Probabilities

$P(A) = 1 - P(A')$

$P(A \cap B) = P(A) \cdot P(B|A)$ dependent

$P(A \cap B) = P(A) \cdot P(B)$ independent

$P(A \cap B) = 0$ mutually exclusive

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = P(A) + P(B)$ mutually exclusive

if $P(A \cap B) > 0$, A, B not mut. exclusive

if A, B are independent, order matters

$P(A|B) = P(A \cap B)/P(B)$ conditional

$P(A|B) = (P(B|A) \cdot P(A))/P(B)$ Bayes'

$\cap$ = intersection = and

$\cup$ = union = or

Statistics

Variance $= \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 = \sigma^2$

avg. squared distance between each data point and the mean

standard deviation $= \sqrt{\text{variance}}$

the spread of data from the mean

standard error $= \frac{\sigma}{\sqrt{n}}$

the spread of data from the mean over multiple samples

MLE =

Probability Mass Functions

Bernoulli

$f(x;p) = \begin{cases} p & \text{if } x=1 \\ 1-p & \text{if } x=0 \end{cases}$

mean $= p$

variance $= p(1-p)$

Binomial

$f(x;n;p) = \binom{n}{x} p^x (1-p)^{n-x}$

mean $= np$

variance $= np(1-p)$

Geometric

$f(x;p) = (1-p)^x \cdot p$

mean $= 1/p$

variance $= (1-p)/p^2$

Poisson

$f(x;\lambda) = (\lambda^x e^{-\lambda})/x!$

mean/variance $= \lambda$

$P(a < X < b) = \sum_a^b P(x)$

Expected Value: $E[X] = \sum_{\forall x} x P(x)$

basically a weighted mean

Probability Density Functions

Normal/Gaussian

$f(x;\mu;\sigma^2) = (1/\sqrt{2\pi\sigma^2}) \cdot e^{\wedge}(-\frac{x-\mu^2}{2\sigma^2})$

mean $= \mu$

variance $= \sigma^2$

exponential

$f(x;\lambda) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$

mean $= 1/\lambda$

variance $= 1/\lambda^2$

uniform

$f(x;a;b) = \begin{cases} 1/b-a & \text{if } x \in [a,b] \\ 0 & \text{otherwise} \end{cases}$

mean $= (a+b)/2$

variance $= (a+b)/12$

$P(a < X < b) = \int_a^b P(x)$

Expected Value: $E[X] = \int_{-\infty}^{+\infty} x f(x) dx$

basically a weighted mean

Variance $= E[X] - (E[X])^2$

$X \xrightarrow{\text{experiment}} x$

random variable → observation

$CI = P(c_l < \theta < c_u) = \gamma = 1 - \alpha$

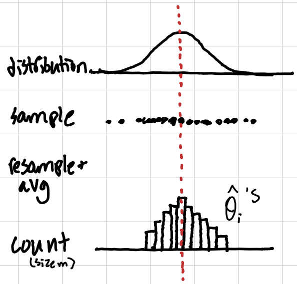
γ usually $= 0.95$

parametric →

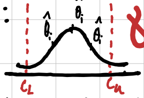
non-parametric → bootstrap

$\hat{\theta}_B = \frac{\sum \hat{\theta}_i}{m}$

$SE = \sqrt{\sum_m \frac{(\hat{\theta}_i - \hat{\theta}_B)^2}{m-1}}$



method 1:



$\gamma = 0.9$

quantile()

(may have coverage issues if dist has long tail)

method 2: BCa in R fixes this

$c_l = \bar{X} - z_{(1-\frac{\alpha}{2})} \times \frac{\sigma}{\sqrt{n}}$

$c_u = \bar{X} - z_{\frac{\alpha}{2}} \times \frac{\sigma}{\sqrt{n}}$

$\hat{\mu} = \bar{X} \pm z_{\frac{\alpha}{2}} \times \frac{\sigma}{\sqrt{n}}$

When σ^2 is not known, use T-dist

$f(x;\mu;S) = (\bar{X} - \mu)/(S/\sqrt{n}) \sim t(n-1)$

$MCI = \bar{X} \pm t_{(1-\frac{\alpha}{2})} \cdot (S/\sqrt{n})$, $df = n-1$

fixes uncertainty when $n \to$ small