

Lecture 17

1/3

A matrix is a rectangular array of numbers, e.g.

$$A = \underbrace{\begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 2 \end{bmatrix}}_{\text{3 columns}} \quad \left. \right\} \text{2 rows}$$

A is 2×3 matrix and $a_{2,1} = 1$, $a_{1,3} = 4$.

Sum of matrices is defined entrywise:

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 2+1 & 3+1 & 4+0 \\ 1+0 & 0+1 & 2+0 \end{bmatrix} = \begin{bmatrix} 3 & 4 & 4 \\ 1 & 1 & 2 \end{bmatrix}$$

This is only defined if A and B have the same sizes.

The product of matrices is more complicated:

Consider the system of equations

$$2x_1 + 3x_2 = 15$$

$$7x_2 + x_3 = 1$$

$$x_1 + 5x_2 - x_3 = -7$$

We rewrite the left-hand side

$$\begin{bmatrix} 2 & 3 & 0 \\ 0 & 7 & 1 \\ 1 & 5 & -1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2x_1 + 3x_2 + 0 \cdot x_3 \\ 0 \cdot x_1 + 7x_2 + x_3 \\ x_1 + 5x_2 - x_3 \end{bmatrix}$$

Another example

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$$\begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 \cdot 1 + 3 \cdot 1 + 4 \cdot 2 & 2 \cdot 2 + 3 \cdot 1 & 3 \cdot 1 + 4 \cdot 1 \\ 1 \cdot 1 + 2 \cdot 2 & 1 \cdot 2 & 2 \cdot 1 \end{bmatrix}$$
$$= \begin{bmatrix} 13 & 7 & 7 \\ 5 & 2 & 2 \end{bmatrix}$$

In general, it looks like

$$\begin{bmatrix} A & \boxed{\quad} \end{bmatrix} \cdot \underbrace{\begin{bmatrix} B & \boxed{\quad} \end{bmatrix}}_{\text{columns}}$$

The order of multiplication is important :

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix} \quad \text{and} \quad B \cdot A = \begin{bmatrix} 2 & 2 \\ 1 & 2 \end{bmatrix}$$

There is an identity matrix $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 2 \end{bmatrix} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{3 rows}} = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 2 \end{bmatrix}}_{\text{2 columns}} = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 2 \end{bmatrix}$$

The transpose:

$$\cdot \begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 2 \end{bmatrix}^T = \begin{bmatrix} 2 & 1 \\ 3 & 0 \\ 4 & 2 \end{bmatrix}$$

Rows become columns,
columns become rows.

$$\cdot \begin{bmatrix} 2 & 3 & 0 \\ 4 & 5 & 7 \\ 1 & -3 & 2 \end{bmatrix}^T = \begin{bmatrix} 2 & 4 & 1 \\ 3 & 5 & -3 \\ 0 & 7 & 2 \end{bmatrix} .$$

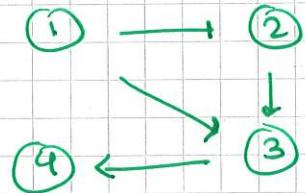
For Boolean matrices, we interpret "0" as False, and "1" as True.

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \vee \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \wedge \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Boolean products are relevant for iterations of relations:

R:



$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Observe that

$$A \odot A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \odot \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and this is exactly the matrix representation
for the square relation R^2 .

Similarly, we can represent R^3 using $A \odot A \odot A = A^{[3]}$.

~~It's a good idea to have a few different types of soil.~~

Chlorophyll Content

10. The following table shows the number of hours worked by 1000 employees of a company.

• 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

10. The following table shows the number of hours worked by 1000 workers in a certain industry.

100

10. *What is the name of the author of the book you are reading?*

2010-2011
Year 1

100

10. *What is the name of the author of the book you are reading?*

1. 2. 3. 4. 5.

10. The following table shows the number of hours worked by 1000 employees in a company.

Table 1. Summary of the main characteristics of the four groups of patients.

J *S* *K* *L* *M*

1. *What is the relationship between the two main characters?*

10. The following table shows the number of hours worked by 1000 employees of a company.

Lesson 1