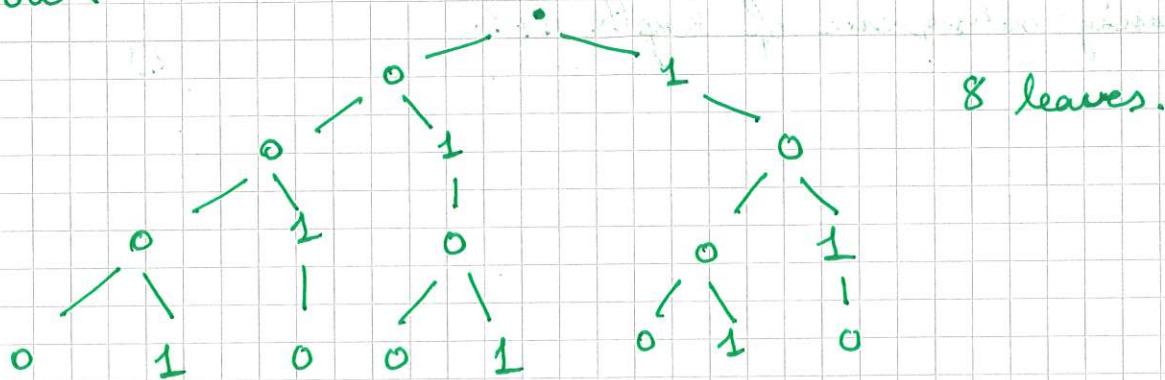
Poll 2



Poll 3

$$\frac{5!}{5} = 24 \quad (\text{Division rule})$$

Poll 4



Claim Any sequence of  $n^2+1$  elements distinct numbers contains a strictly increasing or decreasing subsequence of length  $n+1$ .

IDEA

To each number  $i$  in the sequence, we associate

$\text{dec}_i$ : length of longest decreasing subseq.  
starting at  $i$ 'th element

$\text{inc}_i$ : — increasing —

Example

6	3	5	8
(2,2)	(1,3)	(1,2)	(1,1)

Notice that all pairs are distinct.

Proof

For a sequence of  $n^2+1$  numbers,  
construct the associated sequence

$(\text{dec}_1, \text{inc}_1), (\text{dec}_2, \text{inc}_2), \dots, (\text{dec}_{n^2+1}, \text{inc}_{n^2+1})$ .

→ If two pairs were equal we could extend the increasing or decreasing sequence.

If there were no decreasing or increasing subsequences of length  $n+1$ , then

$\text{dec}_i \leq n$  and  $\text{inc}_i \leq n$  for all  $i = 1, \dots, n^2+1$ .

But there are at most  $n^2$  possible values of such pairs,  
so two of them must be equal by the Pigeonhole principle.

This is not possible. Therefore, there exists an increasing  
or decreasing subsequence of length  $n+1$ . □

Claim For every integer  $n \geq 1$ , there exists an integer  $k \geq 1$  s.t.  $k \cdot n$  consists only of 0's and 1's.

Proof:

Consider the sequence

$$\alpha_1 = 1, \alpha_2 = 11, \alpha_3 = 111, \dots, \alpha_{n+1} = \underbrace{1 \cdots 1}_{n+1}$$

Two of the  $\alpha_i$ 's must have the same remainder modulo  $n$  (by Pigeonhole principle). Suppose this is  $\alpha_i$  and  $\alpha_j$  with  $\alpha_i < \alpha_j$ . Then,

$$\alpha_i \equiv \alpha_j \pmod{n}$$

so there exists  $k \geq 1$  s.t.  $n \cdot k = \alpha_j - \alpha_i$

and this consists only of 0s and 1s.  $\square$

1. What is the difference between a function and a relation?

2. How do you determine if a relation is a function?

3. Graph

4. What is a linear function?

5. What is the slope formula?

6. What is the formula for finding the equation of a line?

7. What is the formula for finding the equation of a line given two points?

8. What is the formula for finding the equation of a line given a point and a slope?

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