DM549/DS(K)820/MM537/DM547

Lecture 1: Propositional Logic

Kevin Aguyar Brix (`kabrix@imada.sdu.dk`)

University of Southern Denmark

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Logical Propositions

Definition (Definition 1.1.1)

A *proposition* (et udsagn) is a declarative statement (that is, a statement that declares a fact) that is true (sand) or false (falsk) but not both.

Remarks:

- We denote a true proposition as T and a false one as F.
- Alternatively, one can also think of bits, where 1 corresponds to T and 0 corresponds to F.

Logical Operators

Using *operators* (operatorer), we can build *compound propositions* (sammensatte udsagn) from other (possibly compound) propositions.

Here, p, q, and r will be variables representing propositions.

Today, we will get to know the following operators:

- the negation $\neg$,
- the conjunction $\wedge$,
- the disjunction $\vee$,
- the implication $\Rightarrow$,
- the bi-implication $\Leftrightarrow$,
- the exclusive or $\oplus$.

The Logical Negation

We can define an operator through a so-called *truth table* (sandhedstabel):

p	$\neg p$
T	F
F	T

For every value of the operand p, the value of the compound proposition is given.

Remarks:

- Read: "not (ikke) p" or "p's negation (negation)"
- In words: $\neg$ "flips" the truth value of p.
- Alternative notation: $\bar{p}$, $!p$.

The Logical And and the Logical Or

We can also define binary (as opposed to unary) operators through truth tables:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

For every combination of values of the operands p and q, the value of the compound proposition is given.

Remarks:

- Read:
  - $p \wedge q$: "p and q" ("p og q")
  - $p \vee q$: "p or q" ("p eller q")
- In words:
  - For $p \wedge q$ to be T, both p and q must be T.
  - For $p \vee q$ to be T, at least one of p and q must be T.

Poll Everywhere

The Logical Implication

Another operator:

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Remarks:

- Read:
  - "p implies q" ("p medfører q")
  - "if p, then q" ("hvis p, så q")
  - "q if p" ("q hvis p")
  - "p only if q" ("p kun hvis q")
  - "p is a sufficient condition for q" ("p er en tilstrækkelig betingelse for q")
  - "q is a necessary condition for p" ("q er en nødvendig betingelse for p")
- p is called the assumption (antagelse), q the consequence (konsekvens)
- In words: for $p \Rightarrow q$ to be T, q may not be F if p is T
- Alternative notation: $\rightarrow$ (book!)

The Logical Bi-implication

A similarly looking operator:

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Remarks:

- Read:
  - "p if and only if q" ("p hvis og kun hvis q")
  - "p iff q" ("p hviss q")
  - "p is a necessary and sufficient condition for q" ("p er en nødvendig og tilstrækkelig betingelse for q")
- In words: for $p \Leftrightarrow q$ to be T, p and q must have the same truth value.
- Alternative notation: $\leftrightarrow$ (book!)

The Exclusive Or

The last operator:

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Remarks:

- Read:
  - "either p or q" ("enten p eller q")
  - "p xor q"
- In words: for $p \oplus q$ to be T, p and q must have different truth values.
- Caution: The book uses
  - "either p or q" to say $p \vee q$ and
  - "either p or q but not both" to say $p \oplus q$.

Poll Everywhere

The Precedence Hierarchy of Logical Operators

Precedence order ("order of evaluation") **of operators:**

- $\neg$, $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$
- There is no consensus on the position of $\oplus$.

Note:

- I (and I believe I speak for a larger group of Mathematicians) would not assume that the reader knows the precedence order among $\wedge$ and $\vee$; neither among $\Rightarrow$ and $\Leftrightarrow$. In these cases, I am putting parentheses.
- Similarly, if you are not sure about the precedence, just put parentheses.