Lecture 2

We consider $p \wedge q \Leftrightarrow \neg q$ and construct the truth table

p	q	$p \wedge q$	$\neg q$	$p \wedge q \Leftrightarrow \neg q$
T	T	T	F	F
T	F	F	T	F
F	T	F	F	T
F	F	F	T	F

So this is a <u>contingency</u>: the truth value depends on the truth values of p and q.

Next, we aim to see that

$$(p \wedge q \Leftrightarrow \neg q) \equiv \neg (q \Rightarrow p) \qquad \text{(logical equivalence)}$$

and we do this by constructing the truth table

$q \Rightarrow p$	$\neg (q \Rightarrow p)$
T	F
T	F
F	T
T	F

and comparing with the above.

We prove the logical equivalence: $\neg (p \Rightarrow q) \equiv p \wedge \neg q$.

$$\begin{aligned}
\neg (p \Rightarrow q) &\equiv \neg (\neg p \vee q) && \text{(slide 9)} \\
&\equiv \neg(\neg p) \wedge \neg q && \text{(de Morgan)} \\
&\equiv p \wedge \neg q && \text{(double negation)}
\end{aligned}$$

An open proposition is when a variable occurs.

$$P(x) : 2x > x.$$

The truth value may change with the variable:

$$P(-1) \equiv -2 > -1 \quad \text{(false)}$$
$$P(0) \equiv 0 > 0 \quad \text{(false)}$$
$$P(1) \equiv 2 > 1 \quad \text{(true)}$$
$$P(2) \equiv 4 > 2 \quad \text{(true)}$$

In fact, we see that $\forall x \in \mathbb{Z}^+ : P(x)$
(this means: the proposition is true for all $x \in \mathbb{Z}^+$).

Universal quantifier: truth value may depend on the domain.

Note: $\forall x \in \mathbb{Z} : P(x)$ is false.

silly example: $\forall x \in \emptyset : 2x > x$ is in fact true.

Existential quantifier: $Q(x) : 2x > x + 4$

Note $\forall x \in \mathbb{Z}^+ : Q(x)$ is not the case.
But there exists a value for x s.t. $Q(x)$ is true:

$$Q(5) \equiv 10 > 9 \quad \text{(true)}$$

We write $\exists x \in \mathbb{Z}^+ : Q(x)$.

Uniqueness: $\exists! x \in \mathbb{Z}^+ : Q(x)$ (this is false)

$\exists! x \in \mathbb{Z}^+ : x \cdot x = x$ (this is true)