Matrices Cheat Sheet

A columnar approach.

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- The numbers are arranged in a column format, with each number being unique. This allows you to easily identify the cell from which a number originates by simply looking at it.
- Columns represent "actions" while rows represent "data". Thus, in a column and row calculation, column's numbers "act" on row's "datum" numbers.

Definitions

This is a matrix :

$$\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

This is a column vector :

$$\begin{vmatrix} 1 \\ 2 \\ 3 \end{vmatrix}$$

This is a row vector :

$$\begin{vmatrix} 1 & 2 & 3 \end{vmatrix}$$

Multiplication

Column Vector * Column Vector

$$\begin{vmatrix} 1 \\ 2 \\ 3 \end{vmatrix} * \begin{vmatrix} 4 \\ 5 \\ 6 \end{vmatrix} = \begin{vmatrix} (1*4) \\ (2*5) \\ (3*6) \end{vmatrix} = \begin{vmatrix} 4 \\ 10 \\ 18 \end{vmatrix}$$

Column Vector * Row Vector

$$\begin{vmatrix} 1 \\ 2 \\ 3 \end{vmatrix} * \begin{vmatrix} 4 & 5 & 6 \end{vmatrix} = \begin{bmatrix} (1*4) & (1*5) & (1*6) \\ (2*4) & (2*5) & (2*6) \\ (3*4) & (3*5) & (3*6) \end{bmatrix} = \begin{bmatrix} 4 & 5 & 6 \\ 8 & 10 & 12 \\ 12 & 15 & 18 \end{bmatrix}$$

Column Vector * Number

$$\begin{vmatrix} 1 \\ 2 \\ 3 \end{vmatrix} * 4 = \begin{vmatrix} (1*4) \\ (2*4) \\ (3*4) \end{vmatrix} = \begin{vmatrix} 4 \\ 8 \\ 12 \end{vmatrix}$$

Row Vector * Column Vector

$$\begin{vmatrix} 1 & 2 & 3 \end{vmatrix} * \begin{vmatrix} 4 \\ 5 \\ 6 \end{vmatrix} = \begin{bmatrix} (4*1) & (5*1) & (6*1) \\ (4*2) & (5*2) & (6*2) \\ (4*3) & (5*3) & (6*3) \end{bmatrix} = \begin{bmatrix} 4 & 5 & 6 \\ 8 & 10 & 12 \\ 12 & 15 & 18 \end{bmatrix}$$

Row Vector * Row Vector

Impossible

Row Vector * Number

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} * 100 = \begin{vmatrix} (1*100) & (2*100) & (3*100) \end{vmatrix} = \begin{vmatrix} 100 & 200 & 300 \end{vmatrix}$$

Matrix * Column Vector

$$\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix} * \begin{vmatrix} 10 \\ 11 \\ 12 \end{vmatrix} = \begin{vmatrix} (10*1+11*4+12*7) \\ (10*2+11*5+12*8) \\ (10*3+11*6+12*9) \end{vmatrix} = \begin{vmatrix} 138 \\ 171 \\ 204 \end{vmatrix}$$

Matrix * Row Vector

Impossible

Column Vector * Matrix

Impossible

Row Vector * Matrix

$$\begin{vmatrix} 1 & 2 & 3 \end{vmatrix} * \begin{bmatrix} 4 & 7 & 10 \\ 5 & 8 & 11 \\ 6 & 9 & 12 \end{bmatrix}$$

$$= \begin{vmatrix} (4*1+5*2+6*3) & (7*1+8*2+9*3) & (10*1+11*2+12*3) \end{vmatrix}$$

$$= \begin{vmatrix} 32 & 50 & 68 \end{vmatrix}$$

Matrix * Matrix

Just split the other matrix into columns and follow *Matrix * Column Vector*!

$$\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix} * \begin{bmatrix} 10 & 13 \\ 11 & 14 \\ 12 & 15 \end{bmatrix}$$

$$= \begin{bmatrix} \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix} * \begin{vmatrix} 10 \\ 11 \\ 12 \end{vmatrix} & \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 7 \\ 7 & 8 & 9 \end{bmatrix} * \begin{vmatrix} 13 \\ 14 \\ 15 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} (10*1+11*4+12*7) & (13*1+14*4+15*7) \\ (10*2+11*5+12*8) & (13*2+14*5+15*8) \\ (10*3+11*6+12*9) & (13*3+14*6+15*9) \end{bmatrix}$$

$$= \begin{bmatrix} 138 & 174 \\ 171 & 216 \\ 204 & 258 \end{bmatrix}$$

Matrix * Number

$$\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix} * 100 = \begin{bmatrix} 100*1 & 100*4 & 100*7 \\ 100*2 & 100*5 & 100*8 \\ 100*3 & 100*6 & 100*9 \end{bmatrix} = \begin{bmatrix} 100 & 400 & 700 \\ 200 & 500 & 800 \\ 300 & 600 & 900 \end{bmatrix}$$

Number * Row Vector

Refer to *Row Vector * Number*

Number * Column Vector

Refer to *Column Vector * Number*

Number * Matrix

Refer to *Matrix * Number*