# DM549/DS(K)820/MM537/DM547

## Lecture 2: Propositional Equivalences and Quantifiers

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# A Joke

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- The second logician says, “I don’t know.”
- The third logician says, “Yes!”

# Last Time

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**Precedence order** ("order of evaluation") **of operators:**

- $\neg$, $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$
- There is no consensus on the position of $\oplus$.

# Tautologies, Contradictions, and Contingencies

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**Q:** How to find out if you are not sure?
**A:** Construct the truth table (or apply rules that we will see later).

# Poll Everywhere

# Logical Equivalences

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See Tables 1.3.6–8 for many useful equivalences! We will now see the most important ones.

# Distributive Laws

## Distributive Laws (Example 1.3.4)

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r), \quad p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

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**Intuition** (first version)**:** For both propositions,

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- if $p$ is F, proposition is T iff both $q$ and $r$ are T.

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$$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r), \quad p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$$

**Intuition** (first version)**:** For both propositions,

- if $p$ is T, full proposition is T.
- if $p$ is F, proposition is T iff both $q$ and $r$ are T.

**Proof** (first version)**:**

| $p$ | $q$ | $r$ | $p \lor (q \land r)$ | $(p \lor q) \land (p \lor r)$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | T | F | T | T |
| T | F | T | T | T |
| T | F | F | T | T |
| F | T | T | T | T |
| F | T | F | F | F |
| F | F | T | F | F |
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# De Morgan's Laws

## De Morgan's Laws (Table 1.3.6, line 8)

$$\neg(p \land q) \equiv \neg p \lor \neg q, \quad \neg(p \lor q) \equiv \neg p \land \neg q$$

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$$\neg(p \wedge q) \equiv \neg p \vee \neg q, \quad \neg(p \vee q) \equiv \neg p \wedge \neg q$$

**Intuition:**

- If not both $p$ and $q$ are T, $p$ must be F or $q$ must be F.
- If not at least one of $p$ and $q$ is T, then both $p$ and $q$ must be F.

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**Proof:** Exercises.

**Note:** This also works for more propositional variables, e.g.:

$$\neg(p \wedge q \wedge r) \equiv \neg p \vee \neg q \vee \neg r.$$

# Equivalences Involving Implications (1)

## Contraposition (Table 1.3.7, line 2)

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**Proof:**

| $p$ | $q$ | $p \Rightarrow q$ | $\neg p$ | $\neg q$ | $\neg q \Rightarrow \neg p$ |
|---|---|---|---|---|---|
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| T | F | F | F | T | F |
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# Equivalences Involving Implications (2)

**Formulation only using $\land$, $\lor$, $\neg$ (Table 1.3.7, line 1)**

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**Proof:**

| $p$ | $q$ | $p \Rightarrow q$ | $\neg p$ | $\neg p \vee q$ |
|---|---|---|---|---|
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**Note:** This justifies the notation of $\Leftrightarrow$ and saying "$p$ if and only if $q$".

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**Other proof:** Blackboard.

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**Remark:** We will talk more about sets and real numbers in later lectures!

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- For now, we will focus on open propositions with a single variable.

# The Universal Quantifier

## Definition

For a propositional function $P(x)$, the statement

$$\forall x \in D : P(x)$$

is equivalent to the statement that $P(x)$ is true for all $x$ in the set $D$. We call $\forall$ the *universal quantifier* (alkvantor).

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- A universal quantification over the empty set is always true.

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- An existential quantification over the empty set is always false.
- The existential quantification is true as long there exists *at least one* $x$ in $D$ with the specified property, not just precisely one.

# The Uniqueness Quantifier

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- How to memorize?
  - for $\forall$ll,
  - there $\exists$xists,
  - "!" looks a bit like "1".

# More on Quantifiers

**Remarks:**

- We say that the quantifier *binds* variables $x$.
- In the above statements, we call $D$ the *domain* (domæne) or universe (univers).
- We also say that we *quantify over* (kvantificerer over) $D$.
- When clear from the context, the domain is sometimes left out.
- Some authors leave out the colon.
- How to memorize?
  - for $\forall$ll,
  - there $\exists$xists,
  - "!" looks a bit like "1".
- Quantifiers have a *higher* preference (i.e., they are evaluated earlier) than the operators $\neg$, $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$, $\oplus$.