Lecture 3

de Morgan for quantifiers.

Let S be the set of students signed up for this class. For $x \in S$, let $P(x)$ be true if x is present. How do we express $\underbrace{\neg \forall x \in S : P(x)}$ using $\exists$?

"Not everyone is here today."

Wrong answers include

- $\exists x \in S : P(x)$ "someone is present today"
- $\forall x \in S : \neg P(x)$ "no-one is here today".

<u>Answer</u>: $\exists x \in S : \neg P(x)$ "someone is not here today".

Why?

$$\neg \forall x \in S : P(x)$$

$$\equiv \neg \big(P(\text{student }1) \wedge P(\text{student}2) \wedge \dots \big)$$

$$\equiv \neg P(\text{student }1) \vee \neg P(\text{student}2) \vee \dots \quad \text{by de Morgan}$$

$$\equiv \exists x \in S : \neg P(x).$$

How do we negate an implication?

Recall from previous lecture: $\neg (p \Rightarrow q) \equiv p \wedge \neg q$.

More variables and nested quantifiers.

Consider $P(x,y)$: $x+y=0$.

Now
$\exists y \in \mathbb{Z} : P(x,y)$ is an open proposition with a single variable. So, it makes sense to write

$\forall x \in \mathbb{Z} \; \exists y \in \mathbb{Z} : P(x,y)$ "Given any $x \in \mathbb{Z}$, we can find $y \in \mathbb{Z}$ such that $x+y=0$".

and this is, in fact, true.

Alternatively, we could say

$\exists y \in \mathbb{Z} \; \forall x \in \mathbb{Z} : P(x,y)$ "There exists a $y \in \mathbb{Z}$ such that given any other $x \in \mathbb{Z}$, we have $x+y=0$"

This is false.

So the order of the quantifiers matters!

Let H be set of all humans, and let B be all 15 min periods. Let $P(x,y)$ be the statement that human x steals a car at 15 min interval y.

The intended proposition: $\forall y \in B \; \exists x \in H : P(x,y)$

Misunderstood proposition $\exists x \in H \; \forall y \in B : P(x,y)$.